CHAPTER 17

WAVE.LENGTH, WAVE VELOCITY AND SHOALING CHARACTERISTICS OF RANDOM WAVES

Akira Kimura* and Yuichi Iwagaki**

ABSTRACT

Fundamental properties of random waves determined by the zero-up-cross method are discussed. Firstly, the wave length and the wave velocity are treated. Experiments are carried out with some supplemental definitions such as the spatial zero-up-cross method in a laboratory wave tank. Average values of K (= a ratio of measured wave length to theoretical one L_e/L_{th}) and R (= a ratio of measured wave velocity to theoretical one C_e/C_{th}) are very close to unity though data are considerably scattered. But the relations between $L_e \sim T_e$ and $C_e \sim T_e$ are slightly different from the theoretical ones where T_e is the measured wave period. Secondaly, breaking characteristics on sloping beaches are treated. The wave height H_b, the wave length L_b and the water depth h_b are measured at a breaking point. Breaking characteristics of random waves are considerably different from those of periodic waves. The measured wave height-water depth ratio H_b/h_b is plotted against the water depth-wave length ratio h_b/L_0 about 30 $\sim$ 40% below the breaker inception curve for periodic waves.

INTRODUCTION

There are two method of analysis for random sea waves. One is a wave spectrum analysis and the other is an analysis by the wave statistics. The zero-up-cross method is the most general one to treat the wave statistics. It has been considered that the method of wave spectrum analysis is supported by the small amplitude wave theory, but analysis by the zero-up-cross method are convenient to some extent. But recently the authers showed that the spectrum analysis method is not theoretically supported for random waves in a shallow-water region [Iwagaki and Kimura (1977)]. And the spectrum analysis method is not applicable also for discontinuous phenomena such as wave breaking, overtopping and etc. On the other hand, the analysis by the zero-up-cross method seems to be empirically usefull for treating these discontinuous phenomena. But this method does not have analytical bases and very few studies have examined the validity of this method. Then fundamental studies on properties of zero-up-crossing waves are desired to know the effectiveness of this method of analysis

* Research Assistant, ** Professor of Department of Civil Engineering Faculty of Engineering, Kyoto University, Kyoto, Japan

WAVE LENGTH

It is very dificult to measure the random wave length because no apparent scale or level line for measurement is available in the real sea condition. Some times oceanographers measured intervals of successive wave crests on ship sides. This definition of wave length is intuitive and uncertain. Pierson(1954) studied the following theoretical relation between the mean wave length and the mean wave period which are determined by the crest-to-crest method:

$$L_m = K \frac{gT_m^2}{2\pi} \tag{1}$$

and concluded that K=2/3 for a uni-directional Neumann spectrum, where g is the acceleration of gravity, L_m and T_m are the mean wave length and the mean wave period of crest-to-crest wave. He assumed that wave length of component waves are equal to those by the small amplitude wave theory. Ewing(1969) studied the theoretical relation between the mean wave length and the mean wave period of crest-to-crest waves for Pierson-Moskowitz spectrum. He introduced directional spectra and concluded that K in Eq.(1) changes from 1/3 to 3/4 according to an increase of truncation frequencies in a high frequency region of the power spectrum. But the crest-to-crest method is not natural to determine the wave length, because there are many maxima on the surface of random waves as seen in Fig.1 if the spectrum width is not nallow [Longuet-Higgins(1957)]. The zero-up-cross method seems preferable for a definition of the wave length. In this section, wave lengths determined by the zero-up-cross method are treated.

DEFINITION OF WAVE LENGTH

Generally the wave period is determined from a record measured with a wave gauge at a fixed point. On the other hand, the wave length is usually determined from a photograph. Then it is difficult to treat the wave length and the wave period as properties of the same wave by the difference of definitions, because the random wave profile changes gradually with propagation. Changes of the wave profile is very slight and wave propagation can be easily followed if the distance of propagation is short. Fig.2 shows wave records measured by three wave gauges installed 1m apart in the wave tank. It can be seen that changes of wave records are very small. Then the wave length and wave period of random waves can be determined as those for periodic waves in case when the travelling interval is very short. But as a nature of random waves, wave profiles change gradually. Then following two supplemental definitions are made to determine the wave length and the wave period.

(1) The wave period is determined from a record measured with the No.2 wave gauge shown in Fig.2 by the zero-up-cross method.
(2) The wave length is determined from a photograph taken at an instance when the water surface crosses the still water level downward at the No.2 wave gauge.

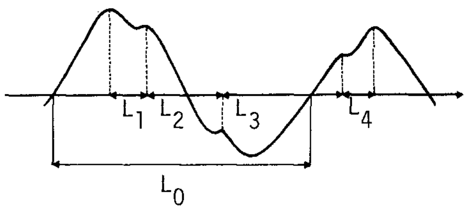


Fig.1 Definition of wave length

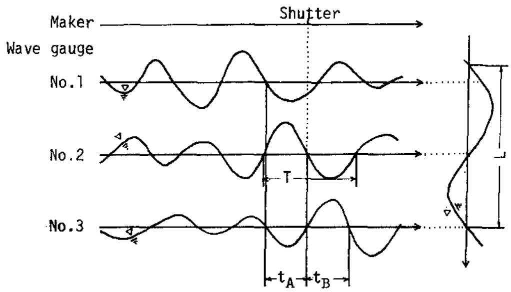


Fig.2 Definition of wave period and wave velocity

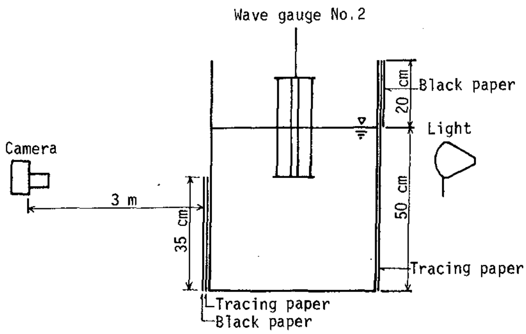


Fig.3 Equipments for photographing

EXPERIMENTS

A random wave generator of electro-hydraulic type is installed at one end of a 27m long, 50cm wide and 75cm deep glass sided wave tank. Measurements were carried out in a 4m long interval apart 12m from the wave generator. On the surface of fore-side glass, a 3mm wide black tape was marked to show a still water level and scale marks were placed with the same black tape. Fig.3 shows illustration of the equipment for photographing. Tracing papers were used to make scattered light. Rest part of the glass was covered with black papers to prevent a camera from direct light of lamps. Random waves were generated with a random wave simulation system [Kimura and Iwagaki(1976)]. Fig.4 shows power spectra of random waves used in this study. A photograph was taken at the instant when the water surface crossed the black tape downward at the No. 2 wave gauge. This instant was automatically marked on a chart of the pen-oscillograph which was connected with a flash terminal socket of the camera. Photo-1 shows an example of the wave profile. Wave lengths were analyzed from these photograpgs by using a film motion analyzer.

EXPERIMENTAL RESULTS

Typical values of the mean wave height H_m, the mean wave period T_m, the significant wave height $H_{1/3}$ and the significant wave period $T_{1/3}$ are shown in Table 1. f_p is the peak frequency of the power spectrum. Measured wave lengths are plotted in Fig.5 (a) ∿ (d). Chaine lines in these figures mean the theoretical relation between the wave length and the waveperiod by the small amplitude wave theory. About 250 data are plotted in each figure. Data are considerably scattered but tendencies of data seem in average to agree fairly well with the theoretical curves in every cases. And the value of $\bar{K}$ which makes D minimum in the following equation is very close to unity as shown in Table 1:

$$D = \sum [L_e - \bar{K} L_{th}]^2 \qquad (2)$$

in which L_{th} is the theoretical wave length by the small amplitude wave theory under the condition that the water depth is 50cm. Then it is known that the wave length and the wave period satisfy the theoretical relation approximately if the discussion is confined only on the mean value. But it can be seen from these four figures that there are little difference between the mean tendency and the theoretical curve if data are examined carefully in detail. The mean tendency of data has slightly mild inclination compared with the theoretical curve. In fig.6 (a) ∿ (d), relations of L_e/L_{th} versus T_e of measured data are plotted in each case. Values of L_e/L_{th} decrease gently with increase of T_e in every figures. Fig.7 shows a results of best fit curves with the 3rd order polynominal expression. The approximated curve in each case has slightly mild inclination compared with the theoretical curve. Ant it is very interesting that each curve does not coincide but shifts upward as f_p decreases.

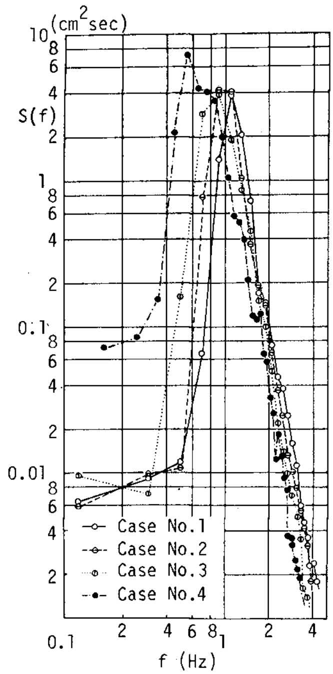


Fig.4 Power spectra of experimental waves

Photo 1 Wave profile of random wave

Table-1 Statistical properties of random waves

Case No.	f_p (Hz)	T_m (sec)	H_m (cm)	$T_{1/3}$ (sec)	$H_{1/3}$ (cm)	$\overline{K}$	T_c (sec)	$\overline{R}$
1	1.2	0.88	5.21	0.89	8.15	0.99	0.92	1.02
2	1.0	0.94	5.28	0.99	8.36	0.98	1.21	1.02
3	0.8	1.00	4.98	1.11	7.82	0.95	1.15	1.00
4	0.6	1.30	5.83	1.50	9.04	0.96	1.32	1.02

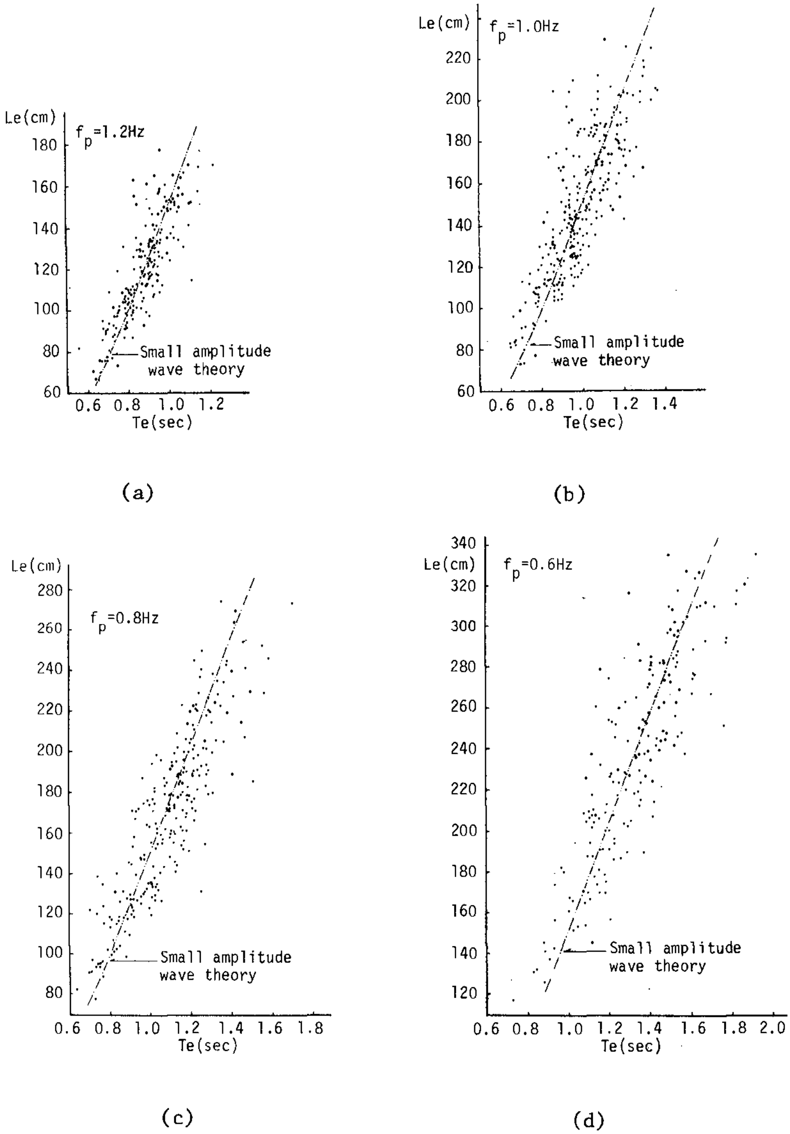


Fig.5 Relations between wave length and wave period for random waves

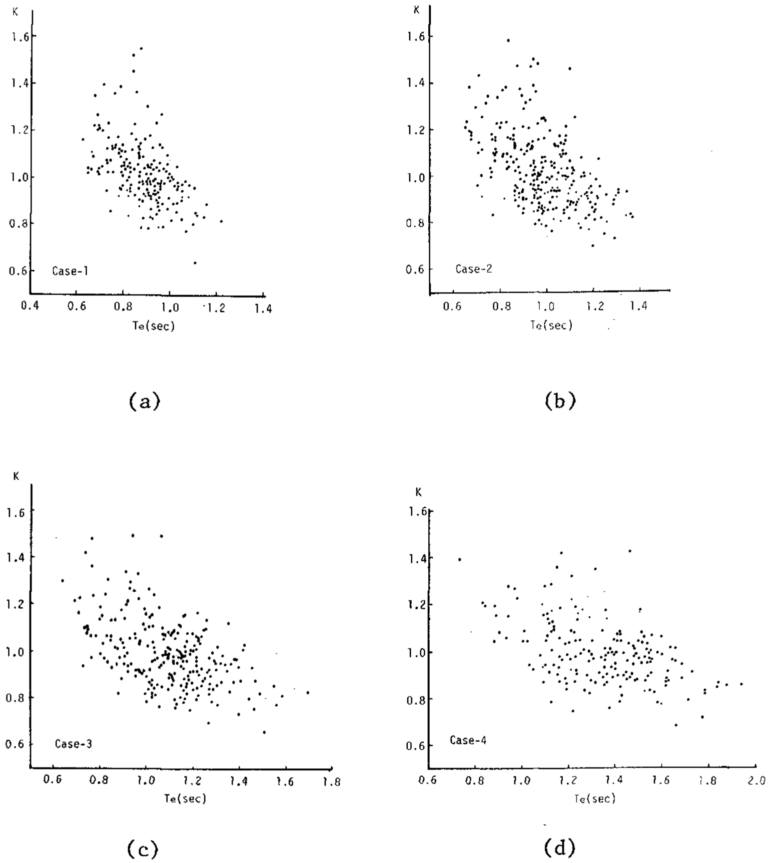


Fig.6 Relations between K ($= L_e/L_{th}$) and T_e

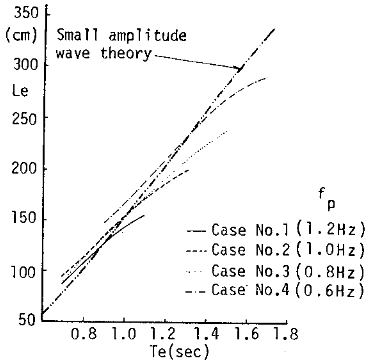


Fig.7 Best fit curves of wave length ∿ wave period relations for measured data

Table-2 Water depth at each wave gauge

Case No.	i	W-1	W-2	W-3	W-4	W-5	W-6	W-7	W-8	W-9	W-10	W-11	W-12
		h(cm)											
I	1/10	50	50	40	30	20	10	-	-	-	-	-	-
II	1/20	50	50	50	50	45	40	35	30	25	20	15	10

Table-3 The mean water depth of measured breaking waves

Case No.	i	f_p (Hz)	h_b (cm)
I-1	1/10	0.8	14
I-2	1/10	1.0	12
I-3	1/10	1.2	13
II-1	1/20	0.8	21
II-2	1/20	1.0	21
II-3	1/20	1/2	19

Then the wave length of random waves cannot be determined only by the water depth, but f_p or the mean wave period is needed in addition to them. T_c in Table 1 shows the period of intersection between the best fit curve and the theoretical curve.

WAVE VELOCITY

It has already explained that the profile of a random wave is conserved with very small change if the distance of propagation is very short. Then the wave velocity can also be determined by measuring the time spent for a wave crest or a zero-crossing point to propagate a fixed interval. In this section, the velocity of zero-down-crossing point is determined as the wave velocity C_e. The wave gauge interval from the No.1 wave gauge to the No.3 wave gauge devided by t_A+t_B is determined as the wave velocity in this study, in which t_A and t_B are the times spent for the zero-down-cross point to propagate the wave gauge No.1 to No.2 and No.2 to No.3 respectively. The wave period T_e was determined by the zero-up-cross method with the No.2 wave gauge record as before. Same waves as used to measure the wave length were sellected to determine the wave velocities. Experimental data are plotted in Fig.8 (a) $\sim$ (d). Data are considerably scattered, but agree farly well with theoretical curve shown by the chaine line. And $\bar{R}$ which makes E in the following equation minimum is very close to unity as shown in Table 1:

$$E = \sum [C_e - \bar{R}\, C_{th}]^2 \tag{3}$$

where C_{th} is the theoretical wave velocity calculated with the wave period T_e by the small amplitude wave theory. Then if discussions are confined only on the mean value, the relation between the wave velocity and the wave period satisfies approximately the theoretical relation by the small amplitude wave theory. But it can be seen from these figures that many of the data are plotted above the theoretical curve in the region where the wave period is small. Fig.9 shows the best fit curves for the plotted data with the 3rd order polynominal expression. The best fit curve is plotted above the theoretical curve in the region where the wave period is small and gradually approaches to the theoretical curve as T_e increases. It is interesting that each curve does not coincide when T_e is small, but shifts slightly upward with increase of f_p. f_p or the mean wave period is needed in addition to the wave period and the water depth to calculate the wave velocity when T_e is small. But when T_e is greater than $1.1T_m$, the wave velocity can be calculated approximately by the water depth and the wave period.

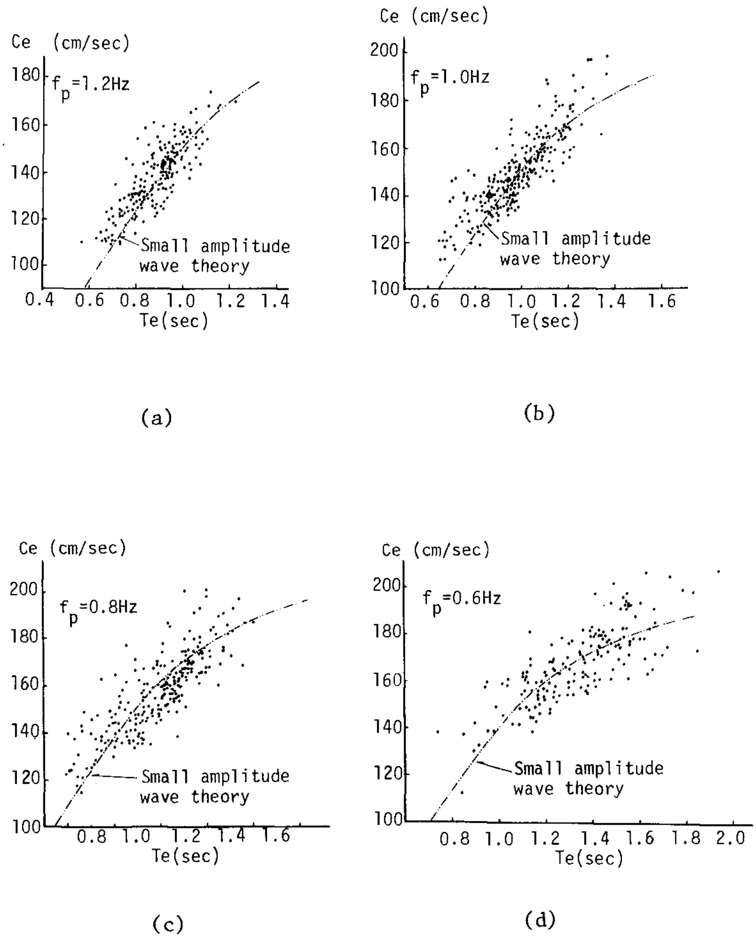


Fig.8 Relations between wave velocity and wave period for random waves

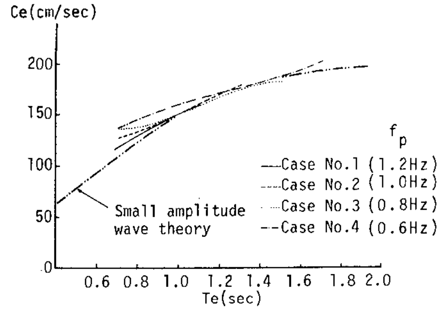


Fig.9 Best fit curves of wave velocity ∿ wave period relations for measured data

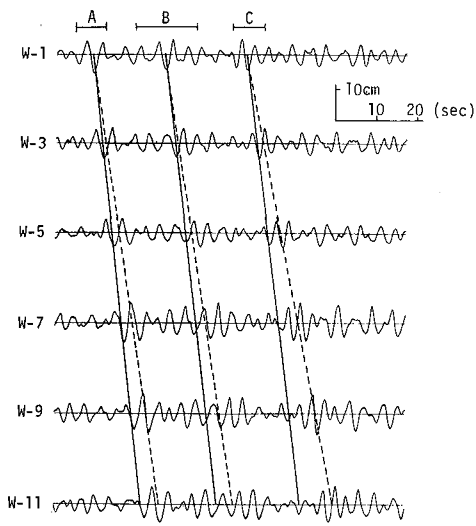


Fig.10 Wave velocity and group velocity over long distance

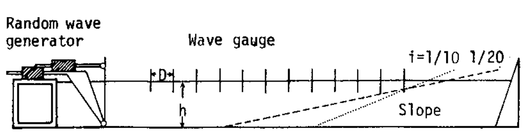


Fig.11 Experimental equipments for wave breaking

The results so far are for the wave velocities which are measured in the short distance. Fig.10 shows the records measured with 6 wave gauges installed 2m apart in the wave tank. It can be seen that wave profiles change gradually with propagation, but clear crests are easily followable. Then mean velocities of random waves over a long distance can be also determined. The solid lines show the theoretical wave velocities which are calculated with the mean periods of the wave groups marked as A,B and C above the W-1 wave gauge record. Each line starts from a clear trough at the middle part of the wave group in the W-1 wave gauge record. The solid lines connect the same wave troughs, then wave velocities over the short distance are very close to the theoretical wave velocities. But each line go over gradually the wave group among propagation. The theoretical wave velocities are a little faster than the velocities of wave groups. Broken lines show the group velocities calculated with the same wave periods as those of the solid lines. These broken lines always pass through middle part of the wave groups. Then over the long distance the wave velocity seems to be equal to the group velocity.

BREAKING CHARACTERISTICS OF RANDOM WAVES

Breaking characteristics of periodic waves have been studied theoretically and experimentally for many years. Many significant results were obtained from these studies. And breaking characteristics of random waves have been treated by Collins(1970), Battjes(1972), Kuo et al(1972), Nath et al(1974) and Goda(1975). They discussed changes of mean and significant wave heights or deformation of probability distribution of the wave height by wave breaking under the assumption that breaking characteristics of the zero-up-crossing waves are equal to those of regular waves. Collins used the breaking criterion by Le Méhauté and Koh.

$$H_b/H_0 = 0.76\ i^{1/7}\ (H_0/L_0)^{-1/4} \qquad (4)$$

where H is the wave height, L the wave length and i the slope of the beach, suffix b means the value at the breaking point and suffix 0 in deep water. Battjes used the following theoretical relation by Miche and Hamada:

$$H_b/L_b = 0.142\ tanh(2\pi h_b/L_b) \qquad (5)$$

where h_b is the water depth at the breaking point. Kuo et al recommended the following criterion obtained from field observations:

$$H_b/h_b = 0.63 \qquad (6)$$

Nath et al used the following equation proposed by Dean:

$$H_b = 0.278\ T_b^2 \qquad \text{(m-sec)} \qquad (7)$$

Goda used the approximate expression of breaker inception proposed by himself for regular waves:

$$H_b/L_0 = A\ [1.0-exp\{-1.5\pi(h_b/L_0)(1.0-1.5i^{4/3})\}] \qquad (8)$$

where A=0.17.

But the common problem to these studies is the fundamental assumption that breaking characteristics of zero-up-crossing waves are same as those of regular waves. In this section, breaking characteristics of zero-up-crossing waves on a sloping beach are examined experimentally to make clear the above assumption.

EXPERIMENTS

The same wave tank and the random wave generator were used in the experiments. Sloping beaches of 1/10(Case-I) and 1/20(Case-II) were installed in the wave tank and 6 wave gauges(Case-I) or 12 wave gauges (Case-II) were provided with 1m interval on the slope as shown in Fig.11. THe water depth at each wave gauge point is shown in Table 2. Measurements were carried out in the interval of 3m long around the breaking point. The water depth changes from 40cm to 10cm in Case-I and from 25cm to 10cm in Case-II in this interval. Same equipments for photographing are provided as shown in Fig.3. A camera and a 16mm movie camera were set 3m apart from the wave tank. Photographs were analyzed and several properties of breaking waves were determined by the following definitions:

(1) The wave height H_b and the wave length L_b at the breaking point are determined from the photograph taken at the instance of breaking as shown in Fig.12. But the water depth h_b at the breaking instant is determined from the 16mm movie film.

(2) The period of breaking wave is determined from a record measured by the nearest offshore wave gauge.

These definitions are different somewhat from those for reguler waves, but very close values to those by the usual method were obtained at the preliminary experiments. Fig.13 shows the power spectra used in the experiments of which f_p are (1) 0.8Hz, (2) 1.0Hz and (3) 1.2Hz. Hereafter the case number will be expressed as I-1, which means that the experiment was carried with 1/10 slope and random waves of the peak frequency 0.8Hz.

REPRESENTATION OF EXPERIMENTAL RESULTS

The mean water depth of wave breaking measured in each case is

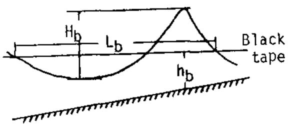


Fig.12 Definition sketch of breaking wave height H_b, wave length L_b and water depth h_b

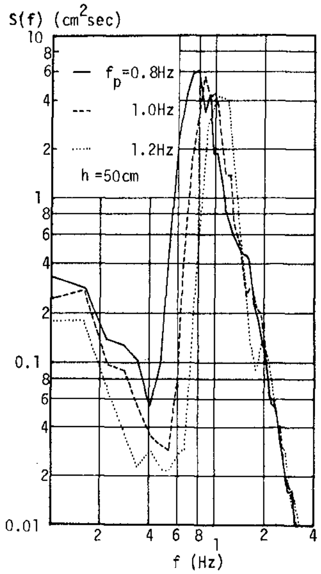


Fig.13 Power spectra of experimental waves for for wave breaking and shoaling

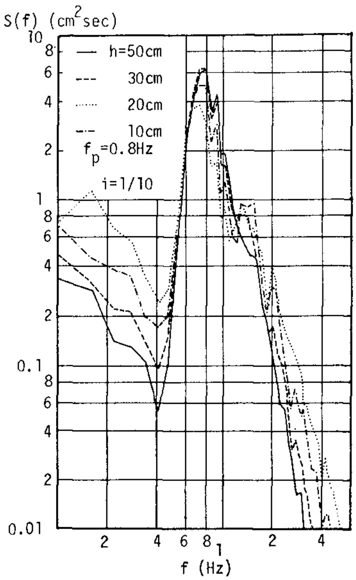


Fig.14 Transformation of power spectrum on 1/10 slope

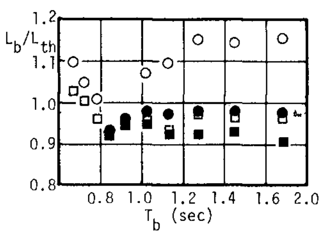


Fig.15 L_b/L_{th} of regular waves

shown in Table-3. There were very few waves which break in the region where h>25cm in Case-I. Most of waves break in the region between the water depth of 25cm and 15cm in Case-II . In both cases very small waves and reproduced waves after breaking break generally in the region where the water depth is less than 5cm. These waves in such shallow region were excluded because they seem to be affected by surf beat phenomena. Breaking type of most waves was of plunging breaker. Spilling type breaking was observed sometimes in the region where h>30cm. Fig.14 shows an example of transformation of the power spectrum. The water depth changes from 50cm to 10cm in this figure. Where the water depth is larger than 30cm there is no apparent change in the shape of power spectrum. But in the region where h is less than 20cm, the energy of power spectrum around the peak frequency decreases and increases in the regions where f<0.4Hz and f>1.0Hz. This means that sudden changes happned on the water surface in this shallow region.

WAVE LENGTH AT BREAKING POINT

Breaking characteristics of reguler waves on 1/10 and 1/20 slopes were examined in the experiments. The breaking wave length L_b is compared with the theoretical one L_{th} in Fig.15. White circles show experimental data on 1/10 slope and black circles show those on 1/20 slope. It can be seen in this figure that the ratio of measured wave length to theoretical one L_b/L_{th} on 1/20 slope are very close to unity but the data on 1/10 slope are plotted considerably above unity. This is because the water depth to calculate the theoretical wave length is determined at the crest phase of the wave as shown in Fig.12. White squares(1/10) and black squares(1/20) show the modified values of L_b/L_{th}, where L_{th} is calculated with the water depth at the center of the wave profile. The data are corrected to some extent. Figs.16(a) and (b) show the experimental results of L_b/L_{th} for random waves in Cases I and II respectively. L_{th} in these figures were calculated with the water depth at the wave crest phase. Many data in Case-I are plotted above the line of unity L_b/L_{th}, but in Case-II many data are plotted below this line as seen in Fig.15. And with respect to the mean tendency of plotted data, L_b/L_{th} decreases as T_b increases in both cases and mean line for each case shifts rightward as f_p decreases. Such tendencies have been observed in Fig.5.

BREAKING CHARACTERISTICS OF RANDOM WAVES

Miche and Hamada gave the breaking criterion for periodic waves shown by Eq.(5). Experimental results of reguler waves on 1/10 and 1/20 slopes are compared with Eq.(5) which is shown by the solid line in Fig. 17. The data on 1/20 slope are plotted below the theoretical curve, but the data on 1/10 slope agree fairely well with this equation. The relation between H_b/L_b and h_b/L_b for random waves is plotted in Figs.18(a) for

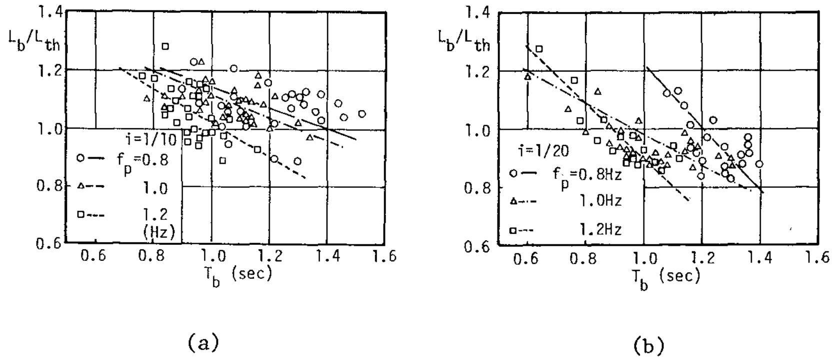


Fig.16 L_b/L_{th} of random waves

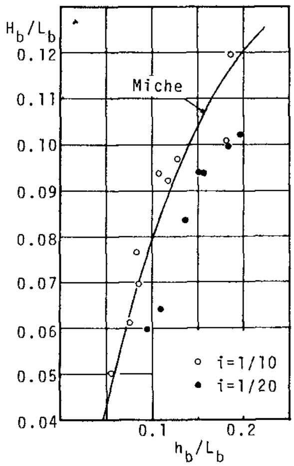
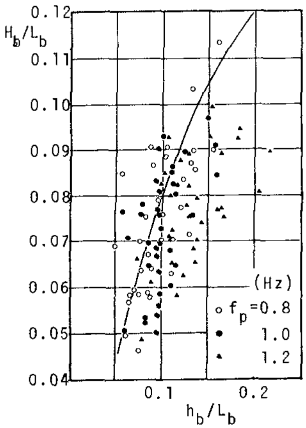
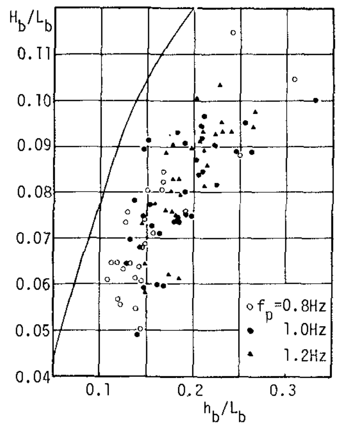


Fig.17 Relations between H_b/L_b and h_b/L_b for regular waves



(a)



(b)

Fig.18 Relations between H_b/L_b and h_b/L_b for random waves

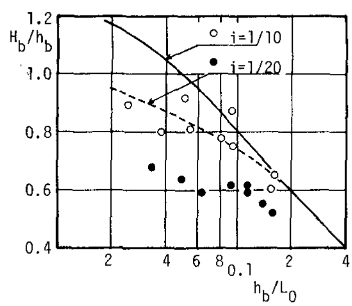


Fig.19 Relations between H_b/h_b and h_b/L_0 for regular waves

1/10 slope,and (b) for 1/20 slope respectively. Mean tendency of data agrees fairly well with the theoretical curve in Case-I, but in Case-II the data are plotted about 30% below the theoretical curve. Then the difference of slope affects largely breaking characteristics of random waves. To estimate the influence of slope, these data are compared with the breaker inception curve by Goda[Eq.(8)]. Firstly, experimental data of regular waves are compared with the breaker inception in Fig.19, in which solid line is for 1/10 slope and the break line is for 1/20 slope. Data are plotted below the lines in both cases. Then experimental results for random waves are plotted in Figs.20(a) and (b). In these figures L_0 is the wave length in deep water calculated with the measured wave period T_b. It is found from both figures that H_b/h_b decreases as h_b/L_0 increases, so that the breaking criterion such that H_b/h_b= constant for cnoidal waves or solitaly wave is not available in this region ($0.03<h_b/L_b<0.4$). Then the effect of slope must be considered for breaking characteristics of random waves as seen in Eq.(8) or Eq.(4). The data are compared with Eq.(8) in Fig.20, in which the data are plotted about 30∿40% below the curves in both cases of 1/10 and 1/20 slopes. Then it can be concluded that random wave break easily compared with regular waves. Dotted lines in these figures are modified breaker inception curves with changing A in Eq.(8) from 0.11 to 0.13. It is known that A=0.12 for Case-I and A= 0.11 for Case-II are adequate.

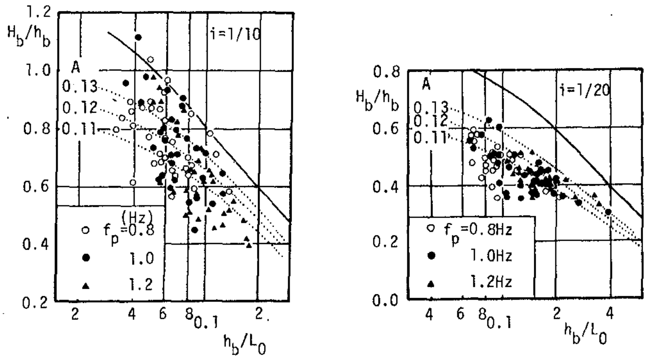
Goda pointed out that changes of the water depth by surf beat affect largely breaking characteristics of random waves. Fig.21 shows the wave records obtained in Case II-1, in which dotted lines express what is called surf beat obtained by cutting off the higher frequency components than 0.35Hz. The mean amplitude decreases as the water depth increases. Apparent surf beat phenomenon cannot be observed at h=25cm. The mean breaking depth of water measured in Case II-1 is 21cm as shown in Table 3, so that the change of water depth by surf beat phenomenon seems to be negligible in this case and also in other cases.

Some other factors will be considered which affect the phenomena of random wave breakings such as unsteady back wask, variation of mean water level due to non-stationary radiation stress and etc. But it is very difficult to estimate the influence of these phenomena because they have not been made clear yet.

CONCLUSION

Fundamental properties of random waves defined by the zero-up-cross method are examined experimentally in a laboratory wave tank, and the following conclusions could be obtained:

The properties such as the wave length and the wave velocity are very close to those of regular waves, but certain properties in wave breaking are considerably different though random waves have their own inherent characteristics. Then it can be finally concluded that the analysis by using the zero-up-cross method is not a conventional mehtod for random waves but an effective method to analyze dynamic properties of random waves. However it is worth mentioning that certain dynamic



(a) (b)

Fig.20 Relations between H_b/h_b and h_b/L_0 for random waves

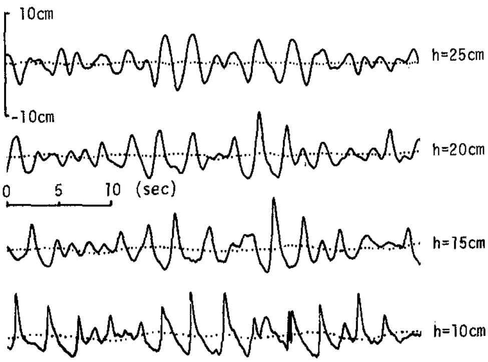


Fig.21 Surf beats at various water depth

properties of zero-up-crossing waves are considerably different from those of regular waves.

ACKNOWLEDGEMENT

The authors wish to acknowledge to Mr. N.Kishida for his assistant throughout the experiments.

REFERENCES

Battjes, J. A. : Set-up due to irregular waves, Proc. 13th Coast. Engg. Conf., 1972, pp.1993-2004.

Collins, J. I. : Probabilities of breaking wave characteristics, Proc. 12th Coast. Engg. Conf., 1970, pp.399-414.

Ewing, J. A. : A note on wave length and period in Confused seas, Jour. of Geoph. Res., Vol.74, No.6, 1969, pp.1406-1408.

Goda, Y. : Irregular wave deformation in the surf zone, Coastal Engg. in Japan, Vol. XVIII, 1975, pp.13-26.

Kimura, A. and Y. Iwagaki : Random wave simulation in a laboratory wave tank, Proc. 15th Coast. Engg. Conf., 1976, pp.368-387.

Longuet-Higgins, M, S. : The statistical analysis of a random, moving surface, Jour. Fluid Mech., Vol.249, A. 966, 1957, pp.321-387.

Nath, J. H. and F. L. Ramsey : Probability distributions of breaking wave heights, Waves'74, 1974, pp.379-395.

Kuo, C. T. and S. G. Kuo : The change of the wave height distribution of wind waves by the breakings, Proc. 19th Coast. Engg. in Japan, 1977, pp.137-142. (in Japanese)

Pierson, W. J. Jr : An interpretation of the observable properties of sea waves in terms of the energy spectrum of the gaussian record, Trans. American Geophysical Union, Vol.35, No.5, 1954, pp.747-757.