

CHAPTER 20

ANALYSES OF HINGED WAVEMAKERS FOR RANDOM WAVES

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ABSTRACT

The theoretical dimensionless frequency response functions for the wavemaker stroke spectrum, $Q_S(\omega)$, and for the wavemaker dynamic pressure moment spectrum, $Q_M(\omega)$, were verified experimentally in the Oregon State University-Wave Research Facility (OSU-WRF) from random waves which were simulated from two types of two-parameter theoretical design wave spectra. The random motions of the wavemaker were first digitally simulated by a unique inverse stacked FFT algorithm which were then used to drive the wavemaker through digital-to-analog converters (DAC). The dimensionless frequency response function for the hydrodynamic pressure moment spectrum was not measured directly but was computed from the dynamic covariance equations for a hinged wavemaker of variable-draft by linear combinations of related dimensionless frequency response functions and cross-spectral estimates. The measured estimates were found to agree reasonably well with the analytical approximations, and these measured stochastic hydrodynamic pressure moment estimates on a hinged wavemaker of variable-draft are believed to be unique.

INTRODUCTION

The elaborate design curves for hinged wavemakers presented by Gilbert, *et al* (1971) have been extended in a minor way (for the case of deterministic waves only) to variable-draft hinged wavemakers by Hudspeth and Chen (1978). Experimental comparisons of these theoretical design curves with deterministic waves which were approximately 25% of the theoretical breaking limit were reported by Hudspeth and Leonard (1978a) for measured data which span one decade of dimensionless relative wave frequency. The squared modulus of the dimensionless wavemaker gain function, S/H , in which S = the wavemaker stroke and H = the wave height, for deterministic waves may be shown to be equivalent to the squared modulus of the dimensionless frequency response function for the wavemaker stroke spectrum; *viz*,

$$\left| \frac{S}{H} \right|^2 = |Q_S(\omega)|^2 = \frac{S(\omega) \overline{S}(\omega)}{\eta \eta} \quad (1)$$

and the squared modulus of the dimensionless wavemaker hydrodynamic pressure moment about the wavemaker hinge, M , may be shown to be equivalent to the squared modulus of the dimensionless frequency response function for the wavemaker hydrodynamic pressure moment spectrum; *viz*,

$$|Q_M(\omega)|^2 = \frac{S(\omega) \overline{M}(\omega)}{W^2 \eta \eta} \quad (2a)$$

in which

$$W = \rho g B h (h - \Delta) \quad (2b)$$

in which ρ = density of fluid; g = gravitational constant; B = width of waveboard; and the dimensions h and Δ are defined in Fig. 1. These theoretical dimensionless frequency response functions for the stochastic design of hinged wavemakers of variable-draft were verified experimentally in the Oregon State University-Wave Research Facility (OSU-WRF) for two types of two-parameter theoretical spectra; *viz*, the relatively narrow-banded Scott spectrum and the relatively broad-banded Bretschneider-Pierson-Moskowitz spectrum.

We shall first give the analytical expressions for the dimensionless frequency response functions required for the stochastic design of hinged wavemakers which were experimentally verified in the OSU-WRF. A brief description will be presented next. Finally, graphical comparisons will be given between the theoretical and experimental dimensionless frequency response functions. The experimental values were computed from measured time series that were synthesized from the two-parameter theoretical design wave spectra. These theoretical dimensionless frequency response functions should be useful for the stochastic design of hinged wavemakers of variable-draft in wave flumes which have water on one side only of the wavemaker.

FREQUENCY RESPONSE FUNCTIONS

From the dynamic covariance equations for a hinged wavemaker of variable-draft in the wave flume shown in Figure 1, it may be shown [cf. Hudspeth and Leonard (1978b)] that the real-valued dimensionless theoretical frequency response function for radial frequency $\omega = 2\pi f$ is given by

$$Q_S(\omega) = \frac{KH}{4 \sinh Kh} \Gamma(\kappa, K) \left[\frac{2Kh + \sinh 2Kh}{K(h-\delta) \sinh Kh - \cosh Kh + \cosh K\delta} \right] \quad (3a)$$

in which

$$\Gamma^2(\kappa, K) = \left[\frac{K}{\kappa} \right] \left\{ \frac{1 + \frac{2\kappa(h-\Delta)}{\sinh 2\kappa(h-\Delta)}}{1 + \frac{2Kh}{\sinh 2Kh}} \right\} \quad (3b)$$

provided that the propagating wave number in the constant depth domain immediately adjacent to the wavemaker of depth, h , is determined from

$$\omega^2 = gK \tanh Kh \quad (3c)$$

and provided that the wave number in the constant depth test section domain of depth, $h-\Delta$, is determined from

$$\omega^2 = g\kappa \tanh \kappa(h-\Delta) \quad (3d)$$

The dimensionless complex-valued frequency response function for the hydrodynamic pressure moment on the wavemaker may be computed from the dimensionless complex-valued moment amplitude given by Hudspeth and Chen (1978) according to

$$Q_M(\omega) = M_p'(\omega) + i M_e'(\omega) \quad (4)$$

in which the dimensionless moment due to the *propagating* mode is given by

$$M_p'(\omega) = \Omega \frac{\{K(h-\delta) \sinh Kh - \cosh Kh + \cosh K\delta\}}{Kh \cdot K(h-\Delta) \cdot K(h-\delta) \sinh Kh} \Gamma(\kappa, K) \quad (5a)$$

and the dimensionless moment due to the *evanescent* modes is given by

$$M_e'(\omega) = \Omega \frac{(Kh)^3 \{2Kh + \sinh 2Kh\}}{K(h-\Delta) \cdot K(h-\delta) \sinh Kh \{K(h-\delta) \sinh Kh - \cosh Kh + \cosh K\delta\}} \Gamma(\kappa, K) \sum_{n=2}^{\infty} \frac{\{K_n(h-\delta) \sin K_n h + \cos K_n h - \cos K_n \delta\}^2}{(K_n h)^4 \{2K_n h + \sin 2K_n h\}} \quad (5b)$$

in which

$$\Omega = (h-\Delta) \frac{\omega^2}{g} \quad (5c)$$

and the evanescent eigenvalues, K_n , are computed from

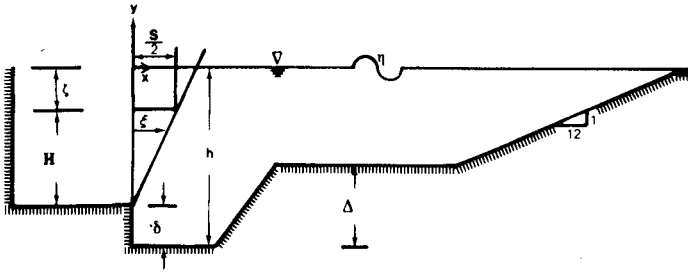


Fig. 1 Definition sketch for wave flume geometry with a hinged wavemaker of variable draft

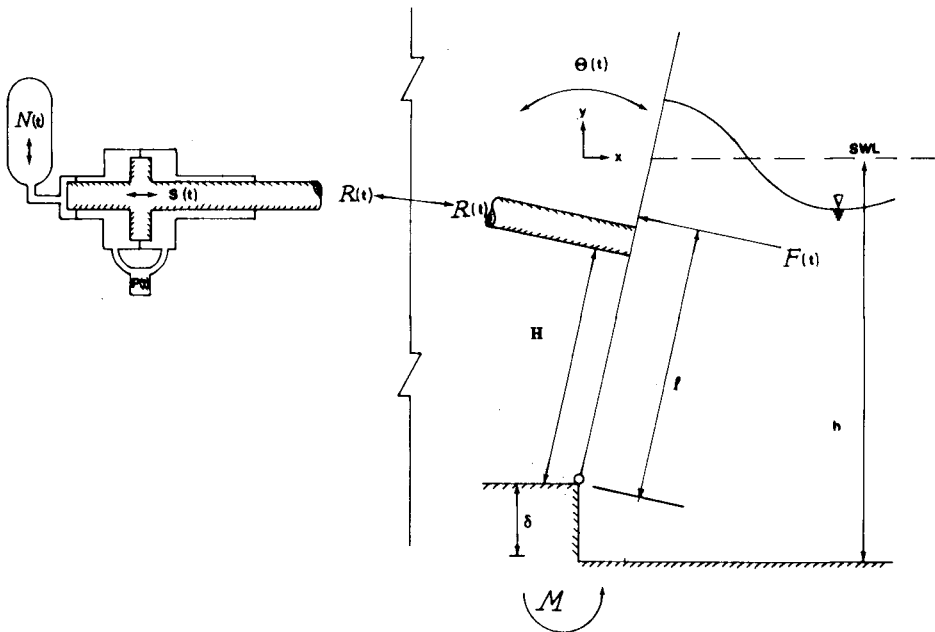


Fig. 2 Schematic representation of hydraulic piston and hinged wavemaker dynamic system in OSU-WRF

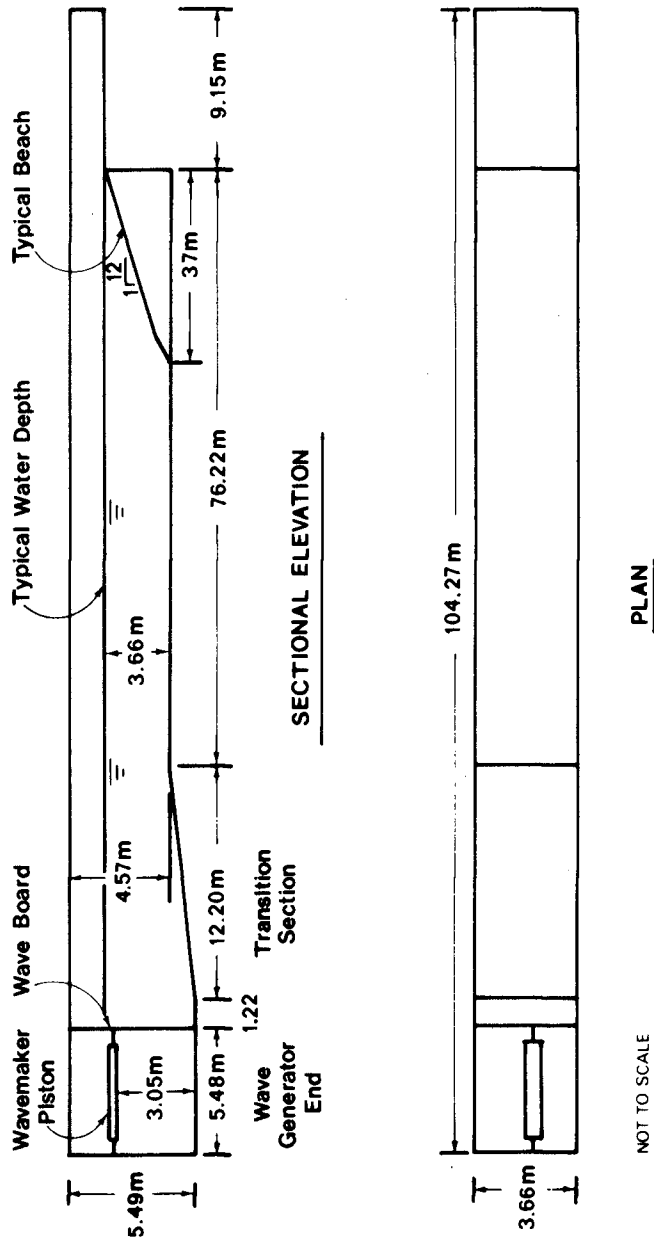


Fig. 3 Schematic representation of OSU-WRF geometry

$$\omega^2 = -g K_n \tan K_n h; n > 1 \quad (5d)$$

The dynamic equation of motion for the translational motion of the wavemaker hydraulic piston displacement, $S(t)$, shown schematically in Figure 2, is given by [cf. Hudspeth and Leonard (1978a)]

$$P(t) = \frac{M(t)}{H} - N(t) + A\ddot{S}(t) + B\dot{S}(t) \quad (6)$$

in which $P(t)$ = hydraulic piston pressure force; $M(t)$ = hydrodynamic pressure moment on the waveboard about the wavemaker hinge; H = hydrodynamic pressure force moment arm about the wavemaker hinge; $N(t)$ = dynamic nitrogen gas pressure force and the coupled translational mass of the hydraulic piston, m , and the rotational inertia of the hinged wavemaker, I/H^2 , is given by

$$A = m + \frac{I}{H^2} \quad (7a)$$

and the coupled translational viscous resistance of the hydraulic piston, μ , and the rotational viscous resistance of the hinged wavemaker, ν/H^2 , is given by

$$B = \mu + \frac{\nu}{H^2} \quad (7b)$$

The time dependent nitrogen gas pressure term, $N(t)$, was not recorded directly in the OSU-WRF and had to be estimated from the measured piston displacement, $S(t)$. The nitrogen gas system in the MTS designed OSU-WRF is used to backpressure the wavemaker piston in order to compensate for the static column of fluid on the wave flume side only of the hinged wavemaker [*vide* Fig. 2]. The time varying nitrogen gas pressure is a result of the small volume changes induced by the motion of the wavemaker piston. The total nitrogen gas pressure on the wavemaker piston may, therefore, be approximated by a static plus a time varying component according to the real gas equation:

$$N = N_s + N(t) = \frac{nZRTA_N}{V_N + A_N S(t)} \quad (8)$$

in which n = mass of the nitrogen gas; Z = the compressibility coefficient for a real gas; R = gas constant for nitrogen; T = temperature; V_N = static volume of nitrogen gas; and A_N = surface area of the terminal end of the wavemaker ram normal to the wavemaker stroke. For the experimental verifications in the OSU-WRF, the ratio of $A_N S(t)/V_N \ll 1.0$; and we may expand Eq. 8 by the binomial theorem to obtain, approximately,

$$N_s + N(t) = \frac{nZRTA_N}{V_N} \left\{ 1 - \frac{A_N S(t)}{V_N} + 0 \left(\frac{A_N S(t)}{V_N} \right)^2 \right\} \quad (9)$$

in which the static nitrogen gas pressure component exactly balances the hydrostatic moment on the wavemaker flap while the dynamic nitrogen gas pressure component acts like a stiffness element in opposing positive displacements of the piston.

Consequently, the experimental dimensionless frequency response functions were estimated by replacing the exact dynamic nitrogen gas pressure force in Eq. 6 with an equivalent linear spring element approximated by

$$N(t) = -KS(t) \quad (10)$$

in which the equivalent spring constant, K , is determined from a static moment balance between the hydrostatic pressure moment on the wavemaker flap and the static nitrogen gas pressure force according to

$$K = nZRT \left(\frac{A_N}{V_N} \right)^2 = \frac{\rho g h^3 B A_N}{6H V_N} \quad (11)$$

in which B = width of the hinged wavemaker flap in Fig. 2. Substituting Eq. 10 into Eq. 6 and evaluating the equivalent nitrogen gas spring constant, K , from the static water depth in the wave flume at the time of experimental testing according to Eq. 11, the coupled dynamic equation of motion for the hinged wavemaker system becomes

$$P(t) = \frac{M(t)}{H} + KS(t) + A\ddot{S}(t) + B\dot{S}(t) \quad (12)$$

The covariance function for a stationary stochastic process such as that described by Eq. 12 may be defined by [cf. Papoulis (1965)]

$$C(\tau) = E\{X(t) \cdot X^*(t+\tau)\} \quad (13)$$

in which $E\{\cdot\}$ = expectation operator and the superscript asterisk $*$ = the complex conjugate value for a stochastic process $X(t)$. The Fourier transform of the covariance function yields the frequency spectrum [cf. Papoulis (1965)]. Solving Eq. 12 for the hydrodynamic pressure moment on the wavemaker, $M(t)$, and forming the covariance function according to Eq. 13, the following dynamic covariance equation is obtained for a variable draft hinged wavemaker [cf. Hudspeth and Leonard (1978b)]:

$$C(\tau) = H^2 \{ C(\tau) + |Q(\omega)|^2 C(\tau) - Q(\omega) C(\tau) - Q(\omega) C(\tau) \} \quad (14)$$

MM PP w SS w PS SP

in which the complex-valued frequency response function for the wavemaker is given by

$$Q(\omega) = Q_I(\omega) + iQ_V(\omega) \quad (15a)$$

$$= (K - \omega^2 A) + i(\omega B) \quad (15b)$$

and $C_{PS}(\tau)$ = cross-covariance function between the hydraulic oil piston pressure, $P(t)$, and the wavemaker stroke, $S(t)$. We note that our definition for the complex-valued frequency response function is the reciprocal of the usual structural definition for the response receptance of a single-degree-of-freedom oscillator [cf. Robson (1963)]. Dividing the Fourier transform of Eq. 14 by the spectrum of the water surface elevation, $S_{\eta\eta}(\omega)$, and a non-dimensional factor defined by Eq. 2b, we obtain the following dimensionless frequency response function for the hydrodynamic pressure moment on the hinged wavemaker:

$$|Q_M(\omega)|^2 = \frac{S(\omega)}{W^2 S_{\eta\eta}(\omega)} \quad (16a)$$

$$= \left\{ \frac{H}{W} \right\}^2 \left\{ Q_P^2(\omega) + |Q_W(\omega)|^2 Q_S^2(\omega) - 2Q_I(\omega) \frac{C_{PS}(\omega)}{S_{\eta\eta}(\omega)} - 2Q_V(\omega) \frac{Q(\omega)}{S_{\eta\eta}(\omega)} \right\} \quad (16b)$$

in which $C_{PS}(\omega)$ and $Q_{PS}(\omega)$ = the co- and quad- spectrum, respectively, for the cross-spectrum between the hydraulic piston pressure and the wavemaker stroke. Graphically, we may now compute the dimensionless frequency response function for the hydrodynamic pressure moment on a hinged wavemaker of variable draft from a linear combination of individual frequency response functions with cross-spectral components. We remark that this method for verifying the dimensionless frequency response functions for the stochastic design of variable-draft hinged wavemakers from the dynamic covariance equations using measured realizations of the hydraulic piston pressure and the piston motion must be considered as an approximation in contrast to the direct measurement of the hydrodynamic pressure moment on the wavemaker. However, for the purpose of verifying the stochastic design method via the dynamic covariance equation, this approximation should be adequate.

OSU-WRF DESCRIPTION

The experimental verification of the theoretical dimensionless frequency response function for the wavemaker gain function, $Q_S(\omega)$, and for the hydrodynamic pressure moment on the wavemaker, $Q_M(\omega)$, were conducted in the Oregon State University-Wave Research Facility (OSU-WRF) shown schematically in Fig. 3. The wave flume is 104.27 m long, 3.66 m wide,

and has a relocatable bottom in the test section. Experimental data were obtained for a test section depth of $h-\Delta = 3.51$ m.

The hinged wavemaker in the OSU-WRF is sealed under pressure along both vertical sides and the horizontal hinged bottom so that water is not allowed in the dry well behind the wavemaker. This dry well reduces the power requirements for the wavemaker and also eliminates the need to place any wave dissipating material behind the wavemaker. The hinged wavemaker is controlled by a 112 kW, 24 MPa hydraulic pump through a hydraulic servo-mechanism mounted 3.05 m above the wavemaker hinge. The forced motion of the wavemaker may be either periodic or random and may be activated by either an electronic function generator or a digital time sequence synthesized on a PDP 11 E10 digital minicomputer through digital-to-analog converters (DAC). A description of a unique inverse finite Fourier transform algorithm (FFT) developed at the OSU-WRF to generate either periodic or random waves of the periodic-random type through a minicomputer is described by Hudspeth and Borgman (1978). The random motions of the hinged wavemaker in the OSU-WRF which were used to verify the linear wave theory dimensionless wavemaker frequency response functions for random waves were synthesized from a stacked FFT digital computer algorithm using the theoretical dimensionless wave-maker frequency response function, $Q_S(\omega)$, and two types of theoretical two-parameter wave spectra. A comparison of two methods used to digitally filter Gaussian white noise to generate random wave spectra in the OSU-WRF is described by Hudspeth and Nath (1978). Similar digital random wave simulations have been described by Weber and Christian (1974), Funke (1974), van Oorschot and Koopmans (1976), and Kimura and Iwagaki (1976). Additional information on the OSU-WRF is presented in Nath (1978) and in Nath and Kobune (1978).

The two types of two-parameter theoretical wave spectra selected to synthesize the random waves used to verify the dimensionless frequency response functions are listed in Table 1. The Pierson-Moskowitz spectrum may be seen from Table 1 to be equivalent to the Bretschneider spectrum when the two parameters of m_0 and ω_0 are employed. The two parameters required to synthesize the wavemaker motion from these spectra are: 1) the peak spectral frequency, ω_0 , and 2) the total area under the spectrum which may be computed from

$$m_0 = \int_0^{\infty} \frac{S(\omega)}{\eta\eta} d\omega \quad (17)$$

for one-sided spectral density functions.

Table 1. Theoretical Two-Parameter Wave Spectra

SPECTRUM	$S_{\eta\eta}(\omega)$
SCOTT	$3.424 m_o \exp - \left\{ \frac{(\omega - \omega_o)^2}{0.065(\omega - \omega_o + 0.265)} \right\}^{\frac{1}{2}}$ $0; 1.65 < \omega - \omega_o < -0.265$
BRETSCHNEIDER	$5 \frac{m_o}{\omega_o} \left(\frac{\omega_o}{\omega} \right)^5 \exp - \left[1.25 \left(\frac{\omega_o}{\omega} \right)^4 \right]$
PIERSON-MOSKOWITZ	$5 \frac{m_o}{\omega_o} \left(\frac{\omega_o}{\omega} \right)^5 \exp - \left[1.25 \left(\frac{\omega_o}{\omega} \right)^4 \right]$

SPECTRAL RESPONSE FUNCTIONS FOR HINGED WAVEMAKERS

The dimensionless frequency response function for the wavemaker stroke spectrum, $Q_S(\omega)$, was estimated from measurements of the wavemaker stroke, $S(t)$, and the instantaneous water surface elevation, $\eta(t)$. The measured wavemaker stroke was recorded through an LVDT mounted just above the wavemaker hinge in the dry well behind the wavemaker [*vide*, Fig. 3]. This LVDT signal was calibrated by rotating the wavemaker through a discrete set of arcs measured at the height of the wavemaker piston and by then correlating the LVDT signal with these measured rotations. The instantaneous water surface elevation was recorded in the test section of the wave flume by a Sonic System Model 86 sonic wave profiler. The sonic wave profiler was calibrated by displacing the sonic transducer through a discrete sequence of vertical distances measured above the still water level in the absence of waves and by correlating the sonic transducer output with these measured displacements.

The two-sided spectral densities for the two data records which are required in order to estimate the dimensionless frequency response function by Eq. 1 were estimated from an integer Finite Fourier Transform (FFT) algorithm which is intrinsic to the SPARTA function library of the Laboratory Peripheral System written by DEC for the OSU-WRF PDP 11 E10 mini-computer. A total of 8192 discrete time values were digitized at even

time intervals $\Delta t = 0.06$ seconds for each of the three realizations synthesized from each of the two theoretical design wave spectra listed in Table 1. The measured data signals were digitized through analog-to-digital converters (ADC) simultaneously with the activation of the motion of the wavemaker piston by the FFT-synthesized wavemaker signal through digital-to-analog converters (DAC). This digital activation of the wavemaker piston motion through a digital signal required a small sending rate time interval, Δt , in order to avoid overdriving the servo-mechanism and results in the small sampling rate for the simultaneous digital recording of the multichannel data signals. The two-sided spectral densities required in the analyses were computed from the discrete complex-valued FFT coefficients, $X(m)$, by the following:

$$S(\omega) = \frac{X(m) \cdot X^*(m)}{d\omega} \quad (18a)$$

in which

$$d\omega = 2\pi df = \frac{2\pi}{(LX) \cdot \Delta t} = \frac{2\pi}{(8192)(0.06)} \quad (18b)$$

Figures 4 and 5 compare the theoretical target wave spectra with a typical smoothed measured wave spectra for the Scott spectra and for the Bretschneider-Pierson-Moskowitz spectra, respectively, which were used in the experimental verifications. Each measured spectrum was smoothed by a box-car filter using 13 discrete spectral estimates which give 26 degrees-of-freedom for discrete spectral estimates computed from FFT coefficients.

Figure 6 compares the squared modulus of the theoretical dimensionless frequency response function for the wavemaker stroke spectrum computed from Eq. 3a with the measured dimensionless frequency response function estimated from the spectral estimates according to Eq. 1.

Figure 7 compares the square modulus of the theoretical dimensionless frequency response function for the wavemaker hydrodynamic pressure moment computed from Eq. 4 with measured frequency response functions which were approximated from spectral estimates of the measured data according to Eq. 16b.

CONCLUSIONS

A minor extension of the elaborate stochastic design curves for hinged wavemakers developed by Gilbert, *et al* (1971) were experimentally verified in the Oregon State University-Wave Research Facility (OSU-WRF) using two types of theoretical two-parameter design wave spectra. The analytical expressions for the dimensionless frequency response functions for the wavemaker stroke spectrum, $Q_S(\omega)$, and for the hydrodynamic pressure moment spectrum, $Q_M(\omega)$, were shown to give quite good agreement with

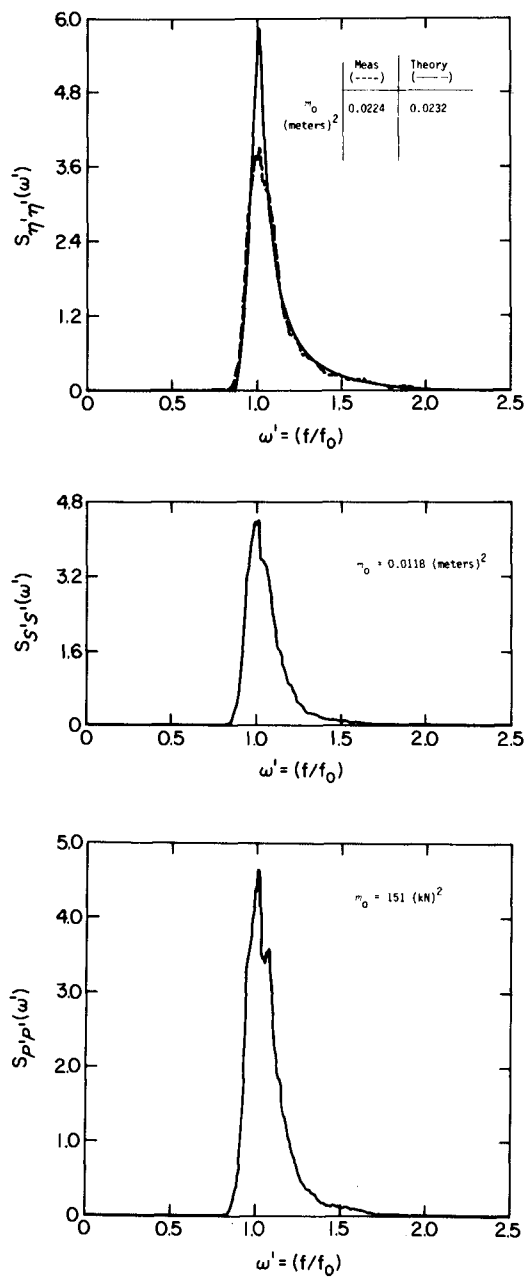


FIG. 4 TYPICAL SCOTT SPECTRA FOR WATER SURFACE ELEVATION, η WAVEMAKER STROKE, S , AND HYDRAULIC OIL PRESSURE, P [$f_0 = 0.2857 \text{ sec}^{-1}$ AND SUPERSCRITPRIME ' INDICATE NONDIMENSIONAL QUANTITIES]

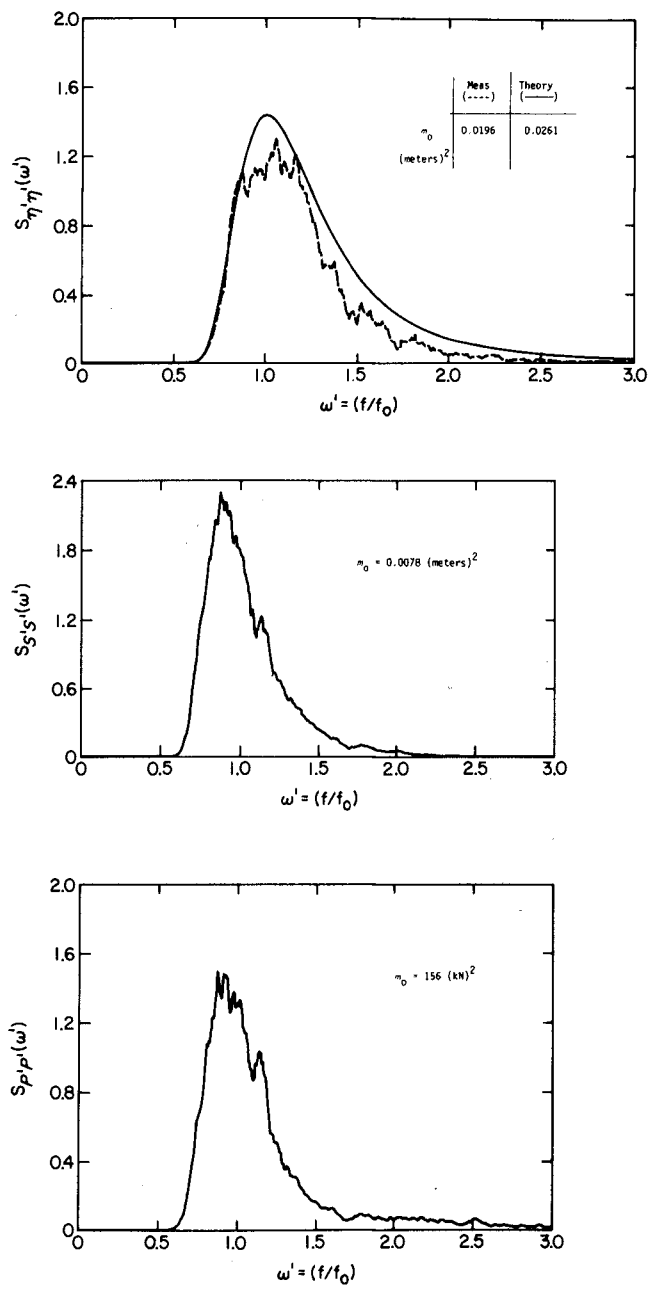


FIG. 5 TYPICAL BRETSCHNEIDER-PIERSON-MOSKOWITZ SPECTRA FOR WATER SURFACE ELEVATION, η , WAVEMAKER STROKE, S , AND HYDRAULIC OIL PRESSURE, P , [$f_0 = 0.3333 \text{ sec}^{-1}$ AND SUPERSCRIT PRIMS INDICATE NONDIMENSIONAL QUANTITIES]

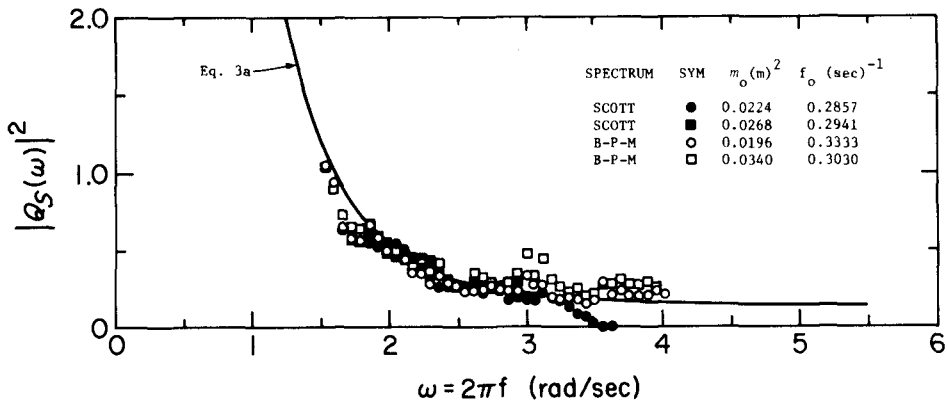


FIG. 6 COMPARISON BETWEEN THEORETICAL AND MEASURED DIMENSIONLESS FREQUENCY RESPONSE FUNCTIONS FOR THE WAVEMAKER STROKE

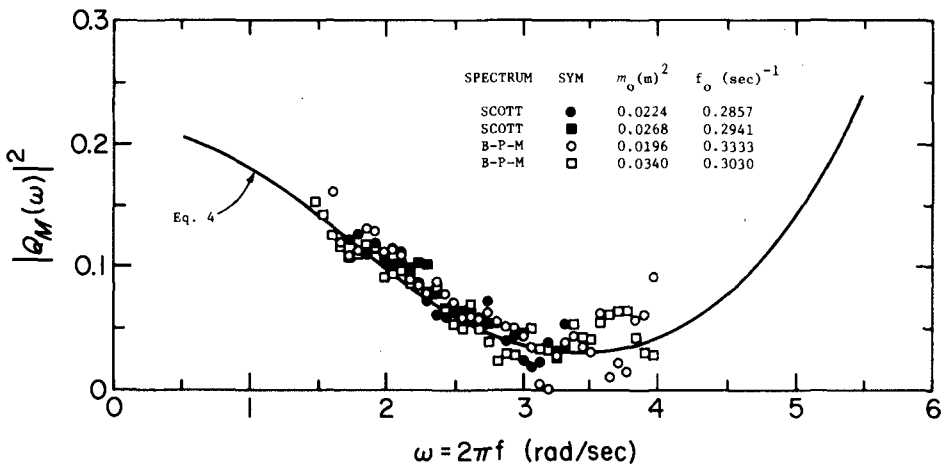


FIG. 7 COMPARISON BETWEEN THEORETICAL AND MEASURED DIMENSIONLESS FREQUENCY RESPONSE FUNCTIONS FOR THE HYDRODYNAMIC PRESSURE MOMENT ON A HINGED WAVEMAKER

the spectral estimates from the measured data. The dimensionless frequency response function for the hydrodynamic pressure moment spectrum was not measured directly but was estimated from the dynamic covariance equation for a hinged wavemaker using a linear combination of related dimensionless frequency response functions and cross-spectral estimates. The dynamic component of the nitrogen gas pressure force in the MTS designed wavemaker was found to be well approximated by an equivalent linear spring in which the spring constant was estimated by a static moment balance between the static nitrogen gas pressure force on the wavemaker flap and the static column of water in the wave flume. The use of the analytical expressions for these dimensionless frequency response functions in the stochastic design of large scale hinged wavemakers of variable-draft appears to be experimentally justified by these stochastic hydrodynamic pressure moment measurements which are believed to be unique.

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