

CHAPTER 78

Beach Fill Stability and Borrow Material Texture

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ABSTRACT

The dependence of beach fill stability on the textural properties of borrow material requires development of quantitative methods for use in selection of borrow areas and in prediction of possible maintenance costs associated with periodic renourishment. If a shore segment is viewed as a sediment mass transfer system, where grains of different size have different transport rates, then termination of natural sediment input to the shore segment will cause the beach to retreat and the materials in the active zone will become coarser. The ratio of retreat rates associated with a given borrow material texture to that associated with native material can be used in optimizing economic factors involved in selection among potential borrow zones.

With certain simplifying assumptions the relative retreat rate associated with a given borrow material texture can be predicted from observations of the modifications in textural properties of native material which occur during the eroding condition following termination of the natural supply of sediment. Further simplifying assumptions result in an analytical expression for relative retreat rates which may not require observations of the natural beach in the eroding condition.

The proposed method is in substantial agreement with qualitative guidelines provided in the Shore Protection Manual [2].

INTRODUCTION

Artificial beach nourishment is a commonly utilized approach for treatment of a beach or shore erosion problem. It is the most direct method for maintaining and improving recreational benefits in shore protection projects. It is also widely used for emergency storm protection in areas where source materials are readily available. This latter use is due primarily to favorable construction costs as compared to alternatives and the simplicity and speed of construction. However, the precise degree of effectiveness of beach fill utilized to stabilize a shore erosion problem is not always predictable. Some projects have suffered severe losses of fill material during the construction phase itself, with an end result of little or no improvement over the pre-existing conditions.

One of the major factors controlling the stability of a beach fill is the texture of the borrow material. Fine, well-sorted borrow material, such as that commonly found in bays, backshore dunes, or the bottom surface of the offshore zone, will generally respond rapidly to wave and current conditions, moving alongshore and offshore out of the project area. Material of this type is generally not suitable for use as beach fill. On

the other hand, coarse, more poorly-sorted material, such as that found in alluvial channels, glacial outwash, and sometimes in offshore shoals, tends to provide more stable beach fills, although the resulting beach is not always ideal for recreational purposes. Clearly, there is a need to develop means of predicting the relative stability of borrow material from potential sources.

DISCUSSION OF PREVIOUS WORK

An early attempt to develop such a method of prediction was made by Krumbein and James [1]. They proposed a simple mathematical model which bases predictions of grain size effects on a direct comparison between the grain size distribution of the native littoral zone materials and that of a potential borrow material. Their basic assumption is that the native material is essentially in equilibrium with local shore processes and hence can be used to predict the stable grain size distribution which will result from sorting modifications to the borrow material. They define a ratio quantity which represents the minimum volume of borrow material required to manufacture, through selective removal of borrow material from individual size classes, a unit volume of material having the same grain size distribution as the native material. Where both native and borrow materials are approximately lognormally distributed an analytical solution is presented, based on the ϕ means and ϕ sortings of the two distributions.

Qualitative and quantitative guidelines based on this model appear in the Shore Protection Manual [2], recently published by the U.S. Army Coastal Engineering Research Center. Figure 1, reproduced from the Shore Protection Manual, shows contours of predicted overfill ratios plotted against dimensionless scales of differences between ϕ means and ϕ sortings of native and borrow materials. (The ϕ grain size measure is the negative logarithm, to base 2, of grain diameter in millimeters, so that larger ϕ means indicates finer sand. The ϕ sorting is the square root of the central second moment of the grain size distribution on the ϕ scale). The horizontal axis represents the relative difference in ϕ means between borrow and native materials scaled by the ϕ sorting of the native material grain size distribution. The vertical axis, plotted on a logarithmic scale, represents the ratio of borrow material ϕ sorting to native material ϕ sorting. In this figure the origin, plotted at the center, represents the point at which ϕ means and ϕ sorting values for both native and borrow material grain size distributions are the same. The area to the right of the vertical axis represents the conditions in which borrow material has a finer, or higher, ϕ mean than the native material. Conversely, the condition in which borrow material is coarser than native material is represented by the region to the left of the vertical axis. The region above the horizontal axis represents the conditions in which borrow material is more poorly sorted, or more well graded, than the native material, and the region below the vertical axis represents the conditions in which borrow material is better sorted, or more poorly graded, than the native material. It should be noted that no curves are plotted for this latter condition. This is because it is not mathematically possible to

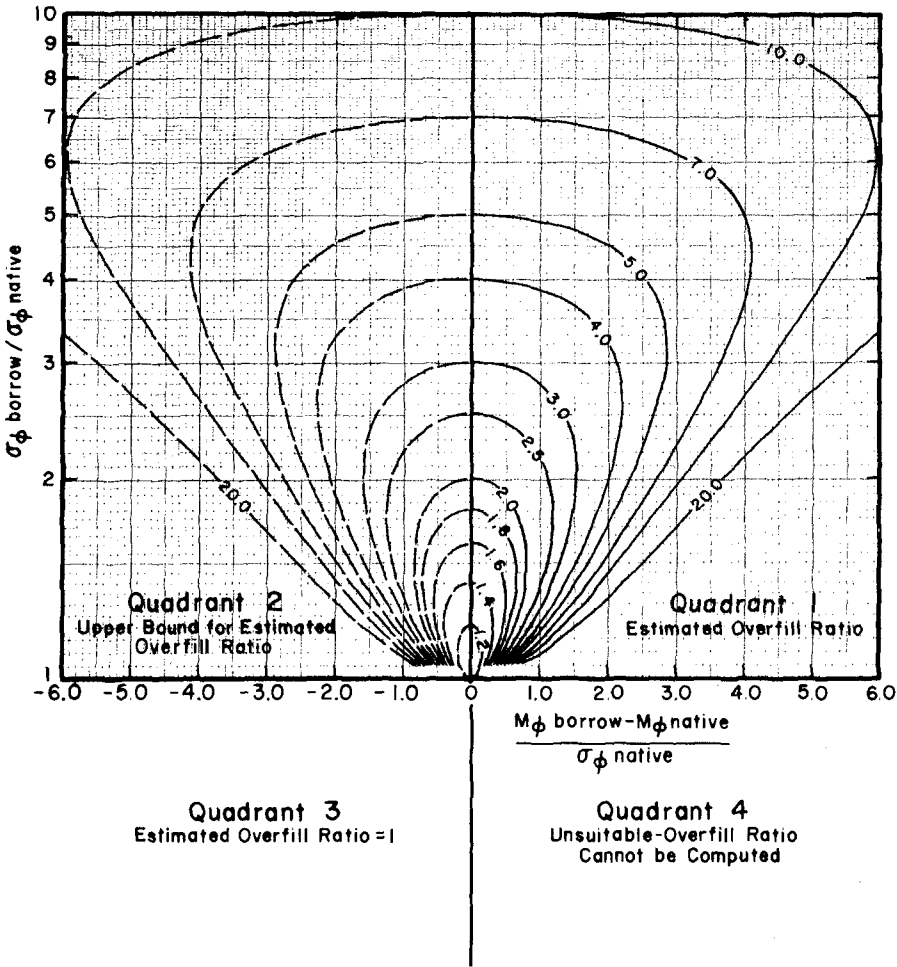


Figure 1. Overfill ratio curves plotted against relative difference in phi means and ratio of phi sorting of borrow and native materials (after SPM, 1974).

produce a lognormal grain size distribution with poorer sorting than that of the original material by selective removal of material. In a more practical sense it means that in the condition of well-sorted borrow material and poorly-sorted native material there will be size classes of the native material grain size distribution for which no material is available in the borrow material grain size distribution.

The inability of this model to predict overfill ratios in this condition represents a major shortcoming of the method. For well-sorted borrow materials the present guidelines are that if the borrow is coarser than native material, an overfill ratio of unity is assumed, and if borrow material is finer than native material it is deemed unstable or unsuitable. A similar difficulty arises in the condition where borrow material is more poorly sorted than native material. Note that the overfill ratio contours are symmetrical about the vertical axis. This means that the calculated overfill ratio is insensitive to the sign of the difference in ϕ means between borrow and native materials. This is to say, the calculated overfill ratio for a coarse borrow material will be the same as that calculated for fine borrow material if the absolute scaled value of the difference in ϕ means between borrow and native materials is the same. This is probably unrealistic. Experience has demonstrated that coarser fill tends to be more stable than finer fill. Note that the curves to the left of the vertical axis are dashed. Present guidelines state that calculated overfill ratios which fall in this quadrant are to be interpreted as upper bounds rather than actual estimates. Unfortunately, in many practical situations, the available borrow materials are either finer and better sorted than native material, or coarser and more poorly sorted, and in these cases the present quantitative guidelines cannot be applied.

The Krumbein and James model is based on the assumption that all of the fill material will be reworked and sorted by wave action. If the resultant fill is stable, this assumption is only valid if the sorting modifications keep pace with the construction phase of the project. This assumption is probably valid in the special case of feeder beach construction, where the fill is stockpiled updrift of the problem area and allowed to move downdrift by natural mechanisms. However, when the entire beach segment in a project is artificially nourished the construction usually occurs at a pace much greater than the natural sorting processes can fully keep up with. The result is construction of an artificial beach mass with textural properties much more similar to those of the borrow material than to those of the native beach material. In such circumstances only a limited portion of the fill will have been exposed to sorting processes if stability is attained. In such circumstances the overfill ratio will overestimate the excess fill required.

In addition to the above mentioned deficiencies of the Krumbein and James model there may be many situations where the basic assumption underlying their model may be quite inaccurate. When beach fill is used as a shore protection measure it is normally used to remedy an erosive condition rather than to improve a stable one. The erosive conditions may occur due to change in water level or wave climate, or due to the termination or sharp reduction of the natural supply of sediment to the eroding beach segment. The latter event may be due to natural causes or to the construction of littoral barriers such as jetties, breakwaters or groins. When a natural beach is in a state of erosion it may be unrealistic to assume that artificial

nourishment will provide a permanent solution. A wiser approach might be to plan on periodic renourishment based on predicted erosion rates.

In this paper a mathematical model is proposed for predicting the ratio of renourishment requirements associated with a given borrow material to those which would be associated with use of native material for a beach fill, given that the natural supply of material to the problem area has been terminated. This ratio, hereafter called the "relative retreat ratio" can be used as a guide for evaluating the relative suitability of alternate borrow sources in the same way as the overfill ratio, as described in the Shore Protection Manual.

THE GENERAL MODEL

One may consider a stretch of beach as a sediment cell, with sediment exchange occurring across the cell alongshore and offshore boundaries. The mass of sediment within such a cell can be considered to lie within one of two distinct zones, an active zone within which material is exposed to wave and current action, and an inactive zone consisting of the backshore and underlying bed materials. The inactive zone serves as a sediment reservoir which is tapped as required to maintain an appropriate mass of material in the active zone in circumstances where the sediment output rate exceeds the input rate. A simple model for the mass-transfer system is based upon the following two assumptions: (1) the mass of material acted upon by waves and currents within the cell is solely determined by the littoral forces. The mass of sediment comprising the active zone does not depend on the grain size distribution of the constituent material. (2) The average amount of time a sediment particle spends in the active zone within the cell depends upon the size of the grain and the littoral forces acting upon it, but does not depend on the distribution of grain sizes.

An equilibrium condition for the mass-transfer system exists when the mass input rate equals mass output rate for each grain size. In this condition the grain size distribution of the active-zone material remains constant, and no material is added to or removed from the backshore reservoir. If this balance is upset, either by reduction in the supply of sediment to the beach area, or by change in the wave climate or water level, the beach will retreat or accrete by removal or addition of material to the inactive reservoir.

In this paper interest is restricted to the condition of erosion, especially in the condition of severe erosion which typically follows the emplacement of beach fill. The placement of fill material on a beach may be viewed as the construction of an artificial headland, which has the effect, at least initially, of terminating the natural supply of sediment to the beach segment, and concentrating erosive wave action on it. The face of the fill will suffer erosion, and the beach will experience a net retreat. The grain size distribution of the active-zone material will coarsen as the finer material is winnowed out at a higher rate than the coarser material.

This initial berm retreat is due in part to a natural profile adjustment during which the offshore foundation of the fill is built by natural

mechanisms. However if no natural supply of sand is available, the fill will continue to experience retreat after the initial profile adjustment.

If littoral processes remain constant, the grain size distribution of the active material will approach a constant form, and the rate of retreat of the beach will become constant. This condition is attained when for each grain size, the system output rate, through winnowing action, exactly matches the rate of incorporation from the reservoir. In order to maintain constant mass in the active zone, the rate of incorporation of material from the reservoir exactly matches the rate of loss through winnowing. Then the rate at which grains of a particular size enter the active zone through incorporation from the reservoir is determined by the total system output rate and the grain size distribution of the material in the reservoir, which for the purposes of this application is the borrow material grain size distribution. The effects of selection of alternative borrow types can be estimated by comparison of the steady retreat rates associated with the respective borrow material grain size distributions.

The general model described above can be mathematically represented in the form of a finite-difference mass balance equation,

$$Mf(\phi, t+\Delta t) = Mf(\phi, t) - w(\phi)\Delta t Mf(\phi, t) + f_b(\phi) \int w(\phi)\Delta t Mf(\phi, t) d\phi, \quad (1)$$

which describes the modification of the grain size distribution of active zone material following emplacement of a fill. If M is the mass of material in the active zone of the cell and $f(\phi, t)$ represents the grain size distribution of this material at any time t , then the product represents the mass of material of a given size in the active zone at time t . During the passage of a time increment of duration Δt , material of various sizes is removed at differing rates due to the winnowing processes. The function, $w(\phi)$, here called the winnowing function, describes the differential rate of erosion associated with particles of various sizes.

This winnowing function may be interpreted probabilistically as follows: the inverse of the winnowing function represents the expected time of residence that a particle of size ϕ remains in the active zone following incorporation from the reservoir, and prior to its passage out of the cell through erosion. The second term on the right-hand side of Equation (1) thus represents the mass of material of a given size in the active zone which is removed during the time increment Δt .

The first assumption in the model requires that the mass of material in the active zone remain constant, under constant littoral forces. Therefore the amount of material which must be incorporated from the reservoir in the time increment Δt is given by the integral of this term over all grain sizes. Inasmuch as the grain size distribution of material in the reservoir is the borrow-material grain size distribution, (which is not the same as the active-zone material grain size distribution) the grain size distribution of the active-zone material will be modified. This grain size distribution will attain a constant form, and the retreat rate will attain a constant value, when the material in the active zone attains the grain size distribution which satisfies the condition given in Equation 2,

$$w(\phi)f'(\phi) = \frac{E}{M} f(\phi), \quad (2)$$

where $f'(\phi)$ is the steady-state active-zone grain size distribution associated with the backshore material having grain size distribution $f(\phi)$, and E is the steady retreat rate. This condition states that the product of the winnowing function and the stable active-zone grain size distribution, is proportional to the borrow or backshore material grain size distribution.

In absence of direct knowledge of the winnowing function, $w(\phi)$, and the active-zone cell mass, M , it is not possible to evaluate Equation 2 for the retreat rate, E , in a practical situation. Of course, it is not the intent here to use this model to predict an actual retreat rate, but rather to provide a basis for selection among various types of borrow material. In many situations where beach fill is considered, the beach area is experiencing erosion. In these cases one can use observations on the performance of the native material as a basis for predicting the performance of a proposed borrow material.

Suppose that in some given situation beach erosion is initiated by sharp reduction in, or termination of the natural supply of sediment to the beach area. This could be caused by the construction of jetties or groins, or by natural causes. If one observes the rate at which the beach retreats and the textural properties of the native backshore material and the active-zone materials following initiation of erosion, then one has sufficient information to use the proposed model to predict the performance of a borrow material.

This can be seen as follows. Using Equation 2, the winnowing function may be expressed in terms of the retreat rate associated with the native material, E_n , the grain size distribution of the native material in the backshore, $f_n(\phi)$, and the resulting modified grain size distribution of active-zone native material, $f'_n(\phi)$, as shown in Equation 3a,

$$w(\phi) = \frac{E_n}{M} \frac{f_n(\phi)}{f'_n(\phi)} \quad (3a)$$

Similarly, Equation 2 may be used to express the steady-state active-zone material grain size distribution associated with a borrow material having grain size distribution f_b as shown in Equation 3b,

$$f'_b(\phi) = \frac{E_b}{M} \frac{f_b(\phi)}{w(\phi)}. \quad (3b)$$

Substituting Equation 3a into 3b, and integrating over grain size, yields the relation expressed in Equation 4,

$$\frac{E_b}{E_n} = R_b = 1 / \int \frac{f_b(\phi) f'_n(\phi)}{f_n(\phi)} d\phi. \quad (4)$$

From this equation it is seen that the ratio of retreat rates associated with borrow and native materials can be determined solely from knowledge of the borrow material grain size distribution, the native material grain size distribution, and the modified grain size distribution of native material in the active zone, observed in the eroding state.

This ratio, here referred to as the relative retreat rate, R_b , can be used for prediction of the economic consequences associated with use of any particular borrow material. If for example, two sources of borrow material are available, but differ greatly in their textural properties, and if one of these sources of borrow material can be utilized with one half the construction costs of the other, this does not necessarily imply that the less expensive material is the most economic. The more costly material may be more stable. Calculation of the relative retreat rates of the two materials may indicate that the less costly material will retreat at three or four times the rate of the more costly material, resulting in the requirement for more frequent renourishment and a higher total maintenance cost.

A SPECIFIC MODEL

In many cases it may not be possible to obtain appropriate data on the performance of the native material. This may occur where the beach fill is planned to accompany construction of jetties or other littoral barriers. In such instances the fill is often planned for the purpose of circumventing initiation of an erosive condition downdrift of the engineering structure, as well as for improvement of recreational facilities and shore protection within the project area. Here the engineer cannot wait to observe the beach in the erosive condition which he intends to prevent.

With certain simplifying assumptions, observation of the natural beach in an eroding condition may not be necessary. If the native and borrow material grain size distributions are approximately lognormal, and it is assumed that the modified active-zone material grain size distribution will also be lognormal, and that coarser particles have longer cell residence times than finer particles, then the winnowing function must be of exponential form on the phi scale. If the winnowing function is exponential, then the relative retreat rate associated with a lognormal borrow material having phi mean $M_{\phi b}$, and phi sorting $\sigma_{\phi b}$ is given by Equation 5,

$$\log(R_b) = \Delta \left(\frac{M_{\phi b} - M_{\phi n}}{\sigma_{\phi n}} \right) - \frac{\Delta^2}{2} \left(\frac{\sigma_{\phi b}^2}{\sigma_{\phi n}^2} - 1 \right), \quad (5)$$

where $M_{\phi n}$, $\sigma_{\phi n}$ are the phi mean and phi sorting of the native material, and Δ is a dimensionless parameter which is a measure of the selectivity of the sorting processes as expressed by the winnowing function. The interested reader can find a derivation of this expression in the appendix.

The parameter Δ . As shown in the appendix, the dimensionless parameter, Δ , represents the scaled difference between phi means of native active-zone material observed both prior to and following the establishment of an eroding condition. Its value reflects the selectivity of the sorting processes. A low value means that the natural sorting processes are not highly

selective, and that consequently the predicted relative retreat rates associated with different borrow materials do not differ greatly from unity. Conversely, high values of the parameter indicate a high degree of selectivity in the sorting processes and yield predicted relative retreat rates which vary greatly over typical sources of potential borrow materials. Hence it is important to estimate the value of this parameter in order to apply the model.

There are several ways in which field data may be used to evaluate this parameter. The most direct method is to observe the textural properties of active zone materials in an equilibrium state, and in an eroding state following termination of natural sediment input. Equation A5 (in the appendix) may then be used to directly estimate the value of Δ . Unfortunately data of this type are not readily available.

An indirect method for evaluating this parameter involves comparison of the grain size distribution of active-zone material in an equilibrium profile condition with that of material caught in a downdrift sand trap. Inasmuch as finer particles have higher transport rates than coarser particles, the mean size of material trapped by a total littoral barrier will be finer than material constituting the active portion of the profile updrift of the trap. The trapped material grain size distribution will be proportional to the product of the winnowing function times the active-zone material grain size distribution. Using this relation it can be shown that

$$\Delta = \frac{M_{\phi t} - M_{\phi n}}{\sigma_n}, \quad (6)$$

where $M_{\phi t}$ represents the phi mean of the trapped material. Data presented in [3, p. C13, C14] were used to estimate the phi parameter of composite grain size distributions of material caught in the sand trap behind the offshore breakwater at Channel Islands Harbor, California, and along the active portion of a profile updrift of this trap. The data are quite sketchy (only three samples from the trap and only 6 from the profile) and hence only give a crude indication of the Δ value. For these data the composite phi sorting is on the order of unity for both profile and trap material, and the composite phi means differ by something between .5 and .8 ϕ , giving a rough estimate of Δ between .5 and .8.

Eulerian tracer theory provides another indirect method for estimating Δ . In an Eulerian sediment tracer experiment the tracer is injected continuously into the surf zone for a period of time sufficiently long to allow thorough mixing of tagged grains with untagged grains at some appropriate distance downdrift of the injection site. Sediment samples collected after mixing is attained are analyzed for tracer concentration as a function of grain size. The grain size distribution of the sampled tracer, normalized by dividing by tracer injection rates for each size, represents the grain size distribution of the material in transit (that which would be trapped by a total littoral barrier). Hence Equation 6 may be used to estimate Δ , comparing the mean grain size of the normalized tracer grain size distribution with the composite mean of the samples from which tracer concentrations were measured, assuming these samples adequately cover the active zone of the transport system.

This technique was applied to the results of such a tracer experiment conducted in April, 1972, at the U.S. Army Coastal Engineering Research Center Prototype Experimental Groin site at Point Mugu, California. This experiment is fully described in [4]. Although sample coverage of the active-zone was not as complete as desired for the purposes of this paper, the estimated value of Δ from these experimental results is 0.66, which is consistent with the range of values indicated by the Channel Island Harbor data.

The two beach locations discussed above are quite similar both in textural properties of native materials and in wave climate. Hence the agreement in estimated Δ values is not overly surprising. Although data of the required type were not found for any other coastal area, an indication of a probable range of Δ values in a very different shore environment can be estimated from data collected in connection with beach fill behavior at Presque Isle Peninsula, on the southeastern shore of Lake Erie. The neck of the peninsula has been breached several times in this century and this has been a source of much concern because the peninsula protects a major Great Lakes port (Erie, Pennsylvania) from wave action. Seawalls, a groin field, and repeated beach fills have been constructed here as shore protection measures. Berg and Duane [5] report results of a beach fill experiment where a coarse, poorly sorted fill ($M_s = 0.4\phi$; $\sigma_s = 1.5\phi$) was placed in one groin cell adjacent to a fine fill ($M_s = 2.2\phi$; $\sigma_s = 0.66$) and the relative behavior of the fills observed. In a later report [6, p. 26] it was reported that on the average over a five year period, sand losses from the fine fill area exceeded those from the coarse fill area by a factor of $3\frac{1}{2}$ to 4. Using these numbers and Equation 5 one can deduce that the appropriate Δ value has the bounds $1.3\sigma_s < \Delta < 1.45\sigma_s$, where σ_s is the phi sorting of the composite native material grain size distribution. Unfortunately the history of repeated beach fill in this area makes it impossible to give an exact value to σ_s . However, the composite phi sorting for natural beaches from a wide variety of coastal environments usually lies between .4 and 1, limiting probable values of Δ to the range 0.5 to 1.5.

Of course it is quite possible that different coastal environments will differ in the selectivity of their associated sorting processes. Variability of wave climate differs radically with relative exposure of shore segments to predominant storm wave attack. Hence it may be necessary to conduct field experiments in a variety of circumstances to determine an appropriate Δ value for application to any given coastal segment. In any case it appears that an appropriate value of delta is on the order of unity. Hence a value of $\Delta = 1$ is adopted for the purposes of further discussions in this paper.

Comparison of Relative Retreat Rates and Overfill Ratios. Figure 2 is a plot of contours of relative retreat rates, (using $\Delta = 1$), plotted using the same abscissa and ordinate as used in Figure 1. The contours of the overfill ratio, as given in the Shore Protection Manual, are overlain in quadrant 1.

Ignoring for the moment the overfill ratio curves, it can be seen that the relative retreat rate can be calculated for any combination of native and borrow material textural parameters; hence contours appear in all four quadrants (relative retreat rates less than 1/7 or greater than 7 are not shown). It can also be seen that the relative retreat rate increases for an increasing difference between the phi means of native and borrow materials,

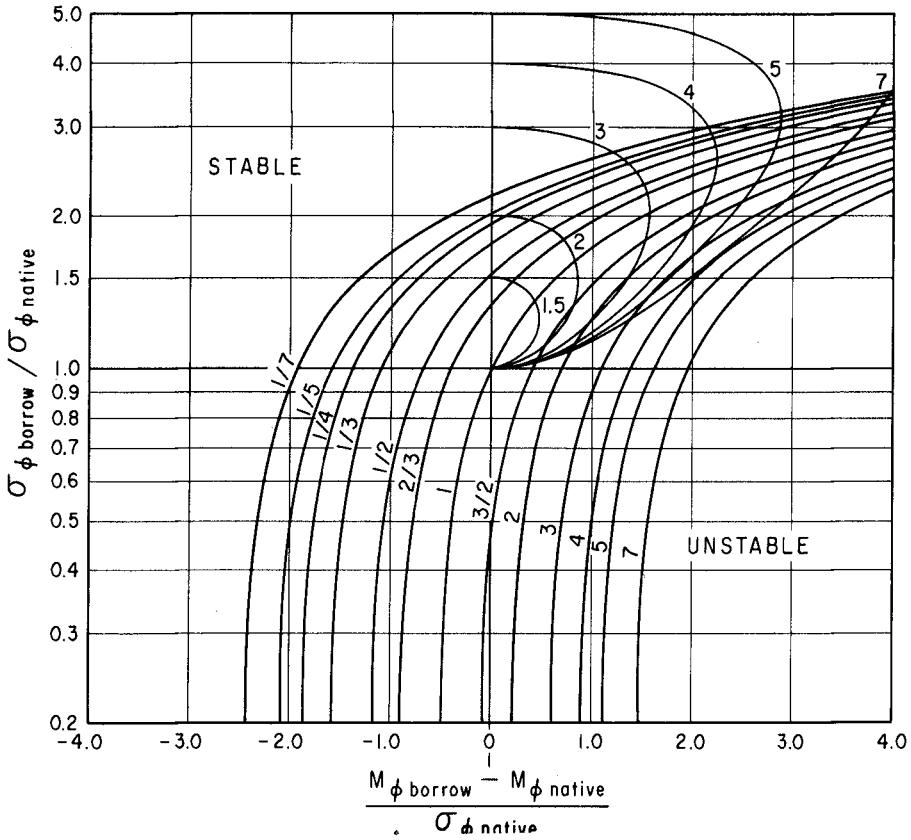


Figure 2. Relative retreat rate curves plotted against relative difference in phi means and ratio of phi sorting of borrow and native materials. For these curves a value of $\Delta=1$ is assumed. In the upper right quadrant overfill ratio curves are also shown for purposes of comparison. Curves for relative retreat rates and overfill ratios exceeding seven are not shown.

and is sensitive to the sign of the difference; the finer the borrow material, the higher the predicted relative-retreat rate. It can also be seen in this figure that predicted retreat rates are sensitive to the ratio of phi sorting between borrow and native material. More poorly-sorted borrow material results in lower steady retreat rates. An intuitive explanation of this relation is that more poorly sorted borrow material contains a larger fraction of coarser material, which provides a more stable armor. The central curve, passing through the origin, shows values of the parameters for which the relative retreat rate is unity. Borrow materials having phi means and sortings which plot on this line have a predicted steady-retreat rate which is the same as that of the native material. Borrow materials plotting to the right and below this curve have higher predicted retreat rates than the native material, and borrow materials plotting to the left and above this curve have lower predicted retreat rates, or are more stable than the native material.

Comparison of relative retreat rate curves and the overfill ratio curves in quadrant 1 indicates that there is very little quantitative agreement between the two techniques. Relative retreat rates are everywhere lower in value than the corresponding overfill ratios. In fact, the only thing the two techniques appear to share, in quadrant 1, is a general tendency to predict lower stability for finer borrow material. Hence it can be said that the model proposed here represents a radical departure, in a quantitative sense, from present guidelines for borrow material plotting in this quadrant.

In contrast, the predictions based on the proposed model are remarkably congruent with the interpretive text which accompanies the graph of this type in the Shore Protection Manual. The Shore Protection Manual states that borrow material plotting in the lower right hand quadrant (quadrant 4) is generally to be considered unsuitable. The proposed model predicts the most unstable fill types are those which plot in this quadrant. An advantage of the proposed model is that it enables calculations to be performed in this quadrant, and hence, in contrast to present guidelines, enables an estimation of the degree of unsuitability. Similarly, the Shore Protection Manual states that material plotting to the left of the vertical axis (quadrants 2 and 3) will be stable, or more stable than predicted by overfill ratio calculations. As can be seen in Figure 2, borrow material plotting to the left of the vertical axis generally have predicted relative retreat rates less than unity: that is, they are more stable than native material. Moreover, the degree of stability can be calculated for borrow materials which are better sorted than native materials.

CONCLUDING REMARKS

In the previous section a direct numerical comparison was presented between relative retreat rates, as predicted by the proposed model, and the overfill ratio, as predicted by the Krumbein and James model. It is important to recall the justification for such a comparison because the conceptual frameworks underlying the two methods differ radically. The overfill ratio is calculated on the assumption that some portion of the borrow material is absolutely stable and hence that a finite proportion of

the original material will remain on the beach indefinitely. The relative retreat rate is calculated on the opposing assumption that no material is absolutely stable, but that finer material is less stable than coarse material, and hence a coarse beach fill will require renourishment less frequently than a fine one. Overfill ratios can never be less than unity because a beach will not retain more material than is added to it. Relative retreat rates can be less than unity because a coarse fill might erode more slowly than native material.

The two methods can be compared only because they both ultimately attempt to predict the economic consequences associated with the utilization of potential borrow materials. In a monetary sense, the engineer can interpret the overfill ratio as a factor to be applied to the actual unit cost of obtaining a given borrow material in quantities sufficient to ultimately establish planned project dimensions. The relative retreat rate can be interpreted as a factor to be applied to the maintenance costs associated with periodic renourishment, when determination of these renourishment requirements is based on natural erosion rates associated with native material.

Both models are quite simple from a conceptual standpoint, thus it seems unlikely that either of them fully describes any real shore situation. However a subtle distinction can be made in the types of uses to which these methods are put. On one hand the engineer wishes to estimate the total cost associated with selection of a given borrow material. Application of the two methods will give some indication of the possible range of such values but strict use of either method seems unjustified, when the inherent simplicity of the models is weighed against the staggering complexities of the physical processes operating in the nearshore and beach environments. On the other hand, the engineer is usually limited to a few economically feasible sources of borrow material and he must choose one, regardless of the absolute accuracy of his predicted costs. The power of these methods to aid in such a decision seems greater than their absolute predictive powers. Indeed it is the very simplicity of the underlying concepts that allow the engineer to exercise independent judgement, based on his experience, in applying the methods and in finding an appropriate compromise between them when they provide very different results.

ACKNOWLEDGMENTS

George M. Watts, of the U.S. Army Coastal Engineering Research Center, provided the motivation for this work, and with David B. Duane, created the atmosphere within which it could be done. William C. Krumbein read two earlier versions of the manuscript and provided a perspective which may yet be inadequately conveyed in the discussion of the relative merits of the two methods. Additional perspective was provided by Limberios Vallianos. Comments and encouragement from Lim and from Neil Parker, Orville Magoon, and Norm Arno, all with the U.S. Army Corps of Engineers, are highly appreciated.

Permission from the Chief of Engineers to publish this material is appreciated. The views expressed in this paper are not to be construed as official Department of Defense policy unless it is expressly so stated in other authorized documents.

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APPENDIX

Derivation of Equation 5

Assumptions:

1. Native and borrow material grain size distributions are lognormal:

$$f_n(\phi) = (1/\sqrt{2\pi}\sigma_n) \exp\{-(\phi - M_{\phi n})^2/2\sigma_n^2\}, \quad (A1a)$$

$$f_b(\phi) = (1/\sqrt{2\pi}\sigma_b) \exp\{-(\phi - M_{\phi b})^2/2\sigma_b^2\}. \quad (A1b)$$

2. The steady-state grain size distribution of active zone material observed in the eroding state is lognormal;

$$f'_n(\phi) = (1/\sqrt{2\pi}\sigma'_n) \exp\{-(\phi - M'_{\phi n})^2/2\sigma_n'^2\}. \quad (A2)$$

3. The winnowing function monotonically increases with ϕ .

Substituting equations A1a and A2 into equation 3a from the text, we have,

$$w(\phi) = \frac{E_n \sigma'_n}{M \sigma_n} \exp\left\{\frac{1}{2} \left(\frac{\phi - M'_{\phi n}}{\sigma'_n} \right)^2 - \left(\frac{\phi - M_{\phi n}}{\sigma_n} \right)^2 \right\}. \quad (A3)$$

Examination of equation A3 shows that assumption 3 above will only be satisfied if $\sigma'_n = \sigma_n$. Hence equation A3 may be rewritten as

$$w(\phi) = (E_n/M) \exp\{(M_{\phi n}'^2 - M_{\phi n}^2)/2\sigma_n^2\} \exp\{\phi(M_{\phi n} - M_{\phi n}')/\sigma_n^2\}. \quad (A4)$$

Let a dimensionless parameter Δ be defined as

$$\Delta = (M_{\phi n} - M'_{\phi n}) / \sigma_n . \quad (A5)$$

Equating A4 and 3a, and using A5 to eliminate the appearance of $M'_{\phi n}$, we can write

$$\frac{f'_n(\phi)}{f_n(\phi)} = \exp\left\{\Delta\left(\frac{M_{\phi n}}{\sigma_n} - \frac{\Delta}{2}\right)\right\} \exp\left\{-\frac{\Delta\phi}{\sigma_n}\right\} . \quad (A6)$$

Multiplying by equation (A6), completing the square in the exponent, integrating over grain size, and taking the inverse yields, (by equation 4 of the text),

$$R_b = \exp\left\{\Delta\left(\frac{M_{\phi b} - M_{\phi n}}{\sigma_n}\right) - \frac{\Delta^2}{2}\left(\frac{\sigma_b^2}{\sigma_n^2} - 1\right)\right\} ,$$

which completes the derivation.