CHAPTER 45

CHARACTERISTICS OF FLOW IN RUN-UP OF PERIODIC WAVES

By Ary Roos[1] and Jurjen A. Battjes[2]

ABSTRACT

An experimental study is presented of some characteristic parameters of the flow in the up-rush and down-rush of periodic waves breaking on a plane, smooth slope. The water layer thickness has been measured as a function of time at four locations above still water level. Discharges and particle velocities have been calculated. The results have been made non-dimensional on the basis of Hunt's formula for the run-up height. They appear to be either independent of the wave steepness H/L_o and slope gradient $\tan\alpha$ or to be a function of a single similarity parameter $\xi = \tan\alpha \,/\, \sqrt{H/L_o}$ only. An hypothesis is stated concerning a relation between the mean rate of overtopping of a dike by waves, and the run-up which would occur under the same circumstances on an uninterrupted slope. On the basis of this hypothesis the overtopping volume per wave can be normalized so as to make it independent of slope angle and wave steepness. A comparison of the result with measurements from other sources indicates a rough agreement.

1. INTRODUCTION

The run-up of waves is an important factor in the design of shore structures. It has been investigated in many studies, both theoretical and experimental. The experimental studies have mostly been confined to the run-up heights (the greatest heights above still water level, reached by the individual waves on the slope). A simple and reliable formula for the run-up height is given by Hunt (4), based on measurements with periodic waves breaking on smooth plane slopes. However, virtually no data are available regarding the characteristics of the flow in the up-rush and the down-rush on a smooth slope, such as layer thickness, particle velocity, wave front velocity, and so on. Such information can be of use in developing or adjusting schemes for numerical calculation of run-up and overtopping, as well as in problems of stability of cover layer material or of seepage of water into the core material of a dike. The purpose of this paper is to present empirical results concerning the above-mentioned flow parameters, obtained from experiments with periodic waves breaking on smooth plane slopes. In the analysis of the data considerable attention is given to similarities in the run-up process; the existence of such similarities had previously been inferred from Hunt's formula for the run-up height.

2. RUN-UP PARAMETERS

In this study periodic waves of perpendicular incidence are considered, breaking on smooth plane slopes. Only the motion of the relatively thin water layer on the slope above the still water level is dealt with.

The independent parameters considered are (see fig. 1) the height of the incident waves H, the wave period T, the water depth d, the acceleration of gravity g, the slope angle α (a smooth plane slope is considered) and the mass density ρ and the dynamic viscosity μ of the water.
A coordinate system is chosen as shown in fig. 1. The origin O is situated at the still water line. The x-axis is directed upwards along the slope. The dependent variables are the layer thickness h, the particle velocity v (averaged over the layer thickness) and the velocity of the run-up wave front c. The variables h and v are functions of the independent variables x and t, in which t is the time; c is a function of x or t.

---

1. Research Engineer, Directorate for Watermanagement and Hydraulic Research, Rijkswaterstaat, The Hague, The Netherlands.
2. Associate Professor of Fluid Mechanics, Department of Civil Engineering, Delft University of Technology, Delft, The Netherlands.

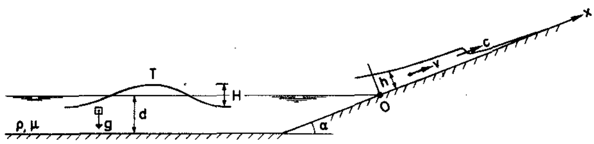

Fig. 1. Definition sketch.

Altogether the following parameter equations are obtained:

$$h = F(x, t, H, T, d, g, \rho, \mu, \alpha) \quad (1)$$
$$v = F(x, t, H, T, d, g, \rho, \mu, \alpha) \quad (2)$$
$$c = F(t, H, T, d, g, \rho, \mu, \alpha) \quad (3)$$

Dimensionless combinations can be formed from the variables of Eq. 1 to 3. It may be possible to reduce the number of relevant independent dimensionless parameters by choosing proper scaling quantities. In this way the variables are not merely made dimensionless; they are normalized. Hunt's formula (3), supplemented by an interpretation given by Battjes (1), provides a clue as to the choice of the appropriate scaling quantities.

Hunt's formula is based on measurements of the run-up of periodic waves breaking on smooth plane slopes. It can be written as:

$$R = \sqrt{HL_o}\,\tan\alpha \quad (4)$$

in which R is the run-up height and L_o is defined by:

$$L_o = \frac{gT^2}{2\pi} \quad (5)$$

By substituting Eq. (5) in Eq.(4) this can be written as:

$$R = 0.4\,T\sqrt{gH}\,\tan\alpha \quad (6)$$

From the Eqs. (4) and (6) it appears that the horizontal distance between the still water line and the point of maximum wave run-up is $\sqrt{HL_o}$ or $0.4\,T\sqrt{gH}$, independent of the slope angle and the wave steepness. This suggests the use of T, $\sqrt{gH}$ and $\sqrt{HL_o}$ as scaling parameters for the time and the horizontal velocities and lengths of the run-up process. No information is available regarding the scale for the layer thicknesses. Tentatively the same quantity will be chosen as for the horizontal lengths, i.e. $\sqrt{HL_o}$. The following relations are obtained in this way:

$$\frac{h}{\sqrt{HL_o}} = F\left(\frac{x}{\sqrt{HL_o}}, \frac{t}{T}, \frac{d}{L_o}, \frac{\rho H^2}{\mu T}, \frac{H}{L_o}, \alpha\right) \quad (7)$$

$$\frac{v}{\sqrt{gH}} = F\left(\frac{x}{\sqrt{HL_o}}, \frac{t}{T}, \frac{d}{L_o}, \frac{\rho H^2}{\mu T}, \frac{H}{L_o}, \alpha\right) \quad (8)$$

$$\frac{c}{\sqrt{gH}} = F\left(\frac{t}{T}, \frac{d}{L_o}, \frac{\rho H^2}{\mu T}, \frac{H}{L_o}, \alpha\right) \quad (9)$$

The parameter $\rho H^2/\mu T$ is a Reynolds number for the incident waves. It is assumed that this does not affect the dimensionless dependent variables of Eqs. 7, 8 and 9. The dimensionless variable d/L_o gives the effect of the water depth. From experiments, such as those reported by Saville (6), it appears that the depth does not affect the run-up heights for periodic waves that break on the slope. Since we are restricting ourselves to breaking waves it is assumed that d/L_o has no influence on the water movement on the slope. In this way the dependent dimensionless variables appear to be determined only by the location on the slope $x/\sqrt{HL_o}$, the phase t/T, the factor H/L_o (which for brevity is called the wave steepness) and the slope angle α. Based on the arguments which led to the choice of the scaling quantities given above, it may be expected that at least some of the dependent normalized variables given in Eqs. 7, 8 and 9 will in fact turn out to be independent of slope angle and wave steepness.

3. RUN-UP EXPERIMENTS

In order to find the effect of the wave steepness and the slope angle on the water movement on the slope, model experiments have been performed.

3.1. Experimental Set-Up

The wave run-up experiments have been performed in a wave flume of the Civil Engineering Department of the Delft University of Technology. This flume is 30 m long, 0.80 m wide and 0.60 m deep. At one end of the flume the waves were generated by a wave board, at the opposite end a plane plywood slope had been installed (see fig. 2).

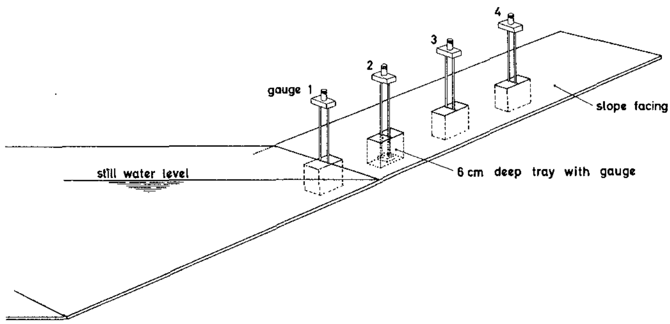

Fig. 2. Slope with gauges for the measurement of layer thicknesses.

On this slope layer thicknesses were measured by means of four wire resistance gauges perpendicular to the slope at equal distances above still water level. They were placed in 6 cm deep trays, which were filled with water and closed by covers flush with the slope facing, in order to maintain the minimum submergence necessary for a linear response. In the constant depth portion of the flume a wire resistance gauge had been placed to measure the incident waves.

3.2. Experimental Procedure

In the experiments the slope angle and the wave characteristics were varied. During each run the incident waves and the layer thicknesses on the slope were registered on recording paper. The measurements on the slope started when the waves reflected from the slope and re-reflected from the wave board arrived at the slope again. The smallest number of waves per run so obtained was thirteen.

The instrumental measurements in each run were supplemented with visual observations of the average run-up length l_u (i.e. the average distance from the still water line to the point of maximum run-up per wave) and the average run-down length l_d (i.e. the average distance from the still water line to the highest point on the slope which appeared to be submerged at all times. During the run-down process the water tongue becomes relatively thin. For this reason the visually determined values of l_d are not as accurate as those of l_u. However, they turned out to be consistent with the instrumental measurements of the water level variations on the slope.

3.3. Experimental Program

Only waves breaking on the slope are considered. The slope gradients which have been chosen are 1:3, 1:5 and 1:7 (vertical : horizontal); they are typical of dikes. The wave steepness (H/L_o) ranged from 0.02 to 0.07 approximately. Reference is made to table 1. The depth in the constant portion of the flume was 0.45 m in all the runs.

Run number (1)	Slope Gradient (2)	Wave height H, in meters (3)	Wave period T, in seconds (4)	Wave steepness H/L_o (5)	$\sqrt{HL_o}$, in meters (6)	l_u, in meters (7)	l_d, in meters (8)	$\sqrt{HL_o}\tan\alpha$ in meters (9)	$R = l_u \sin\alpha$, in meters (10)
1	1:3	0.071	1.26	0.029	0.420	0.45	0.10	0.140	0.142
2	1:3	0.080	1.17	0.037	0.414	0.45	0.13	0.138	0.142
3	1:3	0.106	1.05	0.062	0.426	0.45	0.19	0.142	0.142
4	1:3	0.111	1.01	0.070	0.420	0.45	0.22	0.140	0.142
5	1:5	0.097	1.85	0.018	0.719	0.70	- a)	0.144	0.137
6	1:5	0.116	1.65	0.027	0.702	0.65	-	0.140	0.128
7	1:5	0.131	1.52	0.036	0.687	0.70	-	0.137	0.137
8	1:5	0.156	1.38	0.053	0.680	0.70	-	0.136	0.137
9	1:5	0.169	1.32	0.062	0.678	0.70	-	0.136	0.137
10	1:5	0.183	1.28	0.072	0.685	0.70	-	0.137	0.137
11	1:7	0.116	1.95	0.020	0.828	0.85	0.40	0.118	0.120
12	1:7	0.153	1.63	0.037	0.796	0.80	0.55	0.114	0.113
13	1:7	0.178	1.49	0.052	0.785	0.80	0.60	0.112	0.113
14	1:7	0.213	1.39	0.071	0.801	0.80	0.60	0.115	0.113

a) not recorded

Table 1. Experimental data.

The gauges which measure the layer thickness should cover a constant, fairly large part of the maximum length of the water tongue on the slope in all the runs. Furthermore it was expedient to move the gauges only when the slope angle was changed. For this reason the incident wave characteristics were chosen in such a manner that the run-up length l_u ($\approx \sqrt{HL_o}$ according to Hunt's formula) is about constant per slope angle; see table 1. The value of $\sqrt{HL_o}$ for each slope angle is chosen as large as possible (limited by the flume dimensions) to get the best accuracy of the measurements and the least possible scale effects.

The four gauges on the slope were placed at equal distances, which were constant for each slope angle : 0.125 m for slope 1:3, 0.200 m for slope 1:5 and 0.225 m for slope 1:7. The length covered by the gauges was about 0.9 $\sqrt{HL_o}$ for all the runs.

3.4. Experimental Data

A summary of experimental data is given in table 1. For run 7 (which is used as an example throughout the following in this paper) the recorded signals are shown in fig. 3. The index of h in this figure denotes the gauge number (given in fig. 2).

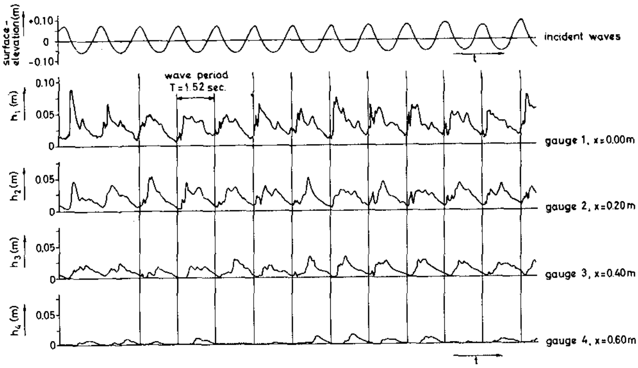

Fig. 3. Recorded waves and layer thicknesses (Run 7).

4. ELABORATION OF THE EXPERIMENTAL DATA

The data available such as those shown in fig. 3 have been elaborated for all the runs presented in table 1. An extensive presentation of the results has been given in ref. (3). In this paper only the results of run 7 are given as an example.

4.1. Phase-Averaging of the Records

From fig. 3 it appears that the records of the layer thicknesses are not quite periodic. To eliminate the irregularities the records have been phase-averaged over ten successive waves at 0.05 sec - intervals. As an example the averaged h(t)-curves of run 7 are presented in fig. 4.

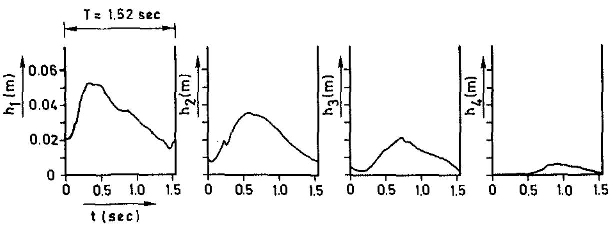

Fig. 4. Phase-averaged h(t)-curves (Run 7).

4.2. Time-History of the Run-Up Front

In the further elaboration the averaged h(t)-curves have been used. The sharp bend in these curves followed by a very quick rise of the water level (see fig. 4) corresponds with the moment of upward front passage. These passage moments have been used to construct an x-t-diagram of the run-up front (see fig. 5, circled points). Through these points a curve has been drawn, which determines the slope of the curve in S. For the completion of the x-t-curve of the wave front the run-up length (l_u) and the run-down length (l_d) are available. The latter of these determines point Q. These data together determine the shape of the dashed curve in fig. 5 reasonably well.

4.3. Determination of the Particle Velocities

To determine the particle velocities (averaged over the layer thickness) on the slope, the profiles of the water tongues were constructed for successive phases of the wave period with an interval of 0.1 sec. The construction of the profile of the water tongue on the slope for an arbitrary instant $t = t_1$ is illustrated in fig. 6. The layer thicknesses h_1, h_2, h_3 and h_4 were read at $t = t_1$ from the h(t)-curves (see fig. 6a, which schematically represents the results given in fig. 4) and were plotted in fig. 6b. This determines the points A, B, C and D.

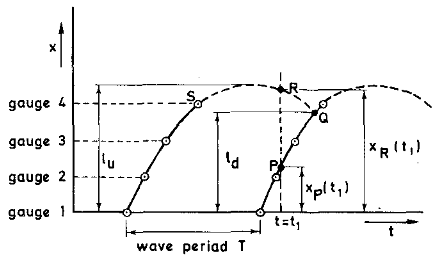

Fig. 5. Construction of the x-t-curve of the wave front on the slope.

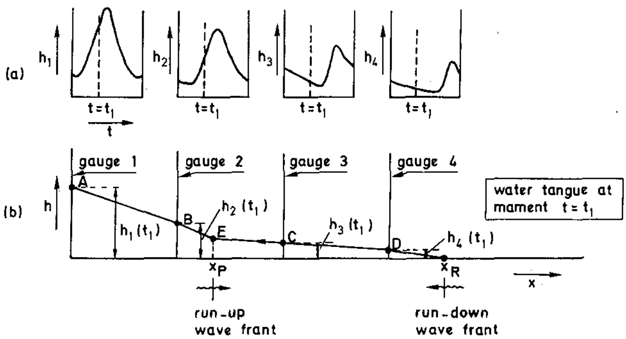

Fig. 6. Construction of the water tongue profile on the slope.

Then the x-coordinates of the run-up wave front (x_P) and the run-down wave front (x_R) were taken at $t = t_1$ from fig. 5 and plotted in fig. 6b. The known points of the profile have been connected by straight lines, except in the area where the run-up wave front is between gauges. In this area the water level of the upper part (beween the gauges 3 and 4 in the example of fig. 6) has been extrapolated to x_P; this gives point E. From this point to point B a straight line has been drawn. An alternative would have been to extrapolate the water level between the gauges 1 and 2 also to x_P, so that a discontinuity in the water level would occur at that location. However, from the h(t)-curves and also from visual observations in the flume it appeared that the run-up front is outlined as a bend in the water tongue profile.

From the constructed water tongue profiles the quantities of water stored above each of the four gauges have been calculated as functions of the time. The curves obtained for these quantities for run 7 are presented in fig. 7a. In this figure B_1 denotes the volume per unit width stored above gauge 1, etc. The particle velocities averaged over the layer thickness have been calculated from these curves according to:

$$v_i(t) = \frac{1}{h_i(t)} \frac{B_i(t+\frac{1}{2}\Delta t) - B_i(t-\frac{1}{2}\Delta t)}{\Delta t} \quad \ldots\ldots\ldots\ldots\ldots \quad (10)$$

in which i denotes the number of the gauge. The time step Δt is equal to the interval between the successive constructed water tongues, i.e. 0.1 sec. Fig. 7b presents the calculated particle velocities for run 7.

The calculated particle velocities v are less accurate than the water layer thicknesses h because of the procedure used in the elaboration.

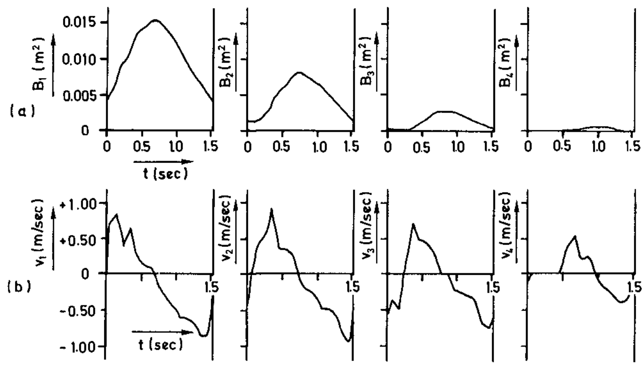

Fig. 7. Calculated volumes of water stored per unit width (a) and particle velocities (b) on the slope (Run 7).

5. EXPERIMENTAL RESULTS

In this section the results of the elaboration as presented in the figs. 4, 5 and 7 are analysed.

5.1. Run-Up Heights

Run-up heights are presented in table 1, column 9 and 10. $\sqrt{HL_0}\tan\alpha$ is the run-up height according to Hunt's formula, while R is the run-up height calculated from the measured l_u-values and the slope angles. Inspection of the data shows that the measured run-up values are in very good agreement with Hunt's formula.

5.2. Water Layer Thicknesses

Eq. 7 gives the parameter equation for the layer thicknesses. With the elaborated data a relation between $h/\sqrt{HL_o}$ and the independent parameters will be sought. To this end it is useful to characterize the h-values by a single value, for which the maximum layer thickness h_{max} has been chosen. Plots of h/h_{max} versus t/T delivered shape functions (not shown here) which were more or less independent of the wave steepness and the slope angle, for which reason h_{max} can be accepted as the only scale parameter characterizing the h-values. The parameter equation for h_{max} can be obtained as a reduced version of Eq. 7:

$$\frac{h_{max}}{\sqrt{HL_o}} = F\left(\frac{x}{\sqrt{HL_o}}, \frac{H}{L_o}, \alpha\right) \quad (11)$$

In fig. 8a the dimensionless layer thickness $h_{max}/\sqrt{HL_o}$ has been plotted versus the wave steepness H/L_o. The layer thickness parameter shows virtually no dependence on the wave steepness. Fig. 8b presents a plot of the layer thickness parameter versus the slope gradient. The dependence on the slope gradient is weak. In fig. 8c the dimensionless layer thickness has been plotted versus the location on the slope. In this figure the point $x/\sqrt{HL_o} = 1.0$ is the location of maximum run-up on the slope according to Hunt's formula (apart from a factor $\cos\alpha$).

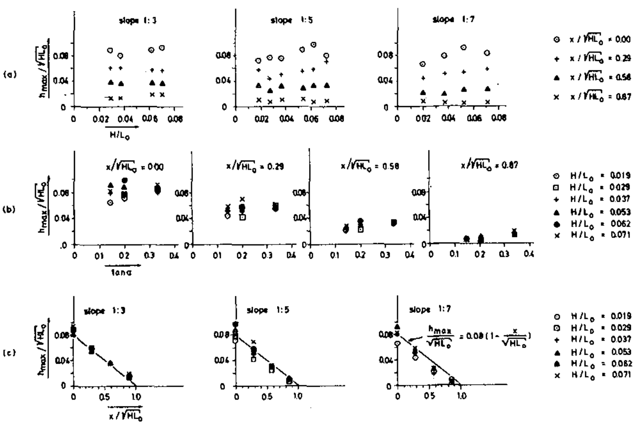

Fig. 8. Normalized layer thickness versus H/L_o (a), $\tan\alpha$ (b) and $x/\sqrt{HL_o}$ (c).

It is concluded from fig. 8 that $h_{max}/\sqrt{HL_o}$ is roughly independent of H/L_o and $\tan\alpha$, and that it is related to the location on the slope approximately as follows:

$$\frac{h_{max}}{\sqrt{HL_o}} = 0.08\left(1 - \frac{x}{\sqrt{HL_o}}\right) \quad (12)$$

This formula is represented in fig. 8c by the straight lines.

5.3. Particle Velocities

Eq. 8 gives the parameter equation for the particle velocities in the water tongue on the slope. The v(t)-curves (see fig 7b) can be characterized by v_{max} (extreme upward velocity) and v_{min} (extreme downward velocity). The shape functions of the v(t)-curves were found to be even more similar than those of the h(t)-curves. In this way the two following parameter equations are obtained from Eq. 8:

$$\frac{v_{max}}{\sqrt{gH}} = F\left(\frac{x}{\sqrt{HL_o}}, \frac{H}{L_o}, \alpha\right) \quad \ldots \quad (13)$$

$$\frac{v_{min}}{\sqrt{gH}} = F\left(\frac{x}{\sqrt{HL_o}}, \frac{H}{L_o}, \alpha\right) \quad \ldots \quad (14)$$

Various cross-plottings of these parameters were made. In each of the three lowermost gauge locations $v_{max}/\sqrt{gH}$ was found to be approximately inversely proportional to $\sqrt{H/L_o}$ and directly proportional to $\tan\alpha$. This means that in those locations $v_{max}/\sqrt{gH}$ is in fact proportional to the similarity parameter ξ (2) defined by:

$$\xi = \frac{\tan\alpha}{\sqrt{H/L_o}} \quad \ldots \quad (15)$$

This is illustrated in fig. 9. The linear dependence of $v_{max}/\sqrt{gH}$ with ξ implies that v_{max} is in fact proportional to $gT \tan\alpha$, independent of the wave height H. This is a rather surprising result, which obviously can be true in a restricted range of the independent variables only. The coefficient of proportionality of $v_{max}/\sqrt{gH}$ with ξ is the same for the three lowermost gauge locations ($x/\sqrt{HL_o} \approx 0.00$, 0.29 and 0.58). It is only in the upper 1/3 of the run-up length that the particle velocities appear to diminish.

Conclusions such as those stated above for v_{max} also hold for v_{min} (see fig 10).

5.4. Front Velocity

The parameter equation for the dimensionless front velocity is given by Eq. 9. In this equation c is the instantaneous derivative of the x-t-curve of the run-up wave front. A characteristic value for c is its average during run-up, defined by $\bar{c} = l_u/t_u$, in which l_u is the run-up length and t_u is the run-up time (see fig. 11). By dealing with the average front velocity $\bar{c}$ Eq. 9 becomes:

$$\frac{\bar{c}}{\sqrt{gH}} = F\left(\frac{H}{L_o}, \alpha\right) \quad \ldots \quad (16)$$

From the analysis of the $\bar{c}$ - data it appeared that $\bar{c}/\sqrt{gH}$ is proportional to $\sqrt{\tan\alpha}$ and inversely proportional to $\sqrt[4]{H/L_o}$. In fig. 12a the parameter $\bar{c}/\sqrt{gH}$ has been plotted versus ξ. From this figure it appears that the following relation holds:

$$\frac{\bar{c}}{\sqrt{gH}} \approx 0.6\sqrt{\xi} \quad \ldots \quad (17)$$

Eq. 17 can also be written as $\bar{c} \approx 0.6 \sqrt{\tan\alpha}\ \sqrt{g\sqrt{HL_o}}$. The term $\sqrt{g\sqrt{HL_o}}$ in this formula can be interpretated as the velocity of a low shock wave in water with a depth $\sqrt{HL_o}$. It has already been pointed out (see Eq. 12)

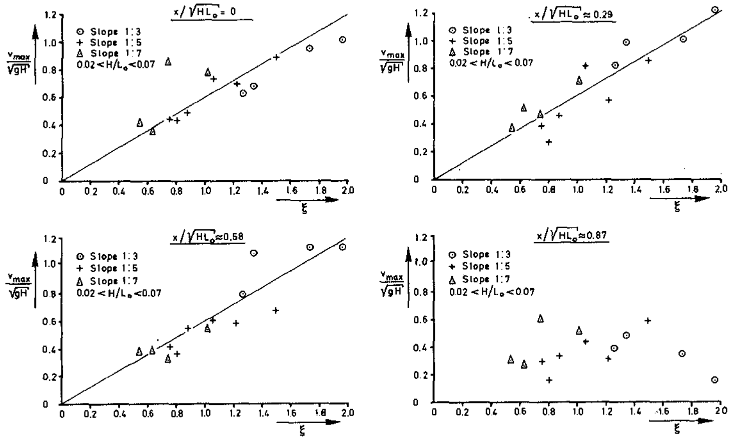

Fig. 9. Normalized upward particle velocity versus ξ.

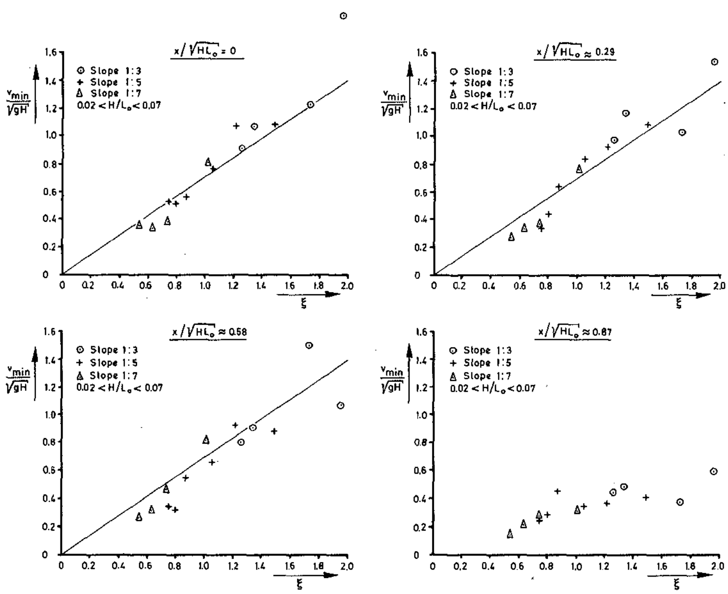

Fig. 10. Normalized downward particle velocity versus ξ .

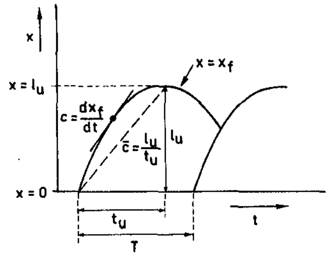

Fig. 11. Definition sketch for the run-up time t_u and the average front velocity $\bar{c}$.

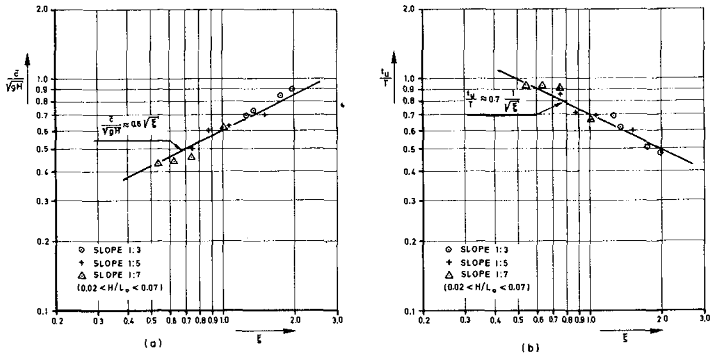

Fig. 12. Normalized average front velocity (a) and run-up time (b) versus ξ.

that the water layer thicknesses on the slope are proportional to $\sqrt{HL_o}$ (although an order of magnitude smaller).

5.5. Run-Up Time

Another characteristic quantity of the x-t-curve of the run-up wave front is the dimensionless run-up time t_u/T. It depends on the wave steepness and the slope angle, expressed by the following parameter equation:

$$\frac{t_u}{T} = F\left(\frac{H}{L_o}, \alpha\right) \quad \ldots\ldots\ldots\ldots \quad (18)$$

It appears from the analysis that t_u/T is proportional to $\sqrt[4]{H/L_o}$ and inversely proportional to $\sqrt{\tan\alpha}$. In fig. 12b the dimensionless run-up time t_u/T has been plotted versus ξ. From this figure it can be seen that the following relation is obtained:

$$\frac{t_u}{T} \approx 0.7\,\frac{1}{\sqrt{\xi}} \quad \ldots\ldots\ldots\ldots \quad (19)$$

5.6. Synthesis to Hunt's Formula

From the expressions for the average front velocity and the run-up time, which have been found independently, Hunt's formula can be reconstructed. From Eqs. 17 and 19 one obtains $\bar{c}t_u/T\sqrt{gH} \approx 0.42$, which can be written as:

$$l_u \approx 1.05\sqrt{HL_o} \quad (20)$$

According to Hunt's formula l_u equals $\sqrt{HL_o}/\cos\alpha$, which varies from $1.01\sqrt{HL_o}$ to $1.05\sqrt{HL_o}$ for the three slopes 1;3, 1:5 and 1:7. This agrees well with Eq. 20.

6. WAVE OVERTOPPING

It was deemed worthwhile to investigate to which extent the run-up data obtained could be used in relation to wave overtopping, although no such measurements have been performed in the present study. To this end the following hypothesis is introduced: The maximum quantity of water stored above a certain location on the uninterrupted slope (B_{max}) as measured in the run-up experiments, is equated to the volume of overtopping per wave (V) which would occur if the crest of the dike would be situated at that location (see fig. 13). B_{max} and V are taken per unit width.

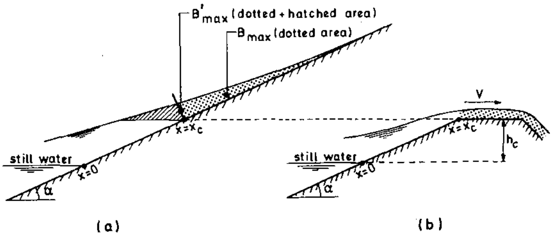

Fig. 13. Definition sketch for the hypothesis on wave overtopping.

It is expected that according to this hypothesis the actual quantity of overtopping is underestimated due to the fact that in the case of overtopping a certain amount of water passes the crest. This implies that the up-rush of the next wave is slowed down less than it would be in the absence of overtopping.

Values for B_{max} are available from the elaboration of the experimental results (see fig. 7a). The variable B_{max} is scaled with HL_o, because the maximum length of the water tongue is about equal to $\sqrt{HL_o}$ and the layer thicknesses are proportional to $\sqrt{HL_o}$. The dimensionless variable B_{max}/HL_o depends on the location on the slope, the wave steepness and the slope angle, expressed by the following parameter equation:

$$\frac{B_{max}}{HL_o} = F\left(\frac{x}{\sqrt{HL_o}}, \frac{H}{L_o}, \alpha\right) \quad (21)$$

From the analysis of the B_{max} - data it appeared that B_{max}/HL_o is independent of H/L_o and proportional to $\sqrt{\tan\alpha}$. The slope influence is ascribed to phase differences between h(t) at various points along the slope, which increase with decreasing α; on the 1:3 slope the maximum values of h(t) at the four gauge locations were attained practically simultaneously. So the normalized volume $B_{max}\sqrt{\cot\alpha}/HL_o$ is independent of H/L_o and $\tan\alpha$ and depends on the location $x/\sqrt{HL_o}$ only. This is shown in fig. 14. A smooth curve has been drawn through the points.

For the case of a dike crest at a finite elevation, a parameter x_c is defined as the x - coordinate of the upper termination of the slope (see fig. 13b). The ratio $x_c/\sqrt{HL_o}$ is about equal to the ratio of crest height h_c (see fig. 13b) and run-up height according to Hunt's formula $\sqrt{HL_o}\tan\alpha$ (apart from a $\cos\alpha$-term).

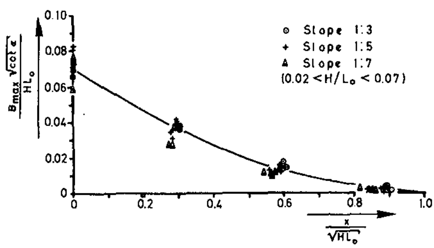

Fig. 14. Normalized maximum volume of water stored on the slope versus $x/\sqrt{HL_o}$

Defining for brevity:

$$R_H \equiv \sqrt{HL_o}\ \tan\alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (22)$$

then $x_c/\sqrt{HL_o} \approx h_c/R_H$.

The curve of fig. 14 has been plotted in fig. 15 on double logarithmic paper (drawn line). Along the horizontal axis $(1-h_c/R_H)$ is given, which represents the relative excess of run-up height (R_H) over crest height (h_c).

An alternative definition of a maximum volume stored above a location $x = x_c$ on the slope is given in fig. 13a as B'_{max}. The behaviour of B'_{max} is similar to that of B_{max}. The smooth curve through the data points of B'_{max} has been plotted in fig. 15 as the dashed line.

To check the hypothesis stated above, overtopping data given by Saville (5) have been used. They were obtained with waves breaking on smooth plane slopes of 1:3 and 1:6; the wave steepness H/L_o varied from 0.026 to 0.066. The data given by Saville have been converted to $V\sqrt{\cot\alpha}/HL_o$ -values and plotted versus $(1-h_c/R_H)$ in fig. 15. The scatter is a known fact in results on wave overtopping and can at least partly be ascribed to the inaccuracy in the measurements, particularly for the relatively small rates of overtopping.

The drawn line in fig. 15, representing $B_{max}\sqrt{\cot\alpha}/HL_o$, is a reasonable lower limit for the overtopping results for the relatively large values of $(1-h_c/R_H)$ (low crest elevations). The dashed line for $B'_{max}\sqrt{\cot\alpha}/HL_o$ seems to be more nearly a mean value.

Quite apart from the fit of the curves to the data points in fig. 15, it can be seen that the normalization which has been adopted, does serve to bring the data for the 1:3 and 1:6 slopes in a common range for all the wave steepnesses which were used. This means that the rate of overtopping of waves breaking on smooth plane slopes, normalized as $V\sqrt{\cot\alpha}/HL_o$, appears to be a function of the relative crest height h_c/R_H only.

7. DISCUSSION

The variables have been normalized with scaling factors which were chosen on the basis of Hunt's formula. The expectation was that the

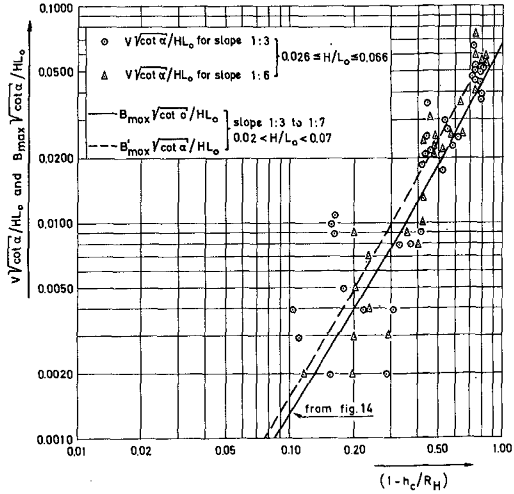

Fig. 15. Normalized maximum volume of water stored on the slope versus $(1-h_c/R_H)$ with plotted data from overtopping measurements.

dimensionless variables so obtained should be independent of the wave steepness H/L_0 and the slope angle α, or nearly so. This turned out to be the case for $h_{max}/\sqrt{HL_0}$, but not for $v_{max}/\sqrt{gH}$, $v_{min}/\sqrt{gH}$, $\bar{c}/\sqrt{gH}$ and t_u/T, all of which were found to vary with ξ. It therefore appears that the detailed relationships of the flow in the run-up of breaking waves are somewhat more complex than previously had been inferred from Hunt's formula (1).

The importance of the parameter ξ has been noted before for other characteristics of waves breaking on slopes, such as Iribarren and Nogales' breaking criterion, the breaker type, the reflection coefficient, and the relative importance of (periodic) run-up and (steady) set-up, and to a smaller extent also for the height-depth ratio at breaking. Furthermore, the normalized run-up height itself, if normalized with the incident wave height H, is a function of ξ only, as long as the waves break. This can be seen by writing Hunt's equation in the form $R/H = \xi$. A more detailed discussion of the role of ξ as a similarity parameter for the waves breaking on slopes has been given elsewhere (2).

REFERENCES

1. Battjes, J.A., "Run-up Distributions of Waves Breaking on Slopes", Proc. A.S.C.E., vol. 97, No. WW1, February 1971.
2. Battjes, J.A., "Surf Similarity", Proc. 14th Conf. on Coastal Eng., Copenhagen, 1974.
3. Battjes, J.A., and Roos, A., "Characteristics of Flow in Run-up of Periodic Waves", Report No. 75-3, Communications on Hydraulics, Department of Civil Engineering, Delft University of Technology, 1975.
4. Hunt, I.A., "Design of Seawalls and Breakwaters", Proc. A.S.C.E., vol. 85, No. WW3, September 1959.

5. Saville, T.Jr., "Laboratory Data on Wave Run-Up and Overtopping on Shore Structures", Beach Erosion Board, Techn. Memo 64, 1955.
6. Saville, T.Jr., "Wave Run-Up on Shore Structures", Proc. A.S.C.E., vol. 83, No. WW2, April 1956.

LIST OF SYMBOLS

B	= volume of water stored above a certain location on the slope (per unit width);
B_{max}	= maximum volume of water stored above a certain location on the slope (per unit width);
B'_{max}	= maximum volume of water stored above a certain level on the slope (per unit width);
c	= velocity of the run-up wave front;
$\bar{c}$	= average velocity of the run-up wave front = l_u/t_u;
d	= constant waterdepth in the flume;
g	= acceleration due to gravity;
H	= height of the incident waves;
h	= layer thickness of the water tongue on the slope;
h_{max}	= maximum layer thickness at a fixed point;
h_c	= crest height of dike above still water level;
L_o	= wave length in deep water = $gT^2/2\pi$;
l_d	= run-down length, average distance from the still water line to the highest point on the slope which appeared to be submerged at all times;
l_u	= run-up length, average distance from the still water line to the point of maximum run-up per wave;
V	= volume of overtopping water per wave (per unit width);
R	= run-up height, or greatest height above still water level reached by an individual wave on the slope;
R_H	= run-up height according to Hunt's formula $=\sqrt{HL_o}\tan\alpha$;
T	= wave period;
t	= time;
t_u	= run-up time, time used by the wave front to run up from $x = 0$ to $x = l_u$;
v	= particle velocity in the water tongue on the slope, averaged over h;
v_{max}	= extreme upward particle velocity at a fixed point;
v_{min}	= extreme downward particle velocity at a fixed point;
x	= coordinate along the slope, positive upwards; origin at still water line;
x_c	= x-coordinate of the crest;
α	= slope angle with respect to the horizontal;
μ	= dynamic viscosity of water;
ξ	= similarity parameter = $\tan\alpha/\sqrt{H/L_o}$;
ρ	= mass density of water.