CHAPTER 157

WAVE OVERTOPPING EQUATION

by

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INTRODUCTION

In the early 1950's the Corps of Engineers' Jacksonville District initiated a series of laboratory tests to investigate the overtopping of proposed levee sections for Lake Okeechobee, Florida. For economic reasons, the alternative to build levees with crest elevations that were at times below the limit of wave runup was investigated and the quantities of water carried over the structures for various freeboard allowances, structure slopes and wave conditions determined. The initial tests were conducted at the Waterways Experiment Station (WES) in Vicksburg, Mississippi for the Jacksonville District at what was taken to be a 1 to 30 model scale. Model wave heights varied from 4.05 cm to 12.2 cm (0.133 to 0.40 ft). In order to expand the range of test conditions investigated, the Beach Erosion Board, currently the Coastal Engineering Research Center (CERC), commissioned an expanded series of tests that considered the overtopping of riprap faced, curved and stepped seawalls as well as the overtopping of "smooth" slopes. These tests, also conducted at WES, were considered to be at a 1 to 17 scale with model wave heights ranging from 5.36 cm to 21.5 cm (0.176 to 0.706 ft). A number of tests were subsequently conducted in CERC's large wave tank to determine the influence scale effects might have on overtopping. These tests are referred to as 1 to 2 1/2 scale tests. The model wave heights investigated ranged from 48.8 cm to 140.2 cm. (1.60 to 4.60 ft).

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Much of the overtopping data obtained during these tests has been presented by Saville (2)* and by Saville and Caldwell (1) and summarized in CERC's TR-4 (3); however, in this latter publication the data were presented in dimensional form making their general application difficult. In keeping with the decision to present information in CERC's Shore Protection Manual (4) in dimensionless form whenever practicable, the overtopping data was reanalysed and an empirical expression derived. The broad range of model scales used in the overtopping experiments also provide an opportunity to investigate the effect of model scale on test results. A summary of overtopping test conditions investigated is given on Table 1.

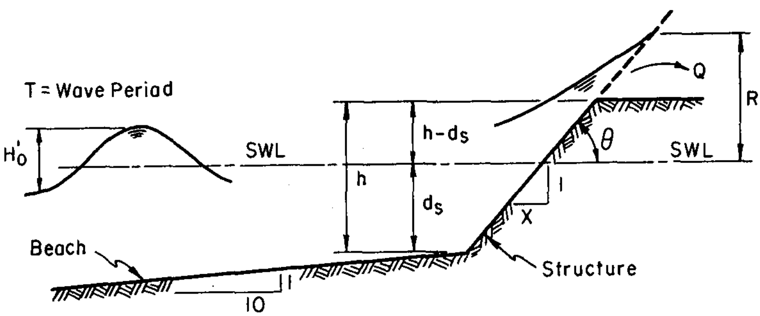

Figure 1. Definition of terms.

*Numbers in parentheses correspond to references listed in Appendix I.

TABLE 1 - SUMMARY OF OVERTOPPING TEST CONDITIONS

"Scale" of Test	Structures Investigated	Flume Dimensions	Type of Generator	Range of Wave Heights (Model Values)	Range of Wave Periods (Model Values)
1 to 30	1 on 3 smooth slope 1 on 6 smooth slope Composite slope Slope with berm	21.3 m long At generator 1.22 m wide 0.88 m deep At test section 0.3Q m wide 0.49 m deep	Flep type	4.05 to 12.2 cm	0.822 to 1.28 sec
1 to 17	Smooth vertical wall 1 on 1-1/2 smooth slope 1 on 3 smooth slope 1 on 1-1/2 stepped slope 1 on 1-1/2 riprap faced slope Curved well Recurved wall	36.6 m long 1.52 m wide 1.52 m deep	Plunger type	5.36 to 21.5 cm	0.717 to 3.64 sec
1 to 2-1/2	1 on 3 smooth slope 1 on 6 smooth slope	193.5 m long 4.57 m wide 6.10 m deep	Bulkhead type	48.8 to 140.2 cm	0.386 to 10.12 sec

TABLE 2

AGREEMENT BETWEEN MEASURED AND CALCULATED OVERTOPPING RATES
(Using SPM published values of α and Q_o^*, based on 1 to 17 scale date)

Structure Type	Number of Points	Correlation Coefficient
Smooth Face		
Vertical	56	0.980
1 on 1-1/2 slope	93	0.996
1 on 3 slope	83	0.992
Riprap Fece		
1 on 1-1/2	43	0.998
Stepped Face		
1 on 1-1/2	60	0.990
Galveston Curved Wall		
on 1 on 10 beach	33	0.995
on 1 on 25 beach	33	0.998
Recurved Wall		
on 1 on 10 beach	5	0.999

DIMENSIONAL ANALYSIS

The variables describing the overtopping of a given structure are depicted on figure 1. They include:

H_o'	= deepwater wave height	$[L]$
T	= wave period	$[T]$
g	= gravitational acceleration	$[L]\ [T]^{-2}$
Q	= overtopping rate (volume per unit time per unit crest length)	$[L]^2\ [T]^{-1}$
R	= runup height measured vertically from the still water level (SWL) (e.g. the height to which the water would runup if the structure were high enough to preclude overtopping)	$[L]$
d_s	= water depth at the structure toe	$[L]$
h	= height of the structure crest above the bottom	$[L]$
ν	= kinematic viscosity	$[L]^2\ [T]^{-1}$
θ	= structure slope	[dimensionless]

plus any other geometric parameters necessary to describe the various structure types. A dimensional analysis of the preceding 9 variables having 2 dimensions gives the following dimensionless terms:

d_s/H_o' = relative water depth at the structure toe

H_o'/gT^2 = wave steepness parameter

$F = (h-d_s)/H_o'$ = relative height of structure crest above SWL

$F_o = R/H_o'$ = relative runup or height of structure crest required to preclude overtopping

$Q^* = Q^2/gH_o'^3$ = relative overtopping rate

θ = structure slope, and

$R_e = \dfrac{H_o'^2}{\nu T}$ = a Reynolds' number.

The phenomenon is scaled primarily according to Froude similarity; however, the Reynolds' number serves as a measure of any scale effects. Other formulations of R_e are possible, the present one having been adopted for its simplicity.

Generally it is not permissible to eliminate dimensionless terms by combining them unless an analytic or empirical relationship between two of the variables is known. If it is assumed that such a satisfactory relationship is available for the runup R, the overtopping rate can be expressed in terms of R and the ratio $F/F_o = (h-d_s)/R$ can be substituted for F and F_o. The preceding dimensionless terms are obviously not the only combinations of terms possible; however, they were selected after considerable trial and error because they provided the greatest possibility for keeping dimensionless variables constant and investigating the variation of Q^* with individual parameters.

DATA ANALYSIS

For a given structure and set of incident wave conditions (e.g. constant d_s/H_o', H_o'/gT^2 and θ), the dimensionless overtopping rate, Q^* was plotted against the dimensionless crest height, $F/F_o = (h-d_s)/R$.

A typical plot showing two data sets differing only in model scale, is shown in figure 2. Generally, all data sets when plotted semi-logarithmically exhibited a linear variation of Q^* with F/F_o for small values of F/F_o; also, the value of Q^* must approach zero as the relative crest height, F/F_o approaches 1.0 (i.e., as the crest of the structure approaches the limit of wave runup). The curve therefore approaches $F/F_o = 1.0$ asymptotically on the semi-logarithmic plot. The hyperbolic tangent function exhibits identical behavior; hence, an equation of the form,

$$\frac{F}{F_o} = \alpha \tanh \left[\log \frac{Q^*}{Q_o^*} \right] \qquad (1)$$

was used to approximate the data. Here α and Q_o^* are empirical coefficients to be established by comparing the equation with the data. The value of α generally establishes the shape of the curve since it is the slope of the curve at $F/F_o = 0$. Q_o^* represents the value of Q^* for a structure with its crest elevation at the SWL. Figure 3 depicts equation 1 for various values of α. To establish values of α and Q_o^* a transparent template was made of figure 3 and used as an overlay to Q^* vs F/F_o data plotted at the same scale on semi-logarithmic graph paper. By moving the template vertically until one of the curves coincided with the trend of the data, the value of α could be directly determined. The value of Q_o^* was determined by reading the value of Q^* where the α curves intersected $F/F_o = h-d_s/R = 0.0$ on the data plot. Thus, by overlaying the template to each data set, values of α and Q_o^* were established for each structure type and set of incident wave conditions. Interestingly, the form of equation 1 is such that it could be used to describe the overtopping of all of the structures for which data were available; consequently, figures similar to figure 4 could be prepared for each structure type. Such figures, which give α and Q_o^* as functions of d_s/H_o' and H_o'/gT^2 for a given structure type, are presented in the SPM (4) for other structure slopes and types.

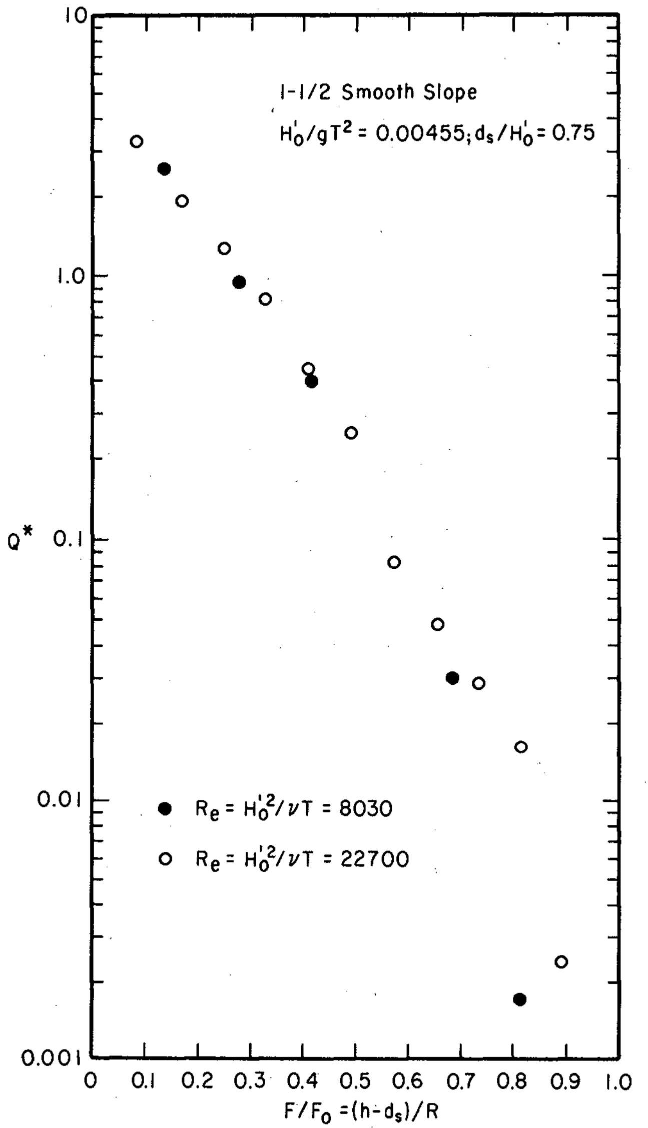

Figure 2. Typical data plot, 1 on 1 1/2 smooth slope, $H_o'/gT^2 = 0.00455$; $d/H_o' = 0.75$.

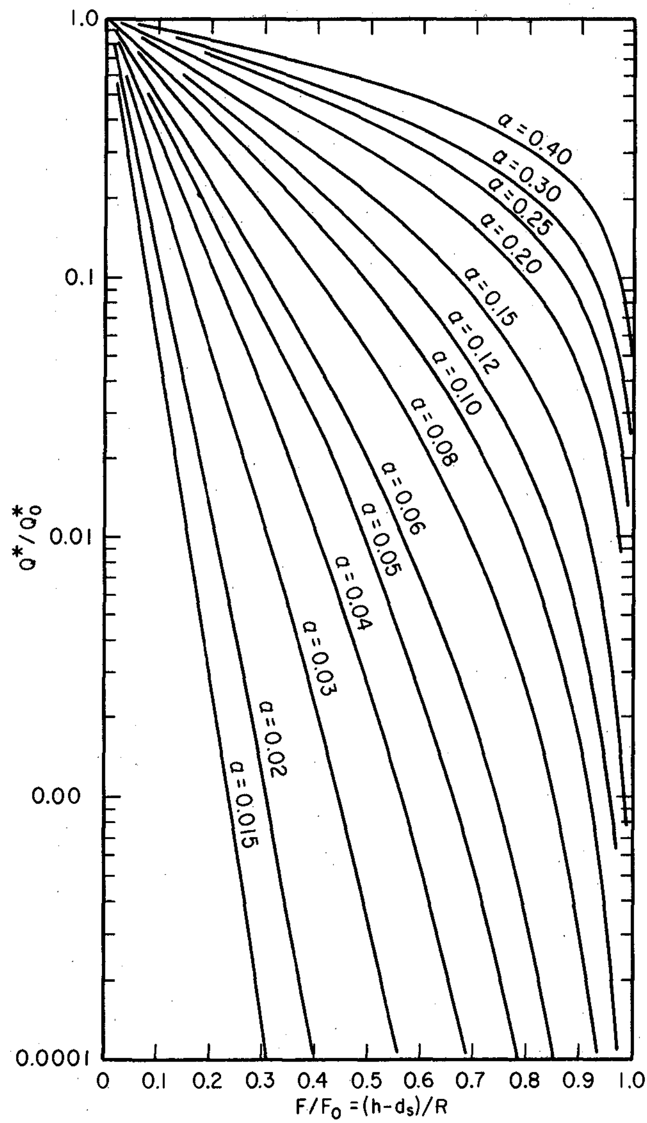

Figure 3. Presentation of equation 2 for various values of α.

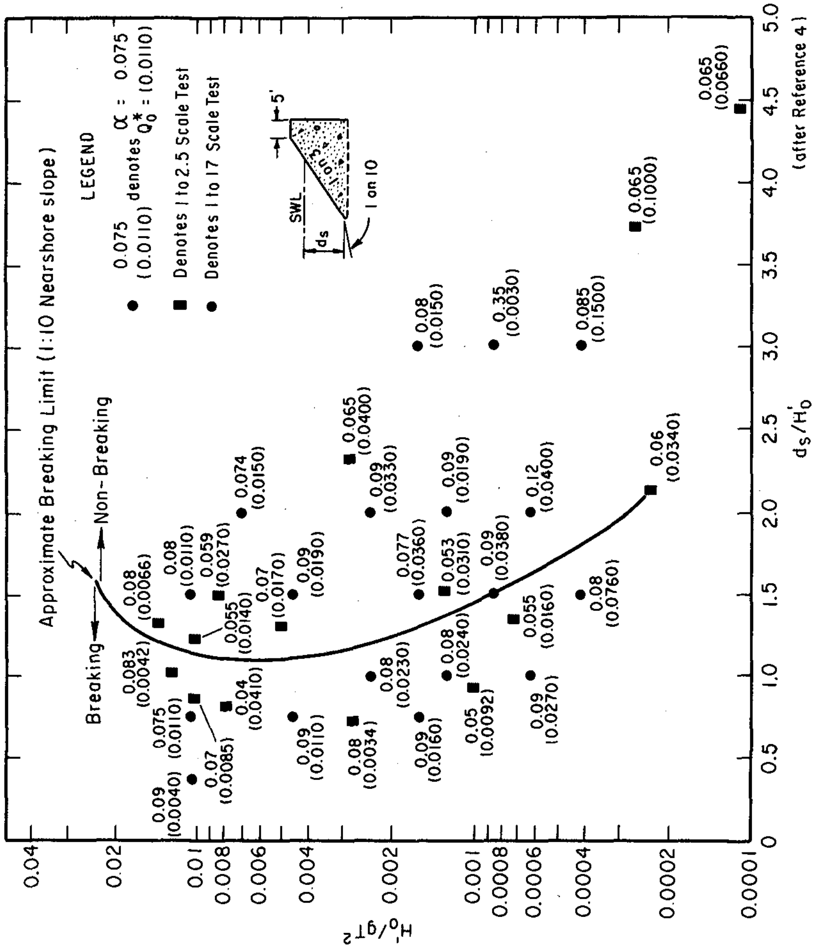

Figure 4. Variation of α and Q_o^* with incident wave conditions - initial data analysis, 1 on 3 smooth slope.

By substituting the dimensionless variables into equation 1 and solving for Q, one finds,

$$Q = \left(gQ_o^* H_o'^3\right)^{1/2} \exp\left\{-\frac{0.217}{\alpha}\tanh^{-1}\left[\left(\frac{h-d_s}{R}\right)\right]\right\} \qquad (2)$$

or equivalently, since $\tanh^{-1}\left(\frac{a}{b}\right) = \frac{1}{2}\log_e\left(\frac{b+a}{b-a}\right)$,

$$Q = \left(gQ_o^* H_o'^3\right)^{1/2} \exp\left\{-\frac{0.217}{2\alpha}\log_e\left[\left(\frac{R+h-d_s}{R-h+d_s}\right)\right]\right\} \qquad (3)$$

Either equation 2 or 3 can be used in conjunction with figures such as figure 4 to determine overtopping rates.

To evaluate the ability of equation 2 or 3 to predict the overtopping rates measured in the experiments, the values of α and Q_o^* as published in the SPM, were used with equation 2 and computed overtopping values compared with measured values. Table 2 presents the correlation coefficients found in the analysis. In general, agreement was excellent; the worst case was for the vertical wall data with $r = 0.980$. (The small number of data points for the recurved wall make the correlation analysis for that structure inconclusive).

Subsequent to publication of the SPM, further analysis of the data for smooth slopes was undertaken in an attempt to relate α and Q_o^* to incident wave conditions and structure slope. For a given slope, the variability of α with incident wave conditions was relatively small, suggesting that an average α could be used and the data reanalysed to establish the Q_o^* value that best fit the data for the average α. The average value, $\bar{\alpha}$, is shown on figure 5 for four smooth structure slopes with data obtained at three different scales.

The effect of decreasing model scale seems to result in a more rapid drop-off of the overtopping rate with increasing structure height. (see figure 2). This effect is also related to the value of R used to compute F/F_o, however, an expression relating $\bar{\alpha}$ with structure slope (smooth

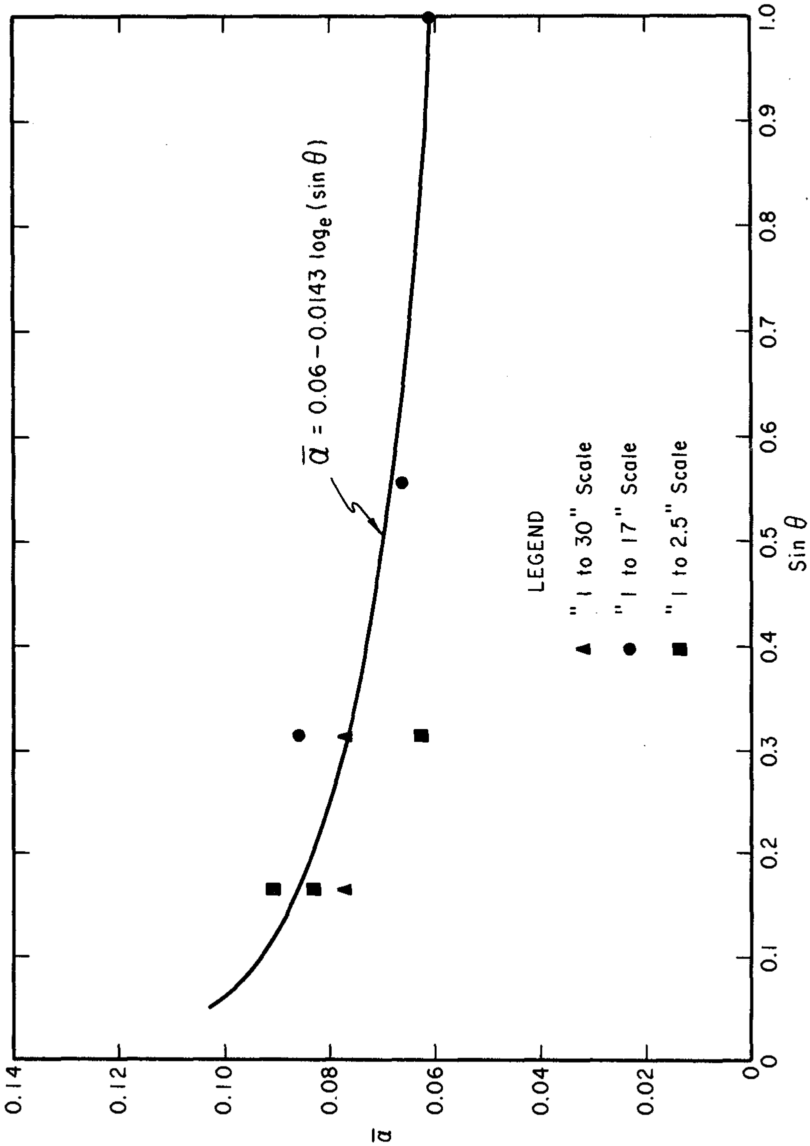

Figure 5. Average values of α as a function of structure slope.

slopes only) is given by,

$$\bar{\alpha} = 0.06 - 0.0143 \log_e (\sin \theta) \qquad (4)$$

The data for smooth slopes was reanalysed using the values of α given by equation 4 for each slope and appropriate best fit values of Q_o^* selected. These Q_o^* values could then be compared with an expression calculated as an upper bound on Q_o^*.

Physically, the value of Q_o^* corresponds to the dimensionless quantity of water transported over the structure if the structure crest were at the SWL (i.e., $F/F_o = 0$). For waves that do not break before hitting a structure, the volume of water above the SWL in a wave profile will define an approximate upper limit for Q_o^*. Defining the volume of water above the SWL as V,

$$V = \varepsilon\, HL \qquad (5)$$

where H = wave height, L = wave length and ε = a dimensionless factor depending on the shape of the wave profile. For a sinusoidal wave, $\varepsilon = 1/(2\pi)$, while for various cnoidal wave profiles, ε can be obtained from figure 6 if the appropriate value of the modulus of the complete elliptic integral k^2 is known. (see Reference 5). Then, the overtopping rate is given by,

$$Q = \frac{V}{T} = \frac{\varepsilon\, HL}{T} \qquad (6)$$

Recalling that Q_o^* is defined by,

$$Q_o^* = \frac{Q^2}{gH_o'^3}$$

and using linear wave theory expressions for H/H_o' and L/L_o,

$$Q_o^* = \frac{\left[\frac{\varepsilon}{2\pi}\right]^2 \left[\frac{H}{H_o'}\right]^2 \tanh^2 \left[\frac{2\pi d_s}{L}\right]}{H_o'/gT^2} \qquad (7)$$

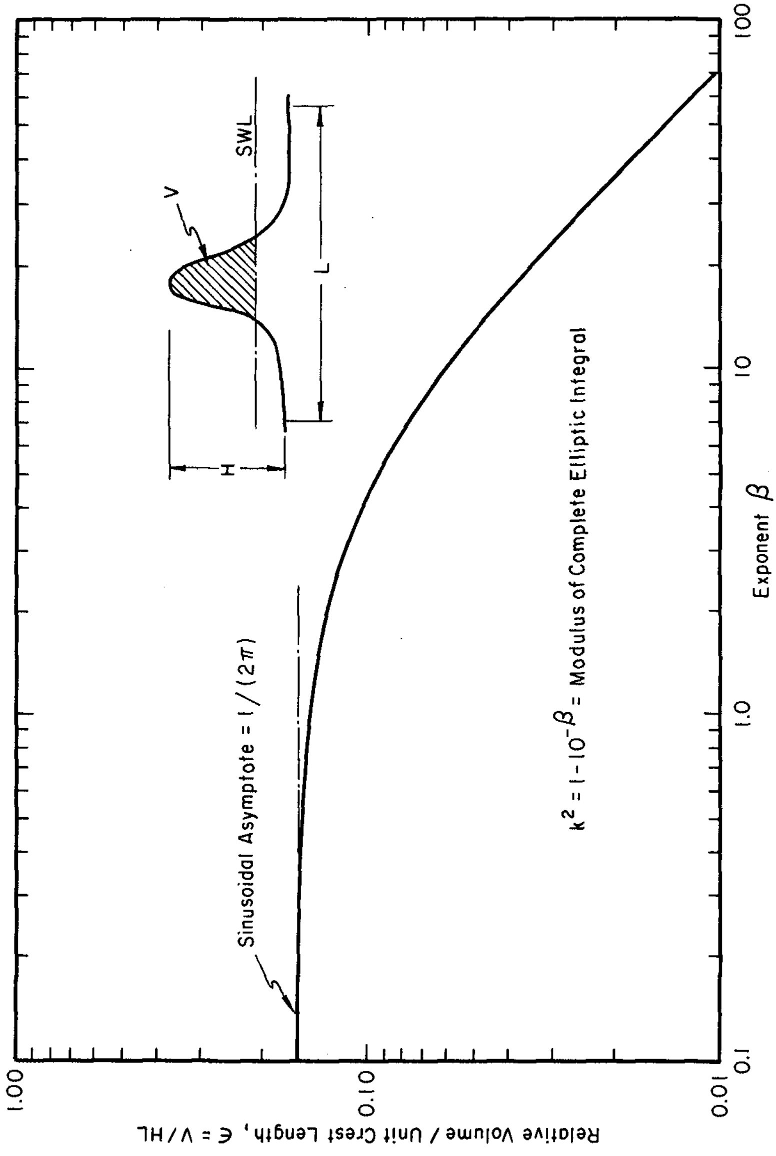

Figure 6. Dimensionless fluid volume per unit crest length above the SWL for cnoidal waves.

Equation 7 is plotted on figure 7 for both $\varepsilon = 1/(2\pi)$ (labelled linear theory) and for ε values determined from cnoidal wave theory. Also shown on figure 7 are the Q_o^* values determined by the analysis for constant $\bar{\alpha}$ described above. In general, data from the 1 to 17 scale tests falls well below the cnoidal wave curves for all slopes for which data were available. Data from the 1 to 2 1/2 scale tests (squares on figure 7), however, are in general conformance with the cnoidal curves when $d_s/H_o' > 1.5$ (non breaking waves). Agreement appears best for long waves of small steepness. Steeper waves, and waves that break have considerably lower values of Q_o^* than predicted by the linear theory or cnoidal theory curves.

SCALE EFFECTS

Scale effects arise because of an inability to model all aspects of a phenomenon simultaneously, usually because of the limited range of fluid properties practically achievable; hence, to achieve both Froude and Reynolds' similarity in a model, the model fluid viscosity would have to vary as the scale ratio to the 3/2 power times the prototype fluid viscosity. Therefore, if as is usually the case, the same fluid is used in both model and prototype, only one similirity law can be satisfied at the expense of the other. If surface tension is also a factor, the problem is even more complex. Since the wave overtopping phenomenon is dominated by wave motion, a Froude modelling law governs; however, turbulent and viscous dissipation also influence overtopping; thus Reynolds' effects must also influence the phenomenon. If viscosity is included in the original dimensional analysis, a Reynolds' number arises from the analysis and serves as a "scale factor" or as a measure of scale effects. The formulation of the Reynolds' number adopted to investigate scale effects was, $R_e = H_o'^2/\nu T$.

The influence of R_e on an individual data set for which all other dimensionless variables are constant is shown in figure 2. Both sets of points on the figure are from what were termed 1 to 17 scale tests; however, the wave height in the one test was 10.76 cm (solid circles, R_e =

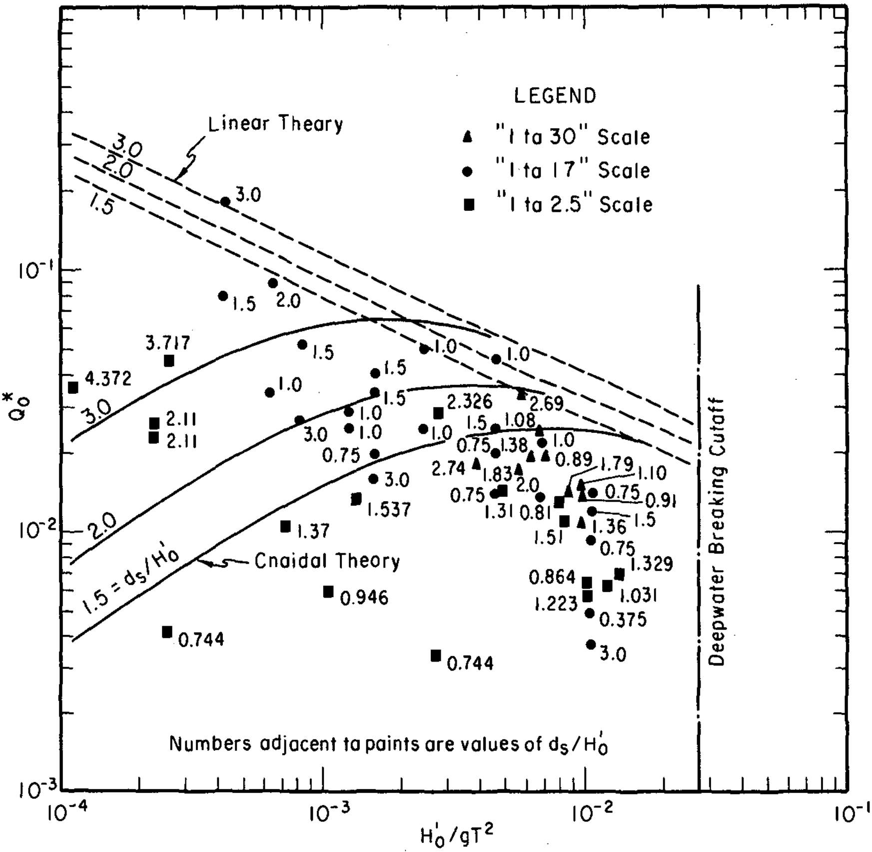

Figure 7. Comparison between measured and predicted values of Q_o^* (Based on reanalysis of data with constant $\alpha = 0.0765$, 1 on 3 smooth slope).

8030) while in the other test, the wave height was 21.52 cm (open circles, $R_e = 22700$). The greatest influence of scale appears when the structure crest is near the limit of wave runup (e.g. at the tail of the curve near $F/F_o = (h-d_s)/R \rightarrow 1.0$). It is in this relatively thin layer of fluid that viscous effects can be expected to increase in importance. Inasmuch as it is the shape of the curve that is affected, it is the value of α that is, to a limited extent, influenced by scale. This slight dependence of α on scale is shown on figure 8. For the 1 on 3 slope, the SPM values of α are plotted against R_e for a broad range of d_s/H_o' and H_o'/gT^2 values. While it is difficult to draw firm conclusions regarding scale effects from figure 8 since d_s/H_o' and H_o'/gT^2 varied, there appears to be a decrease in α with increasing R_e. Note that the test designated as the 1 to 30 scale (triangular points) when characterized by $R_e = H_o'^2/\nu T$ as a "scale factor" are actually not at a smaller scale than the 1 to 17 scale tests (circles) since the smaller wave heights also had correspondingly smaller wave periods.

The effect of scale on Q_o^* appears to be negligible since no systematic variation of Q_o^* with R_e could be found. Since Q_o^* represents the dimensionless overtopping rate for a structure with its crest at the SWL, a relatively thick layer of flow, insensitive to viscous effects, tends to minimize any dependence of Q_o^* on R_e for smooth slopes.

In general, it appears that the primary influence of scale on the overtopping of smooth sloped structures is through scale effects manifest in the runup phenomenon. In the data analysis described herein, the actual measured or "correct" runup was used; consequently, the influence of scale effects on the runup was not considered. Other investigators have demonstrated that small scale runup experiments tend to significantly underpredict runup at larger scales. The observed effect of scale on overtopping itself appears to be relatively small; thus if the runup values used in equations 2 or 3 are corrected for scale effects, the predicted overtopping rates (using appropriate α and Q_o^* values) will provide good estimates of prototype overtopping rates.

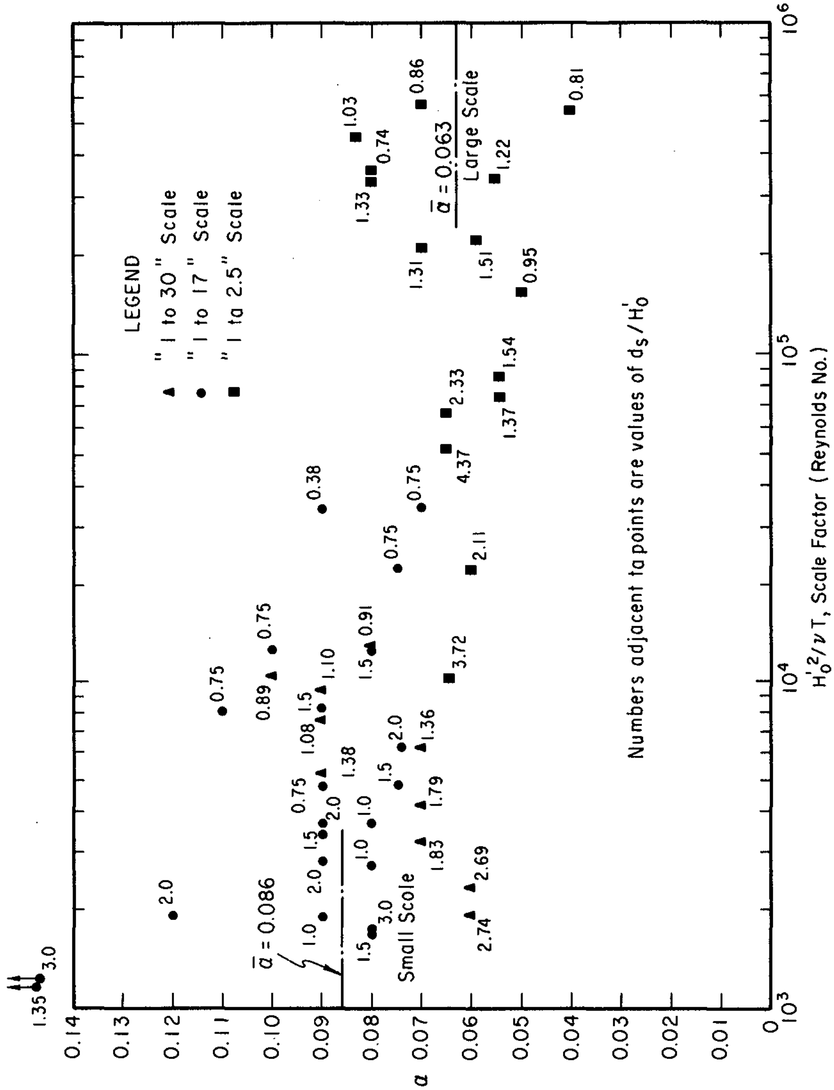

Figure 8. Variation of α with Reynolds' number (scale factor).

EXAMPLE

A levee is constructed with a 1 on 3 slope to just preclude overtopping by a wave 1.05 m high when the water depth at the structure toe is 1.5 m deep. If the design wave period is 5 sec, determine the rate at which water is carried over the structure by the design wave if a storm surge of 0.6 m occurs. The unrefracted deepwater wave height can be estimated from the linear theory; $H/H_o' = 1.073$ for $d_s/L_o = 0.0384$; hence, $H_o' = 1.05/1.073 = 0.98$ m. From reference 4, the relative runup is $R/H_o' = 2.2$, which, when corrected for scale effects, becomes $R/H_o' = 2.2(1.12) = 2.464$. Thus, $R = 2.464(0.98) = 2.41$ m and the height of the structure crest above the bottom is $h = 1.5 + 2.41 = 3.91$ m. To determine the overtopping rate, calculate $d_s/H_o' = (1.5 + .6)/0.98 = 2.14$ and $H_o'/gT^2 = 0.98/9.81(5)^2 = 0.0040$. From figure 4, interpolating, $\alpha \approx 0.08$ and $Q_o^* = 0.024$. From equation 2,

$$Q = \left[(9.81)(0.024)(.98)^3\right]^{1/2} \exp\left\{-\frac{0.217}{0.08} \tanh^{-1}\left[\left(\frac{3.91-2.1}{2.41}\right)\right]\right\}$$

$$Q = 0.471 \exp\left\{-2.713 \tanh^{-1} (0.751)\right\}$$

$$Q = 0.471 \exp\left\{-2.713 (.975)\right\}$$

$$Q = 0.0334 \text{ m}^3/\text{sec-m}$$

If the average α value (equation 4) had been used with the cnoidal curves of figure 7, the value of α would have been 0.0765 and $Q_o^* = 0.033$. Using these values in equation 2 gives $Q = 0.0347$ m^3/sec-m.

CONCLUSIONS

1. Equation 2 or 3 along with the empirically established values of α and Q_o^* of reference 4 can be used to predict overtopping rates for various structure slopes and structure types if accurate predictions of runup are available.

2. Alternatively, for smooth slopes, if conservative (high) estimates are desired, equation 2 or 3 can be used with the average α given by equation 4, and the value of Q_o^* given by equation 7 with ε determined from figure 6. This corresponds to using the Q_o^* values given by the cnoidal curves of figure 7.

3. Scale effects in overtopping tests arise primarily in modifying the runup. If scale effect corrected values of runup are used in equations 2 or 3, the predicted overtopping values will have been corrected for most viscosity induced scale effects.

ACKNOWLEDGEMENTS

Much of the original data analysis discussed herein was performed while the author was in the Design Branch, Engineering Development Division at CERC under the general supervision of Robert A. Jachowski. Philip N. Stoa performed the initial data reduction and analysis and Harold McClung assisted in the computer runs associated with the analysis. Their assistance is gratefully acknowledged.

The research described herein was conducted under the Civil Works Program of the Corps of Engineers. Permission to publish this material is appreciated.

APPENDIX I - REFERENCES

(1) Saville, Jr., T. and J.M. Caldwell, "Experimental Study of Wave Overtopping on Shore Structures," Proceedings, Minnesota International Hydraulics Convention, Minneapolis, Minnesota, September 1-4, 1953.

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