CHAPTER 106

FORCES FROM FLUID FLOW AROUND OBJECTS

By John H. Nath[1] and Tokuo Yamamoto[2]

Abstract

All hydrodynamic forces on submerged objects are shown to be due to the acceleration effects of the fluid flow. However, it is useful to consider separately the influence from the various ambient flow and local flow conditions. At times certain aspects of the flow can be ignored, which simplifies the analysis. Examples are developed for circular cylinders near a plane boundary with a flow direction parallel to the boundary and perpendicular to the cylinder. Potential flow theory predicts that large vertical forces exist away from the boundary when the cylinder is against the boundary and that large negative forces exist toward the boundary when the cylinder is positioned a small distance from the boundary, when the viscous effects are small. When the cylinder is near the boundary, the added mass coefficient is the same regardless of the direction of the flow, providing the flow is perpendicular to the cylinder. In addition, the added mass coefficient is much larger for cylinders near the boundary than when they are in a free stream. Good agreement between theory and laboratory experimentation was obtained for various coefficients with waves on horizontal cylinders near a plane boundary.

INTRODUCTION

There is an increasing need for accurate information on how to analyze the fluid dynamic forces on objects constructed for various types of ocean engineering projects. Structures such as piers, drill rigs, semi-submersibles, pipe lines, cables, buoyancy elements, ships, barges and so forth, require realistic information as to the forces from the major loading activities of the wind, waves and current. Such forces originate from the fluid motion or the object motion, or both. Certain amounts of information exist in the literature on such fluid motions, but much of it is difficult to understand by the average design engineer.

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Ideally, one should consider the submerged object to be a part of the boundary conditions for the flow regime. The general pressure distribution is then determined, if possible, for the entire flow regime and then evaluated on the surface of the object. Closed mathematical solutions are obtained whenever possible, which usually involves determining the velocity potential or stream function in the governing equations of fluid motion, (when viscous effects can be ignored), or a solution to some form of the Navier-Stokes equations is at least tried. However, closed solutions are rarely obtained, so that numerical models may be required. In addition, measurements from a prototype structure and/or physical scale model study in the laboratory give the analyst an opportunity to validate a theoretical model or to establish an empirical model for complicated flows that do not yield to analysis.

OVERVIEW OF ANALYSIS

Models of the physical processes involved can be developed from one or more of the following: closed mathematical solutions, numerical solutions, and physical model studies. The usefulness of all these should be considered for any given problem. In addition, the analyst should try to present the results of his analysis in a form that is relatively easy to understand by the designer, who usually is not the same person.

The flow regimes are sometimes very complicated and depend on many things, some of which are the ambient velocity distribution, free stream turbulence, wake characteristics, boundary and initial conditions, the total acceleration, the phase relationships between the various actions and the motion of the object itself, and the interaction of the object motion with the flow. Consequently, no general design criteria are available and many theoretical works on this subject are based on simplifying assumptions.

In general, an analysis should be concerned with the following:

Potential flow theory (ideal fluids)

1. Uniform steady ambient flow or the steady linear motion of the object.

2. Non-uniform flow due to boundary irregularities, curvilinear motion of the body, free surface conditions, skewed axis bodies, etc.

3. Unsteady, non-uniform flow, such as periodic motion, impulsive motion and aperiodic motion (random fluctuations).

Viscous flow (real fluids)

1. Free stream turbulence.

2. Boundary layer development on the object (skin friction drag).

3. Submergence of the object in a boundary layer developed on a nearby surface.

4. Wake formation (Boundary separation, form drag and vortex shedding).

Topics for differences that occur between simplifying assumptions and the conditions in nature

1. Irregular boundaries.

2. Aperiodic nature of the flow.

3. The shape and motion of the actual wave compared to the wave theory assumed.

4. Three-dimensional aspects of the flow versus a two-dimensional analysis.

5. Unknown oceanographic conditions.

It is sometimes useful to consider the forces from a complicated, time dependent flow to be the result from the superposition of more than one relatively simple terms. Thus, the Morison equation was established as an empirical relationship to explain the phenomena of wave forces on vertical piling. (8). They stated that the wave force was due to an accelerative force (proportional to the virtual mass coefficient and the water particle temporal acceleration evaluated at the pile center) and a drag force (proportional to the drag coefficient and the square of the water particle horizontal velocity). The approach was simple and easy to use, with an attempt at a physical explanation for such forces. However, in subsequent years, evaluations of the coefficients displayed a great deal of scatter. This was sometimes due to the inclusion of several modifying flow effects into the determination of C_D and C_M; whereas, certain effects should be considered separately, as discussed in this paper.

Some basic shortcomings of the Morison equation approach include:

1. The forces from diffracted and reflected waves are not considered, which is necessary when the cylinder diameter to wave length ratio is large (4).

2. The convective acceleration components of the ambient flow are not included.

3. The evaluation of C_D and C_M depends on the wave theory used, and phase relationship between the ambient fluid velocity and the drag force is not taken into account. The importance of this is illustrated in Figure 1 which depicts the surface stress distribution on a circular cylinder oscillating in a harmonic manner, such that laminar flow, without separation, results. The total drag force is $\pi/4$ out of phase with the cylinder velocity. For turbulent flow, including a wake formation, a similar, albeit unknown, phase difference will exist between the drag force and the relative motion between cylinder and fluid. Thus, the evaluation of the virtual mass coefficient may be in error when it is assumed that the drag force is zero when the relative velocity between the cylinder and ambient fluid is supposed to be zero.

4. Another shortcoming is that the Morison equation may give a misconception that drag forces are not due to acceleration effects.

To illustrate the last point for a real fluid, consider Figs. 2 and 3. Figure 2 depicts steady uniform flow around a cylinder such that form drag predominates. Figure 3 depicts shear flow on a thin, flat place such that skin friction drag predominates, a manifestation of which is the development of the boundary layer thickness. In both cases a wake is formed. We assume the pressure gradient is zero in each case (steady unconfined flow) and that shear stresses do not exist on the surface of the control volumes. Thus, the only forces acting on each control volume one due to the cylinder in one case, and the flat plate in the other, which represent the two extremes of form drag and skin friction drag.

The general momentum equation applied to the surface of the control volume will determine the force exerted on the control volume by the object. In general, for any control volume,

$$\Sigma\vec{F} = \frac{d\vec{M}}{dt} = \frac{\partial M}{\partial t} + \vec{u}\cdot\nabla\,\vec{M} \tag{1}$$

or

$$\Sigma\vec{F} = \frac{\partial\vec{M}}{\partial t} + \int_S \rho\vec{u}\;\vec{u}\cdot\vec{n}ds \tag{2}$$

where

$$\vec{M} = \int_V \rho\vec{u}dV \tag{3}$$

is the total momentum within the control volume, $\vec{u}$ is the velocity vector, $\vec{i}u + \vec{j}v + \vec{k}w$, $\vec{n}$ is the unit normal vector directed outward from the control volume, ρ is the fluid mass density, s indicates the surface of the control volume and V is the volume.

The fluid is incompressible so ρ is constant and Eqs. 1 and 2 are readily recognizable in terms of acceleration. The first term on the rhs of Eq. 2 is the temporal term and the second is the convective term. Thus, even for cases where the viscous effects on the force completely predominate, it is seen that they should be considered as acceleration effects. For a circular cylinder in uniform flow where vortices are being shed, the convective term of the acceleration is time dependent, since the ambient velocity, U_∞, is constant. When the ambient velocity is changing with respect to time, the first term on the rhs of Eq. 2 is non-zero.

It is well known that in real fluids the potential flow considerations are quite valid in many conditions except where the viscous effects make modifications through boundary layer development, vortex shedding and other viscous wake manifestations and free stream turbulence.

In many cases the viscous effects are neglibible. For example, in the impulsive motion of a circular cylinder "instantaneously" to the velocity, U, perpendicular to its axis, the viscous effects are initially very small until the wake has had time to build. The drag coefficient for the laminar case is shown in Fig. 4, which appears in Sarpkaya (12) for a Reynolds number of from 0.15 to 1.2×10^5. Thus when the fluid travel is less than about 0.4D the wake has not yet formed. Likewise, it is infered that in waves, if the water particle displacements are less than this amount, separation effects will be negligible.

Another interesting aspect of Fig. 4 is that the drag coefficient reaches the value of 1.55 at $Ut/D = 4$, whereas for steady flow at this Reynolds number it has the value of about 1.1 to 1.2. For this condition twin vortices have been formed in the wake of the cylinder and have not yet been shed. When the vortices are finally shed, a stable alternating of shedding is set up into the familiar Karman vortex trail.

Thus, if considerations for wake formation can be neglected a potential flow solution is possible. If not, we are usually dependent on testing. Theoretically, if the wake shape and time characteristics can be closely approximated, a modified shape of the object can be assumed and a potential flow solution is still possible, using the modified shape for the object. Several recent cases have been presented in the literature where the viscous effects can be neglected, some of which are in (1, 2, 3, 4, 5, 9, 10, 11, 13, and 14).

It is well known that the conditions of nature may not coincide closely with the simplified assumptions made in an analysis. For example, a pipeline placed on the sea bottom may not really approximate the condition of a cylinder near a plane boundary because of bottom irregularities. Thus, the spacing between the pipe and boundary may change with regard to positions on the pipeline. The designer should be aware of the maximum irregularities that can occur and try to conservatively estimate the differences in forces acting so as to obtain an envelope of conditions to which the pipeline is likely to be subjected. This paper will show that small differences in clearance under a pipeline can make large differences in the forces acting on the pipe.

In addition, flow conditions are not likely to be steady, nor periodic. For the case of ocean waves the random nature of the water surface elevation and wave frequencies may be taken into account (10), particularly for linear systems. However, in many areas the waves are multi-directional, although a particular direction may predominate. Considerations must be given to the differences between these simplified analytical conditions and the actual conditions provided by nature.

Laboratory experimentation is viewed with mixed emotions by the profession. The usually large dissimilarity between the Reynolds number in the laboratory and in nature prompt many engineers to feel that the laboratory work is only academic in nature and that realistic forces cannot be obtained. However, when Reynolds number dissimilarity is important, the drag coefficients that are determined from laboratory work are usually larger than would be the case for natural conditions, and the use of them would therefore result in conservatively large forces for design. However, the considerations for the ambient acceleration aspects of the flow may completely predominate the total force condition such that the viscous effects are negligible. Thus, for some pipes subjected to certain wave frequencies and heights, the Reynolds number has negligible influence and, therefore, the undesirable scale effects between laboratory and prototype conditions are non-existant. This is true when Froude number is important and Reynolds is not, indicating that the surface wave generated by the object is important but that the wake formation is not. The results determined from such laboratory experimentation are directly applicable to prototype conditions. In general, for conditions where wake formation is not important, and where surface tension is also not important, laboratory results are directly applicable to prototype conditions providing the load conditions in nature can be reproduced in the laboratory.

THEORETICAL EXAMPLE

Analytical developments for wave forces on cylinders near a plane boundary were compared to laboratory results with some success. First considerations have been given to rigid cylinders with the flow perpendicular to the cylinder axis.

The proximity of the plane boundary will have an influence on the viscous effects. Vortex shedding and wake size are influenced as indicated in Fig. 5, which shows drawings, interpreted from photographs of positions of a 2-inch cylinder near the boundary in a birefringent fluid. The photographs (taken with polarized light) showed that vortex shedding begins to be suppressed when the cylinder is within one diameter of the bottom and that the wake size is considerably increased when the cylinder is on the bottom where vortex shedding is nearly stopped, indicating a possible increase in drag coefficient. The flow Reynolds number was 1.7×10^4. In addition, for conditions where the wake formation is important, as in Fig. 5, the lift forces may always be positive, because of the nearby plane boundary, instead of negative when wake formation is negligible This will be discussed in more detail later.

The free surface effects (or the surface waves generated by the presence of the cylinder) can be examined with the use of potential flow theories. For uniform steady horizontal flow Havelock (5) developed a linearized second-order solution in a complicated series that would not be usable by the usual designer. The results are summarized in (15). However, a translation of the results into graph form is shown in Figs. 6 and 7. They show that the surface effects on the forces on a cylinder submerged in a uniform flow are negligible if the submergence is greater than about four cylinder diameters. Recently Tuck (14) solved the problem in non-linear second-order form and showed that the non-linear effects are significant in some cases.

Likewise, Ogilvie (11) derived the forces acting on a cylinder while it is oscillating near the surface. The solution for the added mass coefficient is again a series solution and is summarized in Fig. 8. It can be seen that if the cylinder is submerged 3 or 4 diameters that the added mass coefficient from the free surface effects (which are frequency dependent) are negligible. Actually, for a cylinder submerged as little as one diameter the forces from the free surface effects are small.

For this experimental work, only oscillatory forces were considered and the cylinders were submerged at least one diameter and usually more. In addition, wave particle motion was calculated to be less than 0.4D so that viscous effects were negligible. Thus, the study reduced to investigating the effects of the nearby plane boundary on the wave forces on cylinders, with potential flow theories. Comparisons were made with laboratory measurements.

The uniform potential flow for a cylinder near a plane boundary is mathematically equivalent to that for two equal cylinders in a uniform flow. The complex potential for two cylinders in a uniform stream was determined with Milne-Thompson's circle theorem. (We have since expanded this to N number of cylinders in a uniform, unsteady stream, each moving individually.) The method of images applied to two cylinders is also reviewed in (1) and (9). An image solution in three dimensions for an ellipsoid travelling parallel to its major axis is presented by Eisenberg (2). The result for this work is detailed in (15). The total forces on the cylinder were then found from the Blasius theorem as

$$X - iY = \tfrac{1}{2}\, i\, \rho \oint \left(\frac{dw}{dz}\right)^2 dz - i\, \rho \frac{\partial}{\partial t} \oint \bar{w}\, d\bar{z} \qquad (4)$$

where X and Y are the forces in the x and y directions and $w = \phi + i\Psi$, the complex potential.

In Eq. 4 the first term on the rhs accounts for the convective acceleration and the second term for the temporal acceleration.

Uniform, steady potential flow, parallel to the plane boundary, was considered first as shown in Fig. 9. No net drag force results parallel to the wall. However, a "lift" force on the cylinder, perpendicular to the wall, due to the convective acceleration effects of the steady local flow, does result. The potential flow solution is in terms of an infinite series of doublets distributed from the center of the cylinder on the radius, toward the wall. It is reviewed in (15) and will be detailed in a future paper. The result for the lift force is, for s > a (see Fig. 9):

$$F_L = -4\pi\rho U^2 a \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} \frac{m_j\, m_k}{\left(\frac{2s}{a} - q_j - q_k\right)^3} \quad , \quad e > a \qquad (5)$$

where

$$m_n = q_2^2\, q_3^2 \ldots q_n^2, \qquad m_1 = 1 \tag{6}$$

$$q_n = \frac{1}{\frac{2s}{a} - q_{n-1}}, \qquad q_1 = 0 \tag{7}$$

The results for this case are also given in terms of bi-polar coordinate variables in Ref. 9. At the wall (s = a), the lift force per unit length of cylinder is given in closed form by Müller (9) as

$$F_L = \rho U^2\, a\, \frac{\pi}{9}\, (\pi^2 + 3). \tag{8}$$

The lift coefficient is defined as

$$C_L = \frac{F_L}{\frac{1}{2}\rho U^2 D}\,. \tag{9}$$

The lift coefficient, C_L, was calculated as a function of the gap between the cylinder and the bottom boundary using the first 40 images and it is plotted in Fig. 10. When the gap, e, is equal to one cylinder diameter, the lift coefficient is very small and may be neglected for many design problems. Figure 10 also shows that (for the theoretical potential flow case) if the gap is very small then a very large negative force exists toward the wall. However, when the cylinder touches the wall a positive lift force is experienced. Therefore, a severe reversal of lift force is possible for some cylinders that are not securely anchored down and where the submerged weight of the pipe is less than the positive force when the cylinder touches the bottom. Boundary layers on both the wall and the cylinder will alter the results from the potential flow theory.

Another simple loading condition of great interest is for uniform ambient flow that is accelerating either perpendicular to, or parallel to, the plane boundary. The added mass coefficients in such cases will be the same as for a cylinder accelerating in the same direction in a still fluid. The result is given below and the derivation, which again is dependent on an infinite destribution of doublets on the cylinder radius, will be detailed in a future publication.

For a cylinder accelerating parallel to a near by plane boundary (the x direction) with a temporal acceleration of $\dot{u}$, the Blasius theorem results in hydrodynamic forces on the cylinder of

$$X = -\rho \pi a^2 C_M \dot{u} \tag{10}$$

and
$$Y = +\rho C_L a u^2 \tag{11}$$

where
$$C_M = 1 + 2 \sum_{j=1}^{\infty} \frac{m_j}{m_1}^2 \tag{12}$$

or
$$C_M = 1 + 2 \sum_{j=2}^{\infty} (q_2^2 \, q_3^2 \, . \, . \, . \, q_j^2)^2 \tag{13}$$

and for C_L the developments in Eqs. 5 through 9 are valid. Thus, for $e > a$ there is an attractive force between the cylinder and the plane boundary.

For a cylinder accelerating perpendicular to the plane boundary,

$$X = 0 \tag{14}$$

and
$$Y = -\rho \pi a^2 C_M \dot{v} + \rho C_L a v^2 \tag{15}$$

The added mass and lift coefficients are determined the same way as for Eqs. 10 and 11. Thus, whether the cylinder is accelerating toward or away from the boundary, there is an attractive force between the cylinder and the boundary which is proportional to the square of the velocity. Actually, for a cylinder accelerating on a line at any angle with the boundary, the lift force will be determined in the same manner and will be proportional to the square of the absolute value of the cylinder velocity and the added mass force will also be the same as before and proportional to, and colinear with, the cylinder acceleration. The added mass coefficient, C_M, as determined from Eq. 13 is plotted in Fig. 11.

The above developments can be applied to cylinders subjected to waves if the velocity and acceleration gradients are negligible over the diameter of the cylinder as indicated in Fig. 12 and if the wake development is negligible. That is, the uniform, potential flow assumptions must be valid in order for these developments to be applicable. This restricts the usefulness of the theoretical developments, but these cases are important. The solutions for non-uniform, unsteady flow, and for cases of significant wake formation, are being developed in order to remove this restriction and they will be presented in a future paper.

EXPERIMENTAL EXAMPLE

Laboratory testing was accomplished on 2", 4½" and 6" diameter cylinders at various spacings from a nearby plane boundary and for various water depths. The test set-up is indicated in plain view in Figure 13. Typical wave and force records are shown in Figs. 14 and 15. A complete summary of the original results is given in Ref. 15.

In most cases the cylinders were submerged at least two cylinder diameters below the free surface. In a few cases, about one diameter submergence was obtained. For oscillatory flow, therefore, the free surface effects should be small, as indicated in Fig. 8. So, as a first approximation the free surface effects were ignored. In addition, the wave heights and lengths, in conjunction with the water depths produced particle displacements that were small with respect to the cylinder diameter, so that the viscous effects could be ignored.

When a wave crest is over a cylinder the water particle acceleration acts downward on the cylinder. In addition, the horizontal velocity is maximum at that point so that the "lift" force from it is maximum and negative, adding to the vertical particle acceleration force. When a wave trough is over a cylinder the water particle acceleration acts upward, in opposition to the lift force. These effects are clearly in evidence in Fig. 14 where the space beneath the cylinder is about 0.02 times the diameter, but they do not appear in Fig. 15 where the space is 0.05 times the diameter. One might predict this result from Fig. 10, although keeping in mind that he is comparing oscillatory flow to steady flow. Evidently they are closely comparable in this case because the lift force was derived from the records from

$$F_L = \frac{1}{2}\left(-\left|F_{crest}\right| - \left|F_{trough}\right|\right) \qquad (16)$$

Likewise, the vertical temporal acceleration force was approximated from

$$F_{VA} = \tfrac{1}{2}\left(- \left|F_{crest}\right| + \left|F_{trough}\right|\right) \qquad (17)$$

Such data were plotted for several runs and examples of some of the results are given in Figs. 16 and 17. The slope of the line in Fig. 16 gives the lift coefficient and the slope of the line in Fig. 17 yields the added mass coefficient. Figure 18 shows similar results for the horizontal temporal acceleration forces, which were obtained from the raw data at times when the horizontal acceleration was computed to be maximum. Linear wave theory was assumed, again as a first approximation, since wave steepness was small.

The results of all the testing are shown in Figs. 10 and 11. In addition, the testing results, replotted from Schiller (11) are shown. Agreement in the lift coefficient between uniform flow theory and experimental wave results were suprisingly close. Considerable scatter exists for the added mass coefficient and is probably due to some influence from drag (or simply a wake formation), the use of Airy wave theory, the inappropriateness of comparing uniform flow to non-uniform flow and experimental error. In addition, the accurate measurement of forces when the cylinder touches the plane boundary is very difficult.

CONCLUSIONS

The influence of a nearby plane boundary on the hydrodynamic forces on a cylinder is clear when there is no wake formation. Figure 11 shows that the added mass coefficient for a cylinder on the boundary is more than twice as large as for a cylinder in a free stream. Similarly the lift coefficient is about +4.5 when the cylinder rests on the boundary and is zero in the free stream (except for viscous flow modifications from vortex shedding). However, when a small gap exists between the cylinder and the boundary a very large and negative lift coefficient exists for the cylinder.

Sometimes relatively simple solutions can yield adequate results for design work providing the constriants implicit with the simplifications are completely recognized. Thus, the diffraction theory approach may not be necessary if surface waves are not caused by the presence of the object, which is the case for cylinders when the submergence is greater than four cylinder diameters. In addition, if velocity gradients are not large over the cylinder diameter, uniform flow solutions may be useful.

Additional work is necessary to include non-uniform flow. In addition, the viscous effects need clarification, particularly with regard to their influence on the potential flow developments, and accurate knowledge of the phase shift between the ambient flow and drag force fluctuations need to be known. The influence of cylinder skew angles and roughness, when near the boundary, are also topics for future investigations.

All forces acting on submerged objects should be considered to be acceleration forces. Analyses should consider the potential flow effects and the viscous effects. Most viscous effects must be determined in the laboratory; however, mathematical solutions for laminar flow may be tractable. In addition, many acceleration (particularly including gravitational) effects can be investigated in the laboratory with little or no deleterious effects from scale dissimilarities. Because the viscous effects were small, the results given here are directly applicable to similar prototype conditions. High Reynolds number testing will be conducted in the future at the OSU Wave Research Facility that is capable of producing waves with heights up to five-feet high and breaking.

It is hoped that the results presented here have been put into a form that is readily useable by designers for problems where the wake development is negligible.

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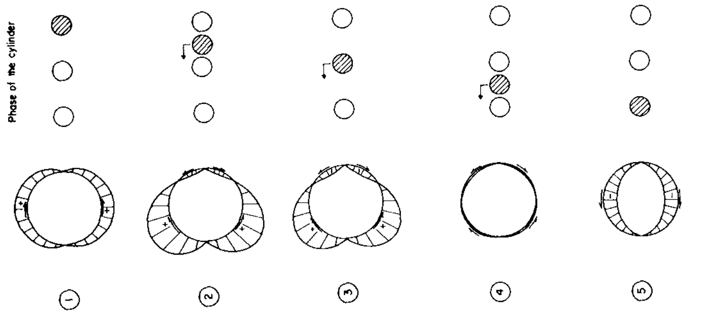

Fig. 1 - Laminar Flow Surface Drag for an Oscillating Cylinder

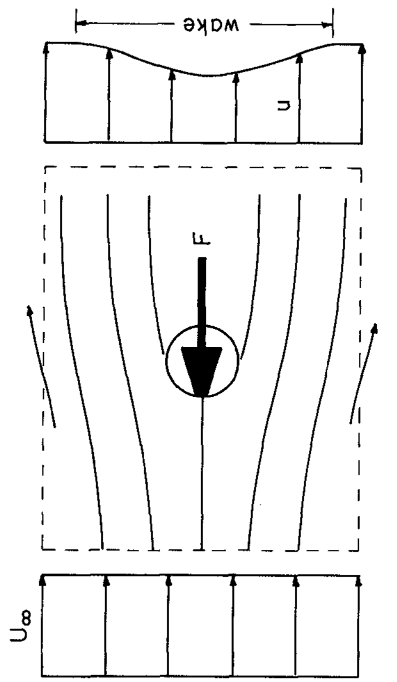

Fig. 2 - Example of Profile Drag

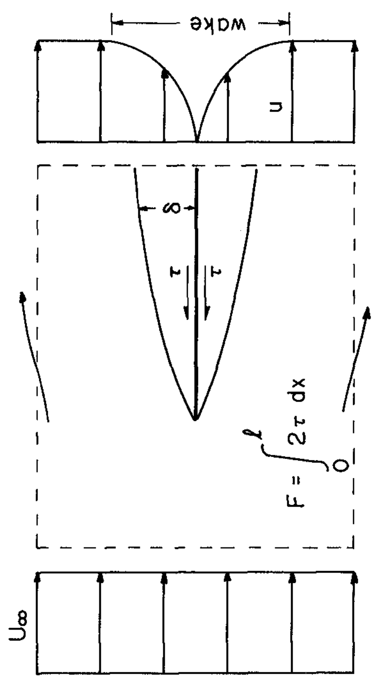

Fig. 3 - Example of Shear Drag

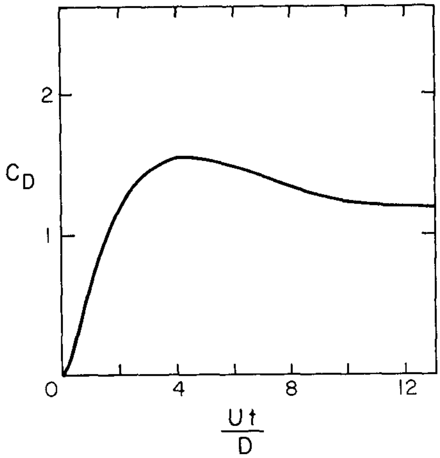

Fig. 4 - Drag Coefficient for Impulsive Motion of a Circular Cylinder (From Rosenhead)

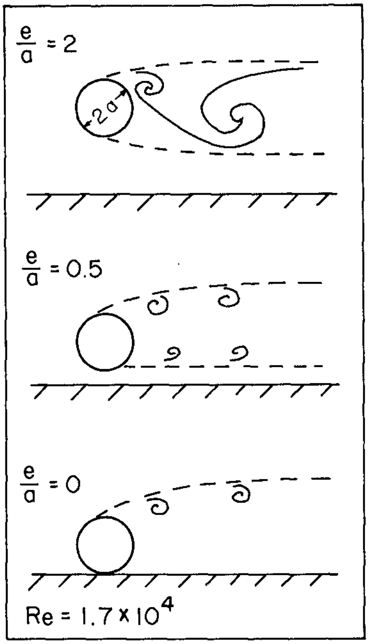

Fig. 5 - Wake Interpretations from Flow Visualization

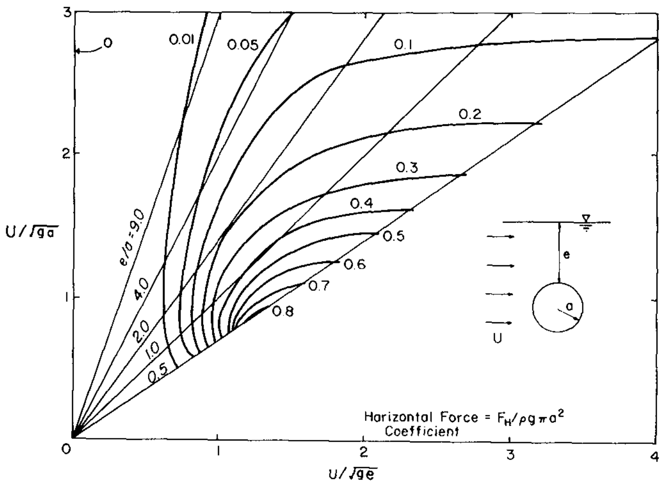

Fig. 6 - Free Surface Effect on Drag Force on a Circular Cylinder (Computed from Havelock(1936)).

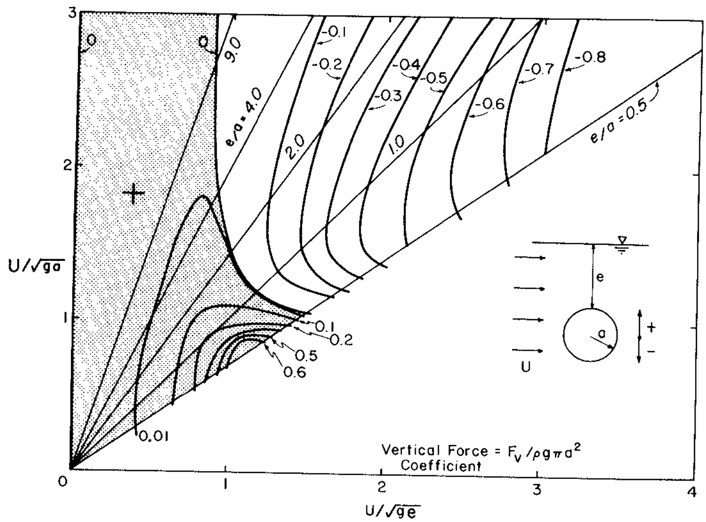

Fig. 7 - Free Surface Effect on Lift Force on a Circular Cylinder (Computed from Havelock (1936)).

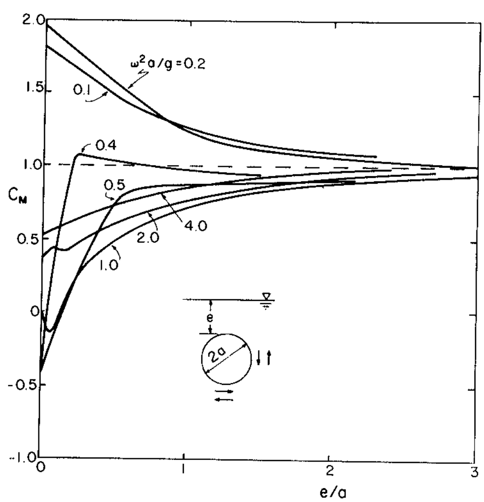

Fig. 8 - Free Surface Effect on Added Mass of a Circular Cylinder (Computed from Ogilvie (1963)).

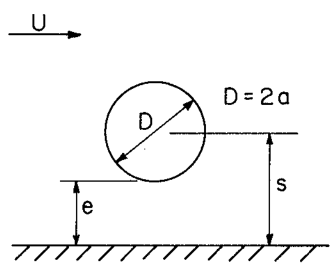

Fig. 9 - Cylinder in Cross Flow with Nearby Plane Boundary

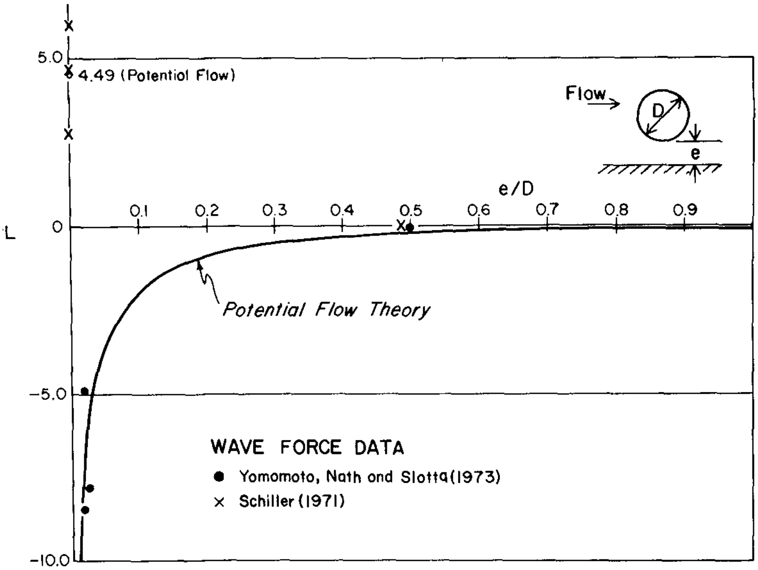

Fig. 10 - Lift Coefficient of Circular Cylinders Near a Plane Boundary

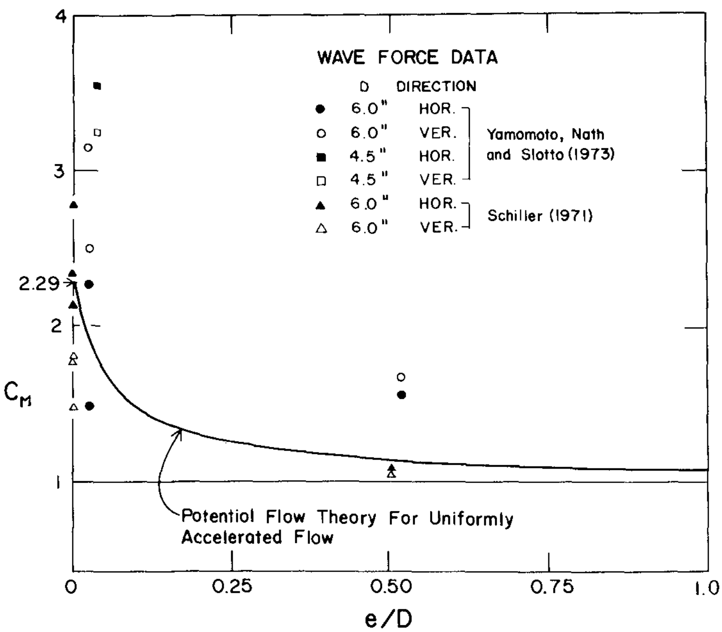

Fig. 11 - Added Mass Coefficients for Circular Cylinders Accelerating Near a Plane Boundary

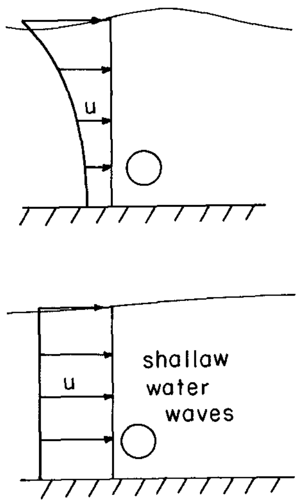

Fig. 12 - Uniform Flow Approximation to Non-Uniform Flow

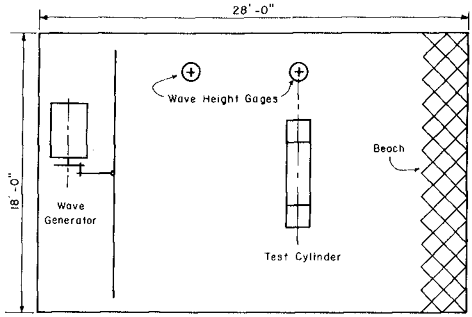

Fig. 13 - Schematic Illustration of Wave Basin and Experimental Setup

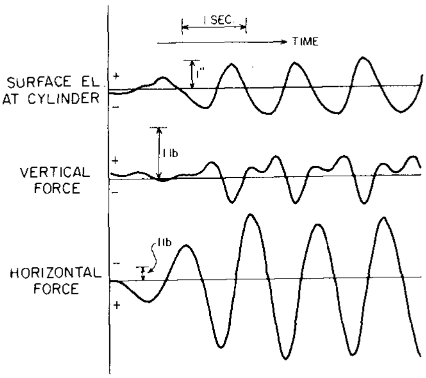

Fig. 14 - Example Data for D = 6", e = 1/8", h = 10"

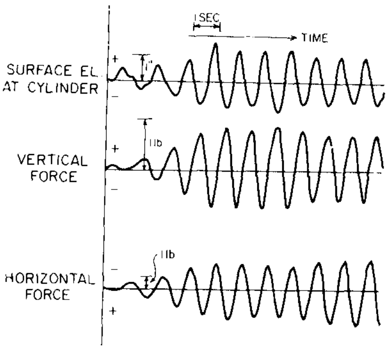

Fig. 15 - Example Data for D = 6", e = 3", h = 14-1/8"

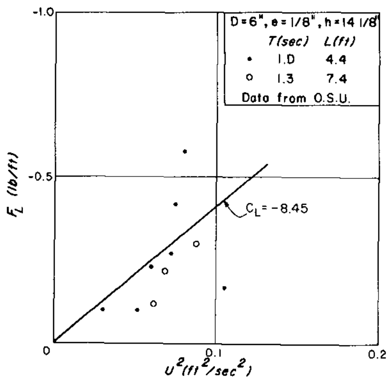

Fig. 16 - Lift Force vs. Horizontal Velocity Squared

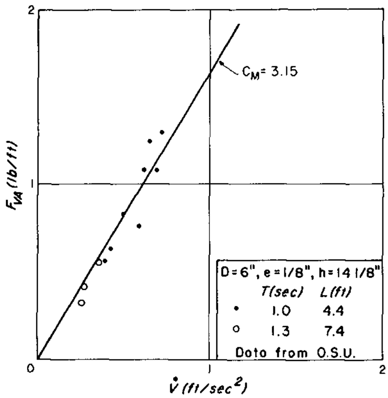

Fig. 17 - Vertical Acceleration Force vs. Vertical Acceleration

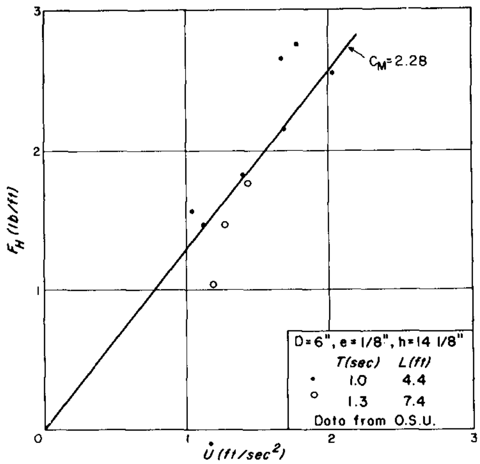

Fig. 18 - Horizontal Force vs. Horizontal Acceleration