# CHAPTER 62

## SCALE LAWS FOR BED FORMS IN LABORATORY WAVE MODELS

by

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ABSTRACT

An experimental study of the variables governing the development of bed forms under wave action has been conducted. A wide range of conditions was covered by the use of a wave flume and an oscillating water tunnel. Scale laws for modelling bed forms are developed using the results of these experiments. It is recommended that the model sediment used should be the same density as in the prototype and its size should be scaled geometrically. If this is not practical, it is shown how the scale laws can be used to minimise the distortion of the bed forms produced in the model.

## INTRODUCTION

The roughness of a sediment bed is dependent on the sediment grain size and the bed forms which occur as a result of the oscillatory fluid motion generated by waves. Thus, the scales for a movable bed wave model should be chosen to reproduce bed forms so that energy losses and wave attenuation in the model are dynamically similar to those in the prototype. In addition, sediment transport rates depend not only on the shear stresses caused by the bottom roughness, but also to a great extent on the shape and size of the bed forms as described by Mogridge (3) and Inman (1). In an attempt to significantly improve the design of movable bed

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wave models, scale laws are developed which govern the reproduction of bed forms.

## SCALE LAWS GOVERNING SEDIMENT TRANSPORT IN MOVABLE BED WAVE MODELS

Yalin (5) has shown that for a movable bed wave model, any quantitative property may be expressed as a function of the following dimensionless variables,

$$\pi_Q = f_a\left\{\frac{v_* D}{\nu}, \frac{\rho v_*^2}{\gamma_s D}, \frac{\rho_s}{\rho}, \frac{a}{D}\right\} \tag{1}$$

where $\rho$ is the density of the fluid,

$\rho_s$ is the density of the sediment,

$\gamma_s$ is the submerged specific weight of the sediment, i.e. $(\rho_s - \rho)g$,

$\nu$ is the kinematic viscosity of the fluid,

D is the sediment grain size,

a is the amplitude of the fluid orbital motion immediately outside the boundary layer,

$v_*$ is the shear velocity, i.e. $\sqrt{\tau/\rho}$,

$\tau$ is the bottom shear stress.

The boundary layer is fully developed rough turbulent, when the magnitude of the first variable in Eq. 1, i.e. Reynolds Number, is very large. Under these conditions which normally occur when bed forms are present, the Reynolds Number has no effect and may be neglected. Therefore, from Eq. 1, the scales for the movable bed model may be obtained from,

$$\frac{n_\rho n_{v_*}^2}{n_{\gamma_s} n_D} = 1, \quad \frac{n_a}{n_D} = 1, \quad \frac{n_{\rho_s}}{n_\rho} = 1 \qquad (2)$$

where the scale n is defined as the prototype value on the model value of the parameter. Then,

$$n_{\rho_s} = n_\rho = 1,$$

$$n_{\gamma_s} = 1,$$

$$n_a = n_D = n,$$

$$n_{v_*}^2 = n_{\gamma_s} n_D = n,$$

or

$$n_{v_*} = n^{1/2} \qquad (3)$$

However, according to Yalin (5), for the mass movement of sediment it is not necessary that $n_{\rho_s} = n_\rho$. Thus, a model may be designed such that $n_{\gamma_s} \neq 1$, and,

$$n_{v_*}^2 = n_{\gamma_s} n_D \qquad (4)$$

If the model has bed forms, then the roughness of the bottom may be considered to be some combination of the roughness due to the bed forms and the sand roughness $k_s$. This effective roughness will be called k. If the model is distorted, or if either or both the sand roughness and bed forms are distorted, then $N_k$ is the distortion of the roughness. The scaling laws for bed forms are developed in order to reduce this roughness distortion, and to reduce the distortion of the bottom shear stress and wave attenuation. Careful modelling of the shape and size of the bed forms also ensures that sediment transport rates will be similar to those in the prototype.

For a fully developed rough turbulent boundary layer such as would normally occur when bed forms are present, the logarithmic velocity distribution is assumed.

$$U_\delta = 2.5 \; v_* \; \ell n \left(\frac{30\delta}{k}\right), \qquad (5)$$

where k is used for the total roughness instead of $k_s$ for the sand roughness alone, and $\delta$ is the boundary layer thickness. Kamphuis (2), used Eq. 5 to derive the relationship,

$$\frac{a}{\delta} = \text{constant} \cdot \ell n \left(\frac{30\delta}{k}\right), \qquad (6)$$

and shows that these equations may be approximated by,

$$\frac{U_\delta}{v_*} = \text{constant} \cdot \left(\frac{30\delta}{k}\right)^{1/6}, \qquad (7)$$

and

$$\frac{a}{\delta} = \text{constant} \cdot \left(\frac{30\delta}{k}\right)^{1/6}. \qquad (8)$$

Since $n_a = n$, Eq. 8 results in,

$$n_\delta = n^{6/7} \; n_k^{1/7} = n \; N_k^{1/7}, \qquad (9)$$

where $N_k$ is the roughness distortion,

$$N_k = \frac{n_k}{n}.$$

From Eq. 7,

$$n_{U_\delta} = n_{v_*} \cdot \frac{n_\delta^{1/6}}{n_k^{1/6}}. \qquad (10)$$

Therefore, since the scale of $U_\delta$ is the same as the velocity scale outside the boundary layer, i.e.,

$$n_{U_\delta} = n^{1/2}, \qquad (11)$$

then substituting for $n_\delta$ in Eq. 10 gives,

$$n_{v_*} = n^{1/2}\, N_k^{1/7}, \qquad (12)$$

and since $n_{v_*}^2 = n_\tau$ if $n_\rho = 1$, then,

$$n_\tau = n\, N_k^{2/7}. \qquad (13)$$

Equation 12 shows that a distortion of the bottom roughness has only a small effect on the scale of the shear velocity. If the bottom roughness is undistorted then,

$$n_{v_*} = n^{1/2}, \qquad (14)$$

and

$$n_\tau = n. \qquad (15)$$

Equation 4 then is,

$$n = n_{\gamma_s} n_D . \tag{16}$$

SCALE LAWS GOVERNING THE REPRODUCTION OF BED FORMS

Quantitative properties of bed forms resulting from wave action may be expressed as a function of the following dimensionless variables

$$X_1 = \frac{\gamma_s D^3}{\rho \nu^2}, \; X_2 = \frac{\rho D}{\gamma_s T^2}, \; X_3 = \frac{\rho_s}{\rho}, \; X_4 = \frac{A}{D}, \tag{17}$$

where A is the orbit length of the flow oscillation immediately outside the boundary layer.

Experiments on bed forms have been conducted by the author (4) at the Coastal Engineering Laboratory, Queen's University at Kingston. These experiments were conducted in a laboratory wave flume and an oscillating water tunnel, in order to cover the range from model to prototype conditions. Natural sand as well as low density materials were used as sediments. The results showed that bed form length $\Lambda$ and bed form height $\Delta$ are independent of the viscosity variable $X_1$, and as an approximation may also be considered independent of $X_3$. Therefore, the bed form length and height may be expressed by the following equations,

$$\frac{\Lambda}{D} = f_\Lambda (X_2, X_4) \tag{18}$$

$$\frac{\Delta}{D} = f_\Delta(X_2, X_4) \qquad (19)$$

The functions $f_\Lambda$ and $f_\Delta$ are given by the curves in Figs. 1 and 2, which are the result of the experiments described in reference (4). From Eqs. 18 and 19, exact similarity of the bed forms is achieved if,

$$n_{X_2} = 1 \text{ and } n_{X_4} = 1. \qquad (20)$$

Thus

$$n_D = n_{\gamma_s} n, \qquad (21)$$

and

$$n_A = n_a = n. \qquad (22)$$

In addition, it must be ensured that similarity laws for sediment movement over the bed forms are satisfied. However, Eqs. 16 and 21 cannot be satisfied simultaneously unless $n_{\gamma_s} = 1$. The scale laws obtained from Eqs. 21 and 22 become then,

$$n_D = n_A = n. \qquad (23)$$

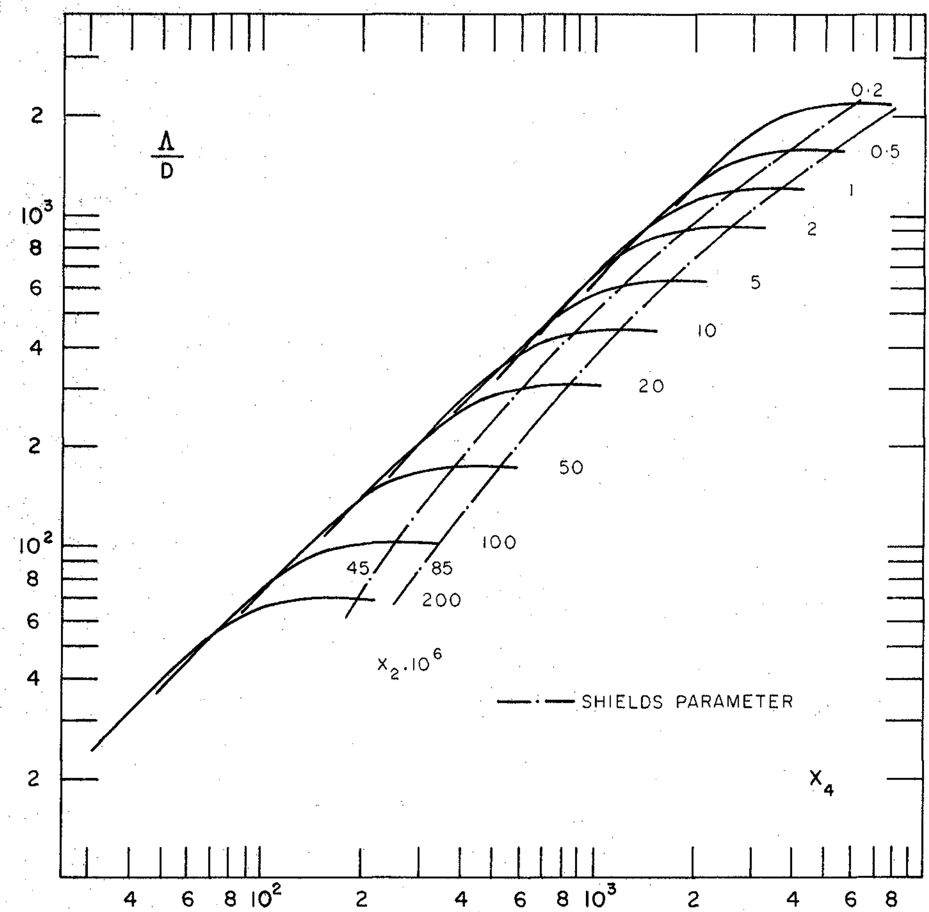
Because there is no distortion of the bottom roughness,

$$n^2_{v_*} = n, \qquad (24)$$

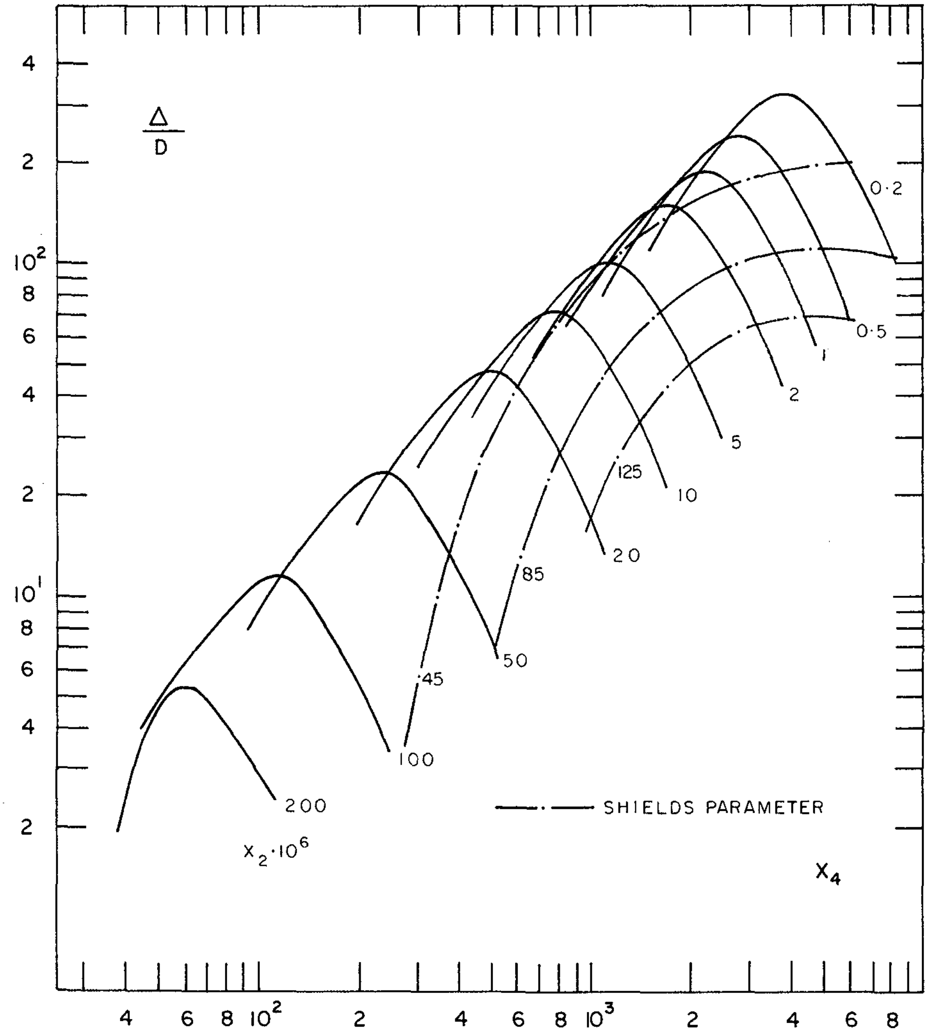
and

$$n_\tau = n. \qquad (25)$$

Thus, if the model sediment has a specific weight identical to that of the prototype sediment, and a grain size scaled




FIG·1 DESIGN CURVES FOR BED FORM LENGTH




FIG·2 DESIGN CURVES FOR BED FORM HEIGHT

geometrically to the general model scale n, the bed forms and the sediment transport over them will be modelled exactly. In many cases such a model is not practical because the model sediment would need to be too fine. The model must therefore be designed with $n_D < n$ and from Eq. 21, $n_{\gamma_s} > 1$. Then, if it can be assumed that the effective roughness is mainly dependent on the bed form roughness, similarity of the shear stress will occur as long as similarity of the bed forms is obtained. This similarity will occur when the bed form length and height ratios are approximately independent of $X_2$ and directly proportional to $X_4$. Then it is not necessary for either $n_{X_2} = 1$, or $n_{X_4} = 1$. In Figs. 1 and 2, for the sections of the curves which are at a slope of 1:1,

$$\frac{\Lambda}{D} \propto \frac{A}{D}, \text{ and } \frac{\Delta}{D} \propto \frac{A}{D} \qquad (26)$$

Therefore,

$$n_\Lambda = n_\Delta = n_A = n \qquad (27)$$

Equation 27 is satisfied approximately in the range defined by,

$$6.5 < \frac{\rho U_\delta^2}{\gamma_s D} < 40. \qquad (28)$$

For the Shields parameter in Eq. 28 less than 6.5, there is no sediment movement. When it is greater than 40, the bed forms have lengths and heights which are not directly proportional to the fluid orbit length A, and are dependent on the sediment grain size. Here, a distortion of the grain size will usually cause a distortion of the bed form length and height, so that,

$$n_D \neq n,\ n_\Lambda \neq n \text{ and } n_\Delta \neq n.$$

From Eq. 17,

$$n_{X_2} = \frac{n_\rho n_D}{n_{\gamma_s} n_T^2} = \frac{n_D}{n_{\gamma_s} n}, \tag{29}$$

and

$$n_{X_4} = \frac{n_A}{n_D} = \frac{n}{n_D}. \tag{30}$$

And for similarity of sediment movement Eq. 4 must be satisfied, i.e.

$$n_{V_*}^2 = n_{\gamma_s} n_D, \tag{4}$$

where

$$n_{V_*} = n^{1/2}\ N_k^{1/7} \tag{12}$$

However, until $n_{X_2}$ is determined, the distortion of the bed roughness $N_k$ cannot be known, so as an initial approximation $N_k$ is taken as unity, so that,

$$n = n_{\gamma_s} n_D. \tag{31}$$

To model bed forms and the sediment movement occurring over them, the model should be designed as follows. A value of $\gamma_s$ is chosen for the model sediment material, which gives $n_{\gamma_s}$. From Eq. 31, the sediment size scale is obtained, i.e.,

$$n_D = \frac{n}{n_{\gamma_s}}.$$

From Eq. 29, $n_{X_2}$ is obtained and thus $X_2$ for the model. Equation 30 gives $n_{X_4}$ and $X_4$ may be determined for the model. The distortion of the bed form length may be obtained by first determining the constant $m_1$ from Fig. 1.

$$m_1 = \frac{(\Lambda/D) \text{ for } X_2, X_4 \text{ in prototype}}{(\Lambda/D) \text{ for } X_2, X_4 \text{ in model}} = \frac{n_\Lambda}{n_D} \qquad (32)$$

Substituting $n_D$ from Eq. 30 into Eq. 32,

$$m_1 = \frac{n_\Lambda}{n} \cdot n_{X_4},$$

from which,

$$n_\Lambda = \left[\frac{m_1}{n_{x_4}}\right] \cdot n. \qquad (33)$$

Therefore, the distortion of the bed form length is

$$N_\Lambda = \left[\frac{m_1}{n_{X_4}}\right]. \qquad (34)$$

If the slope of the line joining $X_2$, $X_4$ for the prototype and the model is exactly 1:1, then,

$$m_1 = n_{X_4},$$

and there is no distortion, i.e.,

$$n_\Lambda = n.$$

As can be seen in Fig. 1, curves for constant values of the Shields parameter defined in Eq. 28 are approximately at a slope of 1:1. And because the Shields parameter is held constant in the model design, then the distortion of the bed form length will never be very large. However, the curves for constant Shields parameter in Fig. 2 display considerable curvature so that severe distortions of bed form height are possible, particularly if a very low density sediment is used in the model.

If, as a result of the first choice for the model sediment $\gamma_s$, the distortion of the bed form length is too large, then an improved design may be obtained by choosing a different sediment density. An exactly similar calculation can be made using Fig. 2 to obtain,

$$m_2 = \frac{n_\Delta}{n_D}, \tag{35}$$

and to determine the distortion of the bed form height,

$$N_\Delta = \left[\frac{m_2}{n_{X_4}}\right] \tag{36}$$

Now knowing the sediment size and bed form size in the model from the initial calculation using Eq. 31, the bottom roughness k and thus $N_k$ should be determined. Then Eq. 4 becomes,

$$n\ N_k^{2/7} = n_{\gamma_s} n_D$$

from which a more accurate calculation of $n_D$ can be made. However, this calculation is dependent on the use of a reliable method for determining bottom roughness. Different methods have been proposed, for example by Yalin (6) and Mogridge (3), but confirmation with experimental data is necessary. Until such data is available it must be assumed that bottom roughness distortion is minimised by reducing the bed form distortion.

Now as an example of the model design method outlined above, assume prototype conditions as a wave period equal to 10 seconds, a wave height of 2 metres in a water depth of 9 metres. Assume the sediment diameter to be 1 mm and a sediment of coal with specific gravity of 1.2. Therefore,

$$n_{\gamma_s} = \frac{1.65}{0.2} = 8.25$$

Substituting $n_D$ from Eq. 31 into Eq. 29,

$$n_{X_2} = \frac{1}{n_{\gamma_s}^2} = 1/68.$$

Therefore,

$$X_{2_m} = 42.1 \times 10^{-6}.$$

Substituting $n_D$ from Eq. 31 into Eq. 30,

$$n_{X_4} = n_{\gamma_s} = 8.25.$$

Therefore,

$$X_{4_m} = 2910/8.25 = 353.$$

Assuming a general model scale of n = 50,

$$n_T = n^{1/2} = 7.16.$$

Therefore, $T_m$ = 1.4 seconds, $d_m$ = 18 cm, $H_m$ = 4 cm. From Eq. 30,

$$n_D = \frac{n}{n_{X_4}} = \frac{n}{n_{\gamma_s}} = \frac{50}{8.25} = 6.06.$$

Therefore, the sediment diameter in the model $D_m$, is 0.165 mm. Now $m_1$ is obtained from Fig. 1.

$$m_1 = \frac{1400}{230} = 6.09.$$

Therefore, the distortion of the bed form length is,

$$N_\Lambda = \left[\frac{m_1}{n_{X_4}}\right] = \frac{6.09}{8.25} = 0.737,$$

i.e. $$n_\Lambda = 0.737\ n.$$

If such a distortion is not considered satisfactory, then the calculation should be repeated for a different model sediment density. A similar procedure should also be followed to design the model according to the bed form height criterion.

The design procedure has been carried out for a single water depth. If the depth varies, as is normally the case in a mobile bed wave model, then although $n_{X_2}$ and $n_{X_4}$ remain constant, $m_1$ and $m_2$ vary. Therefore, the distortion can vary with the water depth. This difficulty does not occur however, if, $m_1 = m_2 = n_{X_4}$.

## CONCLUSIONS

In movable bed wave models, bed forms should be produced to scale to ensure similarity of wave attenuation and sediment transport rates. To model bed forms it is recommended that the sediment used should be the same density as in the prototype and its size should be scaled geometrically. On a smooth bottom resulting from using a fine sediment, it may be necessary to initiate the generation of bed forms by an artificial roughness, after which the bed forms will spread rapidly. If, by scaling geometrically, the sediment size becomes too fine to be practical, an alternative scaling procedure can be used. The method which has been described uses a low density sediment, larger in diameter than that resulting from scaling geometrically. When the Shields parameter is held constant in model and prototype, distortion of the bed form length will usually be small, but a large distortion of the bed form height is possible if care is not exercised in the model design.

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