CHAPTER 96

HYDRAULICS OF GREAT LAKES INLET - HARBORS SYSTEMS

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and

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Abstract

Revereing currents in inlets on the Great Lakes are generated primarily by long wave seiching modes of the lakes rather than by the tide. In order to investigate the nature of long wave excitation and the generating mechaniem for significant inlet velocitiee, to establish techniques for predicting inlet-harbor system reaponae, and to develop base data for future planning and design studies, field measurements were conducted in 1974-75 at several harbora on the Great Lakes. Data collected includes continuous harbor water level measurements at all sites, inlet velocity measurements at the primary site (Pentwater, Michigan), and channel hydrographic surveys at the eites where more recent data were needed. Historic water level and velocity data for some of the harbor sites was also available.

Amplification of harbor oscillations and generation of the highest inlet velocities are caused by the Helmholtz resonance mode which has a period of 1 to 3 hours for the inlet-harbor systems studied. A recently developed simple lumped-parameter numerical model is shown to be quite effective in predicting inlet-harbor response over the range of excitation periods encountered. Selected data from Pentwater are presented to demonstrate the hydraulic responee of the inlet harbor system and the applicability of the lumped-parameter numerical model.

Introduction

Situated along the coastlines of the U.S. - Canadian Great Lakes are several inlet-harbor systems that consist of lakes or other embayments which are connected to the adjacent Great Lake by one or more artifical jettied inlets. There are also a few inlet-harbor systems that heve natural uncontrolled inlets, but most inlets are jettied and periodically dredged to maintain adequate navigation channels.

Higher velocity reversing currents at these inlets are generated in response to etorm-generated seiching of the individual Great Lakes. Of particular importance to the generation of higher velocitiea is the resonant amplification of Great Lakes seiches by the inlet-harbor systeme. Although high

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velocities can be generated by excitation of resonant conditions, a cumulative frequency distribution of channel velocities at most Great Lakes inlets will show that velocities in excess of 1 ft/sec typically occur less than 5 percent of the time.

A field data collection program was conducted at nine inlets on the U.S. coast of the Great Lakes. Also, historic field data were available for some of these nine inlets as well as for three additional inlet sites. Historic and project field data includes continuous measurements of Great Lake and harbor water levels and inlet velocities over periods of a few weeks to nearly a whole ice free season, and hydrographic surveys at inlet channels where insufficient hydrographic data were available. The primary site for field data collection was at Pentwater, Michigan.

The three objectives of this project were: (1) to define the hydraulic mechanisms that generate the dominant inlet currents and related harbor oscillations, (2) to establish analytical techniques for prediction of inlet-harbor response to Great Lakes oscillations, and (3) to develop background data that will demonstrate the hydraulic behavior of these inlet-harbor systems as well as the validity of analytical prediction techniques, and that will provide base data for guidance in future project design studies.

This paper will summarize the hydraulic behavior of Great Lakes inlet-harbor systems and the techniques used herein to predict the hydraulics of these systems; it will outline the field data collection program; and it will present results from the data collection program at the primary study site in order to demonstrate details of the hydraulic response of these inlet-harbor systems as well as the ability of analytical techniques to predict this response.

Inlet-Harbor System Hydraulics

Figure 1 schematically depicts a prismatic inlet channel that connects a large body of water such as a sea or one of the Great Lakes to a much smaller bay, lake or harbor (herein referred to as the harbor). The inlet channel has a length, L, width, B, average depth, d, cross-sectional area, A_c, and time dependent instantaneous horizontal velocity, V. A_h is the surface area while η_s and η_h are the time dependent instantaneous sea and harbor surface elevations. The harbor surface is assumed to remain horizontal as it rises and falls in response to excitation from the sea. This assumption requires that the harbor surface response period be long compared to the time required for a shallow water wave to propagate from the inlet to the farthest point in the harbor.

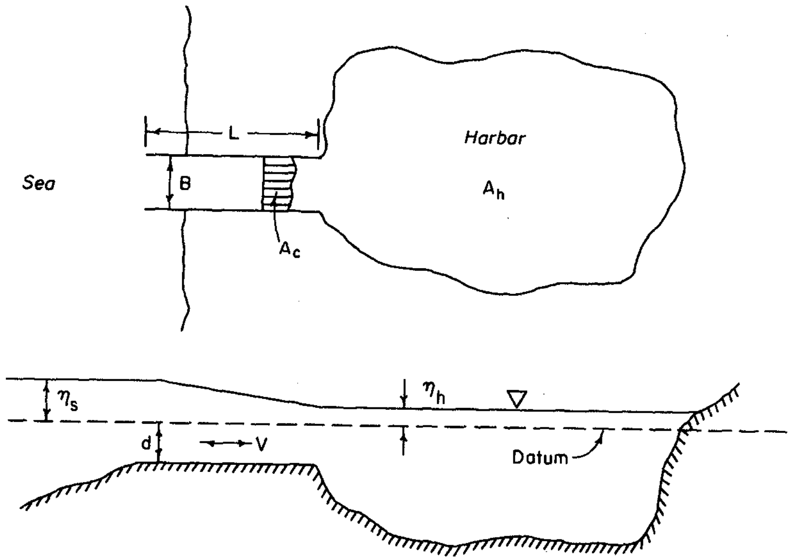

Fig. 1 Inlet - harbor system

For purposes of discussion it is helpful to write a simple inlet-harbor continuity equation and the one-dimensional equation of motion for flow in the inlet channel. The continuity equation

$$Q = VA_c = A_h \frac{\partial \eta_h}{\partial t} , \qquad (1)$$

where Q is the instantaneous channel discharge, equates the volumetric flow through the inlet to the harbor water surface rise or fall needed to balance that flow. An accurate record of the harbor surface elevation time-history can thus be used to calculate the instantaneous channel discharge and velocity using Eq. 1 (provided that the harbor surface remains horizontal as it oscillates).

The one-dimensional equation of motion along the inlet channel axis can be written

$$- g \frac{\partial \eta}{\partial x} = f \frac{V|V|}{8R} + V \frac{\partial V}{\partial x} + \frac{\partial V}{\partial t} \tag{2}$$

where R is the channel hydraulic radius, x the distance along the channel axis from some reference point, f the channel friction factor, and g the acceleration of gravity. Eq. 2 equates the horizontal driving force due to the water surface slope with the three terms on the right which are the channel frictional resistance, the convective acceleration owing to velocity variation along the channel axis, and the temporal acceleration (or inertia) owing to velocity variation at a point with time. In nearly prismatic channels the convective acceleration is often negligible.

At U.S. tidal inlets, because of the magnitude of common tidal periods and amplitudes, friction strongly dominates the effects of inertia. The result is that at most tidal inlets inertia can be neglected in hydraulic calculations, the harbor tidal range is less than the range at sea, and the phase lag between the sea and harbor tides is much less than 90°.

On the Great Lakes, the components of the long wave energy spectrum that excite the dominant inlet-harbor system response modes typically have amplitudes in the order of 0.1 to 0.2 ft and periods of less than 3 hours. This causes the inertia term to be larger than the friction term throughout most of the cycle of oscillation. (Friction will dominate only around times of peak ebb and flood velocity, which are also the times of minimum temporal acceleration.) As a result, the harbor response is amplified and its phase lag can exceed 90°.

The inlet-harbor system response is analagous to the response of a slightly damped spring-mass system or its acoustic counterpart, the Helmholtz resonator (see Kinsler and Frey, 1950). The motion of the mass of water in the inlet channel corresponds to the motion of the mass of the spring-mass system, and the action of gravity on the rising and falling harbor water surface corresponds to the restraining force of the spring. Details of the response characteristics of this mode of oscillation (usually called the Helmoltz mode) are demonstrated by Figure 2 which depicts the classical behavior of a single-degree of freedom oscillating system.

In Figure 2, the phase lag between the sea and harbor surface elevations as well as the amplification of the harbor surface response are plotted as a function of excitation period divided into the frictionless resonance period, T_H. The series of curves represents different degrees of frictional damping of the system. Note that increased friction tends to shift the resonant or Helmholtz period to slightly higher values. For excitation at periods much longer than the Helmholtz period (A) the harbor amplitude equals the sea amplitude and the phase lag is small (pumping condition). With decreasing excitation periods and significant friction (B) the ratio of

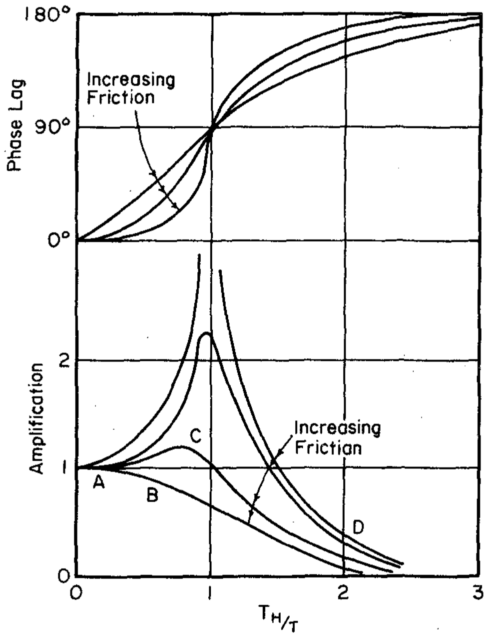

Fig. 2 Amplification, phase lag for inlet - harbor system

harbor to sea amplitude is less than unity and the phase lag less than 90^{o}. This is the typical response of most tidal inlets. As the Helmholtz period is approached, particularly with lesser frictional damping (C), the harbor response is amplified and the lag increases toward (or above) 90^{o}. A significant decrease in the harbor response occurs for all levels of frictional dissipation when the excitation period is much less than the Helmholtz period (D). At a harbor on the Great Lakes, excitation will occur at a number of periods in the long wave energy spectrum (typically $\gtrless T_H$) and cause a range of responses for these different periods as indicated in Figure 2.

The Helmholtz period is longer than the fundamental free seiching period of the harbor. A simple equation for the Helmholtz period can be derived (Carrier, Shaw and Miyata, 1971) by neglecting the friction and convective acceleration terms in Eq. 2 and solving this equation and continuity for the conditions depicted by Figure 1. This yields

$$T_H = 2\Pi \sqrt{\frac{L\ A_h}{g\ A_c}} \tag{3}$$

Eq. 3 ignores the water masss that is just outside each end of the inlet channel but is part of the mass of water set in motion at resonance. Miles (1948), using an acoustic analogy, developed an equation for the added channel length, L', necessary to account for this additional water mass where

$$L' = \frac{-B}{\Pi} \ln \left[\frac{\Pi\ B}{\sqrt{gd}\ \ T_H} \right] \tag{4}$$

Thus, an improved form of Eq. 3 can be written as

$$T_H = 2\Pi \sqrt{\frac{(L + L')\ \ A_h}{g\ A_c}} \tag{5}$$

The inclusion of end effects is particularly important for short channels (i.e. $L \lesssim L'$). For the inlet-harbor systems encountered in this study Eq. 5 proved to be reasonably effective for determining the Helmholtz period. A numerical method for determining the Helmholtz period for a harbor with more than one entrance channel is given by Freeman, Hamblin and Murty (1974).

Note from Eq. 5 that the Helmholtz period increases as the channel length or the bay surface area increases and as the channel cross-sectional area decreases. The Helmholtz mode of oscillation is independent of the harbor depth which is not the case for the free seiching modes of harbor resonance.

A recently developed simple lumped-parameter numerical model for inlet hydraulic calculations has been used at CERC to predict inlet-harbor response to Great Lakes long wave motion. Details of the development and application of this model are given by Seelig et al. (1977).

In the lumped-parameter model the oscillating harbor water surface is assumed to remain horizontal and continuity of flow is defined by Eq. 1

written in finite difference form. The equation of motion is integrated along the channel axis to yield

$$\frac{\partial Q}{\partial t} = \frac{FQ^2}{2}\left(\frac{1}{A_{cs}^{\;2}} - \frac{1}{A_{ch}^{\;2}}\right) + gF(\eta_s - \eta_h) - \frac{Fg}{2}\sum_{i=1}^{S}\frac{1}{\sum\limits_{j=1}^{c} A_{ij}}\sum_{j=1}^{c}\frac{n_{ij}\,|W_{ij}Q|\,W_{ij}\,QB_{ij}L_{ij}}{(d_{ij})^{1/3}\,A_{ij}^{\;2}} \qquad (6)$$

The inlet channel is divided into C subchannels and S sections along its length by construction of a flow net to yield SXC grid sections. Each grid section is assigned a Manning's n (n_{ij}), depth, d_{ij}, width, B_{ij}, length, L_{ij}, area normal to flow, A_{ij}, and flow weighting factor, W_{ij}. The flow weighting factor determines what fraction of the total flow, Q, passes through each grid. It may be selected (1) to distribute flow equally in all subchannels (2) to distribute flow across subchannels so friction is mimimized at each section or (3) to distribute flow in each subchannel (no cross over between subchannels) so friction is minimized in each subchannel. F is a channel geometry factor that develops from integration of the equation of motion and A_{cs} and A_{ch} are the inlet channel cross-sectional areas at the sea and harbor ends respectively.

Given the harbor surface areas, inlet hydrography, and $\eta s = f(t)$, the channel flow net is drawn, n_{ij} and W_{ij} distributions are established and the model is solved in time steps by a fourth order Runge-Kutta-Gill technique to yield η_h at t, t+1,... and Q (or V distribution) at t + 1/2, t + 3/2,... If (η_s, η_h and V) = f(t) data are available for an inlet-harbor system the model can be calibrated by adjusting n_{ij}.

The lumped parameter model is particulerly appropriate for Great Lakes inlet-harbor system hydraulic calculations because of the nature of the harbor water level response during Helmholtz resonance and because the model allows input of irregular sea level time-histories common to Great Lakes long wave spectra. Also, because it has the capacity to handle harbors with more than one inlet channel as are found at some locations on the Great Lakes.

Field Data Sites and Collection Program

A field data collection program was conducted during October and November 1974 and July through November 1975. The harbor sites where data were collected are shown in Figure 3 along with the sites where usable historic data are available.

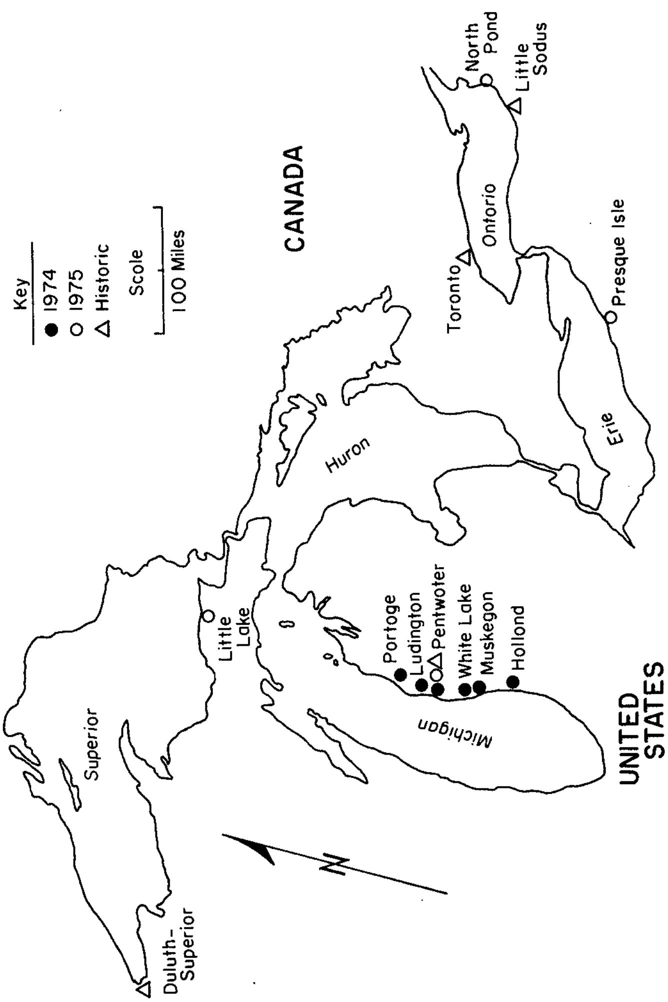

Fig. 3 Field data collection sites

The field program included harbor water level and inlet channel velocity measurements as well as inlet channel hydrographic surveys. The water level measurements were taken at 2 or 5 minute intervals on a continuous basis at the harbors indicated in Figure 3. Continuous current velocity measurements were made in the Pentwater inlet channel at a point middepth, midlength and 7 ft from the north jetty.

Historic lake and harbor water level and channel velocity data at Pentwater are available (Duane and Saylor, 1967) for July and August 1967. Figure 4, a map of Pentwater Lake, shows the locations where water level and

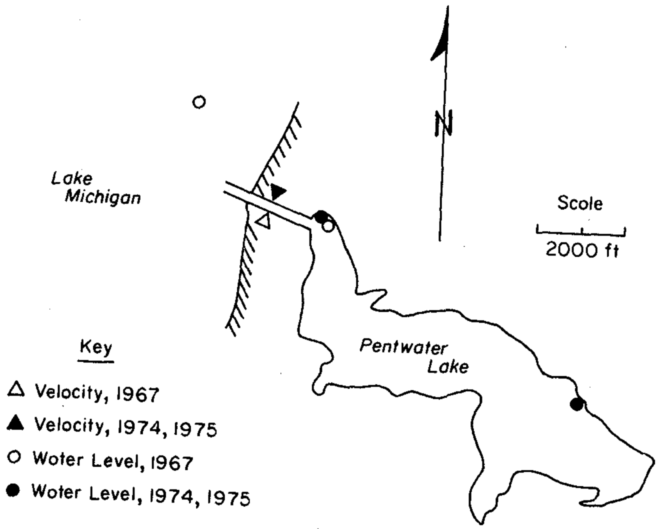

Fig. 4 Pentwater Lake, Michigan

current data were measured. Pentwater was chosen as the primary site for this study (and paper) because of its uncomplicated inlet channel geometry, the availability of the 1967 data, and the existence of concurrent field studies at Pentwater by other CERC personnel.

Pentwater Lake has a surface area of 1.81 x 10^7 ft^2 and is connected to Lake Michigan by a jettied inlet channel 2000 ft long and 145 feet wide. The average inlet channel depth at the time of the field data collection was 12.8 ft while Pentwater Lake has an average depth of 30 ft. A report by Seelig and Sorensen (1977) gives additional information on the field data collection sites, measurements made, instrumentation used, and results obtained.

Table 1 lists the Helmholtz period for each of the six field study sites located on the eastern shore of Lake Michigan. The Helmholtz period was calculated by Eq. 5 so the true period will be somewhat higher than listed owing to frictional resistance in the channel. Also listed in Table 1 are the fundamental seiching periods, T_1, in each harbor calculated by the classical Merian equation.

Table 1 - Helmholtz and fundamental seiching periods

Location	Helmholtz period (hrs)	Fundamental seiching period (hrs)
Portage	2.17	0.16
Ludington	1.22	0.32
Pentwater	1.39	0.13
White Lake	2.55	0.37
Muskegon	3.80	0.27
Holland	2.23	0.63

At most locations on the Great Lakes the amplitude of the astronomical tide is less than 0.3 ft. The amplitudes of the various components of the longwave spectrum on the Great Lakes are of the same order of magnitude but their periods (particularly for higher harmonics) are significantly lower and closer to the resonant period than are the tidal periods. Thus, Great Lakes seiching is more likely to generate noticeable inlet velocities.

Table 2 lists the periods of the longitudinal free seiching modes in Lake Michigan (information is from a variety of field data and numerical model analyses). Owing to the geometry of Lake Michigan, the transverse seiching modes are hard to generate. Note that the 6th through 9th longitudinal modes have periods around the Helmholtz period for Pentwater. It appears (see Mortimer, 1965) that the 7th and 9th modes have nodes near Pentwater while the 6th and 8th modes have antinodes and thus should cause greater hydraulic activity at Pentwater.

Table 2 - Longitudinal free seiching periods, Lake Michigan

Mode:	1	2	3	4	5	6	7	8	9
Period (hrs):	9.0	5.2	3.7	3.1	2.5	1.85	1.58	1.44	1.25

Field Data Measurement and Analysis

Water level data were measured by stilling well-float type gages and recorded digitally (at 5-minute intervals) on punch tape with a Fisher-Porter Model 15-42 level recorder. Vertical resolution of the water level records was to the nearest 0.01 ft. Extreme care must be taken in the design of a stilling well-recorder combination when digital water level measurements are made because the signal of interest is of much lower amplitude than that of the higher frequency "noise" owing to wind waves. Special linear damping stilling wells (Seelig, 1976) with a design based on the work of Noye (1974) were used in this study. The punch tape water level records were machine converted to standard computer punch cards for spectral and other analyses.

Inlet current speed and direction data at Pentwater were measured with a Bendix Q-9 current meter and recorded on strip chart. The velocity records were digitized for analysis at 5-minute intervals timed to coincide with the nearby water level data. Owing to the uniform geometry of the Pentwater inlet channel, measured current velocities should give a good indication of the average channel velocities.

Spectral analyses of water level and velocity data were conducted using the Cooley and Tukey Fast Fourier Transform algorithm with a cosine bell window (Harris, 1974). As a compromise, a record length of 1.78 days (512 data points) was used to maintain the assumption of a weakly stationary system and still give good resolution for the range of periods of interest.

Field Data Results

Selected data from Pentwater are presented in this section to demonstrate the response of the inlet-harbor system to Lake Michigan water level oscillations. Figure 5 shows records of storm generated water levels in Lake Michigan at Pentwater along with the resulting Pentwater Lake levels and channel velocities. These data are typical of storm conditions at Pentwater. High wind waves at Pentwater will often occur when the wind is from the west but during this time inlet velocities will remain low. When the wind shifts to parallel the axis of Lake Michigan and generate seiching action in the Lake, harbor oscillations and inlet velocities increase.

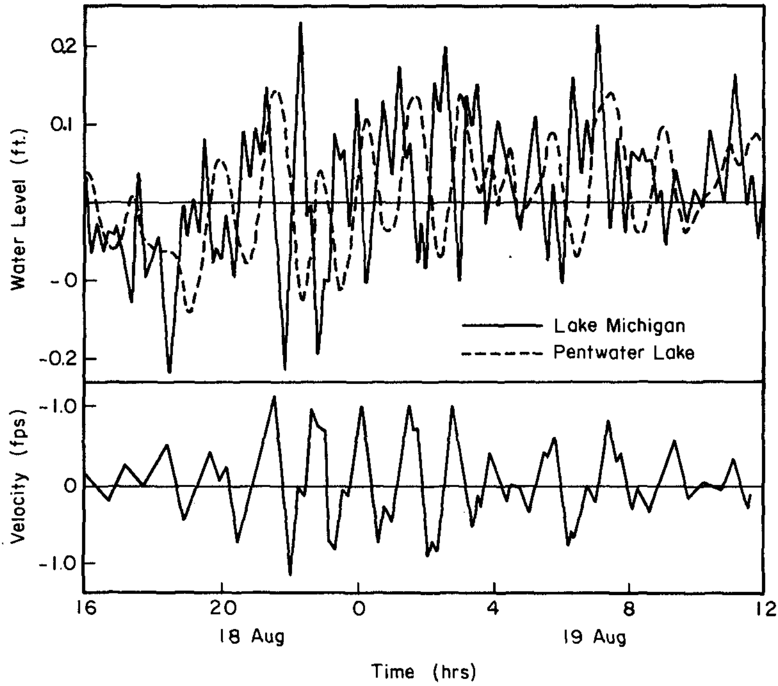

Fig. 5 Lake Michigan water level and resulting Pentwater channel velocity and harbor water level

As demonstrated by Figure 2, higher frequency oscillations are strictly damped by the inlet-harbor system so the harbor water level record is smoother than the Lake Michigan record. The harbor water level oscillation in Figure 5 has a predominant period of 1.5 to 2 hours and is approximately 180° out of phase with the oscillation of Lake Michigan. Harbor response like that shown in Figure 5 typically lasts for a period of 2 to 3 days.

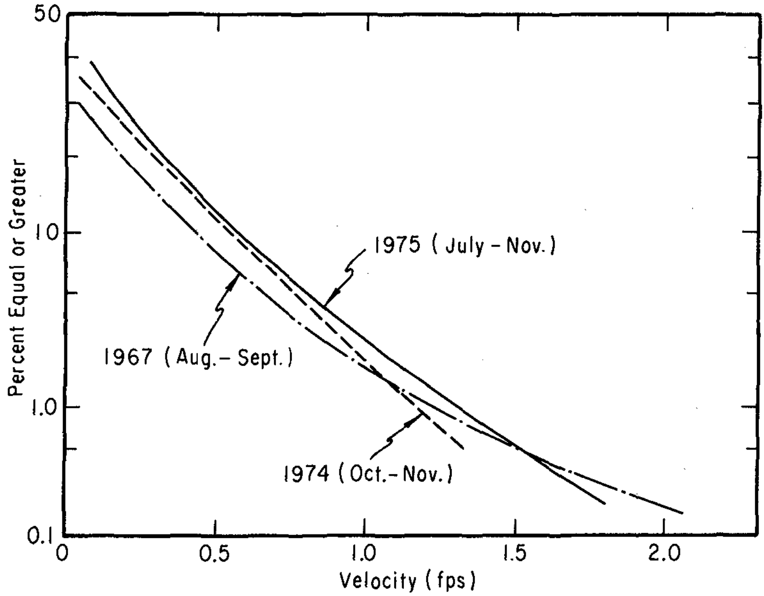

Fig. 6 Velocity cumulative frequency distributions at Pentwater inlet

The cumulative frequency distributions of velocities measured at Pentwater in 1967, 1974 and 1975 are plotted in Figure 6. Although the data were collected during the summer and fall they should be somewhat representative of a typical year at Pentwater. Because of the low frequency of occurrence of significant velocities, inlet currents should only rarely cause concern for navigation. Also, velocities high enough to flush sediment from the inlet (say $V > 1$ fps) occur only about 1 percent of the time so continuous inlet maintenance would likely be required.

Inlet velocity cumulative frequency distributions at all of the other inlets studied (except Duluth-Superior) were approximately the same or lower than the distribution at Pentwater. These distributions were constructed from velocity measurements, when available, or from velocities calculated (by Eq. 1) from harbor water level records. A comparison of measured and calculated velocity cumulative frequency distribution from Pentwater showed the distributions to be quite compatible.

The Duluth-Superior harbor is located at the end of Lake Superior (see Figure 3) where the shorelines converge and seiche antinodes occur. Oscillation with amplitudes greater than 0.5 ft and periods close to the resonant period have been measured at Duluth-Superior. Inlet velocities up to 7 fps were generated.

Lake Michigan and Pentwater Lake water level spectra for a period bracketing that in Figure 5 are shown in Figure 7. Spectral peaks in the Lake Michigan record occur at 5.3, 1.8, 1.44, 1.0 and 0.85 hours - the second, sixth and eight longitudinal seiching modes plus two undetermined higher frequency modes. As expected, the 5.3 hour mode is only very slightly amplified, the 1.8 and 1.44 hour modes which are near the Helmholtz period are strongly amplified, and the 1.0 and 0.85 hour modes are strongly damped.

Water level spectra for several of the storms that occurred during the periods of record at Pentwater showed responses similar to Figure 7. Quite often the 1.44 hour period was dominant although occasionally other periods such as the 5.3 hour mode would dominate. Of course, the 5.3 hour mode would not be significantly amplified and lower velocities could be expected because of the lower amplitude of harbor oscillation and the longer oscillation period.

Application of Lumped Parameter Numerical Model

The lumped parameter model was calibrated for Pentwater (with $C = 1$, $S = 5$, $n_{ij} = n$, $W_{ij} = 1$) by adjusting the friction term (Manning's n) until there was agreement with the field data. This was accomplished by comparing the amplification predicted by the model for incident sinusoidal waves

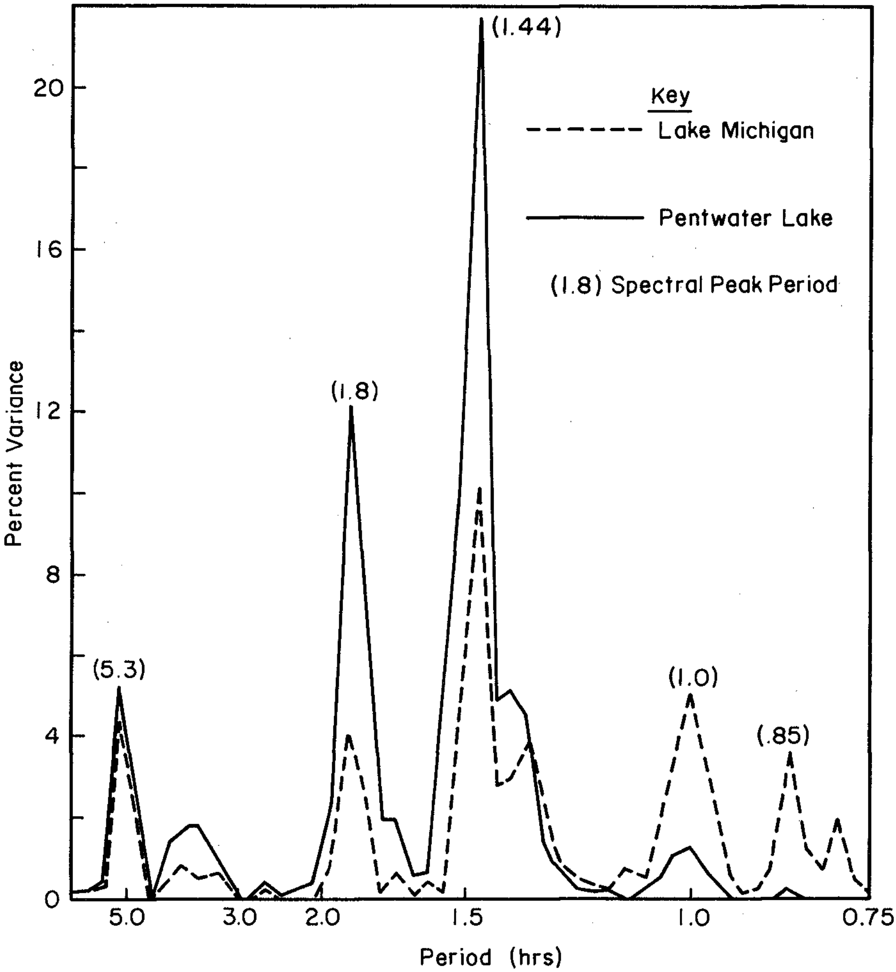

Fig. 7 Lake Michigan and Pentwater Lake spectra, 18 - 20 August 1967

having a 0.1 ft amplitude and a range of periods, to the amplification observed in the field. The calculated amplification for the calibrated condition (solid line) and the amplification determined from the field data (circles) are plotted in Figure 8. The numerical model usually had to be run for two or three cycles for the harbor response to build to equilibrium. In the prototype harbor it is likely that equilibrium (full amplification) is never fully achieved. Thus, the calibration curve in Figure 8 forms the upper envelope of the field data.

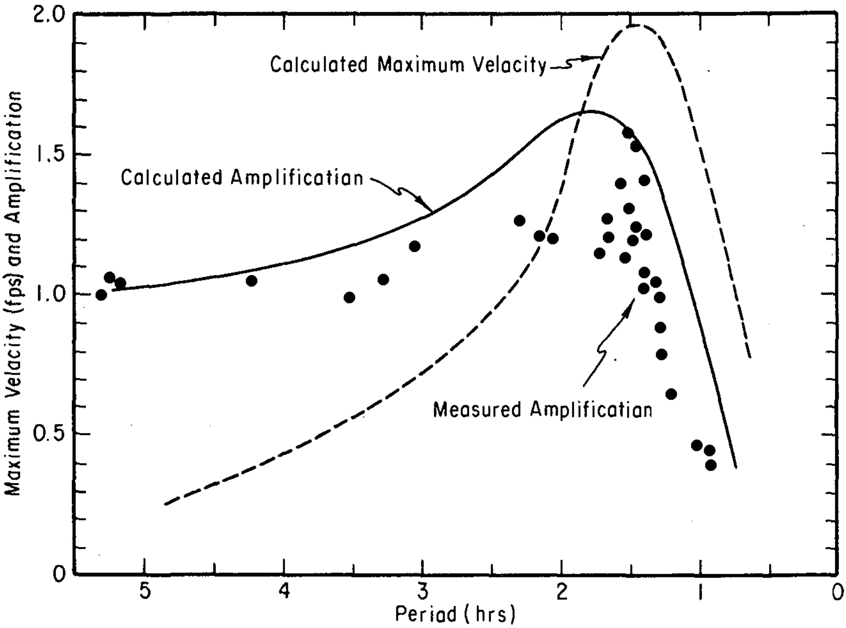

Fig. 8 Response to long wave excitation at Pentwater (wave amplitude = 0.1 ft)

A Manning's n (see Eq. 6) of 0.036 was required to calibrate the model. This value is probably somewhat higher than the true prototype value because Eq. 6 is applied from the sea to the harbor and inlet channel entrance and exit losses thus become incorporated in the friction term.

The peak resonant period in Figure 8 ia 1.8 hours as compared to a period of 1.39 hours predicted by Eq. 5; the difference bsing due to the effects of friction. Also plotted on Figure 8 are ths calculated maximum channel velocities for the renge of excitetion periods and a Lake Michigan excitation amplitude of 0.1 ft. Beceuse inlet velocity is dependent on both the period and amplitude of the herbor oscilletions, the peak velocity occurs at the lower period of 1.4 hrs (which is coincidentelly close to the resonant period predicted by Eq. 5).

The harbor level and inlet velocity time-historiss computed for an incident sinusoidal wave of 0.1 ft amplitude and 1.5 hour period are plotted in Figure 9. As indicated by Eq. 1 and shown in Figure 9 the inlet velocity is maximum when the gradient of the harbor water level time-history curve is maximum and the velocity is zero at the instent of high and low slack water in the harbor. The phase lag bstween the sea and harbor water levels is about 0.35 hours or 84^{o} which conforms to Figure 2.

Also plotted in Figure 9 are the magnitudes of the channel friction, head differential and temporal acceleration terms of the equation of motion, normalized by dividing by the highest instantaneous velue among the three. Note that the temporel acceleration or inertia term exceeds the friction term over more than half of the cycle.

Figure 10 shows a selected water level record from Lake Michigan along with the meesured and calculated (n = 0.036) water level response in Pentwater Lake. After three to four hours of rscord the numerical model stabilized and quite accuretely predicted the remaining portion of the measured harbor water level record. This further confirms the model calibration based on Figure 8. The high frequency oscillations in the measured harbor water level record wsre not predicted by the numerical model. Perhaps these oscillations are due to harbor seiching (T_1 = 0.13 hrs) which is not accounted for in the model.

Conclusions

Field and historic hydraulic data for several Great Lakes inlet-harbor systems were collected and analyzed. The primary conclusions from this effort are:

Reversing inlet currents are generated in response to various modes of low amplitude (<0.4 ft) seiching of the Great Lakes. The strongest currents develop when Great Lake seiching excites ths Helmholtz resonance mode of the inlet-harbor system and whsn the seiching mode has an antinode in the vicinity of the system.

Other than a few percent of the time, at moat inlets on the Great Lakes current velocities are less than 0.5 fps.

The simple lumped-parameter numerical model used in this study is effective in predicting the hydreulic response of most inlet-harbor systems of the type found on the Great Lakes.

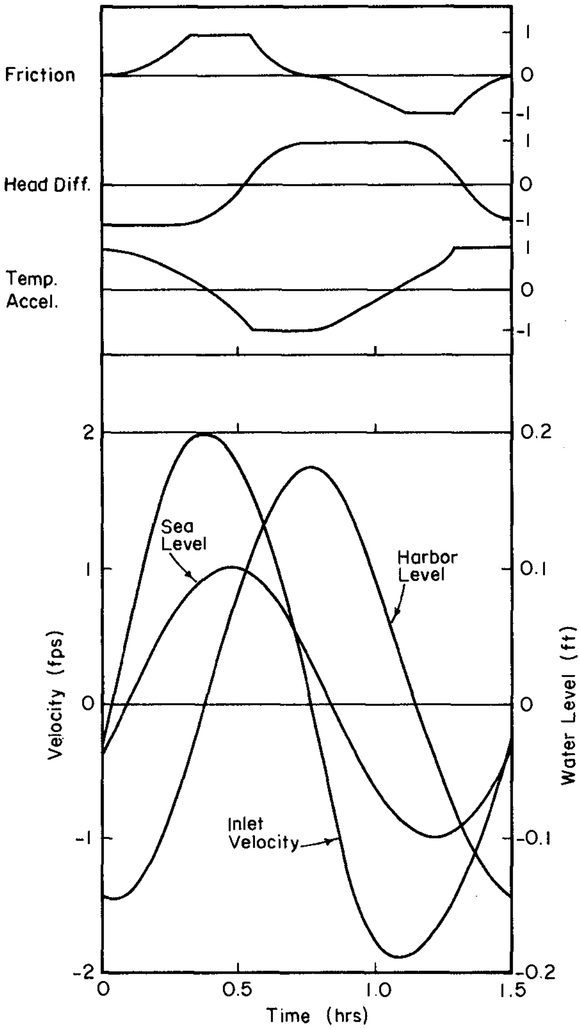

Fig. 9 Pentwater response to sinusoidal wave
(period = 1.5 hrs, amplitude = 0.1 ft)

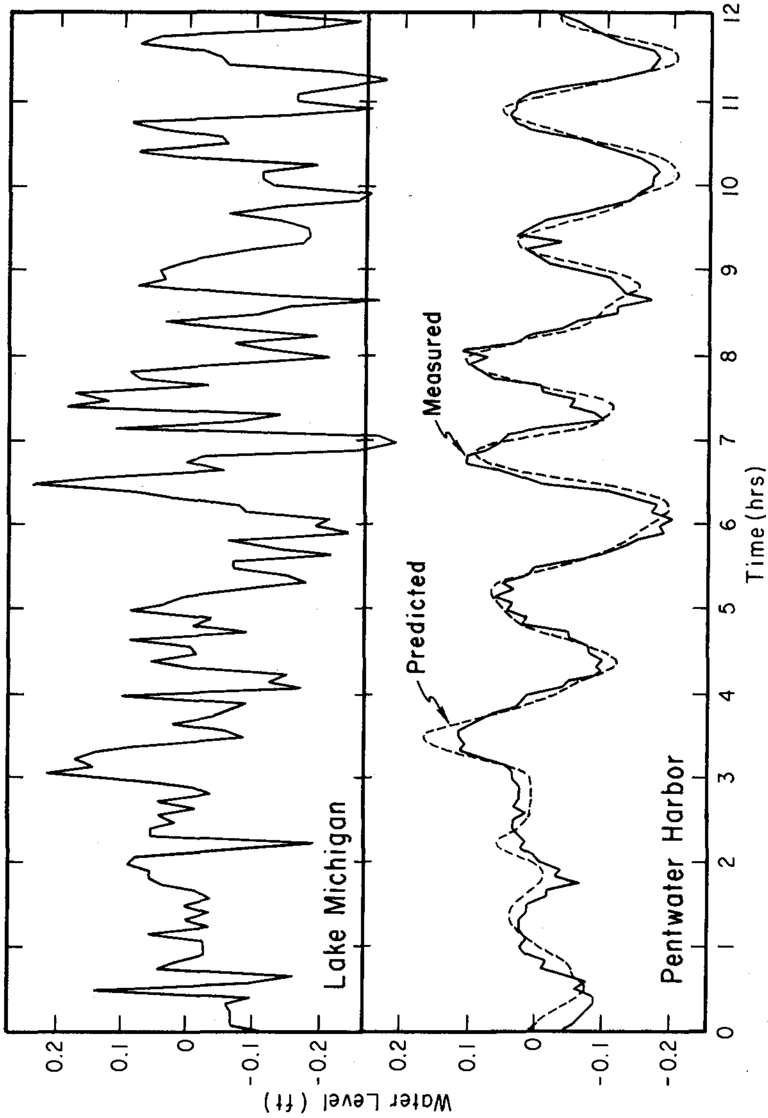

Fig. 10 Numerical model water level predictions at Pentwater

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