CHAPTER 129

NUMERICAL SIMULATION OF OIL SLICK TRANSPORT IN BAYS

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ABSTRACT

A computer model for simulating oil spreading and transport has been developed. The model can be utilized as a useful tool in providing advance information and thus may guide decisions for an effective response in control and clean-up once an accidental spill occurs. The spreading motion is simulated according to the physical properties of oil and its characteristics at the air-oil-water interfaces. The transport movement is handled by superimposing the spreading with a drift motion caused by winds and tidal currents. By considering an oil slick as a summation of many elementary patches and applying the principle of superposition, the model is capable of predicting the oil size, shape, and movement as a function of time after a spill originates. Field experiments using either cardboard markers or soybean oil to simulate a spill were conducted at the Long Beach Harbor. Computer predictions showed good agreement with the field traces.

1. INTRODUCTION

The rapid growth of offshore oil production and marine oil transport has led to an increasing danger of oil contamination of the coastal environment. Consequently, there is a growing worldwide concern over possible environmental damage caused by accidental oil spills.

The present study has been aimed at developing a computer model which is capable of predicting oil slick transport in harbors and bays. By means of numerical approximation, the model is able to simulate the spreading and movement of oil slicks on the ocean surface and to predict their destinations.

The movement of an oil slick is simulated as a combination of the various phenomena which affect the spreading and transport. The simulation of the spreading process is governed by the physical properties of oil and its characteristics at the air-oil and oil-water interfaces. The spreading motion is then superimposed on the drift motion caused by winds and tidal currents to give the total movement. By considering the slick as a summation of many elementary patches in its numerical scheme and applying the principle of superposition to each individual patch, the model is capable of simulating the slick's shape distortion as a result of the relative shear motion produced by the non-uniformity of tidal currents and wind drifts on the water surface.

Sample computations have been conducted using the Long Beach Harbor-San Pedro Bay area as a test case. Several field tests using either cardboard markers or soybean oil to simulate a spill were also conducted in this area. Computer predictions using input obtained from field information show good agreement with the field traces and the validity of the computer model for field operation is substantially confirmed.

A dedicated small computer system is useful to accommodate the model for use in port offices. The geographical data and the oceanographic environment of the harbor can be pre-fed into the system. Wind information may be fed constantly to the system from remote sensors in the field. In the event of an accidental spill occurring in this area, one may press the keyboards to supply very simple data such as the time of the spill, type of oil, location and volume of the spill, and from this input the model will predict the oil size, shape, and movement as a function of time. The model may provide a moment-to-moment update of critical conditions. The model may predict the arrival time of the oil slick at various portions of the shoreline along with its spreading and transport well ahead of the true slick's arrival. This advance information of arrival time, location, and size would be extremely useful in guiding the decisions of engineers and operational personnel for an effective response in control and clean-up, and therefore, the model can be a very effective tool to minimize the environmental impact on shorelines.

2. OIL SPREADING ON CALM WATER SURFACE

Only a sudden release of oil is considered in this paper. The site of spill is considered inside a harbor or inside the breakwater protection of a bay where the effect of the waves are generally negligible and the water may be regarded as calm.

The basic mechanism affecting the spreading of oil slicks over calm water includes inertia, gravity, viscosity and surface tension. At the initial stage, the primary driving force is due to gravity and the rate of spreading is governed by a balance between the gravitational pressure and the oil inertia. As the spreading proceeds, the oil slick becomes thinner and the viscous effect becomes more evident. Quickly, the oil thickness becomes thin enough so that the inertia effect becomes negligible and the spreading enters into its second phase while the gravitational spreading force is primarily balanced by the water's viscous retardation. The gravity oriented body force gradually becomes less important as compared to the air-water-oil interfacial effect when the film thickness becomes even thinner, typically on the order of milimeters. Finally, surface tension becomes dominant as the major driving mechanism and responsible for the final phase of spreading. The spreading rate at this phase is therefore determined by a balance between the surface tension spreading force and the viscous drag.

By balancing the various combination of forces, Fay [1] derived the governing equations for various regimes. The actual spreading processes at various regimes were also investigated experimentally and results were compared with analytical predictions (Hoult [2]). While the experiments show that the theory predicts correctly in terms of the functional relationship among the governing parameters, discrepancies exist on the magnitude of the proportionality constants. Waldman et al [3], also recommended their values of these proportionality constants based upon an analytical prediction performed by Fannelop and Waldman [4]. Their values in some cases differ as much as 50% from Fay's recommendation. In the following, the governing equations for radial spreading derived by Fay are summarized. With the understanding that large discrepancies exist between theories and experiments, the proportionality constants were chosen mainly based upon fitting of empirical data.

1. Inertial Spreading

$$r = 1.14\,(g\Delta V)^{1/4}\,t^{1/2} \tag{1}$$

2. Viscous Spreading

$$r = 1.45\left[\frac{\Delta g\,V^2\,t^{3/2}}{\nu^{1/2}}\right]^{1/6} \tag{2}$$

3. Surface Tension Spreading

$$r = 2.30\left[\frac{\sigma^2\,t^3}{\rho^2\,\nu}\right]^{1/4} \tag{3}$$

where

r = radius of circular patch of oil

g = gravitational constant

$\Delta = \dfrac{\rho_{water} - \rho_{oil}}{\rho_{water}}$

V = volume of spill

t = time

ν = kinematic viscosity of water

σ = spreading coefficient (net surface tension at air-oil-water interface)

ρ = density

For a small volume spill, only the third phase is of significance as the first two phases last only a very short period in the entire spreading history. For instance, in a 10,000 gallon spill the inertial phase lasts only 20 minutes and the viscous spreading process should complete in 40 minutes. Consequently, Equation (3) alone is sufficient to describe the spreading almost entirely except in the very first hour after the spill starts.

Equation (3) also shows that within the surface tension process, the leading edge of the oil spreading is independent of the volume spilled. There is no question, however, that the final area of the film does depend on the volume of the oil spill. Theoretical determination of the final area would involve the knowledge of various physical properties of oil in water, such as the change of the spreading coefficient, the diffusivity, and the solubility. For practical purpose, however, an overall estimate proposed by Fay [5] is used; the relationship is given as follows:

$$A = 10^5 V^{3/4} \tag{4}$$

Here the final area A is in square meters and the spill volume is in cubic meters. The expression seems much over-simplified; it relates with only one single parameter, the spill volume. Nevertheless, it does fit the field data very closely.

Nominal values for the physical properties of water and crude oil are used in the entire study; they are listed as follows:

$\rho_{water} = 1.94 \text{ slug/ft}^3$

$\rho_{oil} = 1.84 \text{ slug/ft}^3$

$\nu = 1.296 \times 10^{-5} \text{ft}^2/\text{sec}$

$\sigma = 0.65 \times 10^{-3} \text{lb/ft}$

Based upon these nominal values, calculations of the slick growth for spill volumes of 10^3, 10^4, 10^5, and 10^6 gallons are presented as a function of time in Figure 1. Since the slick size is independent of volume in the surface tension phase of spreading, all patches propagate along the same path line until they reach their final dimensions and cease to spread. This figure clearly shows that in small volume spills the surface tension phase is dominant during the entire spreading history.

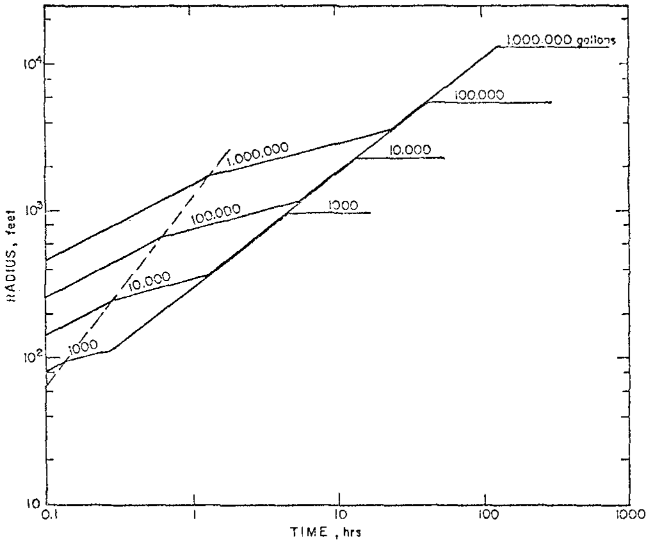

Figure 1. Slick Radius Increase as a Function of Time for Various Size Spills

The thickness of the oil slick varies from its center to its edge. The exact thickness distribution of a slick can only be obtained through proper modeling of the spreading dynamics during its growth. In order to give an order of magnitude idea about the slick thickness during the various phases of spreading, however, Figure 2 has been prepared by assuming uniform thickness distribution for all cases at all times. The figure shows the nominal thickness variation as a function of time for the same four spills.

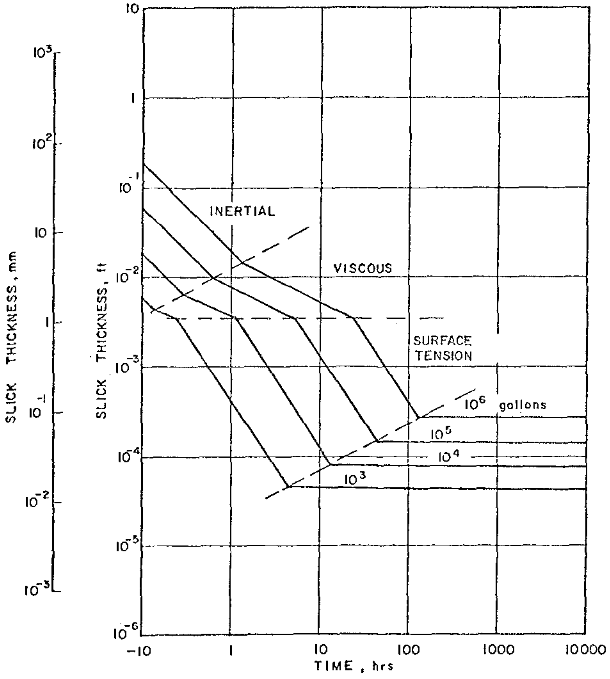

Figure 2. Nominal Thickness Decrease as a Function of Time

3. OIL TRANSPORT IN HARBOR

The primary factors affecting the movement of an oil slick are winds, tidal currents and waves. In the foregoing discussion on the spreading process, the water surface has been assumed calm without any effect due to water circulation or perturbation. In describing the transport process, the assumption is simply extended that the drifting motion of an oil slick caused by winds, currents and waves can be superimposed on the spreading motion of the slick on calm water as described in the previous section.

The direct drift caused by the wind shear stress over the water surface is generally agreed to be on the order of 3% of the over-water wind speed. There is disagreement as to the direction, however. For instance the experiments of Teeson and Schenck [6] and the analysis of Warner, et al [7] suggests that an angular displacement on the surface drift due to the effect of earth rotation must be considered, whereas the analysis of the Torrey Canyon Oil Slick Movement [8] shows good agreement by assuming the oil drift always aligned with the wind direction.

It is anticipated that Coriolis force will affect the course of transport if an oil slick is in movement under a steady wind over a large distance on an open sea. While inside a harbor as opposed to an open sea, the winds are highly local and unsteady in some cases and the Coriolis effect can hardly be developed fully in a short time scale; nevertheless, a partial development of the Coriolis effect may be significantly responsible for oil transport.

The effects of surface waves are generally negligible because of their oscillatory nature which produces little net force affecting the spreading motion. Certainly, nonlinear effects of wave interaction may produce surface drift due to second order mass transport. On the other hand, however, there is a retardation effect caused by waves on surface drifts because of the presence of wind shadow on the lee side of the wave crest. Schwartzberg [9] found that the enhancement and the retardation in surface drift caused by waves are approximately of the same order of magnitude and cancel each other. The apparent wind drift can factually be regarded as a constant, and no net effect caused by waves needs to be considered.

The magnitude and direction of the tidal current are assumed, in the present work, either calculable or measurable with required accuracy. In the numerical example given in the later part of this report, the tidal currents were calculated as a function of time and space with the tidal stage as the input data.

In summary, the transport process of an oil slick is handled as follows:

(1) The effects of wind and current are assumed uncoupled with the spreading dynamics and they are superimposable on the spreading motion of an oil slick on calm water.

(2) The effects of waves on surface drift are considered negligible.

(3) The tidal currents are obtained from either calculations or measurements. They are input data varying as a function of time and space.

(4) The wind induced surface drift is assumed to have a magnitude of 3% of the wind speed. Its angle of deviation from the wind direction is to be provided as a part of the input information.

4. NUMERICAL SIMULATION

The two-dimensional field of a harbor is constructed by a set of perpendicular grids of equal grid spacings. At each grid point, the field characteristic, ie., land or water, is identified by information fed from input data. Each grid point represents a rectangular area of $\Delta s \times \Delta s$, where Δs is the grid spacing. Whenever the grid point represents water in the field, the local water depth should also be specified. In the two-dimensional field, the tidal current and the wind driven current are each represented by a two-dimensional velocity vector at each grid point. The variation in magnitude and direction of these vectors from one grid point to another signifies the surface current distributions. Similarly, if there are oil slicks in the field, the local film thickness (averaging over a Δs square) will be specified at the corresponding grid points.

The spreading of an oil slick consists of three distinct phases as discussed in Section 2. The relationships governing the spreading phenomenon in different phases presented in Section 2 provide fundamental information of circular spreading for an oil slick on a calm water surface. When the oil slick is on water of uniform current, the effect of the water movement may simply be superimposed to the spreading motion. When the film is subject to non-uniform disturbances resulting from tide, wind, and others, the slick shape would be distorted and appear irregular. To solve the exact problem concerning spreading and transport in an arbitrary velocity field requires a detail analysis of forces on elementary slick segments. The analysis would very likely result in a set of differential equations similar to the convective diffusion type equations with a tensor of air-oil-water interface spreading, equivalent to a tensor of turbulent mixing relating mass transfer between streamlines in the pollutant transport problem. Assuming that this spreading coefficient could be specified as a function of space and time, there should be no difficulty in general to write a computer program to determine the movement and spreading of an oil slick in a given environment as well as the slick thickness distribution as a function of space and time. Nevertheless, even with modern computers such an attempt would be too expensive to be practical in terms of computing time, if not just ambitious.

Instead of solving the exact problem, the present model simulates the spreading and transport phenomena numerically by means of superposition. The oil film is fictitiously divided into patches according to the already established two-dimensional grids; each patch occupies an area of Δ s square centered at the grid point. The concept is to follow the motion of the center of these patches and to superimpose their motion together with a spreading. The process is carried out in appropriate time intervals. At the end of each time step, the average thickness of each patch is obtained by summing up the overlapping layers, resulting from spreading and transport at each grid point. The concept is similar to that adopted by Fischer [10] in predicting pollutant transport in water. As a result of carrying out this concept, the center of the slick is thicker initially, but the gradient of the film surface decreases gradually until a limiting thickness governed by the spreading mechanism is reached.

In numerical computations dealing with the diffusion type equations, the time step and grid spacing must be kept in appropriate limits, depending upon the magnitude of the diffusion coefficient, in order to assure the solution to be stable. Similarly, in the present numerical scheme, the correct simulation of the spreading mechanism requires an appropriate choice of the grid size and the time step. The selection of them depends mainly on the spreading coefficient and the spill volume.

The model is designed for predicting oil slick movement in a harbor or nearshore area. The primary input, therefore, is the information of the land boundaries. Land and water are identified by different code numbers. Wherever the region is identified as water, a water depth must also be specified. For a given bay area the above mentioned data are constant and seldom changed. It is therefore convenient that these data are stored on a magnetic tape file in a prearranged order.

The tidal current data should be input as a function of time and space. They can be obtained either through numerical computation from a separate tidal program or by compiling the observed information. In the present study, a separate hydrodynamic program is used for current data generation. This program takes the same boundary specification as the oil slick movement model. It requires the temporal information of the tidal stage as input. The current magnitude and direction are computed for every half hour interval and given as a function of location. These data are stored on a magnetic tape in a prescribed order and chronological sequence.

Other information, such as the site of spill, the quantity of spill, the oil properties, the wind speed and direction, etc. , varies from case to case. They must be updated each time the program is operated.

5. AN EXAMPLE PROBLEM

This section presents some sample computations of oil slick transport using San Pedro Bay as an example. A typical diurnal tide used for this study is shown in Figure 3. The 25 hour cycle tidal current was generated external to the system, but stored on a magnetic tape in sequence of every half hour intervals. Figure 4 shows only one plot of a continuous series of the calculated current velocity vectors in the bay, with a wind of 5 knots blowing from the northwest being imposed.

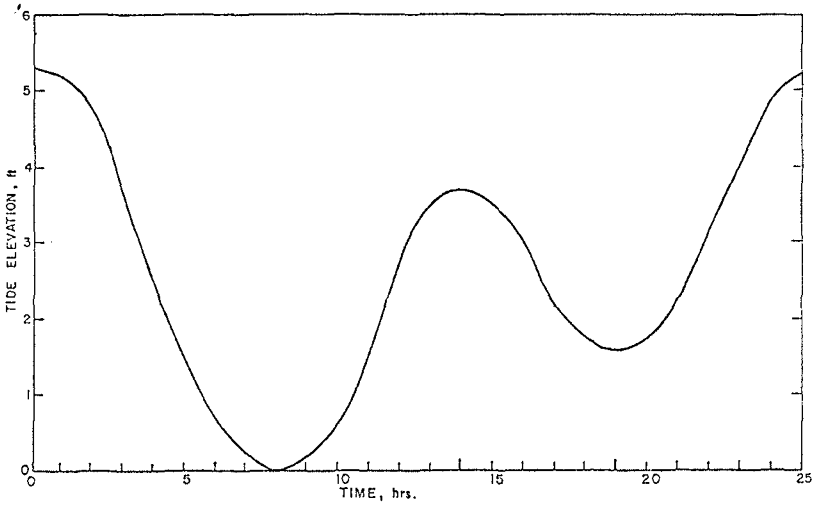

Figure 3. Typical Tidal Fluctuation in San Pedro Bay

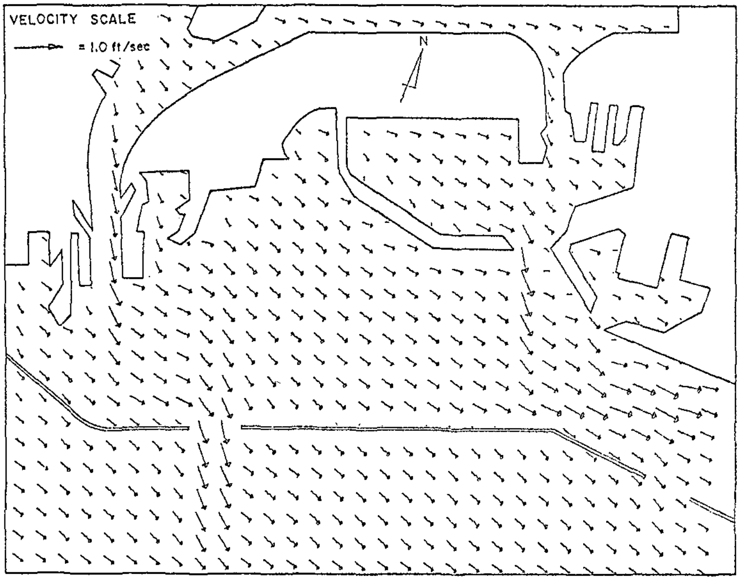

Figure 4. Current Field in San Pedro Bay at Referenced Hour 3.5, Wind Speed 5 Knots, Direction 284.5°

Numerical experiments were conducted in the system to simulate spreading and transport for a sudden release of 100,000 gallons of crude oil. The results have been reported in [11]. Figure 5 shows only a typical computer output corresponding to 36 hours after the 100,000 gallons of oil is released at the location (12, 16) with no wind being taken into account. The asterisks in the plot stand for land or ocean boundaries; the numbers at each grid point indicate the thickness of the oil film in 10^{-2} mm, averaged over an area of 10^6 ft^2 around that point. The entire grid space under these numerals indicate the area covered by oil on the water surface at a particular time.

6. FIELD TESTS AND MODEL VERIFICATION

In order to verify the numerical model there were, in total, five field tests and demonstrations conducted at San Pedro Bay. The first two tests were made with 3' x 3' cardboards to simulate the oil transport. In the last three tests, soy bean oil was dumped at various locations. These tests covered a wide range of wind speed and direction and they were conducted at various stages with regard to tidal circulations. The complete test results and their comparisons with their numerical predictions have been reported in [11]. In the following, only the test procedures and two typical sets of the results are presented.

(1) CARDBOARD TESTS

The floating markers were made of 3' x 3', 1/16" thick posterboard and were sealed with paraffin on their edges in order to prevent them soaking between the layers. They were coated with florescent paint of a bright red-orange color and each board was identified by a number painted in black. In one test conducted on October 5, 1973, three cardboards were launched approximately 1000 feet apart from each other from a 25-ft. Bertram class motor boat, which has a fiberglass hull and wooden deck finishing. The cardboard markers behaved extremely well on the water surface; they were flexible and moved harmonically with the surface waves. The positions of the markers were fixed by a Cubic Autotape DM-40 electronic positioning system. The system includes a two-range interrogator on board the boat and two responders at fixed locations on shore. The Autotape employs microwave to measure the ranges between the moving boat and the two fixed sites where the two responders were located. The interrogator visually displays these ranges once per second in 5 metric units. These ranges were also simultaneously recorded on a paper printer.

In the October 5th test, one responder was put at the east end of Terminal Island and the second one was at Pier F

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THICKNESS OF OILFILM IN CM/1000, AT END OF HOUR   36
 OIL INSIDE THE BAY=     100000 GALLONS

M N = 1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18  19  20  21  22  23  24  25  26  27  28  29  30  31  32
 1 ********************************************************************************************************************************
 2 ********************   0   0   0   0********   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0************
 3 ************************   0   0   0   0   0   0   0****************************************   0   0   0************************
 4 ************************   0   0   0   0   0****************************************************   0****************************
 5 ********************   0   0   0   0************************************************************   0****************************
 6 ********************   0   0   0****************************************************************   0****************************
 7 ********************   0   0************************   0   0****   0   0   0   0   0   0   0****   0****   0   0****************
 8 ********************   0****************************   0   0****   0   0   0   0   0   0   0****   0   0   0   0****************
 9 ********************   0****   0   0************   0   0   0********   0   0   0   0   0   0   0   0   0   0   0****************
10 ********************   0****   0   0****   0   0   0   0   0   0********   0   0   0   0   0   0   0****************************
11 ********************   0****   0********   0   1   0   0   0   0   0************   0   0   0   0   0****************************
12 ********   0********   0****   2   3   5   4   1   2   0   0   0   0   0   0****************   0****   0   0********   0********
13 ****   0   0********   0****   1   5   8   8   3   3   2   1   0   0   0   0   0   0   0   0   0********   0********   0********
14 ****   0   0****   0   1   1   2   3   9  15   9   6   4   1   0   0   0   0   0   0   0   0   0   0****   0   0   0   0   0****
15 ****   0   0****   0   0   1   0   2  10  15  17  11   4   1   0   0   0   0   0   0   0   0   0   0   0   0****   0   0********
16 ****   0   0   0   0   0   0   0   0   2  15  14  12   5   1   0   0   0   0   0   0   0   0   0   0   0   0********************
17 ********   0   0   0   0   0   0   1   4   8  23  10   4   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0********
18 ****   0****   0   0   0   0   0   0   4  12  20  19   2   1   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0****
19 ****   0   0****   0   0   0   0   4   4  10  23  14   7   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0****
20 ****   0   0   0********************   4   0********************************************************   0   0   0   0   0   0****
21 ****   0   0   0   0   0   0   0   1   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0********   0   0   0   0****
22 ****   0   0   0   0   0   0   1   2   4   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0********   0   0****
23 ****   0   0   0   0   0   0   0   3   2   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0****
24 ****   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0****
25 ****   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0****
26 ********************************************************************************************************************************
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Figure 5. Computer Output of Simulated Results for Oil Slick Distribution at End of Hour 36, No Wind
Grid Spacing = 1000 ft

of the Long Beach Harbor. During the test, the locations of the markers were traced and the distances of each marker from the two responders were recorded as a function of time. The paths of the markers were then determined by fixing their positions using two range lines.

Wind information was obtained by means of an anemometer and a magnetic compass on shore. Wind speed and direction were recorded every 15 minutes; the records are shown in Figure 6. Taking wind speed as a constant of 8 knots but updating the wind direction every half hour, the routes of the three markers were calculated and compared with the field traces as shown in Figure 7. It is noted that because Marker 2 was generally following the same path as Marker 3, Marker 2 was relocated to a new location at 2:15 PM. The computation used a 500 ft. grid space and a one hour time step. The total simulation time was four hours for Markers 1 and 3, and two hours for Marker 2 beginning at the relocated position. The computation considered zero angle deflection with regard to the wind drift current. The agreement of the calculated results with the field data appears to suggest that the markers were essentially moving along the direction of the wind.

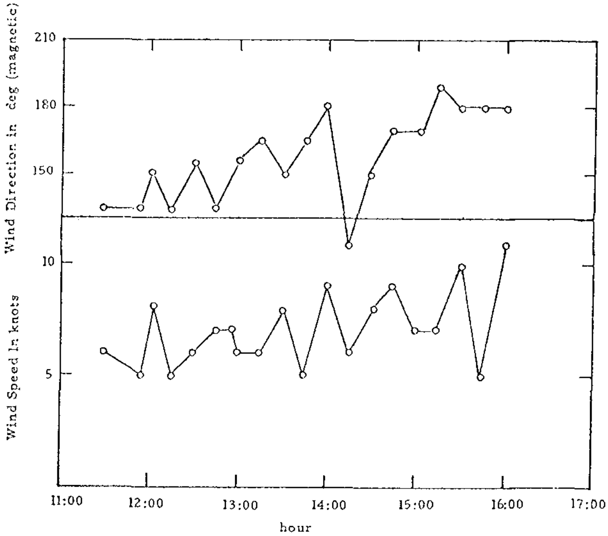

Figure 6. Wind Records

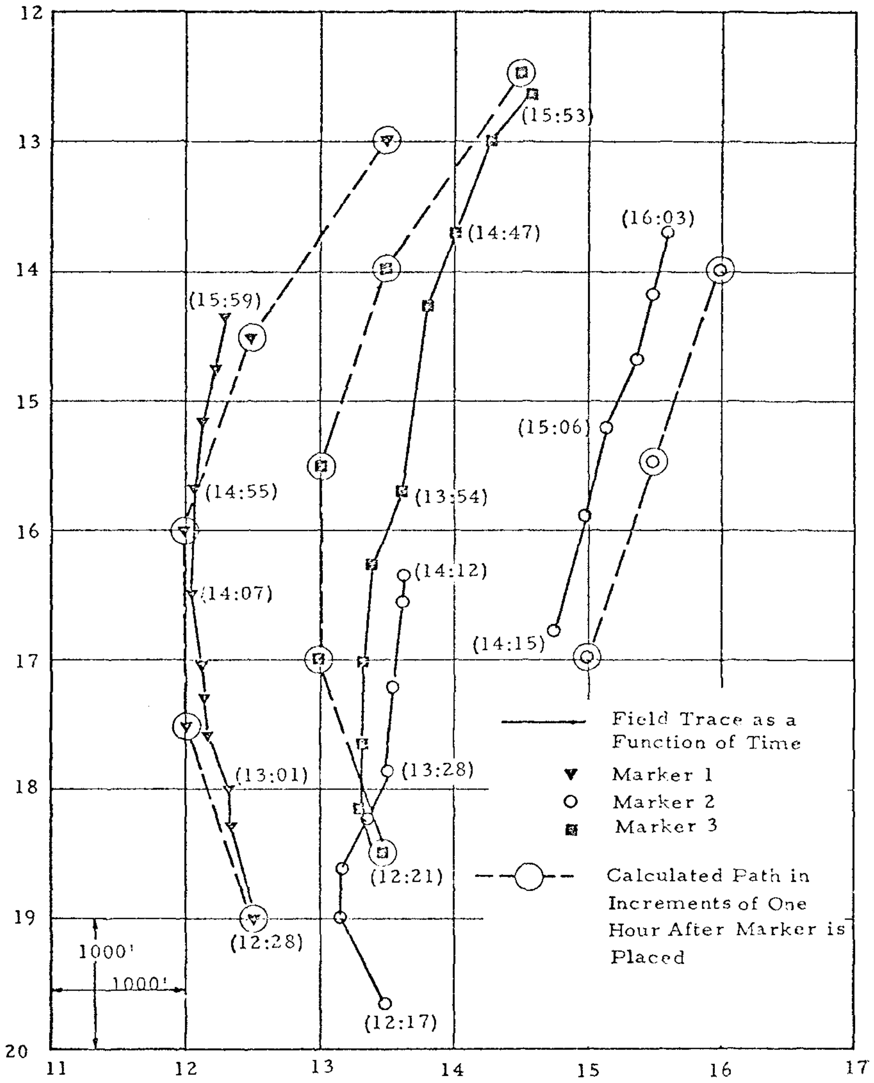

Figure 7. Calculated Paths and Field Traces of Markers (Refer to Figure 5 for Grid Locations)

(2) SOYBEAN OIL SPILL TESTS

While the cardboard tests were essentially to show the credibility of the model for analysis using field information, the next three tests with soybean oil were to show the capability of the model for forecasting with the initial information and the capability of updating when better information becomes available.

Instead of using the Cubic Autotape, a Motorola Mini-Ranger System was used for oil slick positioning. Four responders were stationed onshore. The oil dumping position fixing group was on a Coast Guard 40-ft. steel-hull boat. The forecasting group was stationed onshore and equipped with a portable teleprinter linked to the Control Data Kronos Time-Sharing System by means of a regular telephone line.

In one test conducted on February 11, 1974, five gallons of soybean oil were dumped at 12:15 PM inside the west end of the middle breakwater close to the Los Angeles Harbor Main Channel. Because of its closeness to the main channel traffic, the slick was unfortunately disturbed by a passing ship, supertanker ESSO BERLIN, and no accurate trace could possibly be made. Despite this unsuccessful start, a second dumping of about ten gallons was made at a new location at 2:13 PM. The dumping site and the initial wind condition were transmitted from boat to shore through walkie-talkie type radio communication. The shore group immediately fed these data into the computer and obtained a two-hour prediction.

The wind information received at 2:13 PM was 9 knots and 215° (magnetic). Prediction based upon this information showed an approximate 10 degree discrepancy with the traced data obtained later on. It appeared that a 10 degree wind drift deflection had to be included in the computation for better agreement. This correction was also confirmed in a subsequent test conducted on the following day as reported in [11]. With this correction included in the computation model, the predicted oil movement is shown in Figure 8 together with the field traces.

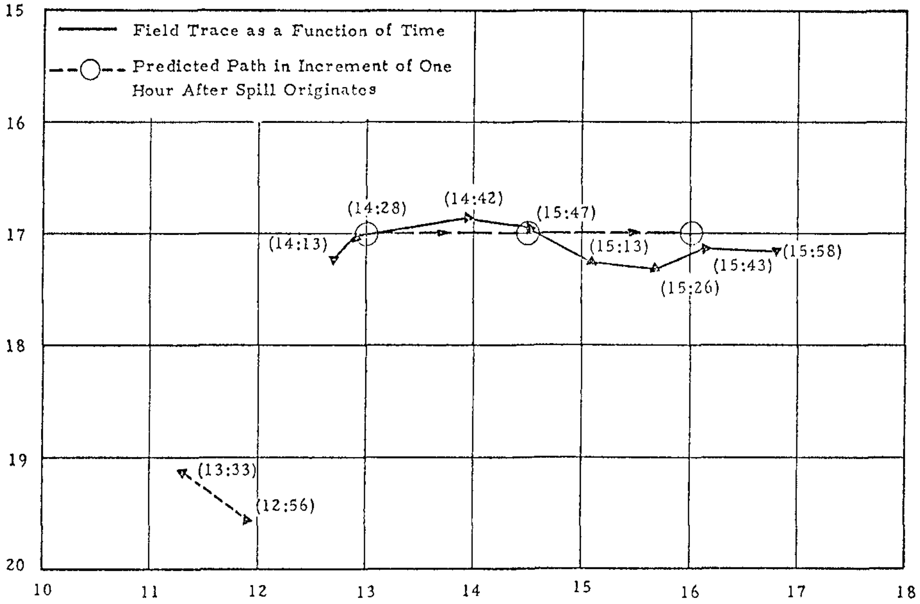

Figure 8. Predicted Path and Field Traces of the Slick Centroid (10° Wind Drift Deflection Included in Prediction)

7. DISCUSSION AND CONCLUSION

While field observations of drift markers and small scale oil slick movements have essentially validated the present model, some important assumptions applied herein deserve more detailed discussions. The most important approximation is the spreading process simulation. The numerical procedure is an attempt to represent the effect of the pressure gradient over the slick boundary of the elementary patches. The concept assumes that each patch of oil slick will locally spread and diffuse over a circular field whose radius is stretched approximately by one grid space during one time step before the final area of the slick is reached. At the end of each time step, a superposition of the overlapping patches takes place. It is obvious that the grid size and the time step are necessarily governed by the spill volume and the spreading characteristics in order that the spreading process is properly simulated. This is equivalent to the grid size and the time step being controlled by the diffusion coefficient in solving convective-diffusion equations by a finite difference technique.

While the necessity of adjusting the grid size and the time step is fundamental for a proper simulation of spreading, the application of the numerical program is not limited by these restrictions. From an operational point of view, the total spreading process is generally too short to be important. In other words, the impact of slick transport is generally more significant than the consequence of spreading in affecting decision on directing protective actions and control. Consequently, the grid size and time step may not necessarily tie up to the type and volume of the oil spill; they may be chosen essentially for convenience. The program then adopts a simple procedure to provide advance information on the slick's area and shape as a function of time.

The adopted concept of superimposing a transport motion on a spreading motion in calm water signifies that the mechanism for spreading and dispersion is essentially isotropic, which is known as not supported by observations. Field observations often find oil slicks pronouncedly stretched in the direction of drift. This phenomenon may be taken into consideration in the numerical scheme by including the directional property in the spreading coefficient, provided that sufficiently reliable data are available in this regard.

In regard to the wind drift information, it is anticipated that the Coriolis force may play an important role in altering the course of an oil slick only if it is transported over a long distance. Inside a harbor over a short time span, one should expect the wind drift deflection to be negligible. Nevertheless, the field test of soybean oil in San Pedro Bay showed an approximate 10 degree clockwise deflection in wind induced drift. There are no sufficient data available with regard to the wind drift deflection in harbors. Field surveys are necessary to obtain this information.

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