CHAPTER 19

DIRECTIONAL SPECTRA OF OCEAN SURFACE WAVES

by

H. Mitsuyasu and S. Mizuno

Research Institute for Applied Mechanics

Kyushu University, Fukuoka, Japan

ABSTRACT

From 1971-74 seven cruises were made to measure the directional spectrum of ocean waves by using a cloverleaf buoy. Typical sets of wave data measured both in open seas and in a bay under relatively simple conditions have been analyzed to clarify the fundamental properties of the directional spectrum of ocean waves in deep water.

It is shown that the directional wave spectrum can be approximated by the product of the frequency spectrum and a unimodal angular distribution with mean direction approximately equal to that of the wind. The normalized forms of the frequency spectrum show various forms lying between the Pierson-Moskowitz spectrum and the spectrum of laboratory wind wave which has a very sharp energy concentration near the spectral peak frequency. The form of the JONSWAP spectrum is very close to that of laboratory wind waves. The concentration of the spectral energy near the spectral peak frequency seems to decrease with increasing the dimensionless fetch and the spectral form finally approaches to the Pierson-Moskowitz spectrum which can be considered as the spectrum with the least concentration of the normalized spectral energy. However, the definite relation between the shape of the normalized spectrum and the dimensionless fetch has not been obtained.

Concerning the angular distribution, it is shown that the shape of angular distribution of the single-peaked wave spectrum in a generating area can be approximated by the function $G(\theta,f) = G'(s)\,|\cos(\theta-\bar{\theta})/2|^{2s}$ proposed orinally by Longuet-Higgins et al. (1963). Here $G'(s)$ is a normalizing function, $\bar{\theta}$ is the mean direction of the spectral component, and s is a parameter which controls the concentration of the angular distribution function.

The significant results of this study are that $\bar{\theta}$ is very close to the wind direction irrespective of the wave frequency, and that the parameter s can be uniquely determined from the dimensionless frequency. The angular distribution is very narrow for the frequencies near the dominant peak of the frequency spectrum, whereas it widens rapidly towards high and low frequencies. Thus, the major energy-containing frequency components of the ocean waves propagate in almost the same direction as the wind with the least angular spreading.

An idealized form of the angular distribution function of the ocean wave spectra is proposed for practical purposes. Qualitatively, the idealized form proposed here has a property which is similar to that of the SWOP spectrum, although they are different quantitatively.

INTRODUCTION

To a first approximation, ocean surface wave can be regarded as a linear superposition of statistically independent free waves and is consequently described by two-dimensional wave spectrum (directional spectrum). During the last twenty years great many studies have been made to clarify the properties of ocean wave spectra. However, in contrast with numerous studies of the one-dimensional (frequency) spectrum, only a few studies have been made of the two-dimensional (directional) spectrum of ocean waves. Particularly, reliable data for estimating the directional spectra of ocean waves are remarkably lacking except for the data reported by Coté et al. (1960), Longuet-Higgins et al. (1963), Ewing (1969) and recently by Tyler et al. (1974). These situations may be partly attributed to the technical difficulties for the measurement of the directional wave spectrum as compared to that of the one-dimensional spectrum. Therefore, many studies on the directional wave spectrum have been concentrated on the measuring techniques and their accuracies rather than the measurements and analysis of the directional wave spectra in various conditions.

In order to save these situations, in 1971 we have developed the cloverleaf buoy which is almost the same as that of the National Institute of Oceanography (Cartwright and Smith 1964). From 1971 to 1974 seven cruises were made to measure the directional spectrum of ocean waves by using the cloverleaf buoy. Typical wave data measured both in open seas and in a bay have been analyzed to clarify the fundamental properties of the directional spectrum of ocean waves in fairly simple generating conditions. A new standard form of directional wave spectrum is proposed on the basis of typical sets of measured data. The proposed form of the directional spectrum is further verified by the additional data obtained in the East China Sea during the period of AMTEX'75.*

WAVE DATA AND ANALYSIS

Waves were measured by using a cloverleaf buoy which is almost the same as that of the National Institute of Oceanography (Cartwright and Smith, 1964). It can measure the

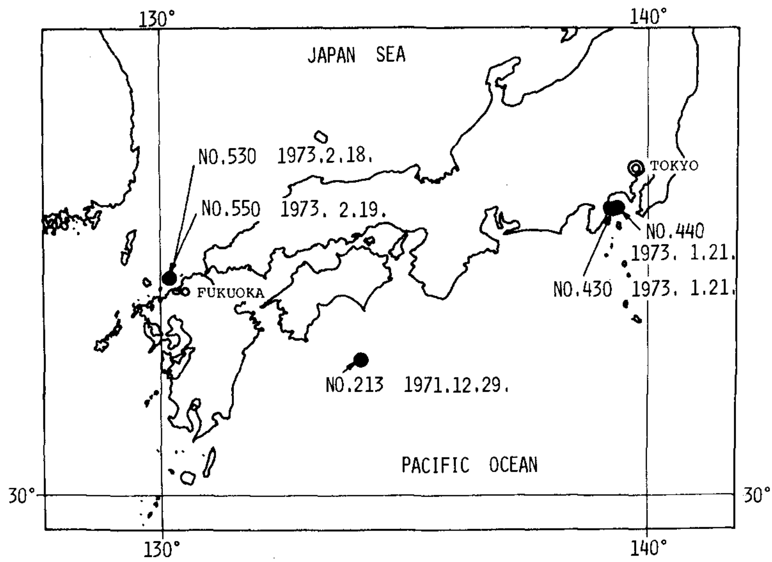

Fig. 1. Locations and dates of the wave observations.

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* AMTEX'75 (Air-Mass Transformation Experiment in 1975) is one of the international sub-program of GARP (Global Atmospheric Research Programme).

vertical acceleration η_{tt}, slope η_x, η_y, and curvature η_{xx}, η_{yy}, η_{xy} of the wave surface $\eta(t,x,y)$. The detailed description of the instrumentation including the buoy and of the observational procedures has been given elsewhere (Mitsuyasu et al., 1973a,), and will not be repeated here.

From approximately 50 cases, only five typical data sets (identified as Nos.213, 430, 440, 530 and 550) have been analyzed; each is a continuous record of about 1 h. These data were obtained under the fairly simple generating conditions. That is, the wind speeds and directions are fairly constant within the duration time. Fig.1 shows the locations and dates of the wave observations.

Data sets 213, 530 and 550 correspond to wave data in open seas and data sets 430 and 440, to those for a bay.

Table 1 gives the wind direction θ_w, speed U, and approximate duration t_d, which have been measured from the tending ship near each observation station; H and T are the significant wave height and period, respectively, measured by the cloverleaf buoy.

Data set	θ_w	U ($m\,s^{-1}$)	t_d (h)	H (m)	T (s)	H/L
		parameter				
213	E-NE	10	26	1.49	6.20	0.025
430	ENE-NE	7	2	0.80	4.45	0.026
440	ENE-NE	7	4	0.74	4.11	0.028
530	NNE	9	4	0.84	4.50	0.027
550	N-NE	10	24	2.34	8.30	0.022

Table 1. Wave parameters from the five data sets.

The wind speeds measured at the observation stations have been determined by taking the mean of the measured wind speeds during the duration. Table 2 gives similar kind of data which have been measured in the East China Sea (28°19'N ~ 28°36'N, 125°17'E ~125°42'E) and are used for veryfying the conclusions derived from the first series of data shown in Table 1.

Original wave data recorded on magnetic tapes in analogue form were digitized by using a high-speed A-D converter. The cross spectral analysis of the data was done on the computer system FACOM 270-20 using a standard program based on fast Fourier transform procedures. Fundamentals of the mathematical procedure of the data analysis are almost the same as those used by Longuet-Higgins et al. (1963) and by Cartwright and Smith (1964). Condensed descriptions of the data analysis have been given also in our previous papers (Mitsuyasu et al. 1973a, and 1975).

		parameter				
Data set	θ_ω	U ($m\,s^{-1}$)	t_d (h)	H (m)	T (s)	H/L
801	0°	10	55	2.33	7.80	0.025
804	340°	13	18	2.79	8.00	0.028
805	350°	14	23	2.67	7.80	0.028

Table 2. Wave parameters from the additional three data sets

Cross spectra $C_{\ell m}(f) - Q_{\ell m}(f)$ were computed using the measured six signals from the ocean waves. The directional spectrum $E(f,\theta)$ is conveniently expressed in the form

$$E(f,\theta) = \phi_1(f)G(f,\theta), \quad (1)$$

where $\phi_1(f)$ is the one-dimensional wave spectrum and $G(f,\theta)$ an angular distribution function. The one-dimensional spectrum $\phi_1(f)$ is determined from the acceleration spectrum $C_{11}(f)$, i.e.,

$$\phi_1(f) = (2\pi f)^{-4}C_{11}(f). \quad (2)$$

The coefficients A_n and B_n in the Fourier expansion of an angular distribution function $h(f,\theta)$ [$= \pi G(f,\theta)$] can be determined up to $n = 4$, from the measured cross-spectral elements $C_{\ell m}$ and $Q_{\ell m}$. From these Fourier coefficients the angular distribution function $h(f,\theta)$ can be expressed approximately as

$$h_4(f,\theta) = \frac{1}{2} + \sum_{n=1}^{4} w_n(\ A_n\cos n\theta + B_n\sin n\theta\ ), \quad (3)$$

for $w_1 = 8/9,\ w_2 = 28/45,\ w_3 = 56/165,\ w_4 = 14/99$

where the weights w_n are introduced to make $h_4(f,\theta)$ non-negative. The partial Fourier sum with the weights w_n corresponds to the smoothed average of $h(f,\theta)$ by the weighting function $W_4(\theta)$ which is very approximately proportional to $\cos^{16}\theta/2$ and has an rms width $\pm$ 29°.

RESULTS

a. One-dimensional spectra of ocean waves

In a previous study one of the present authors studied the similarity of the one-dimensional spectra of wind-generated waves by using accurately measured wave data for three different groups A, B and C (Mitsuyasu et al., 1973c). Wave data for groups A and B were obtained in two quite different wind-wave facilities under various conditions. The wave data of group C were obtained in Hakata Bay, where the water depth is approximately 5 m and the fetch is approximately 5 km to the north.

It was found that wave spectra normalized in the form

$$\frac{\phi_1(f)\,f_m}{E} = \phi\left(\frac{f}{f_m}\right) \qquad (4)$$

were quite similar and stable within each the groups; here f_m the spectral peak frequency and E the total energy of the wave spectrum defined by

$$E = \int_0^\infty \phi_1(f)\,df = \int_0^\infty\int_0^{2\pi} E(f,\theta)\,d\theta\,df \;. \qquad (5)$$

Moreover, there were no differences in the normalized spectra between groups A and B. However, the normalized spectra of group C were slightly different from those of groups A and B. These results can be clearly seen in Fig.2 which is reproduced from the previous paper (Mitsuyasu et al., 1973c), and shows the mean form of the normalized spectra of each group together with the normalized spectrum of Pierson and Moskowitz (1964). As can be seen from Fig. 2, the concentration of the normalized spectral energy near the spectral peak is lower in the data of Group C than

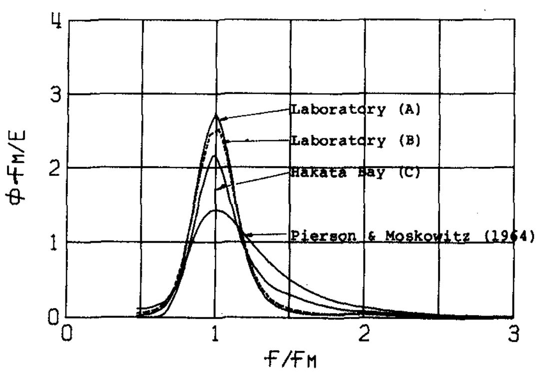

Fig.2 Similarity of the one-dimenaional wave spectrum.

in those of groups A and B. The concentration of the normalized spectral energy of the Pierson-Moskowitz spectrum is much smaller than that of group C. The dimensionless fetch gF/u_*^2 is approximately 10^2-10^3 for the wave data of groups A and B, 10^5-10^6 for those of group C, and approximately 10^7 for the Pierson-Moskowitz spectrum. Therefore, within the range of our previous data, it was concluded that the concentration of the normalized spectral energy of wind waves seems to decrease with increasing dimensionless fetch gF/u_*^2.

The normalized spectra shown in Fig.2 are used as a kind of reference for discussing the present data of one-dimensional spectra measured under various conditions. All of the measured spectra were normalized in the form (4) and compared with the previous results. Some typical examples of the normalized forms of the measured wave spectra are shown in Figs. 3-5.

The dashed line in each figure shows the spectral form which is selected from the typical spectra shown in Fig.2 and which has the closest resemblance to each measured spectrum. It can be seen from these figures that the wave spectrum of No. 430/1 is quite similar to the spectrum of laboratory wind waves, the wave spectrum of No. 550/1 is quite similar to the spectrum observed in Hakata Bay, and the wave spectrum of No. 213/1 is quite similar to the Pierson-Moskowitz spectrum. All of the other spectra were scattered between the spectral form for laboratory wind waves and that of Pierson-Moskowitz (1964), though they are not shown in the figures. Furthermore,

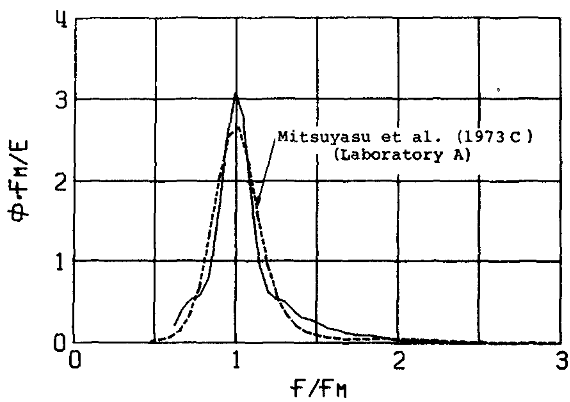

Fig.3. Normalized form of the one-dimensional wave spectrum: Continuous curve, ocean wave data set No. 430/1; dashed curve, laboratory wave data (group A).

within the range of our observed spectra, it was shown that the Pierson-Moskowitz spectrum is the spectrum with the smallest energy concentration, in other words, the spectrum with the largest spectral width. Within the range of the present data, however, a definite relation between the normalized spectral form and the dimensionless fetch has not been obtained. Recently, a new spectral form, the JONSWAP spectrum, has been reported (Hasselmann et al. 1973). The JONSWAP spectrum has very high concentration of normalized spectral energy, which is comparable to that for laboratory wind waves. Fig.6 shows the comparison of the JONSWAP spectrum, the Pierson-Moskowitz spectrum and the idealized form of laboratory wind wave spectra (Mitsuyasu 1973b). As shown in Fig.6, the JONSWAP spectrum is quite similar to the spectra of laboratory wind waves in the normalized form (4). Therefore, we

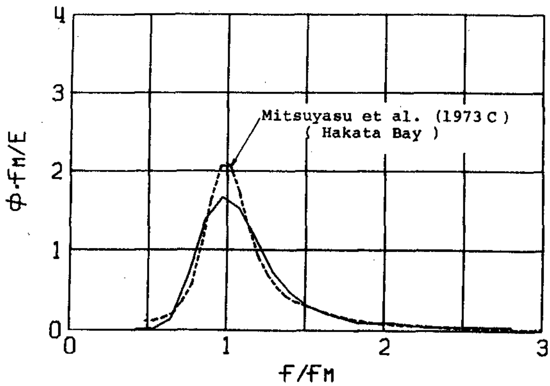

Fig.4. As in Fig.3 except for ocean wave data set No. 550/1, and the wave data of Hakata Bay (group C).

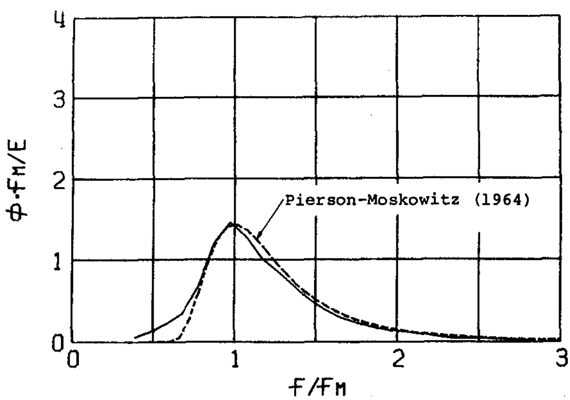

Fig.5. As in Fig.3 except for ocean wave data set No. 213/1, and the Pierson-Moskowitz spectrum.

can say that many of the ocean wave spectra scatter between the JONSWAP spectrum and the Pierson-Moskowitz spectrum. In fact, almost every wave spectra of our additional data sets 801, **804**, 805 were just between these two extem forms of the wave spectra. As an example, normalized spectra of wave data No. 805 is shown in Fig.7.

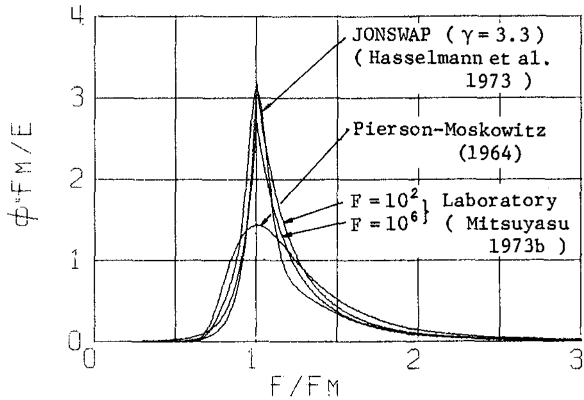

Fig.6. Similarity of the one-dimensional wave spectrum

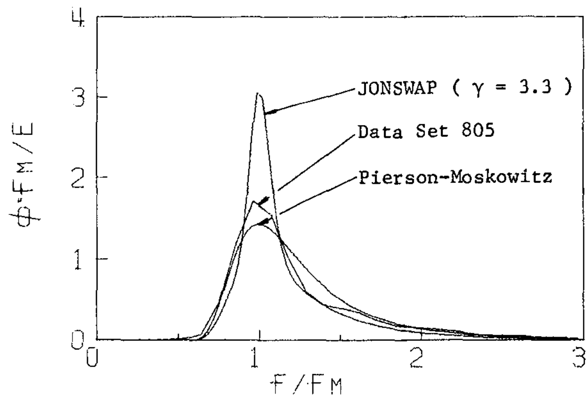

Fig.7. Normalized form of one-dimensional wave spectrum: dats set No. 805, the Pierson-Moskowitz spectrum and the JONSWAP spectrum.

b. The angular distribution function

Fig.8 shows an example of the measured angular distribution $h_4(f,\theta)$ together with $h_2(f,\theta)$ which is given by

$$h_2(f,\theta) = \frac{1}{2} + \sum_{n=1}^{2} w_n' \ (A_n\cos n\theta + B_n\sin n\theta) \qquad (6)$$

$$\text{for } w_1' = \frac{2}{3}, \quad w_2' = \frac{1}{6}.$$

Here, $h_2(f,\theta)$ is an angular distribution which can be determined only from the signals of η_{tt}, η_x and η_y, and corresponds

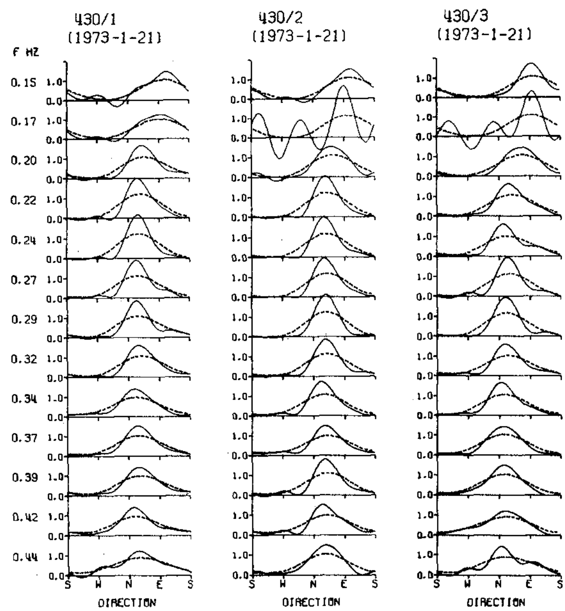

Fig.8. Angular distribution functions of the directional spectrum (wave data case 430/1, 2, 3): continuous curve, $h_4(f,\theta)$; dashed curves, $h_2(f,\theta)$.

to the data measured by a "pitch-roll buoy". Eq. 6, the partial Fourier sum with weights w_n', corresponds to the smoothed average of $h(f,\theta)$ by the weighting function $W_2(\theta)$ which is proportional to $\cos^4\theta/2$. Since the width of $W_2(\theta)$ is approximately twice as large as that of $W_4(\theta)$, $h_2(f,\theta)$ is smoother than $h_4(f,\theta)$ as shown in Fig.8. In the figure $h(\theta)$ for each frequency component is shown separately. It can be seen from Fig.8 that $h(f,\theta)$ can be well approximated by an unimodal distribution function except for some exceptional cases which are considered to be affected by some errors.

In order to investigate $h(f,\theta)$ more quantitatively, the following function, originally proposed by Longuet-Higgins et al. (1963), is fitted to the measured angular distributions;

$$h(\theta) = G'(s) \left| \cos\frac{1}{2}(\theta-\bar{\theta}) \right|^{2s}, \qquad (7)$$

where $G'(s)$ is a normalizing function to make

$$\int_0^{2\pi} h(\theta)d\theta = \pi ,$$

i.e.,

$$G'(s) = 2^{2s-1} \times \frac{\Gamma^2(s+1)}{\Gamma(2s+1)} . \qquad (8)$$

The parameter s is, in a general case, a function of the wave frequency f, and $\bar{\theta}$ is the mean direction defined by

$$\bar{\theta} = \tan^{-1}B_1/A_1 . \qquad (9)$$

In Eq. (8) Γ is the gamma function.

The form of $h(\theta)$ given by the above equation is shown in Fig.9 for different values of s. As can be seen, $h(\theta)$ tends to a narrower distribution with an increase of the parameter s. That is, s can be

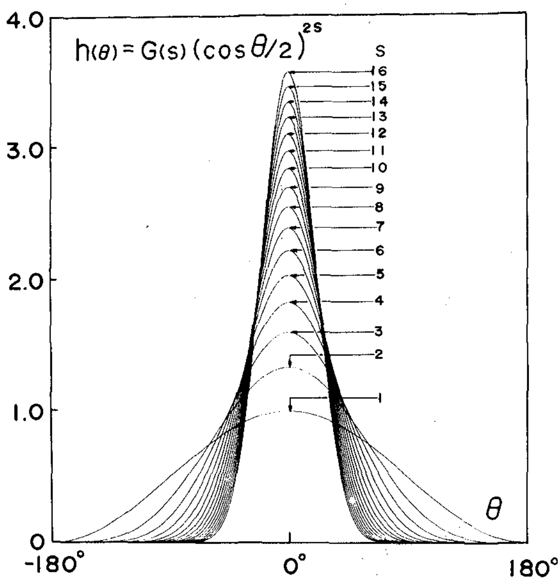

Fig.9. An idealized angular distribution function.

considered as a parameter which controls the concentration of the directional distribution of the wave energy.

If Eq. (7) were a perfect fit to the measured angular distribution, the parameter s would have to satisfy all of the expressions

$$C_n = \frac{s(s-1)\cdots(s-n+1)}{(s+1)(s+2)\cdots(s+n)}, \quad n = 1, 2, 3, 4 \quad (10)$$

where

$$C_n = (A_n^2 + B_n^2)^{\frac{1}{2}} \quad (11)$$

By using Eqs. (10) and (11) the parameter s_n can be determined from the corresponding Fourier coefficients A_n and B_n, respectively, for n = 1, 2, 3, 4. Furthermore, if Eq. (7) were a perfect fit to the measured angular distributions, all s_n (n = 1, 2, 3, 4) should be equal. In fact, s_2 was found to be approximately equal to s_1, but s_3 and s_4 were somewhat different from s_1 and s_2. These might be attributed to the fact that the higher terms of the Fourier coefficients of angular distribution function contain non-negligible errors, due to the poor accuracy in the measurement of wave curvature. Therefore, the parameter s_1 and s_2

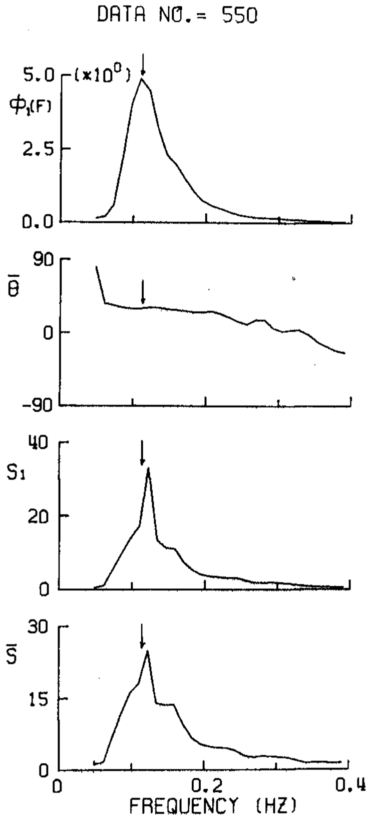

Fig.10. The one-dimensional wave spectrum $\phi_1(f)$, mean direction $\bar{\theta}$, and the spreading parameters s_1 and $\bar{s}$ for wave data set 550.

will be used for the following discussions.

Fig.10 shows typical set of values of $\phi_1(f)$, $\bar{\theta}$, s_1 and s, where s is the mean of the values of s_1 and s_2. The location of the arrow shows the frequency of the maximum spectral density. It can be seen from this figure that the value of s_1 or s is very large near the maximum spectral density frequency. However, it decreases rapidly toward higher and lower frequencies. In other words, the angular distribution is very narrow for the frequency components near the spectral peak frequency, whereas it widens rapidly toward higher and lower frequencies. It can also be seen that the mean direction of the dominant spectral component is almost the same as that of the wind (cf. Table 1). Similar results have been obtained from the other sets of data.

Therefore, it can be said that in a generating area, frequency components near the dominant peak of the frequency spectrum, i.e., the most energy-containing frequency components, propagate in almost the same direction as the wind direction and their angular spread is very narrow. However, the angular spreading increases toward higher and lower frequencies as the spectral energy decreases.

AN IDEALIZED FORM OF THE ANGULAR DISTRIBUTION FUNCTION

In order to find the relation between the parameter s and the dimensionless frequency $\hat{f}$[$= 2\pi f U/g = U/C$], s[$= (s_1+s_2)/2$] and $\hat{f}$ are plotted in Fig.11 on a log-log scale. It can be seen that values of the parameter s are almost uniquely determined from the dimensionless frequency $\hat{f}$ on the high-frequency side of the spectrum, although the data are somewhat scattered due to the insufficient accuracy of the measurements. In Fig.11 the arrow marks the location of the dimensionless frequency $\hat{f}_m$ that corresponds to the spectral peak frequency f_m. It can also be seen from Fig.11 that in each spectrum the parameter s shows its maximum value near $\hat{f}_m$ as already shown in Fig.10, and s decreases rapidly with decreasing $\hat{f}$ at $\hat{f} < \hat{f}_m$. Similar results can also be obtained from the data on s_1.

These facts suggest that for the high-frequency side of the spectrum the parameter s shows an equilibrium form which is given approximately by

$$s = 11.5\ \tilde{f}^{-2.5} \ . \qquad (12)$$

In the frequency range below the spectral peak, $\tilde{f} < \tilde{f}_m$, an approximate expression for s can be given by

$$s = k\,\tilde{f}^{m}, \qquad (13)$$

where k and m are dimensionless constants. The power m in the above expression seems to depend somewhat on $\tilde{f}_m$. That is, as shown in Fig.11, m is slightly larger in the cases of smaller $\tilde{f}_m$ than in those of larger $\tilde{f}_m$. At this time, however, since the scatter of the data are considerable, these differences in m are neglected and m is assumed to be

$$m = 5.$$

The constant k can be determined from the condition that s has its maximum value s_m at $\tilde{f}_m$ where Eq. (13) coincides with Eq. (12).

Approximate expressions for s for $\tilde{f} \leq \tilde{f}_m$ and for s_m are

$$s = 11.5\,\tilde{f}_m^{-7.5}\,\tilde{f}^{5}, \qquad (14)$$

$$s_m = 11.5\,\tilde{f}_m^{-2.5}. \qquad (15)$$

On the other hand, the dimensionless frequency $\tilde{f}_m$ of the spectral peak can be determined from the dimensionless fetch $\tilde{F}[\,= gF/U^2\,]$ by using the so-called fetch relation. For example, the fetch relation (Mitsuyasu 1968)

$$\hat{f}_m = 1.00\,\hat{F}^{-0.330}, \qquad (16)$$

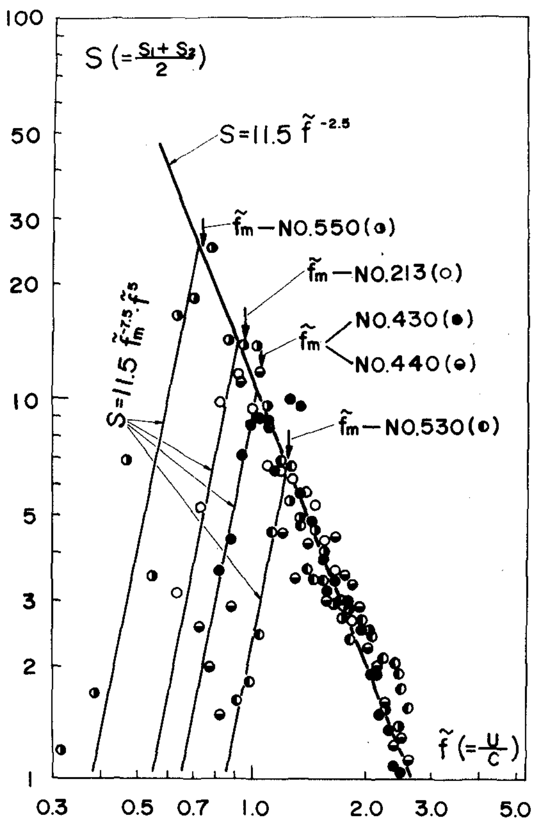

Fig.11. The parameter s as a function of the dimensionless frequency $\tilde{f}$. $\tilde{f}_m$ is the dimensionless frequency of the maximum density of the one-dimensional wave spectrum.

where

$$\hat{f}_m = u_* f_m/g, \quad \hat{F} = gF/u_*^2 \tag{17}$$

where u_* is the friction velocity of the wind, can be transformed into

$$\tilde{f}_m = 18.8\,\tilde{F}^{-0.330} \tag{18}$$

by using the approximate relation

$$25\,u_* = U_{10} = U\,. \tag{19}$$

From Eqs. (7), (12), (14) and (18) we can derive an idealized form of the angular distribution function that can be used for practical purposes. That is, the angular distribution is given by

$$G_1(f,\theta) = G_1(s)\,|\cos\theta/2|^{2s} \tag{20}$$

where

$$G_1(s) = \frac{1}{\pi}\,2^{2s-1}\,\frac{\Gamma^2(s+1)}{\Gamma(2s+1)}\,. \tag{21}$$

The parameter s is given by Eq. (12) for $\tilde{f} > \tilde{f}_m$ and by Eq. (14) for $\tilde{f} < \tilde{f}_m$, and $\tilde{f}_m$ can be determined from Eq. (18). Fig.12 shows further check of the proposed relations (12) and (14) by the data sets 801, 804 and 805. Coincidence between the prediction and observation is generally satisfactory.

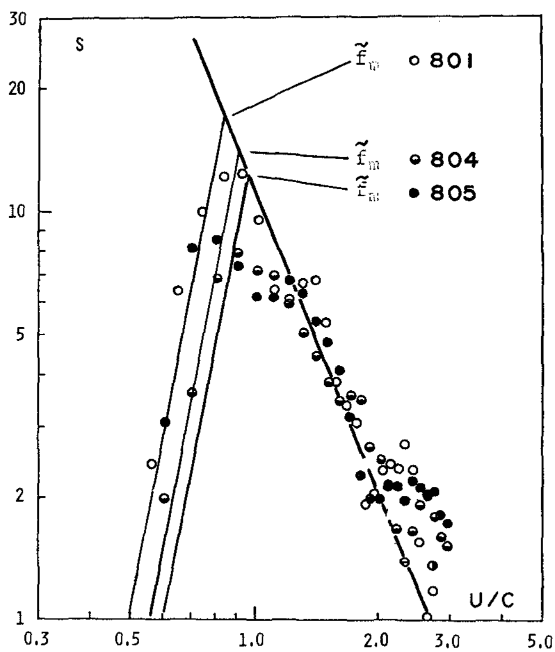

Fig.12. As in Fig.11 except for wave data sets No.801, 804 and 805.

From Eqs. (12), (14) and (15) the other expressions for s are as follows:

$$s/s_m = (\tilde{f}/\tilde{f}_m)^{-2.5}, \quad \text{for } \tilde{f} \geq \tilde{f}_m, \tag{22}$$

$$s/s_m = (\tilde{f}/\tilde{f}_m)^{5}, \quad \text{for } \tilde{f} \leq \tilde{f}_m. \tag{23}$$

It should be mentioned here that the parameter s at the spectral peak frequency $\tilde{f}_m$ increases with an increase in the dimensionless fetch $\tilde{F}$. In other words, the angular distribution becomes narrower as the dimensionless fetch increases. This is a rather unexpected result, because it would appear that the angular distribution should broaden with increasing fetch due to the fluctuation of the wind direction in a generating area.

COMPARISON OF VARIOUS ANGULAR DISTRIBUTION FUNCTIONS

Finally, the proposed form of the angular distribution

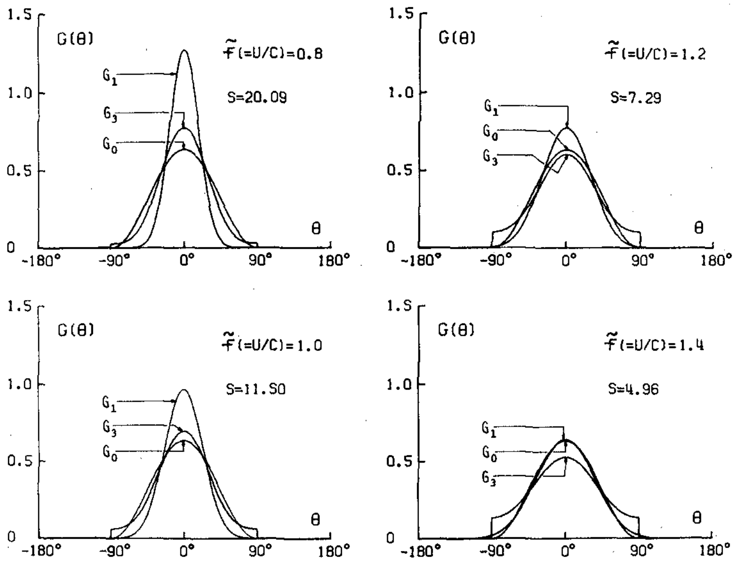

Fig.13. The angular distribution functions $G_0 = (2/\pi)\cos^2\theta$; G_1, proposed spectrum and G_3, SWOP spectrum, as a function of the dimensionless frequency $\tilde{f}$.

function, $G_1(f,\theta)$, is compared with the other forms which have been used frequently for practical purposes. Typical forms of the angular distribution are

1. PNJ type (Pierson et al., 1955)

$$G_0(\theta) = \frac{2}{\pi}\cos^2\theta \qquad (24)$$

2. SWOP spectrum (Coté et al., 1960)

$$G_3(f,\theta) = \begin{cases} \frac{1}{\pi}(1 + a\cos 2\theta + b\cos 4\theta), & \text{for } |\theta| \le \pi/2 \\ 0, & \text{for } |\theta| > \pi/2 \end{cases} \qquad (25)$$

where

$$a = 0.50 + 0.82\exp\left(-\frac{1}{2}\tilde{f}^4\right) \qquad (26)$$

$$b = 0.32\exp\left(-\frac{1}{2}\tilde{f}^4\right). \qquad (27)$$

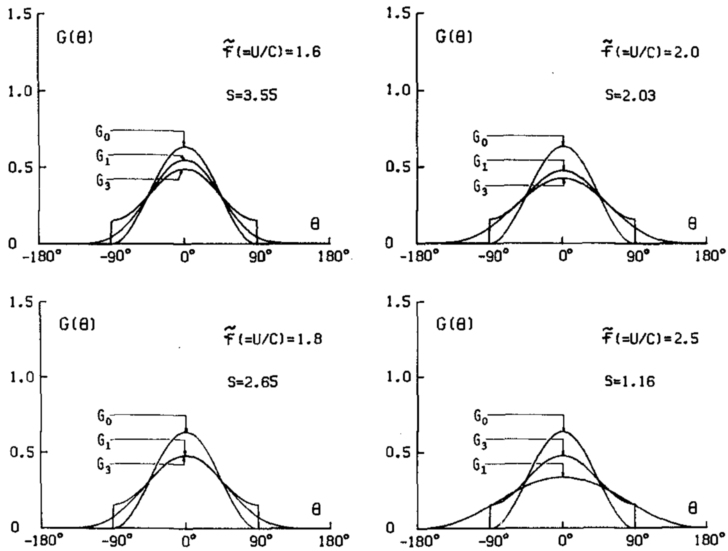

Fig.14. As in Fig.13 for other values of $\tilde{f}$.

Figs. 13 and 14 show the comparisons of the various forms for the angular distribution functions, $G_0(\theta)$, $G_1(f,\theta)$ and $G_3(f,\theta)$, by taking $\tilde{f}[= U/C]$ as a parameter. Here, $G_1(f,\theta)$ shown in the figures corresponds to angular distributions of the spectral components in a frequency range $\tilde{f} > \tilde{f}_m$. It can be seen from Figs. 13 and 14 that $G_0(\theta)$ is an appropriate form for the spectral components near the frequency $\tilde{f}[= U/C] \doteqdot 1.5$, but it has a wider angular distribution for the frequency components $\tilde{f} < 1.5$ and narrower angular distribution for the frequency components $\tilde{f} > 1.5$, as compared $G_1(f,\theta)$; $G_3(f,\theta)$ is very close to $G_1(f,\theta)$ for the frequency components $\tilde{f} \doteqdot 1.8\text{-}2.0$, although there exist some discontinuities of the distributions at $|\theta| = \pi/2$. Furthermore, $G_3(f,\theta)$ shows a wider angular distribution for the frequency components $\tilde{f} < 1.8$ and a narrower angular distribution for $\tilde{f} > 2.0$.

In some cases the following form of the angular distribution function has been used for practical purposes:

$$G_2(f,\theta) = G_2'(n)\cos^n\theta \,, \qquad (28)$$

where

$$G_2'(n) = \frac{1}{\pi^{\frac{1}{2}}} \frac{\Gamma(1+\frac{n}{2})}{\Gamma(\frac{1}{2}+\frac{n}{2})} \,. \qquad (29)$$

It can be shown by comparing $G_2(f,\theta)$ with $G_1(f,\theta)$ that $G_2(f,\theta)$ becomes very close to $G_1(f,\theta)$ if we assume

$$n = 0.46\, s \,, \qquad (30)$$

and use Eqs. 12 and 14 for the values of s. The relation (30) is only an approximate relation which gives consistent results for the frequency components $\tilde{f} < 1.6$ or $s > 4$.

CONCLUSIONS

Using typical wave data measured by the cloverleaf buoy we have determined the directional spectrum of ocean waves, in particular, the form of the one-dimensional frequency spectrum and that of the angular distribution function. While final conclusions have not been obtained for the frequency spectrum, an idealized form of the angular distribution function has been determined, which can be used for practical purposes.

The most important findings of the present study are the following properties of the angular distribution of wave energy. The angular distribution is very narrow for the frequency components near the spectral peak, whereas it widens toward high and low frequencies. Furthermore, the angular distributions of the frequency components near the spectral peak become narrower with increasing dimensionless fetch and, thus, with the growth of the wind waves. These properties are included in the proposed form of the idealized angular distribution function. Finally, it should be mentioned that, in many cases, the variabilities in the angular distribution functions are fairly large. Particularly, the scatter of the parameter s is considerable for the low frequency components ($\tilde{f} < \tilde{f}_m$) of the spectrum. This may be attributed to the effects of background swell and nonlinear wave-wave interactions. It should be also mentioned that the idealized form of the angular distribution function is based only on the ocean wave data and not on the laboratory wave data. Therefore, the extension of the formula to the laboratory wind wave is unrecommendable.

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