CHAPTER 34

Consideration on Friction Coefficient for Sea Bottom

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ABSTRACT

A bottom friction is an effective factor which will prove the deformation of progressive waves in shallow water, and many investigators have obtained the friction coefficients from field observations. However, they have not considered the effect of turbulent loss due to sand ripple at a sea bottom.

The authors, first of all, study on the friction coefficient for artificial fixed ripple by using the boundary layer theory of rough turbulence, and a new formula on the friction coefficient is proposed. The proposed friction coefficients are compared with Zhukovets' experimental results which were performed on a movable bed, and it is found that the theoretical friction coefficients for artificial fixed ripple have to be modified in order to apply for the natural beach. Lastly, the wave deformation due to the bottom friction on the movable bed is calculated by the modified friction coefficient and the effect of bottom friction on the wave deformation is discussed.

1. INTRODUCTION

It is very important to predict a wave height in shallow water for a design of coastal structure. A bottom friction is an effective factor which will prove the deformation of progressive waves in shallow water. Bretschneider[1], Iwagaki and Kakinuma[2] and other many investigators have obtained the friction coefficient at the sea bottom from the measuring results of wave height in the field. However, the friction coefficient given by Iwagaki and Kakinuma is larger than 0.01 which was given Bretschneider. We don't know the reason even now. Many investigators have not considered the effect of turbulent loss due to a sand ripple at the sea bottom. But we know that a boundary layer at the sea bottom in shallow water is turbulent in rough state, and that the sand ripple develops at the sea bottom.

As a clue to eclucidate the wave deformation due to the bottom friction, the authors, first of all, study on the coefficient for artificial fixed ripple by using the boundary layer theory on rough turbulence which was given by Kajiura[3], and a new formula on the friction coefficient is proposed.

Secondarily, the friction coefficient given by the new formula is verified by experimental results at a movable bed, and it is found that the friction coefficient by assuming fixed ripple have to be modified in order to apply for the natural beach.

Lastly, the deformation of shallow water waves on a constant beach slope is calculated by the modified friction coefficient, and the effect of bottom friction on the wave deformation is clarified.

2. FRICTION COEFFICIENT FOR ARTIFICIAL FIXED RIPPLE

The bottom friction under the ripple formation has been investigated by Bagnold[4], and Putnum and Johnson[5] have obtained $f \fallingdotseq 0.01$ from the field investigation under the consideration of Bagnold's investigation results. This value of friction coefficient was verified by Bretschneider's field observation. However, these studies were not considered the effect of boundary layer.

According to Kajiura theory[3] on the boundary layer which expanded Bagnold's experimental results, the ranges of transition are given by the following equations.

$$\left.\begin{aligned} &0.4 \leq \delta^*/D \leq 5.0, \text{ for laminar-turbulent transition} \\ &0.4 \leq D / D_L \leq 5.0, \text{ for smooth-rough transition} \end{aligned}\right\} \quad (1)$$

furthermore,

$$\left.\begin{aligned} &D/D_L = \hat{C}^{1/2} M/N \quad , \qquad \delta^*/D_L = \hat{C}^{3/2} R^2 N \\ &M = 30 \hat{U}_b Z_o/\nu \quad , \qquad R = \hat{U}_b \sqrt{\sigma/\nu} \end{aligned}\right\} \quad (2)$$

where, $\hat{C}$ is an amplitude of friction coefficient, ν is a coefficient of kinematic viscosity, σ is an angular frequency of wave, $\hat{U}_b$ is an amplitude of horizontal velocity at the sea bottom, Z_o is a roughness length, D is Nikuradse's equivalent sand roughness, and N=12.

The authors assume the sand ripple to be fixed artificial ripple, and Nikuradse's equivalent sand roughness D equals $30Z_o$. Andmore, it is already revealed that D equals 4η by Motzfeld[6].

On the other hand, some equations about the sand ripple are given by Homma, Horikawa and Kashima[7] as follow,

for criterion of ripple formation : $\hat{U}_b / \sigma\lambda > 0.5$ (3)

for height of sand ripple : $\eta = 0.175 \left(2\hat{U}_b/\sigma\right)^{-0.915} \lambda^{1.19}$ } (4)

for length of sand ripple : $\lambda = \alpha T^{\gamma} \left(2\hat{U}_b/\sigma\right)^{1-2\gamma}$ }

where, α and γ are given by the function of sand grain size as shown in Fig. (1) and the authors approximate them by the following equations.

$$\left.\begin{aligned} \alpha &= -389.4\, d_{50}^3 + 729.5\, d_{50}^2 - 313.0\, d_{50} + 40.7 \\ \gamma &= -82.7\, d_{50}^3 + 87.2\, d_{50}^2 - 28.6\, d_{50} + 3.3 \end{aligned}\right\} \quad (5)$$

where, d_{50} is a mean diameter of bed material, in centimater.

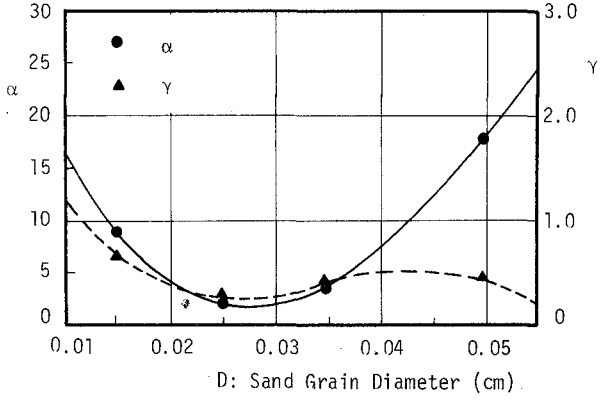

Fig.1 Relation between α, γ and sand grain size.

From the above Eqs.(1) (5), it is known that the state of flow over the ripple due to wave shows almost rourh turbulence. Futhermore, the relation between a bottom shear stress and a friction coefficient, C, is given by Eq.(6) due to Kajiura's theory.

$$\tau_b / \rho = C \cdot \hat{U}_b \cdot U_b \quad , \quad C = \hat{C}\, e^{i\theta} \quad (6)$$

$$\hat{C} = 1.7 \left(30\lambda / 4\eta\right)^{-2/3} \quad , \quad \text{for} \quad 0.5 < \hat{U}_b / \sigma\lambda < 1.0$$

$$\hat{C} = A \left(U_b / \sigma Z_0\right)^{-B} \quad , \quad \text{for} \quad 1.0 < \hat{U}_b / \sigma\lambda \quad (7)$$

where, τ_b is a bottom shear stress, $\hat{C}$ is an amplitude of friction coefficient as shown in Fig.2, θ is a phase of friction coefficient which is given by Kajiura as shown in Fig.3.

Fig.2 and Fig.3 are approximated by the dotted line. From the approximated line, A,B in Eq. (7) and θ in Eq. (6) are given by Table 1.

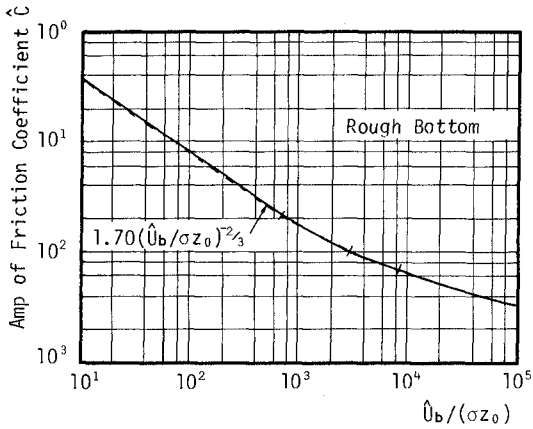

Fig. 2 Change of $\hat{C}$ with $\hat{U}_b/(\sigma z_0)$

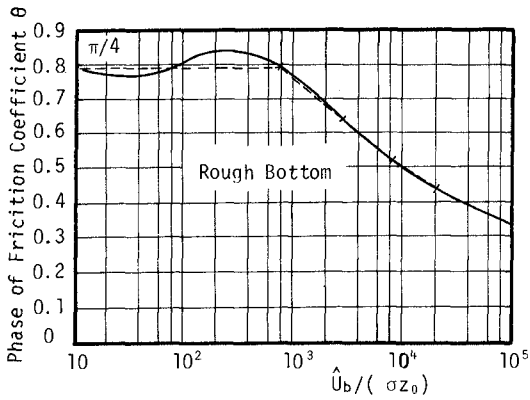

Fig. 3 Change of θ with $\hat{U}_b/(\sigma z_0)$

$\hat{U}/\sigma z_0$	$C=A(\hat{U}_b/\sigma z_0)^{-B}$	θ
~685	$1.7(\hat{U}/\sigma z_0)^{-0.667}$	$\approx \frac{\pi}{4}$
$685\sim 3\times10^3$	$0.711(\hat{U}/\sigma z_0)^{-0.536}$	$1.558-0.266\log_{10}(\frac{\hat{U}}{\sigma z_0})$
$3\times10^3\sim 8\times10^3$	$0.240(\hat{U}/\sigma z_0)^{-0.4}$	$1.524-0.257\log_{10}(\frac{\hat{U}}{\sigma z_0})$
$8\times10^3\sim 2\times10^4$	$0.068(\hat{U}/\sigma z_0)^{-0.26}$	$1.287-0.196\log_{10}(\frac{\hat{U}}{\sigma z_0})$
$2\times10^4\sim 10^5$	$0.068(\hat{U}/\sigma z_0)^{-0.26}$	$1.092-0.150\log_{10}(\frac{\hat{U}}{\sigma z_0})$

Table 1. Approximated value of A, B and θ

In terms of friction coefficient $\hat{C}$, a mean energy dissipation $\langle E \rangle$ is given by Eq.(8).

$$\langle E \rangle = \rho/2\, \hat{C}\hat{U}_b^3 \cos\theta \tag{8}$$

On the other hand, in the ordinary definition of friction coefficient, f, which is used by many investigators, the energy dissipation, $\langle E \rangle$ is given by Eq.(9).

$$\langle E \rangle = 4/3 \cdot \rho \cdot f \hat{U}_b^3 \tag{9}$$

So, the coefficient, f, is represented by Eq.(10) from the comparison of Eqs.(8) and (9).

$$f = 3/8\, \hat{C}\cos\theta \tag{10}$$

Summarizing the above relation, the friction coefficient, f_{fix}, provided that the sand ripple is assumed to be fixed artificial one, is given by Eq. (11).

$$\left.\begin{aligned} f_{fix} &= 0.116\,(\frac{\pi}{a})^{0.127}\, Re_T^{-0.127} \quad , \quad \text{for} \quad (\frac{1}{a})^{\frac{1}{\gamma}} < Re_T < \pi\,(\frac{2}{a})^{\frac{1}{\gamma}} \\ f_{fix} &= 1.18\, A\cos\theta \left\{ 0.0467\,(\frac{\pi}{a})^{1.19\gamma} \right\}^B \cdot Re_T^{-1.19B\gamma} \quad , \\ &\qquad\qquad \text{for} \quad Re_T > \pi\,(\frac{2}{a})^{\frac{1}{\gamma}} \\ a &= (\frac{\nu}{\pi})^{\gamma}/\alpha \quad , \qquad Re_T = \hat{U}_b^2 T/\nu \end{aligned}\right\} \tag{11}$$

where, subscript "fix" shows that the sand ripple is assumed to be fixed artificial ripple. α and γ are given by Eq.(5) and A, B and θ are given by Table 1.

Using the above relations, the friction coefficient for an arbitrary

sand grain size is obtained. The relation between f_{fix} and Re_T of Eq.(II) for 4 kinds of sand grain size are shown by the full line in Fig.4.

In Fig.4 the friction coefficients in the field data obtained by many investigators are plotted, and they are scattered and indicate different values according to sand grain sizes. The field data contain the effect of wave directional dispersion, wave breaking, etc., so it is difficult to compare the theoretical coefficient with tbe field data.

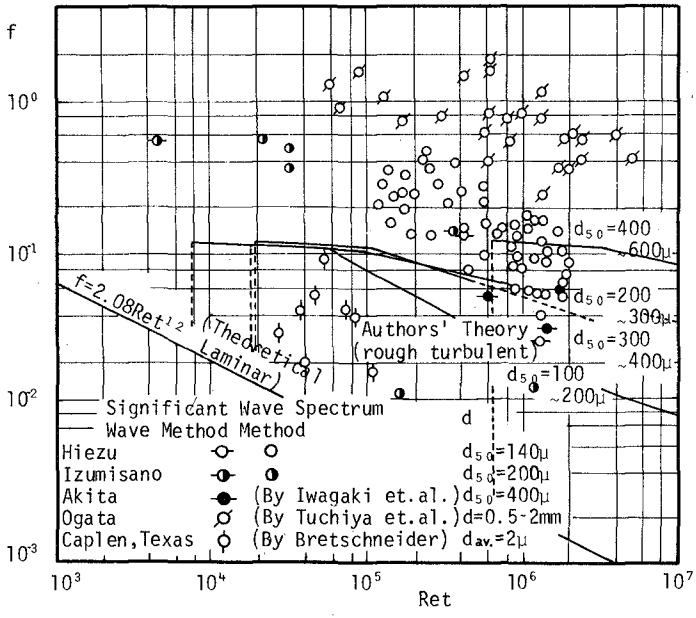

Fig. 4 Relations between f_{fix} and Re_T

3. VERIFICATION OF THE FRICTION COEFFICIENT BY THE EXPERIMENTAL ON THE MOVABLE BED

The friction coefficient of Eq.(II) is compared with the experimental values which were required by Zhukovets[8] from the wave deformation on horizontal movable bed in a laboratory. In this experiment, the wave height ranged from 3 to 18 cm, and the wave steepness varied from 0.1 to 0.04. As the bed material, the sand which the mean diameter of grain size is 0.25 mm was used.

The comparison of the experimental data within the range of rough turbulence with the theoretical coefficient is shown by Fig.5. In Fig.5, f_{exp} is the experimantal friction coefficient and K_{sm} indicates the relative roughness of the bottom and is defined by Eq.(12). Re in this figure shows the Reynolds number which is given by Eq.(13).

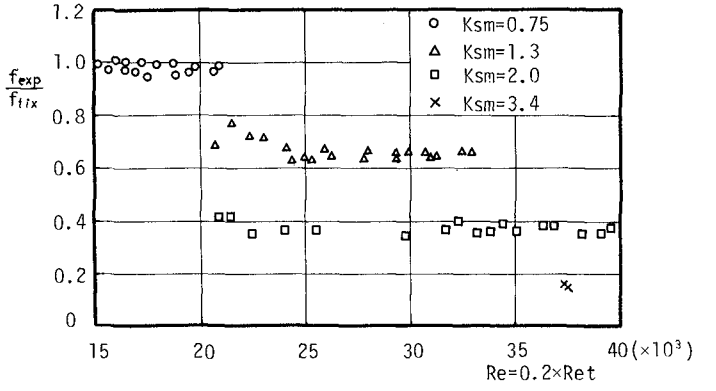

Fig.5 Comparison of f_{fix} and f_{exp}.

$$K_{sm} = V_g^2 \Big/ \left\{ 2\,g\,d_{50} \sinh^4 \sqrt{1 + \eta / d_{50}} \right\} \tag{12}$$

$$R_e = V_g H / \nu \sinh kh \tag{13}$$

where η is the height of sand ripple, and V_g is the velocity of water particle motion at bottom averaged over a quarter period of wave and is given by Eq.(14).

$$V_g = (\sigma H / \pi) \operatorname{cosech} kh + (3k\sigma H^2 / 8) \operatorname{cosech}^4 kh \tag{14}$$

where k is a wave number $k = 2\pi / L$, L is a wave length and H is a wave height.

Then, Re is transformed into Re_T by the following equation.

$$Re = \frac{V_g H}{\nu \sinh kh} \simeq \frac{2}{\pi^2} \frac{\hat{U}_b^2 T}{\nu} = 0.2\, Re_T \tag{15}$$

From Fig.5, when K_{sm} is 0.75, f_{fix} has a fairly good agreement with f_{exp}, but when K_{sm} becomes larger, f_{fix} becomes larger than f_{exp}. The reason of this fact is not evident, but it is considered as follow; when K_{sm} becomes larger, V_g becomes large and the bed material seems to have a tendency of suspension. Generally, the friction in suspended state is less than the friction of non-suspended state, so the assumption of the fixed ripple model

for the theoretical friction coefficient is not able to apply for the case of large K_{sm}. Therefore, the theoretical friction coefficient f_{fix} has to be modified with regard to K_{sm} which indicates the characteristics of the movable bed. According to Fig.5, the ratio of f_{exp} to f_{fix} depends only on K_{sm}, and it is independent of the Reynolds number.

Now, the ratio is called a modification factor, F_c. The modification factor varies with K_{sm} as shown in Fig.6. Then, the theoretical friction coefficient for fixed ripple has to be modified into f_m by the modification factor F_c.

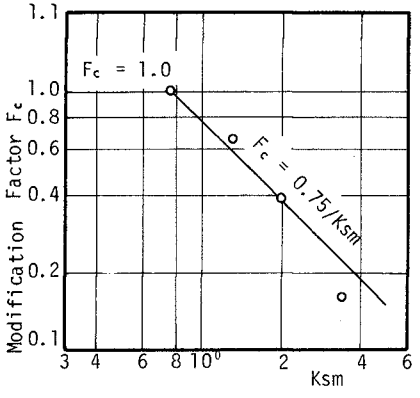

Fig.6 Change of the modification factor with Ksm.

The modified coefficient f_m is approximated by Eq.(16).

$$\left.\begin{array}{lll} K_{sm} \leq 0.75 & f_m = & f_{fix} \\ K_{sm} > 0.75 & f_m = & (0.75 / K_{sm})\ f_{fix} \end{array}\right\} \quad (16)$$

However, when K_{sm} becomes very large, f_m becomes smaller than the friction coefficient in case of laminar boundary layer, f_L, which is represented by Eq.(17).

$$f_L = 2.08\ Re_T^{-1/2} \quad \text{for laminar boundary layer}^{9)} \quad (17)$$

In this case, f_m is assumed to be f_L for convenience.

4. EFFECT OF BOTTOM FRICTION TO WAVE DEFORMATION

The deformation of shallow water wave due to the bottom friction on a constant beach slope is calculated by the modified friction coefficient f_m.

In this calculation, Eq.(18) derived from the law of energy conservation is used.

$$H_{i+1} = \left(\frac{C_{g\,i}}{C_{g\,i+1}} H_i^2 - \frac{32 \Delta x}{3\pi g C_{g\,i+1}} f_{mi} \hat{U}_{bi}^3 \right) \qquad (18)$$

where C_g is a group velocity of wave, the subscript "i" indicates the i-th step, and g is a gravity acceleration.

The calculation is started from the point where a relative water depth 1 and is stopped at the breaking point which was given by Kishi and Saeki as follows:

$$H_b / h_b = 5.68\, S^{0.4} \qquad (19)$$

where, S is a beach slope.

The calculation is carried out by the iterative method, that is, the authors evaluate the friction coefficient with the mean value of H_i and the assuming value of H_{i+1}. With the friction coefficient, the calculated value of H_{i+1} will be obtained by Eq.(18). The iteration is continued till the difference between the assuming value of H_{i+1} and the calculated value of H_{i+1} becomes small.

For the wave condition of wave period which equals to 4.0 sec, beach slope of 1/120 and 4 kinds of wave height at deep water, the deformation of wave height is obtained as shown in Fig.7.

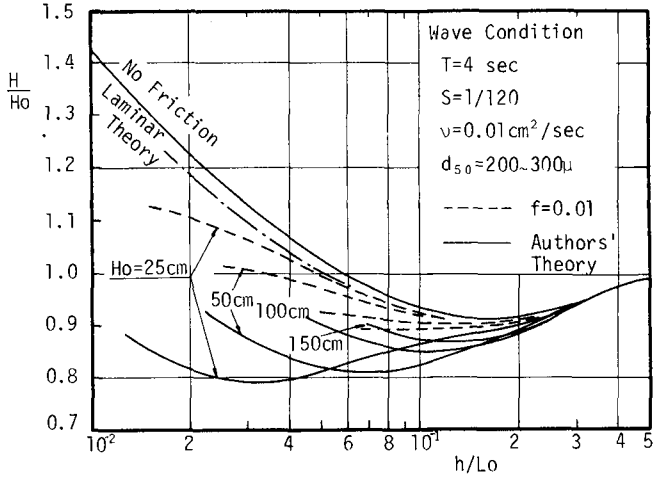

Fig.7 Comparison of wave decay with various friction coefficient

In this figure, the full lines show the change of wave height by the authors' theory, the chain line is the result due to the laminar boundary layer theory and the dotted lines indicate the calculation results in case that the friction coefficient is 0.01. Furthermore, no friction curve, that is, the shoaling curve is drawn in this figure.

From these curves, it is found that the decay of wave height is considerably larger than the laminar case, and the decay curves indicate different tendency of the calculation results for f=0.01. According to the authors' theory, when the wave height becomes larger, the effect of bottom friction becomes smaller.

Fig. 8 shows the effect of wave period on the wave deformation due to the bottom friction under the condition that the wave height is 50 cm and the beach slope is 1/120. From this figure, it is recognized that as the wave period becomes shorter, the wave decay becomes larger.

Fig. 9 shows the effect of beach slope upon the wave decay. From this figure, even if the wave condition at deep water is the same,as the beach slope is gentle, the degree of wave decay becomes larger.

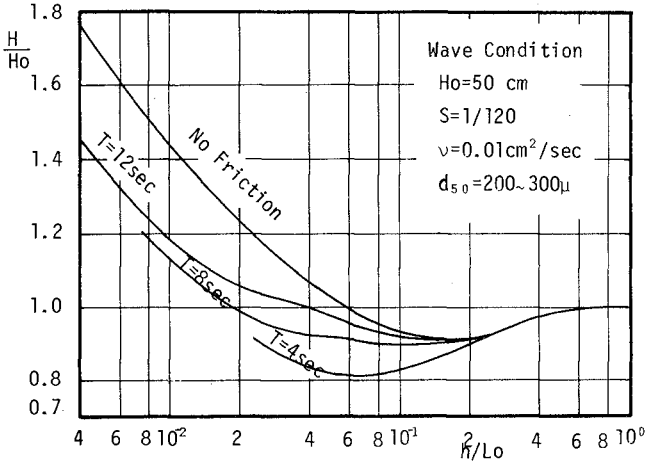

Fig. 8 Effect of wave period for wave decay

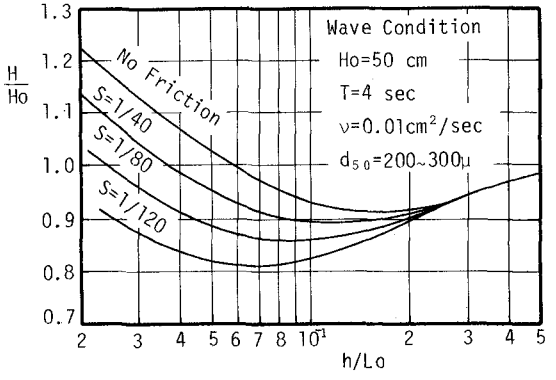

Fig. 9 Effect of beach slope for wave decay

5. CONCLUSION

Considering the effect of sand ripple which is assumed to be fixed artificial ripple, the new formula on friction coefficient is proposed by using the boundary layer theory on turbulence. The proposed friction coefficients are compared with Zhukovets' experimental results which were performed on the movable bed in the laboratory. From the comparison, it is clarified that the proposed friction coefficient for sand ripple has to be modified by the modification factor for a movable bed, and the wave deformation due to the bottom friction on the movable bed is calculated by using the modified coefficient, f_m.

From the calculation, it is pointed out that the wave decay which is considered to be due to the effect of sand ripple under the turbulent condition, is considerably larger than one at laminar case and the wave decay curve indicates the different tendency from that in the friction coefficient f=0.01.

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