CHAPTER 204

WAVE INDUCED OSCILLATIONS OF HARBORS WITH VARIABLE DEPTH

by

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ABSTRACT

A numerical method is presented to treat the wave-induced oscillations of a harbor with a variable depth and width. A two-dimensional finite difference approach is used inside the harbor matched at the entrance to a solution for the open-sea based on the Helmholtz Equation which includes incident, reflected, and radiated wave energy. Examples of the response and the modal shapes of the water surface are presented for harbors with simple and complex shapes.

INTRODUCTION

In recent years significant progress has been made in developing analytical models to determine the response of harbors to incident waves. The ultimate objective in such research is to be able to treat a harbor with variable depth, planform, and coastline configuration, and with a variable interior reflectivity. Such a model would be extremely useful in preliminary design work and guiding laboratory studies of the phenomenon.

Early theoretical investigations of harbor resonance concentrated on harbors with constant depth and simple geometric shapes. Examples of these studies are: Miles and Munk (1961) and Ippen and Goda (1962). One of the major results of these investigations was the realization that the open-sea was important in allowing for the loss of energy radiated from a harbor. For the steady-state excitation of a harbor the radiated energy from the harbor to the open-sea is an important aspect of the response problem, and provides a form of "dissipation" in an otherwise inviscid theoretical approach. Methods were presented which demonstrated, quite well, (particularly the study of Ippen and Goda (1962)) the effect on the harbor response of geometric characteristics of the harbor, such as the ratios of: width to length and entrance width to harbor width.

Lee (1969), Hwang and Tuck (1970), and Lee and Raichlen (1971) investigated the problem of the wave-induced oscillations of constant depth harbors of arbitrary shape. Numerical methods were developed to treat the problem of a complex harbor with perfectly reflecting interior boundaries. In the study of Hwang and Tuck (1970) the open-sea and the harbor were treated as one region; Lee (1969) and Lee and Raichlen (1971) treated the harbor and the open-sea separately, then matched the solutions at the harbor entrance. (This difference leads to the study reported herein.)

Several approaches have been proposed to determine the response of harbors with variable shape and depth to incident waves, e.g., Raichlen (1965), Wilson et al. (1965), Olsen and Hwang (1971), and Chen and Mei (1974). Comments are made by Miles (1974) relative to the applicability of certain of these methods, and the interested reader is directed to that publication.

A two-dimensional approach was presented by Raichlen (1965) to treat the oscillations of long waves in closed basins of arbitrary planform and variable depth. This was extended, in an approximate fashion, to the case of a harbor

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connected to the open-sea. The major assumption in the case of the harbor was that a node existed at the harbor entrance. It was realized at the time that this was a serious limitation to the analysis, especially in view of the work of Ippen and Goda (1962) where it was shown, for rectangular harbors, that only for small ratios of width to length and large ratios of entrance width to harbor width is this assumption reasonable. Suggestions were made for improving this, but these were not attempted at the time.

The objective of the study reported herein is to eliminate this imposed entrance condition and develop a simple method which allows the wave amplitude at the entrance to adjust naturally to the external and internal waves similar to the analyses presented by Ippen and Goda (1962) and Lee (1969) for simple and complex shapes, respectively. The harbor and the ocean are considered as two separate regions with the wave amplitude and the water surface slope obtained in each region matched at the harbor entrance. In this manner, it is possible to use analytical approaches which are different in each of the two regions.

ANALYTICAL CONSIDERATIONS

In this section the analytical method used will be discussed considering in order: the solution inside the harbor, the solution in the open-sea, and the solution for the combined region. There are certain limitations which are placed on the solution which may or may not restrict its application. These are:

(1) Only shallow water waves are considered.
(2) The problem is reduced to a two-dimensional problem; no surface variations are allowed in the harbor in a direction perpendicular to the *talweg* of the harbor.
(3) The open-sea has a constant depth equal to the average depth at the entrance.
(4) The coastline is assumed to be straight.

Harbor Region

The solution for the interior of the harbor follows the method proposed by Raichlen (1965), and it will be summarized here. A body of water is considered with a length ℓ measured along the natural coordinate direction s, see Fig. 1. The natural coordinate is directed along the *talweg* which is the line which smoothly connects the deepest parts of the body of water. The variation of surface width and cross-section area perpendicular to the s-direction are denoted as b(s) and A(s), respectively.

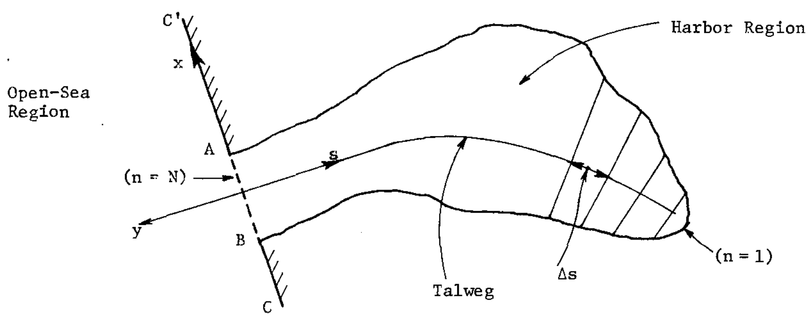

Fig. 1. Definition Sketch of Harbor and Open-Sea Regions.

For long waves with small amplitude the equation of continuity for the fluid body may be written as:

$$\frac{\partial}{\partial s}(Au) + b\frac{\partial \eta}{\partial t} = 0 \tag{1}$$

where u is the water partical velocity in the s-direction averaged over the cross-section and η is the amplitude of the water surface relative to the mean water level.

Neglecting friction, the equation of motion in the s-direction for two-dimensional flow (without body forces) is:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial s} = -\frac{1}{\rho}\frac{\partial p}{\partial s} \tag{2}$$

where ρ is the density of the fluid and p is the local pressure. For small amplitude shallow water waves, the pressure can be assumed hydrostatic:

$$p = \gamma(\eta - z) \tag{3}$$

where γ is the specific weight of the fluid and the coordinate direction z is positive upwards. Neglecting the convective acceleration in Eq. 2 (which is reasonable for this small amplitude approach), Eqs. 2 and 3 can be combined to give:

$$\frac{\partial \eta}{\partial s} = -\frac{\partial u/\partial t}{g} \tag{4}$$

Eq. 4 states that the local water surface slope in the s-direction is given by the ratio of the local acceleration to the acceleration of gravity. It is assumed that the amplitude variation of the free surface can be expressed as a separable function of space and time as:

$$\eta = \eta(s)\cos\sigma t \tag{5}$$

where σ is the circular wave frequency, 2π/wave period. Differentiating the continuity equation (Eq. 1) with respect to time, substituting for the local acceleration from Eq. 4 and using Eq. 5 in the resulting expression, the following is obtained:

$$\frac{\partial}{\partial s}\left[A\frac{\partial \eta}{\partial s}\right] + \frac{b\sigma^2}{g}\eta = 0 \tag{6}$$

Eq. 6 can be expanded into an equation of the Sturm-Lionville form:

$$A\frac{d^2\eta}{ds^2} + \frac{dA}{ds}\frac{d\eta}{ds} + \lambda_m b\eta = 0 \tag{7}$$

where $\lambda_m = \frac{\sigma_m^2}{g}$ $\qquad m = 1,2,\ldots, M$

and λ_m is a characteristic or an eigenvalue of the problem, and A and b are functions of s alone. Wilson et al. (1965) describe two boundary conditions that may exist at the end of the basin opposite to the entrance:

(1) The cross-section area tends to zero or,
(2) there is perfect reflection from the end.

From Eq. 7, these conditions can be expressed respectively as:

(1) $A = 0$ $\qquad \frac{dA}{ds}\frac{d\eta}{ds} + \lambda_m b\eta = 0$ (8a)

$$(2) \quad \frac{d\eta}{ds} = 0 \qquad A\frac{d^2\eta}{ds^2} + \lambda_m b\eta = 0 \tag{8b}$$

If both conditions exist simultaneously, the trivial case of $\eta = 0$ is obtained from Eq. 7. In addition, inherent in the assumption of $A = 0$ at the end is the fact that $b \neq 0$ at that point; if not, Eq. 7 reduces to $\frac{dA}{ds}\frac{d\eta}{ds} = 0$. Thus, the two boundary conditions at the closed end of the harbor are somewhat restrictive, although they do cover most of the interesting problems.

To obtain a numerical solution to Eq. 7, the basin is divided into N cross-sections spaced a distance Δs apart, where $\Delta s = \ell/(N-1)$; the cross-sections are perpendicular to the *talweg*. The section $n = 1$ is the closed end of the basin and $n = N$ is at the entrance which communicates with the open-sea. The first and second derivatives are expressed in finite difference form using central differences, and Eq. 7 becomes:

$$a_{n,n-1}\eta_{n-1} + a_{n,n}\eta_n + a_{n,n+1}\eta_{n+1} = \Lambda_m\eta_n \tag{9}$$

where:

$$a_{n,n-1} = -\frac{1}{b_n}\left[A_n - \frac{1}{4}(A_{n+1} - A_{n-1})\right] \qquad a_{n,n} = 2A_n/b_n$$

$$a_{n,n+1} = -\frac{1}{b_n}\left[A_n + \frac{1}{4}(A_{n+1} - A_{n-1})\right] \qquad \Lambda_m = \lambda_m\Delta s^2$$

For the boundary condition where the area at the end section $(n = 1)$ goes to zero, Eq. 8a, forward differences are used to define the derivatives and Eq. 8a becomes:

$$a_{1,1}\eta_1 + a_{1,2}\eta_2 = \Lambda_m\eta_1 \tag{10a}$$

where:

$$a_{1,1} = \left(A_2/b_2\right) = -a_{1,2}$$

Therefore, the wave amplitude at the next section $(n = 2)$ can be expressed in terms of the amplitude at section $n = 1$ from Eq. 10a as:

$$\eta_2 = \left(1 - \frac{b_2\sigma^2\Delta s^2}{gA_2}\right)\eta_1 = E_2\eta_1 \tag{10b}$$

In the case of perfect reflection from the end boundary $(n = 1)$ when the depth does not go to zero, the water surface slope becomes zero in accordance with Eq. 8b. This equation could be written in difference form similar to that shown in Eqs. 10a and 10b. However, to obtain a solution a zero water surface slope can be forced to occur at the end by using the "mirror-image" method proposed by Raichlen (1965). In that method the basin is extended one cross-section beyond the end wall to construct a mirror image; for example, for reflections at $n = 1$ a cross-section at $n = 0$ is defined with $A_0 = A_2$, $b_0 = b_2$ and $\eta_0 = \eta_2$. In this manner, by symmetry about $n = 1$, a zero water surface slope is forced at $n = 1$. The difference equations written for this are:

$$a_{1,2}\eta_2 + a_{1,1}\eta_1 = \Lambda_m\eta_1 \tag{11a}$$

where $\quad a_{1,1} = (2A_1/b_1) = -a_{1,2}$

Hence, the wave amplitude at $n = 2$ can be expressed in terms of the amplitude at the end of the basin as:

$$\eta_2 = \left(1 - \frac{b_1 \sigma^2 \Delta s^2}{2gA_1}\right) \eta_1 = E_2' \eta_1 \tag{11b}$$

It should be recalled, the analytical method used is to obtain a solution inside the harbor that can be matched at the entrance to a solution obtained by a different method for the outside region (the open-sea). To accomplish this, an inside solution must be evaluated at the entrance, $n = N$. Eq. 9 and the appropriate boundary condition (Eq. 10b or 11b) is used for this. In this regard, it is instructive to look first at the wave amplitude at the cross-section $n = 3$.

$$\eta_3 = \frac{1}{A_2 + \frac{1}{4}(A_3 - A_1)} \left\{ \left[\frac{1}{4}(A_3 - A_1) - A_2\right] + \left[2A_2 - \frac{b_2 \sigma^2 \Delta s^2}{g}\right]\left[E_2 \text{ or } E_2'\right] \right\} \eta_1 = E_3 \eta_1 \tag{12}$$

Therefore, knowing the wave amplitude at the end of the basin, $n = 1$, and the cross-section geometry and wave period, the wave amplitude at $n = 3$ can be evaluated. Proceeding iteratively, the amplitudes at arbitrary cross-sections are expressed as:

$$\eta_{n-1} = E_{n-1} \eta_1 \tag{13a}$$

and

$$\eta_n = E_n \eta_1 \tag{13b}$$

where E_n is a coefficient which can be evaluated easily.

Using backward differences, the water surface slope at the entrance ($n = N$) is approximated as:

$$\frac{d\eta_N}{ds} \approx \frac{\eta_N - \eta_{N-1}}{\Delta s} = F_N \eta_1 \tag{14}$$

where

$$F_N = \frac{E_N - E_{N-1}}{\Delta s} .$$

For any cross-section, E_n (and ultimately F_N) is determined by arbitrarily setting $\eta_1 = 1$ and the calculating $\eta_2, \eta_3, \ldots. \eta_N$ iteratively. Dividing η_N and η_{N-1} by η_1, the desired values of E_N and E_{N-1} are obtained, and the slope of the water surface (or the velocity) at the entrance is defined.

The wave amplitude η is a complex quantity which, at the entrance ($n = N$), can be written as:

$$\eta_N = \eta_N^{(R)} + i\, \eta_N^{(I)} \tag{15}$$

where $\eta_N^{(R)}$ and $\eta_N^{(I)}$ are the real and imaginary parts of the entrance wave amplitude, respectively, and thus provide both amplitude and phase information.

Open-Sea Region

The solution for the open-sea region is taken from Lee (1969) and will be summarized here; the interested reader is directed to the publication cited for a more comprehensive description. As mentioned previously, for this development the open-sea is assumed to be a constant depth equal to the average depth at the harbor entrance. If a separable solution of the velocity potential Φ is sought, within the limitations of small amplitude wave theory, the following expression can be used for the velocity potential:

$$\Phi(x,y,z;t) = -\frac{1}{i\sigma}\,\frac{a_i g \cosh k(h+z)}{\cosh kh}\,f(x,y)e^{-i\sigma t} \tag{16a}$$

and hence:

$$\eta(x,y;t) = a_i f(x,y)e^{-i\sigma t} \tag{16b}$$

wherein a_i is the wave amplitude, k is the wave number, and h is the depth. From Laplace's Equation ($\nabla^2\Phi = 0$) it is found that the spacial wave function, f, must satisfy the Helmholtz Equation:

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + k^2 f = 0 \tag{17}$$

In the outside region the wave function is defined as:

$$f_{out} = f_{inc} + f_{ref} + f_{rad} \tag{18a}$$

where f_{inc} is the incident wave function, f_{ref} is the wave function for the reflected wave, and f_{rad} is the wave function for the wave which is radiated outward from the harbor entrance. From Eq. 16b the wave amplitude can be expressed as:

$$\eta = a_i\,(f_{inc} + f_{ref} + f_{rad})e^{-i\sigma t} \tag{18b}$$

Referring to Fig. 1, the following boundary conditions are imposed on f_{out}.

(1) $$\frac{\partial f_{out}}{\partial n} = 0 \text{ on } \overline{AC'} \text{ and } \overline{BC} \text{ at } y=0 \tag{19a}$$

(2) $$\frac{\partial f_{out}}{\partial n} = -C \text{ on } \overline{AB} \text{ at } y=0 \tag{19b}$$

(3) $$f_{out} \to (f_{inc}+f_{ref}) \text{ as } (x^2+y^2)\to\infty \tag{19c}$$

The first boundary condition implies a zero velocity along the impermeable coastline where n is the normal to the boundary. The second boundary condition expresses the average velocity across the entrance in terms of the solution of the velocity obtained from inside the harbor. (This does not pertain to the immediate solution sought, but it is presented here for its usefulness later.) The third boundary condition ensures the radiated wave system, f_{rad}, disappears with increasing distance from the harbor entrance and the wave becomes simply a standing wave at infinity.

Lee (1969) obtains the radiated wave function using Weber's solution for the Helmholtz Equation (Eq. 17) letting the standing wave amplitude at infinity be a_i, i.e., $(f_{inc} + f_{ref}) = 1$. Thus the wave function along the entrance is written as:

$$f_{out}(x,0) = 1 - \frac{i}{2}\int_{\overline{AB}} C(x,0)H_o^{(1)}(kr)\,dx \tag{20}$$

where $H_o^{(1)}(kr)$ is the Hankel function of the first kind and zero order chosen as a fundamental solution of the Helmholtz Equation. If variations in amplitude and velocity across the entrance in the x direction are considered small, f_{out} and $C(x,0)$ in Eq. 20 can be replaced by their values averaged over the width of the entrance: $\overline{f_{out}}$ and $\overline{C}$. Hence, from Eq. 20 one obtains

$$\overline{f_{out}} = 1 - \frac{i}{2}\left[J_c + i\,\frac{2}{\pi}\,Y_c\right] b_N \overline{C} \tag{21}$$

where:

$$J_c = 1 - \frac{1}{6}\left(\frac{kb_N}{2}\right)^2 + \frac{1}{60}\left(\frac{kb_N}{2}\right)^4 - \frac{1}{1008}\left(\frac{kb_N}{2}\right)^6 + \frac{1}{25920}\left(\frac{kb_N}{2}\right)^8 + \ldots$$

$$\begin{aligned} Y_c = &\left[\ell n\left(\frac{kb_N}{2}\right) + \gamma - \frac{3}{2}\right] - \frac{1}{6}\left(\frac{kb_N}{2}\right)^2\left[\ell n\left(\frac{kb_N}{2}\right) + \gamma - \frac{19}{12}\right] \\ &+ \frac{1}{60}\left(\frac{kb_N}{2}\right)^4\left[\ell n\left(\frac{kb_N}{2}\right) + \gamma - \frac{55}{30}\right] - \frac{1}{1008}\left(\frac{kb_N}{2}\right)^6\left[\ell n\left(\frac{kb_N}{2}\right) + \gamma - \frac{353}{168}\right] \\ &+ \frac{1}{25920}\left(\frac{kb_N}{2}\right)^8\left[\ell n\left(\frac{kb_N}{2}\right) + \gamma - \frac{826}{360}\right] + \ldots\ldots \end{aligned}$$

and: $\gamma = 0.5772157$ (Euler's constant). (To handle relatively large ratios of entrance width to wave length, the numerical computations for J_c and Y_c are carried to terms of order $(kb_N/2)^8$.)

Solution for the Combined Regions

Eq. 21 provides a first approximation to the outside wave function, f_{out}, and hence the wave amplitude in terms of an unknown average water surface slope, $\overline{C}$, at the harbor entrance. Since the $\overline{C}$ can be complex, it is described by the derivative of Eq. 15 with respect to the s-direction, i.e., $\overline{C} = (1/a_i)(d\eta_N/ds)$. This substituted into Eq. 21 provides a general expression for the wave function at the entrance averaged over the entrance width. Separating the resulting expression into the real and imaginary parts the following are obtained:

$$\eta_N^{(R)} = a_i + \frac{1}{2}\left[J_c\,\frac{d\eta_N^{(I)}}{ds} + \frac{2}{\pi}\,Y_c\,\frac{d\eta_N^{(R)}}{ds}\right] b_N \tag{22a}$$

$$\eta_N^{(I)} = -\frac{1}{2}\left[J_c\,\frac{d\eta_N^{(R)}}{ds} - \frac{2}{\pi}\,Y_c\,\frac{d\eta_N^{(I)}}{ds}\right] b_N \tag{22b}$$

Substituting Eqs. 13 and 14 obtained from the interior solution into Eqs. 22, the latter can be rewritten as:

$$\eta_1^{(R)} = \frac{a_i}{E_N} + \frac{1}{2}\left[J_c F_N \eta_1^{(I)} + \frac{2}{\pi}\,Y_c F_N \eta_1^{(R)}\right]\frac{b_N}{E_N} \tag{23a}$$

$$\eta_1^{(I)} = -\frac{1}{2}\left[J_c F_N \eta_1^{(R)} - \frac{2}{\pi} Y_c F_N \eta_1^{(I)}\right]\frac{b_N}{E_N} \tag{23b}$$

Eqs. 23a and 23b can be solved simultaneously for the real and the imaginary parts of η_1, the amplitude at the end of the harbor ($n=1$):

$$\eta_1^{(R)} = a_i\left(E_N - \frac{1}{\pi} Y_c F_N b_N\right)/\alpha_N \tag{24a}$$

$$\eta_1^{(I)} = a_i\left(-\frac{1}{2} J_c F_N b_N\right)/\alpha_N \tag{24b}$$

where:

$$\alpha_N = \left(E_N - \frac{1}{\pi} Y_c F_N b_N\right)^2 + \left(\frac{1}{2} J_c F_N b_N\right)^2 .$$

If the response function for the harbor is defined as the ratio of the wave amplitude at a position in the harbor (e.g., at the backwall, $n=1$) divided by the amplitude at the entrance if the entrance were closed, then from Eqs. 24 this becomes (for the backwall):

$$R_1 = \frac{1}{a_i}\left[\left(\eta_1^{(R)}\right)^2 + \left(\eta_1^{(I)}\right)^2\right]^{1/2} \tag{25}$$

The amplification factor, R_1, can be evaluated by setting the standing wave amplitude to unity ($a_i = 1$) and substituting Eqs. 24a and b into Eq. 25. With reference to Eq. 13, the response at any other location becomes:

$$R_n = E_n R_1 \tag{26}$$

where E_n is defined, as before, from an expression similar to Eq. 12. Therefore, from Eqs. 24, 25, and 26 the response of a variable depth harbor to incident waves can be investigated keeping in mind the restrictions imposed on the solution by the assumptions stated previously.

DISCUSSION OF RESULTS

In evaluating this method of analysis, a harbor of simple geometry was considered first. This was a rectangular harbor with a width w, a linearly varying depth, and a fully open entrance connected directly to the open-sea. The depth of the open-sea region was set equal to the depth at the harbor entrance. The ratio of the depth at the backwall of the harbor to that at the entrance varied from zero to unity. The ratio of the harbor width to the length for the case considered was: $w/\ell = 0.194$; the same ratio as the constant depth case considered by Ippen and Goda (1962) and Lee (1969).

The response curves obtained are presented in Fig. 2 for four ratios of depths at the backwall to the entrance: $h_1/h_2 = 0, 0.67, 0.33, 1.0$. The ordinate is the response function as defined by Eq. 25 and the abscissa is the product of the harbor length and the wave number based on the wave period and the depth of water at the harbor entrance. The response for $h_1/h_2 = 1$ is the same as obtained by Ippen and Goda (1962) and Lee (1969). As the slope of the bottom increases the amplification at resonance increases significantly and the maximum response shifts to smaller values of $k_2\ell$. Note that as $k_2\ell$ tends to zero (the wave length tends to infinity) the response tends to unity, i.e., the wave length is so large that the wave essentially does not "see" the harbor and the amplitude is the same as the standing wave with the harbor entrance

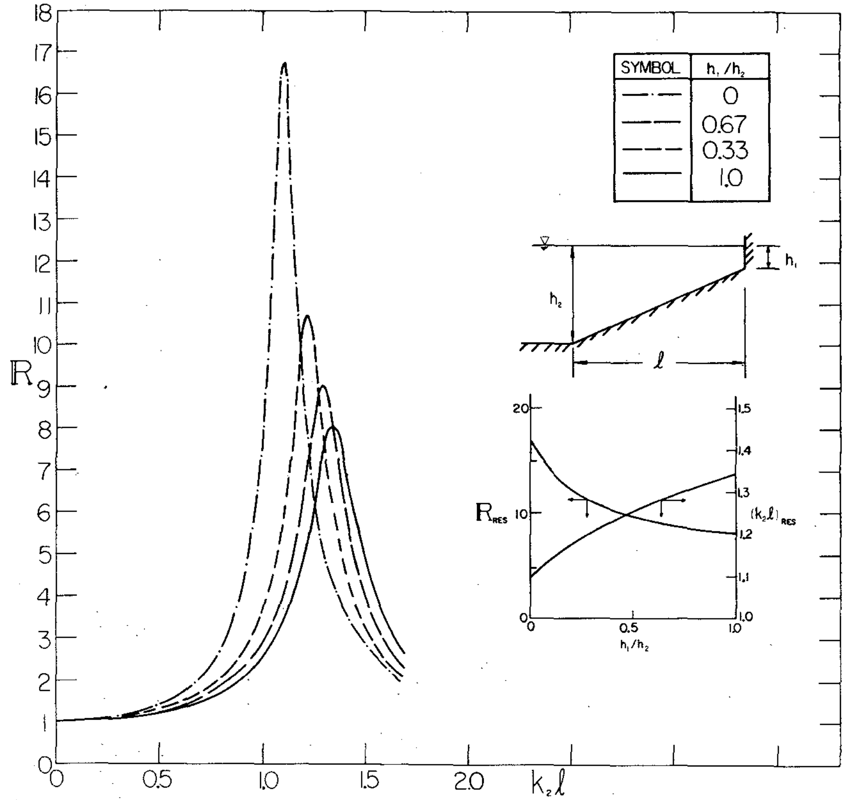

Fig. 2. Response at the Backwall of a Rectangular Harbor with a Linearly Varying Depth; the Fundamental Mode (length = 31.15 cm, width = 6.03 cm, h_2 = 7.62 cm).

closed. The variation of the amplification and the product $k_2\ell$ at resonance with the depth ratio are presented in the inset to Fig. 2; it is apparent that the effect of the change in depth on the amplification at resonance is much more significant than the effect on the resonant wave number, k_2. The former can be considered an effect of shoaling; the latter also is an effect of shoaling and represents a change in the resonant wave length caused by the change in mean depth.

In Fig. 3 the response of a fully open rectangular harbor with a linearly varying depth is presented for two cases: a width to length of 0.1 and 1.0; the fundamental mode is shown for each case. (It should be noted, the response function is presented logarithmically along the ordinate.) The amplification at resonance is reduced by nearly an order of magnitude by increasing the ratio of width to length in the same amount. In addition, the resonant wavelength increases as the width of the harbor increases (the resonant wave number decreases). In fact, it appears that if the width were great enough compared to the length, the response at the backwall would be equal to unity independent of wave number. This is the case where the harbor is so wide compared to the length that it simply becomes like the coastline.

In Fig. 4 the variation of the relative amplitude with relative length is shown for the fundamental mode of three harbors with a varying ratio of width to length. For the three cases the ratio of the depth at the backwall to the depth at the entrance is zero. The results of the numerical analysis are shown along with the exact solution of Lamb (1945) for a long wave in a canal with a linearly varying depth, and the agreement is good. At $x/\ell = 1$ the entrance effect can be seen, and as the ratio of width to length, w/ℓ, decreases, the relative amplitude at the entrance also decreases. In fact, it appears that if $w/\ell \gg 1$ the amplitude in the harbor would be approximately uniform in the x-direction; this is the same trend observed in Fig. 3 as w/ℓ increases.

An example of the response of a harbor with a more general shape is the case of Apra Harbor, Guam, M. I. This harbor was chosen because of the availability of the results of hydraulic model tests which were conducted in the late 1940's to evaluate certain inner harbor modifications and the design of a proposed breakwater (see Knapp and Vanoni (1949)).

A schematic drawing of the harbor is presented in Fig. 5. In the analysis, the outer harbor was limited near the East end by a series of shoals; in the hydraulic model one proposed method to protect the inner harbors was to build a series of breakwaters connecting certain of the shoals. In the analytical model the shoal-breakwater location and the LST berth were considered to be closed ends and the results were compared to the experimental results with the breakwaters in place. The general direction of the *talweg* is shown in Fig. 5 and the areas and surface widths of sections perpendicular to the *talweg* are shown in Fig. 6. The depth averaged over the cross-section varies from approximately 60 ft at the entrance to a maximum of 113 ft about one-third of the harbor length from the entrance and 18 ft near the LST berth.

A response curve obtained for this harbor is presented in Fig. 7; the ordinate is the ratio of wave amplitude at the LST berth (see Fig. 5) to that at the harbor entrance with the entrance closed. The abscissa is the product of the wave number and the harbor length along the *talweg*; the length used is 16,000 ft with the harbor divided into 33 sections. The theoretical response curve exhibits six modes of oscillation for the range of $k\ell$ shown. This response curve must be somewhat in error, since the oscillations corresponding to the response shown in Fig. 7 are two-dimensional and three-dimensionality would probably become important in such a harbor for the higher modes of oscillation. Nevertheless, this does show certain aspects of the response in a variable depth, arbitrary shaped harbor.

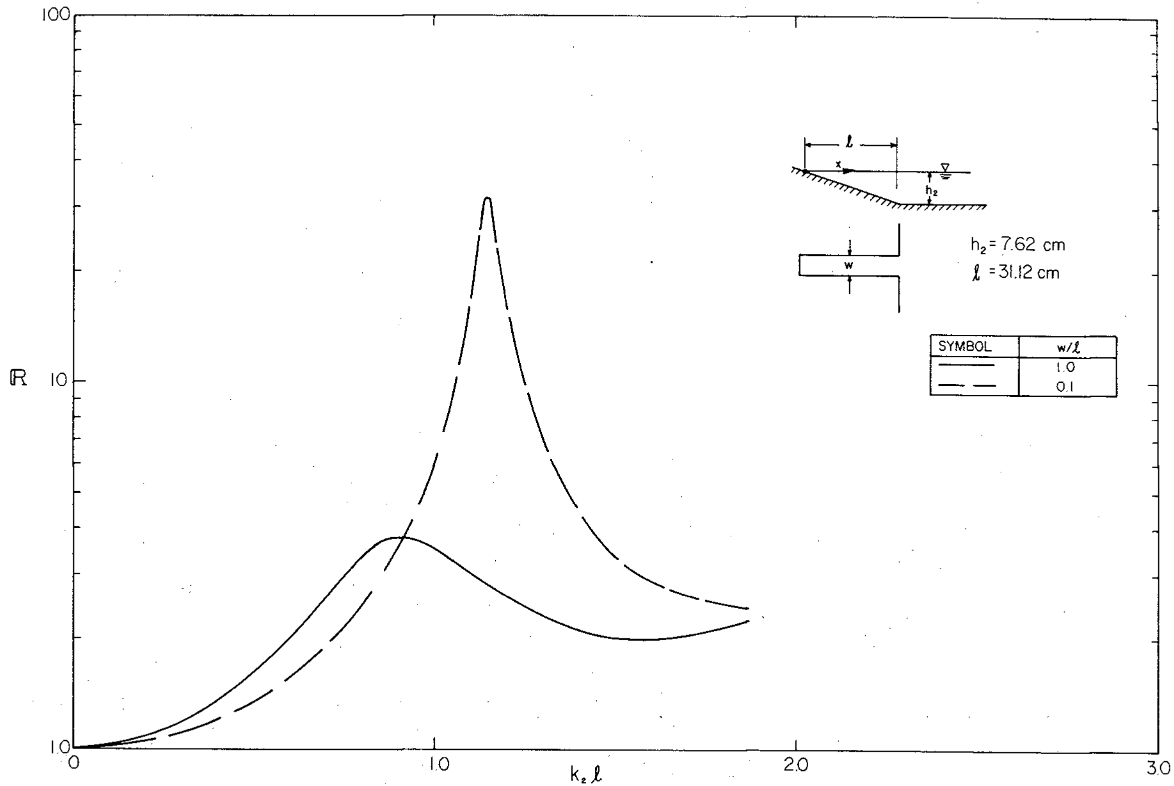

Fig. 3. Response at the Backwall of a Rectangular Harbor with a Linearly Varying Depth ($h_1/h_2 = 0$); the Fundamental Mode.

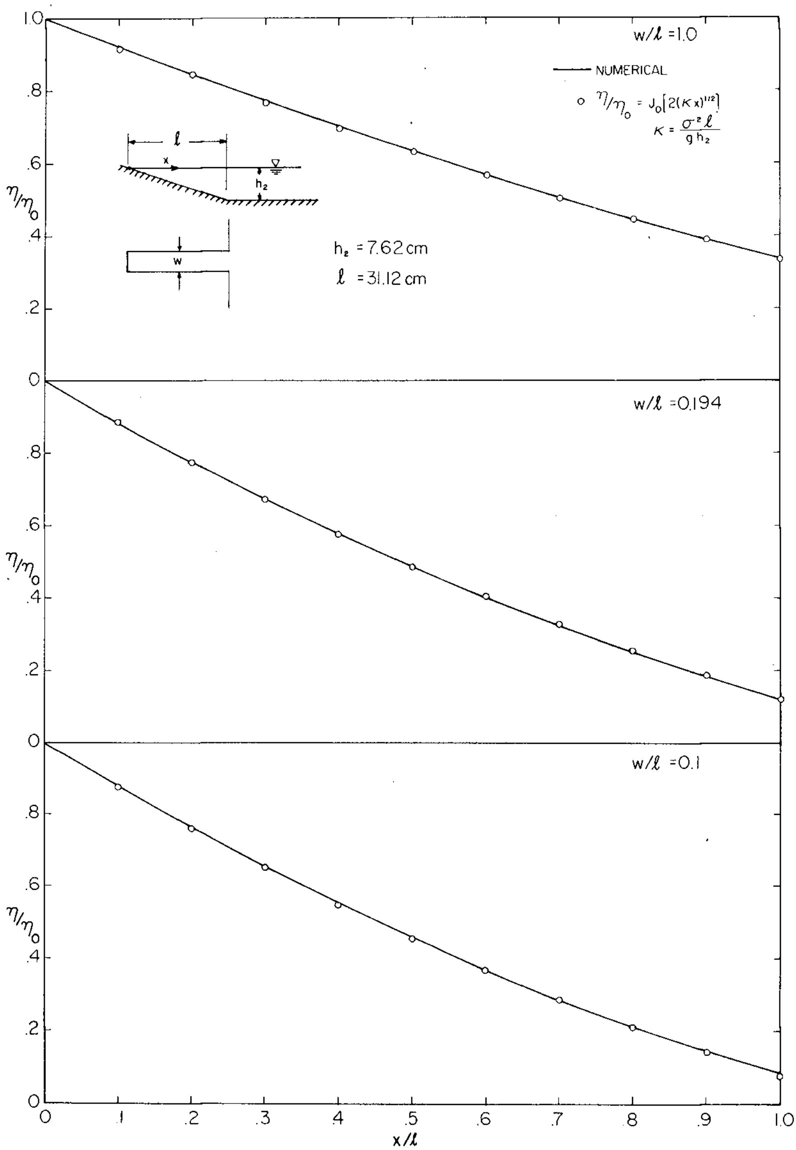

Fig. 4. Relative Wave Amplitude vs. Relative Length for the Fundamental Mode.

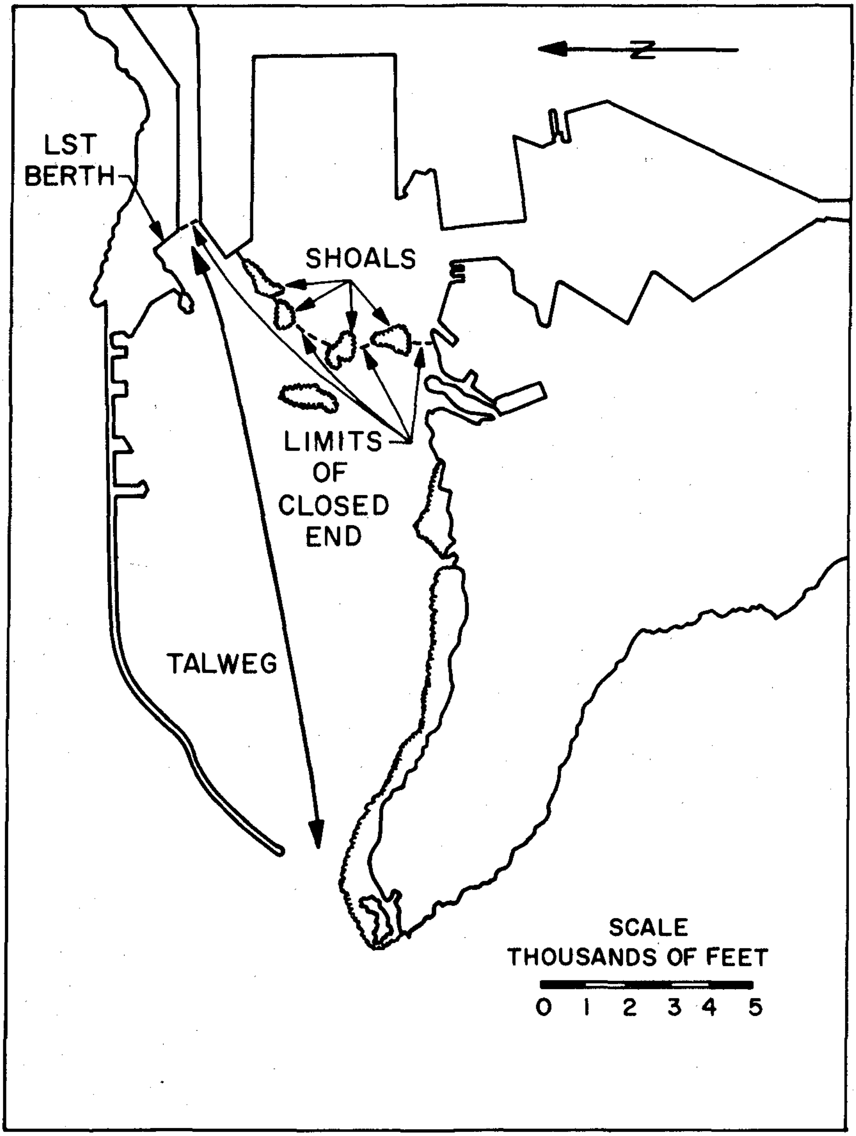

Fig. 5. Schematic Drawing of Apra Harbor, Guam, M.I.

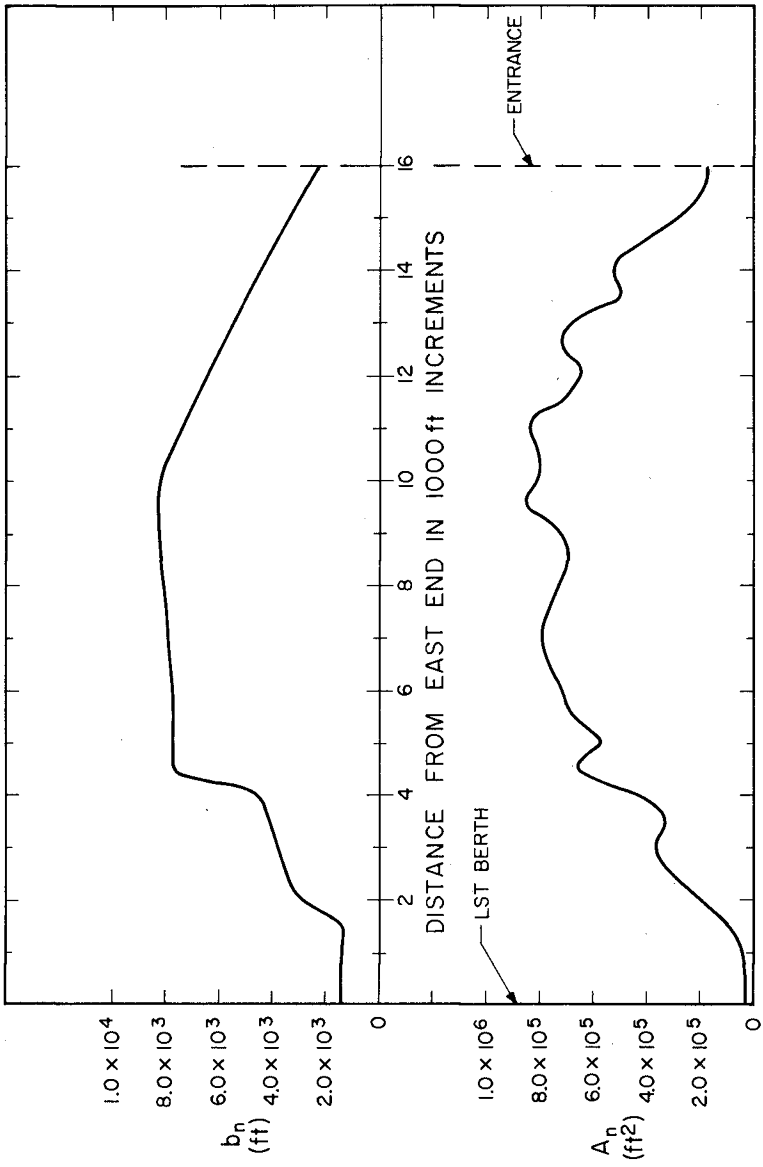

Fig. 6. Variation of Surface Width and Cross-Section Area, Apra Harbor, Guam, M.I.

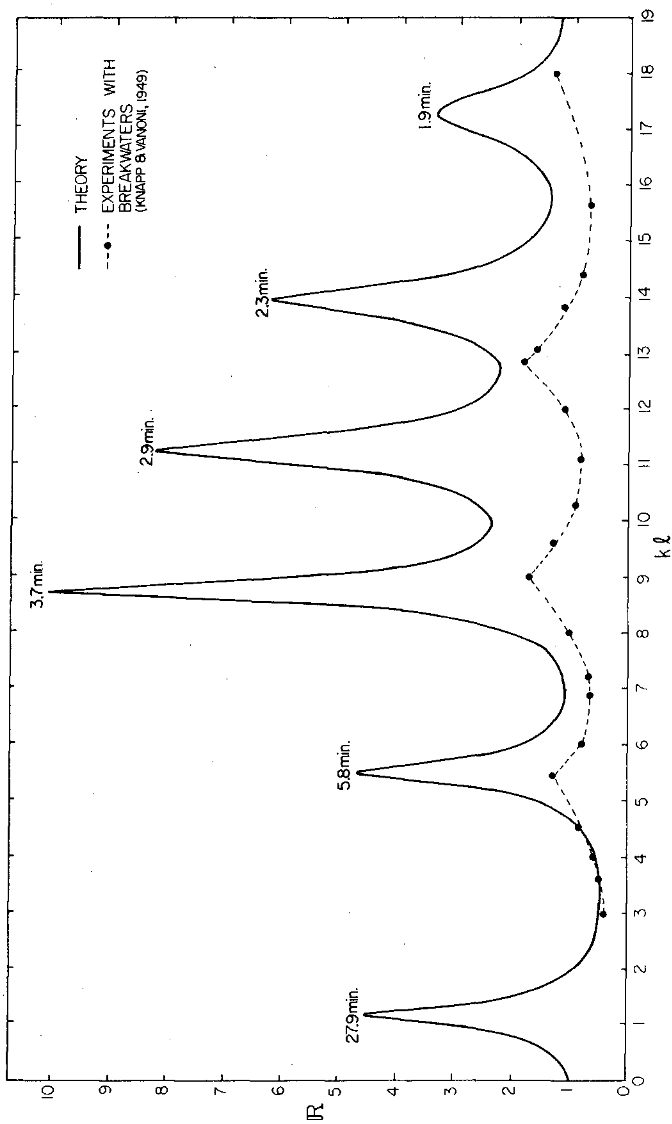

Fig. 7. Response Curve at the LST Berth, Apra Harbor, Guam, M.I.

The experimental data presented in Fig. 7 were obtained by Knapp and Vanoni (1949); the definition of the response function used in that study was not precisely the same as that used in the theory. In their experiments the amplification factor was defined as the ratio of the maximum amplitude at the location of interest to the maximum amplitude outside the harbor with the entrance open. Since there must be an effect on the outside wave due to the harbor, in comparing the experiments to the theory one would expect differences. In Fig. 7 the agreement in the shape of the response curve perhaps is reasonable for the second and third modes but considerably poorer thereafter. This may be due to the failure of a two-dimensional theory in describing the higher modes of oscillation where three-dimensionality must be important as well as the differences mentioned between the theory and the experiments.

The periods of the various modes are shown in Fig. 7 next to the peaks. It is noted that the lowest mode, the fundamental, has a period of 27.9 min; the maximum amplification for $0 < k\ell < 19$ is exhibited by the third mode with a period of 3.7 min. (These periods are based on an average depth of 84.7 ft inside the harbor.) The shape of the water surface for the first three modes are shown in Fig. 8 where the local amplification factor (R_n in Eq. 26) is plotted as a function of distance from the LST berth along the *talweg*. By plotting in this manner, the relative importance of mode shape at a given location can be observed. An interesting feature of Fig. 8 is the water surface amplitude at the entrance ($n = N$). For these three modes the classical condition of a node at the entrance certainly is not met. Thus, the approach proposed by Raichlen (1965) and Wilson et al. (1965) would be in error in predicting both the resonant periods and the mode shapes.

For a rectangular harbor with a similar geometry ($2b/\ell \cong 0.33$ and $d/b = 0.5$) the value of $k\ell$ at resonance for the first mode would be 1.15. (It should be noted, the width of both the harbor and the entrance used for this computation are approximate.) Referring to Fig. 7, the value of $k\ell$ at resonance for the first mode, the pumping mode, is about the same as that predicted for the corresponding mode for the rectangular harbor. Hence, the fact that a node does not exist at the entrance is partly due to the aspect ratio of the harbor and partly due to the partially closed condition.

The second basin investigated was Monterey Bay, California. This bay has experienced problems due to long period oscillations in the past, and in connection with an approximate numerical model presented by Raichlen (1965) and Wilson et al. (1965) it is of some interest. This is a large bay with a length along the *talweg* of approximately 56,000 ft from the shore to the entrance, a maximum width near the entrance of 138,000 ft and an average depth which decreases from 581 ft at the entrance to nearly zero at the shore. The longitudinal variation of the surface width and cross-section area are presented in Fig. 9. For the numerical calculations the harbor was divided into 20 sections.

The response curve for a location at the shoreline ($n = 1$) is presented in Fig. 10. It is interesting that due to the shape of the basin, the modes of oscillation are not defined as distinctly as those for Apra Harbor shown in Fig. 7. Nevertheless, the first three modes of oscillation are evident in Fig. 10 and are defined therein. For this harbor the ratio of width to length is nearly 2.5, thus three-dimensional effects would be expected for higher modes of oscillation. The results of the two-dimensional analysis of Raichlen (1965) and Wilson et al. (1965) for this case are indicated along the abscissa; in those analyses a node was assumed at the entrance. That this is not so is evident from Fig. 10 and is emphasized even more in Fig. 11 where the shape of the water surface is shown for the first three modes of oscillation. It is seen that nodal conditions

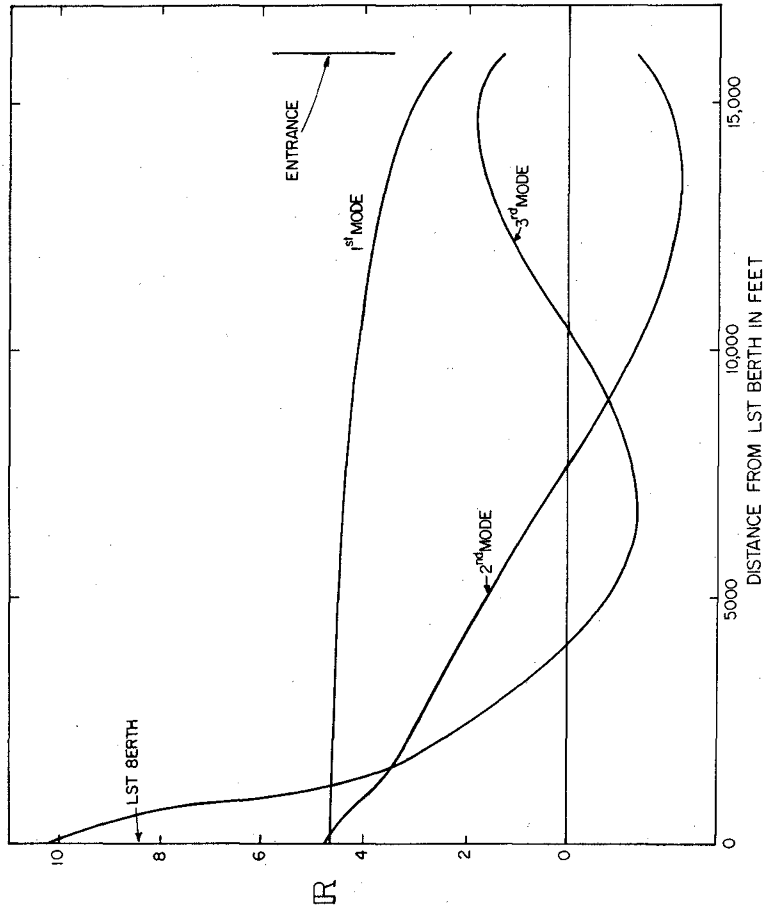

Fig. 8. The Shape of Three Modes of Oscillation for Apra Harbor, Guam, M.I.

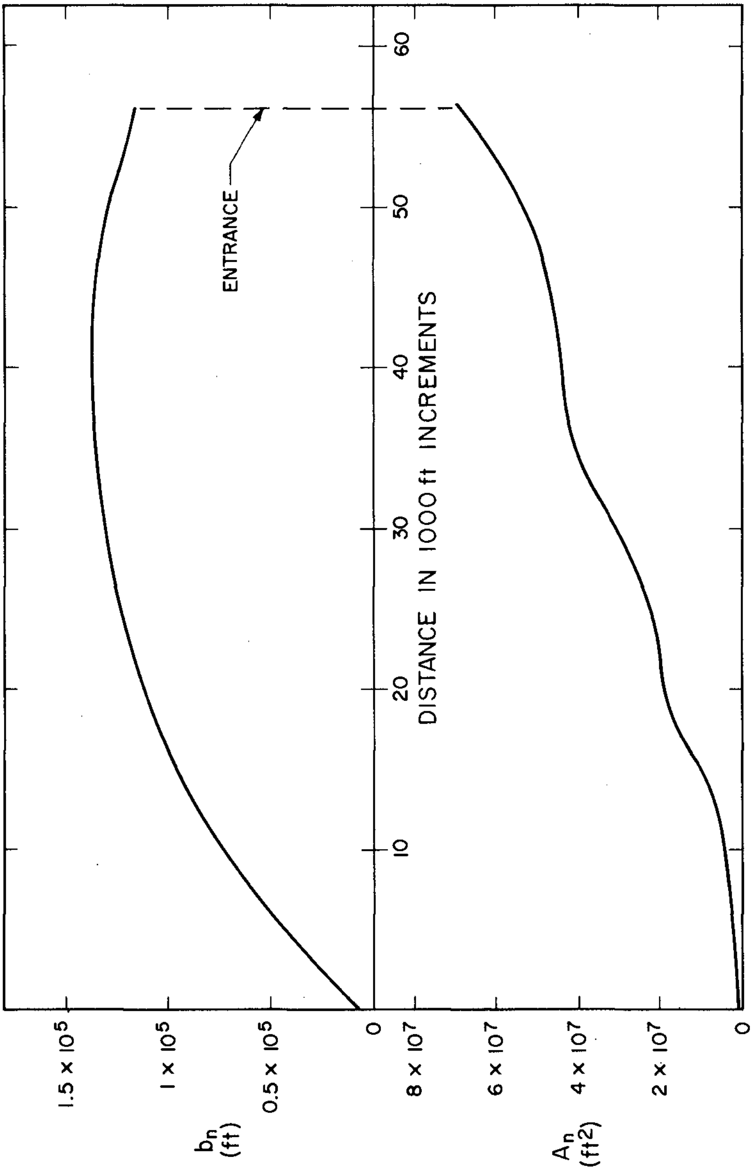

Fig. 9. Variation of Surface Width and Cross-Section Area, Monterey Bay, California.

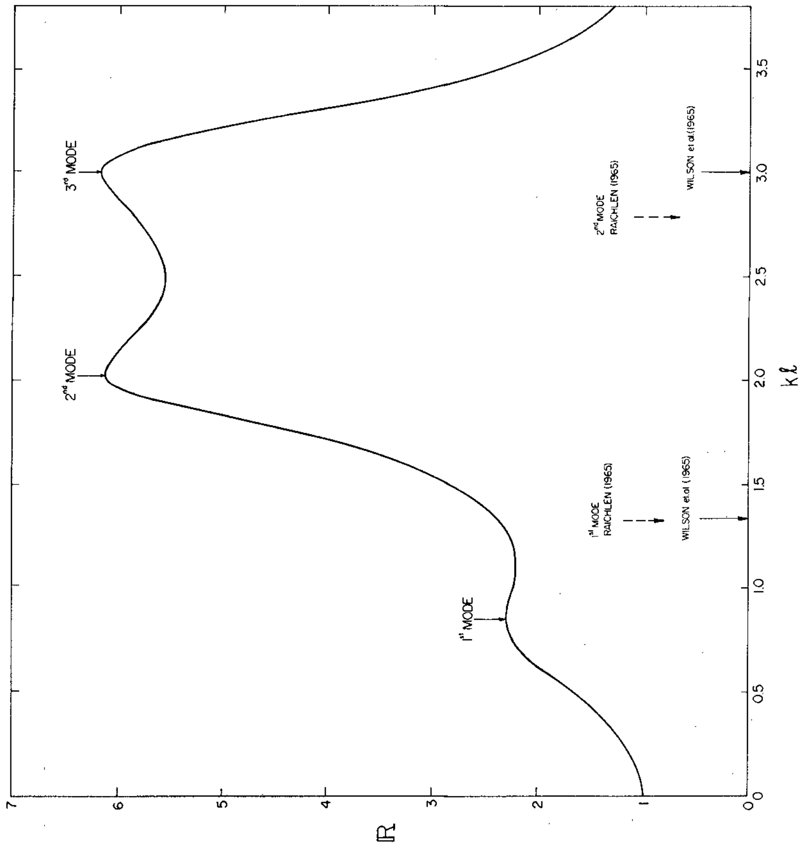

Fig. 10. Response Curve at the Backwall of Monterey Bay, California.

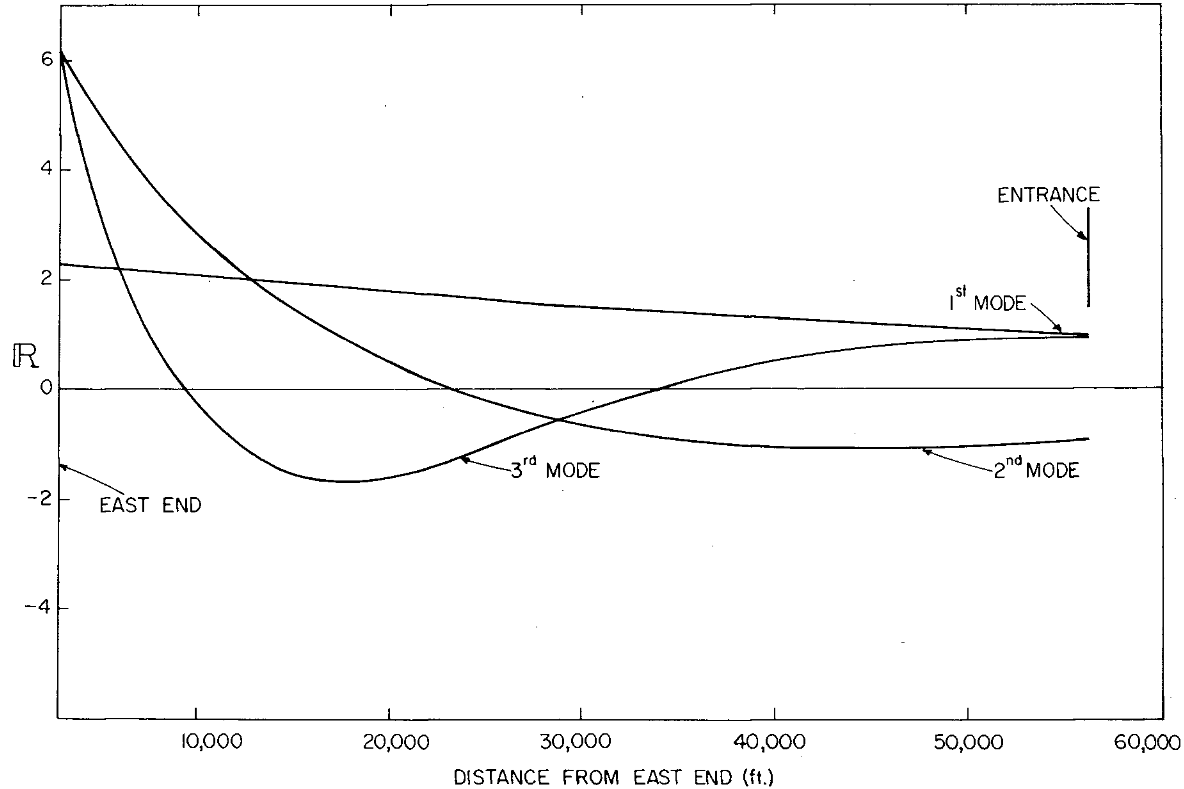

Fig. 11. The Shape of Three Modes of Oscillation for Monterey Bay, California.

are not met for any of these modes of oscillation. This emphasizes the importance of not imposing entrance boundary conditions for a harbor. The conditions at the entrance must respond naturally, depending upon the shape of the basin and the period of the incident waves.

CONCLUSIONS

The following major conclusions may be drawn from this study:

1. Using matching conditions at the entrance to a harbor it is possible to use two completely different methods of analyses in the two domains: the harbor region and the open-sea region.
2. Reasonably good agreement is found for the period of the lower modes of oscillation measured experimentally compared to the results of the numerical analysis for a harbor with a relatively complex shape.
3. The boundary condition at the entrance of a harbor must be allowed to develop naturally and a particular amplitude such as a node cannot be forced to occur. In the event this is done, the response of a harbor determined numerically may be considerably in error.

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