AN INCLINED-PLATE WAVE GENERATOR

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ABSTRACT

A numerical method for determining the characteristics of waves generated by a hinged inclined-plate wave generator operating in a constant depth channel is discussed. The analysis is in reasonably good agreement with experimental results. The results indicate the sloping generator operating under certain conditions is completely inefficient, i.e., for a given stroke and depth-to-wave-length ratio, very small waves are produced; for other wave periods for the same conditions significantly larger waves are generated.

INTRODUCTION

The inclined-plate wave generator discussed herein consists of a plate with one edge hinged and attached to the bottom and mounted, in its at-rest position, at a given angle in a constant depth wave tank; the plate is moved about this position with a periodic motion. The purpose of this investigation was to determine, theoretically and experimentally, the characteristics of such a wave-maker.

Two different problems served as incentives for this study. An understanding of the characteristics of such a generator may be important in predicting waves which could arise from the impulsive movement of a sloping nearshore region due to an earthquake. This is a transient problem, but an understanding of the characteristics of an inclined wave generator, i.e., the transfer function, would assist in the solution of this type of problem. (Linear aspects of the generation of impulsive waves have been explored and presented by Hammack (1973) and the interested reader is referred to that publication.) The steady state problem is related to a proposed offshore mobile breakwater constructed from a group of moored partly sunk barges. By mooring each barge in an inclined position with one end resting on the bottom and the other end near the free surface pointing in the direction of wave incidence, a temporarily protected site may be created. An understanding of the waves generated by an inclined wave-generator is important to the understanding of the transmission characteristics associated with this type of breakwater.

Numerous investigators have studied analytically the wave generating characteristics of vertical wave generators, among these are Havelock (1929), Biesel and Suquet (1951), Ursell *et al.* (1960), Madsen (1970), and Gilbert *et al.* (1971). Several of these investigators have conducted experiments or used experimental data of others to compare to

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their theory. Gilbert *et al.* (1971) and Hammack (1973) studied the behavior of more unusual generators; the former investigating the characteristics of a moving wedge and the latter studying the waves generated by the motion of a section of the bottom of a constant depth tank. In addition to these, theoretical studies have been conducted by Tuck and Hwang (1972) relating to the impulsive generation of long waves by the movement of a continuously sloping beach. These studies provide insight into the general problem of wave generation, and suggest certain differences which might be realized for the inclined-plate generator.

THEORETICAL CONSIDERATIONS

For the analysis of waves generated by the periodic oscillations of an inclined plate hinged at the bottom, the fluid is assumed inviscid and the flow irrotational. The velocity potential, Φ, is expressed in a separable manner as:

$$\Phi(x,y;t) = \phi(x,y)e^{-i\sigma t} \tag{1}$$

where x is in the horizontal direction and y represents the depthwise direction as shown in the definition sketch, Fig. 1, $i = \sqrt{-1}$, and σ is the circular wave frequency defined as $2\pi/T$ where T is the wave period. The spatial variation of the velocity potential, $\phi(x,y)$, must satisfy the two-dimensional Laplace's equation:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \tag{2}$$

with the following boundary conditions:

(i) $\frac{\partial\phi}{\partial y} = 0$ at the bottom, $y = 0$ (3a)

(ii) $\frac{\partial\phi}{\partial n} = -i\sigma\xi\varepsilon$ on the surface of the inclined plate (The n-direction is normal to the boundary, ξ is the distance from the hinge point to the position (x,y) at the plate surface, and ε is the maximum angular displacement of the plate.) (3b)

(iii) $\frac{\partial\phi}{\partial n} = \frac{\sigma^2}{g}\phi$ on the free surface, $y = h$ (This is a linearized free surface condition; thus, small amplitude wave motion is assumed.) (3c)

(iv) $\frac{\partial\phi}{\partial n} = ik\phi$ at $x = x_{o_1}$, to approximately represent the radiation condition. (This condition for Region II at $x = x_{o_2}$ is shown also in Fig. 1.) (3d)

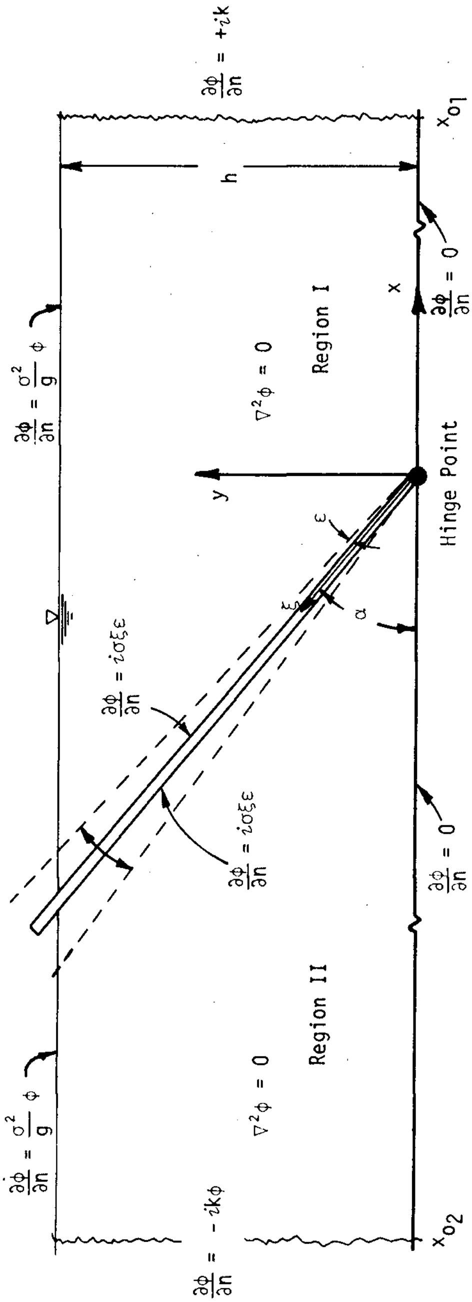

Figure 1 Definition Sketch for Theoretical Analysis.

Since the two regions cannot be represented by a separable coordinate system, it is advantageous to seek the solution of Eq. 2 by the boundary integral method. (A description of this method can be found in a number of texts on partial differential equations, e.g. Carrier and Pearson (1976).) Thus, the velocity potential can be expressed as:

$$\phi(\vec{x}) = \frac{1}{2\pi} \int_C \left[\ell n\left(\frac{1}{r}\right) \frac{\partial \phi}{\partial n} - \phi \frac{\partial}{\partial n} \ell n\left(\frac{1}{r}\right) \right] ds \qquad (4)$$

where r represents the distance between the field point $\vec{x}$ and the boundary points, and ds is a differential length along the boundary. The boundary integration is to be performed in a counter-clockwise direction, and the position vector $\vec{x}$ of a field point is in the interior of the region to be considered. Eq. 4 implies that if one knows both ϕ and $\frac{\partial \phi}{\partial n}$ at the boundary of the region, the solution ϕ at the interior can be found readily. However, as indicated in the boundary conditions (Eqs. 3), the value of ϕ along the boundary needs to be determined before the value of the velocity potential in the interior of the fluid can be obtained. For this purpose, the following integral equation is obtained from Eq. 4 by allowing the field point $\vec{x}$ to approach the boundary point $\vec{x}_j$:

$$\phi(\vec{x}_j) = \frac{1}{\pi} \int \left[\phi \frac{\partial}{\partial n} (\ell n\, r) - \ell n(r) \frac{\partial \phi}{\partial n} \right] ds \qquad (5)$$

An approximate solution of ϕ in Eq. 5 is obtained numerically by dividing the boundary into a finite number of segments, N. In this manner Eq. 5 can be written in the following discrete form:

$$\phi(\vec{x}_j) = \frac{1}{\pi} \sum_{i=1}^{N} \left[\phi(\vec{x}_i) \frac{\partial}{\partial n} \ell n(r_{ij}) - \ell n(r_{ij}) \frac{\partial \phi}{\partial n} (\vec{x}_i) \right] \Delta s_i \qquad (6)$$

where Δs_i represents the boundary segment length and r_{ij} is the distance between the i^{th} and the j^{th} segment. Eq. (6) can be expressed as a matrix equation as:

$$\begin{bmatrix} \phi(\vec{x}_1) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \phi(\vec{x}_N) \end{bmatrix} = \frac{1}{\pi} \begin{bmatrix} \\ (G_n)_{ij} \\ \\ \end{bmatrix} \begin{bmatrix} \phi(\vec{x}_1) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \phi(\vec{x}_N) \end{bmatrix} - \frac{1}{\pi} \begin{bmatrix} \\ (G)_{ij} \\ \\ \end{bmatrix} \begin{bmatrix} \frac{\partial \phi}{\partial n}(\vec{x}_1) \\ \cdot \\ \cdot \\ \cdot \\ \frac{\partial \phi}{\partial n}(\vec{x}_N) \end{bmatrix} \qquad (7)$$

The elements in matrics G_n and G are defined as:

$$(G_n)_{ij} = \frac{1}{r_{ij}} \left[- \frac{(x_i - x_j)}{r_{ij}} \left(\frac{\Delta y}{\Delta s}\right)_j + \frac{(y_i - y_j)}{r_{ij}} \left(\frac{\Delta x}{\Delta s}\right)_j \right] \Delta s_j$$
$(i \neq j)$

$$(G_n)_{ii} = \left[\frac{(-x_{ss}y_s + x_s y_{ss})_i}{2}\right]\Delta s_i$$

$$(G)_{ij} = \left[\ell n(r_{ij})\right]\Delta s_j \quad (i \neq j)$$

$$(G)_{ii} = \left[\ell n\left(\frac{1}{2}\Delta s_i\right) - 1\right]\Delta s_i$$

The vector $\frac{\partial\phi}{\partial n}(\vec{x}_i)$ in Eq. 7 involves the plate velocity, the bottom boundary condition, the radiation condition at either x_{o1} or x_{o2} and the free surface boundary condition and is given by:

$$\frac{\partial\phi}{\partial n}(\vec{x}_i) = \Big[\underbrace{\frac{\partial\phi}{\partial n}(\vec{x}_1)\ldots\frac{\partial\phi}{\partial n}(\vec{x}_p)}_{\text{"p" elements on the plate}},\ \underbrace{0\ldots0}_{\text{elements at bottom}},\ \underbrace{ik\phi(\vec{x}_\ell)\ldots ik\phi(\vec{x}_m)}_{\text{elements at the radiation boundary}},\ \underbrace{\frac{\sigma^2}{g}\phi(\vec{x}_n)\ldots\frac{\sigma^2}{g}\phi(\vec{x}_q)}_{\text{elements at the free surface}}\Big]$$

It is clear that the vector $\frac{\partial\phi}{\partial n}(\vec{x}_i)$ contains values of the velocity potential along both the radiation boundary and the free surface which are to be evaluated in the solution. Therefore, to obtain a solution, the vector $\frac{\partial\phi}{\partial n}(\vec{x}_i)$ is represented by the linear superposition of vectors involving the known values of the velocity of the plate and the unknown velocity potentials on the boundaries. After some manipulation Eq. 7 can be rearranged into a form which can be solved readily:

$$\left\{\underbrace{\left[\frac{1}{\pi}(G_n)_{ij} - I\right]}_{Ⓐ} - \frac{1}{\pi}\underbrace{\begin{bmatrix} 0\cdot(G_{ij}) \\ \text{-----} \\ ik(G_{ij}) \\ \text{-----} \\ \frac{\sigma^2}{g}(G)_{ij}\end{bmatrix}}_{Ⓑ}\right\}\begin{bmatrix}\phi(\vec{x}_1)\\ \phi(\vec{x}_2)\\ \vdots \\ \phi(\vec{x}_N)\end{bmatrix} = \underbrace{\left[\frac{1}{\pi}G_{ij}\right]}_{Ⓓ}\cdot\underbrace{\begin{bmatrix}\frac{\partial\phi}{\partial n}(\vec{x}_1)\\ \vdots \\ \frac{\partial\phi}{\partial n}(\vec{x}_p)\\ 0 \\ \vdots \\ 0\end{bmatrix}}_{Ⓕ} \qquad (8)$$

and from Eq. 8 one can obtain the following matrix expression for the velocity potential at the boundary $\phi(\vec{x}_i)$:

$$\phi(\vec{x}_i) = (A - B)^{-1} \cdot D \cdot \underset{\sim}{f} \qquad (9)$$
$$= M\underset{\sim}{f}$$

From the linearized dynamic free surface condition the wave amplitude at the free surface is given as:

$$\eta = -\frac{1}{g}\frac{\partial\Phi}{\partial t}\bigg|_{y=h} \qquad (10)$$

In obtaining the numerical solutions described by Eqs. 6, 7, 8, two aspects of the problem should be mentioned. The first deals with the distribution of the elements on the boundary. The basic requirement is that the length of a boundary-element must be small compared to the wave length. Such requirement is particularly important in regions near the plate where the water depth is changing.

For the experimental conditions of $h = 15.24$ cm and $\alpha = 21.8°$ the entire boundary of Region I was divided into 85 segments which include 20 segments on the wave-generating plate and 20 segments for the portion of the free surface which is located directly above the inclined plate; the remaining segments were distributed around the rest of the boundary. It should be noted the segments are not of equal length but are considerably smaller in the region over the plate on the free surface compared to the free surface in the constant depth region. The segment length was determined from numerical experiments. Numerical experiments were also conducted with the "radiative" boundary (at $x_{01} = 1.22$ m for the experimental arrangement) extended to $1.5\ x_{01}$ and $2.0\ x_{01}$. It was found that the results from these extended regions were nearly identical to those when the "radiative" boundary was set at 1.22 m. In other words, progressive waves of the same amplitude were found at $x_{01} = 1.22$ m, 1.83 m, or even at 2.44 m, i.e., for 8, 12, or 16 depths from the toe, respectively.

The second aspect of the numerical solution which must be given special attention is the treatment of the "corner point" above the plate where the water depth goes to zero. To avoid difficulty in this region the "corner" point was eliminated in the computation and it was replaced with a very small vertical segment. The numerical solution indicated that without such an approximation the results would be very sensitive to the distribution of the boundary elements. However, if a small portion of the "tip" of the fluid volume above the plate was eliminated, the solution was quite stable and the results tended to converge. The ratio of the length of the portion which was eliminated to the length of the inclined plate varied from 0.5% to 4%. The numerical results presented in the following section are obtained by eliminating a portion which is approximately 1% of the length of the plate. (Of course, a more sophisticated treatment of the region where the water depth tends to zero would be a welcome addition to the theoretical treatment of the problem.)

EXPERIMENTAL EQUIPMENT AND PROCEDURES

The experiments were conducted in a wave tank which is 18.3 m long, 84 cm wide, and 30 cm deep. The wave machine is located at one end of the tank and a rock-covered beach was placed at the opposite end for purposes of wave dissipation thereby substantially reducing the amplitude of reflected waves.

The wave generator could be inclined in the at-rest position at a specified angle and consisted of an inclined plate with a hinge at one end which was attached to the bottom of the wave tank. The top of the plate was restrained to be in contact with two horizontal arms which were connected at their opposite ends to a carriage which was mounted to horizontal rails by linear ball bushings. Ball bearings were attached to the end of each arm at the point of contact between the arm and the wave plate. Thus, as the carriage moved the hinged plate was forced to undergo a periodic motion with a maximum angular excursion, ε. A rod approximately 1.8 m long connected the carriage to an eccentric mounted to the output shaft of a variable speed motor; hence, varying the eccentricity varied the stroke. In this manner the inclined plate was forced to move with minimum modification to existing wave generating equipment.

The wave plate was constructed of plywood with a very small clearance (less than 1 mm) where it was hinged at the bottom. Along each sidewall of the wave tank the clearance was approximately 1.5 mm. Only waves were measured for the region for which the included angle between the wave machine and the bottom was an obtuse angle (Region I in Fig. 1). Various methods were used to measure both the angle of the plate and the stroke of the wave machine to ensure accuracy; both of these measurements were important in the comparison of the experimental results to the theory.

Mounted directly to the output shaft of the motor was a circular disk with 150 holes uniformly spaced near its circumference. A photocell and a light source were independently mounted on either side of the disk and aligned with the holes so as it rotated the interrupted light generated a pulsed voltage output. The frequency of these pulses was determined using an electronic counter; hence, the rotational speed of the motor could be accurately set and measured. In this manner the wave period could be determined to within less than 0.1 per cent.

The waves were measured approximately 6 m from the toe of the inclined-plate ($x = 6$ m); for the experiments conducted, this corresponds to 24 to 38 depths away from the generation region. All measurements were made using conventional parallel-wire-resistance wave gages which were calibrated before and after each experiment. The wave heights were measured before any reflections from the beach could return to the wave gage. Thus, a reasonably accurate determination of the incident wave could be obtained.

RESULTS AND DISCUSSION OF RESULTS

In this section the results that have been obtained from the theory and the experiments are presented and discussed. In Figure 2 the

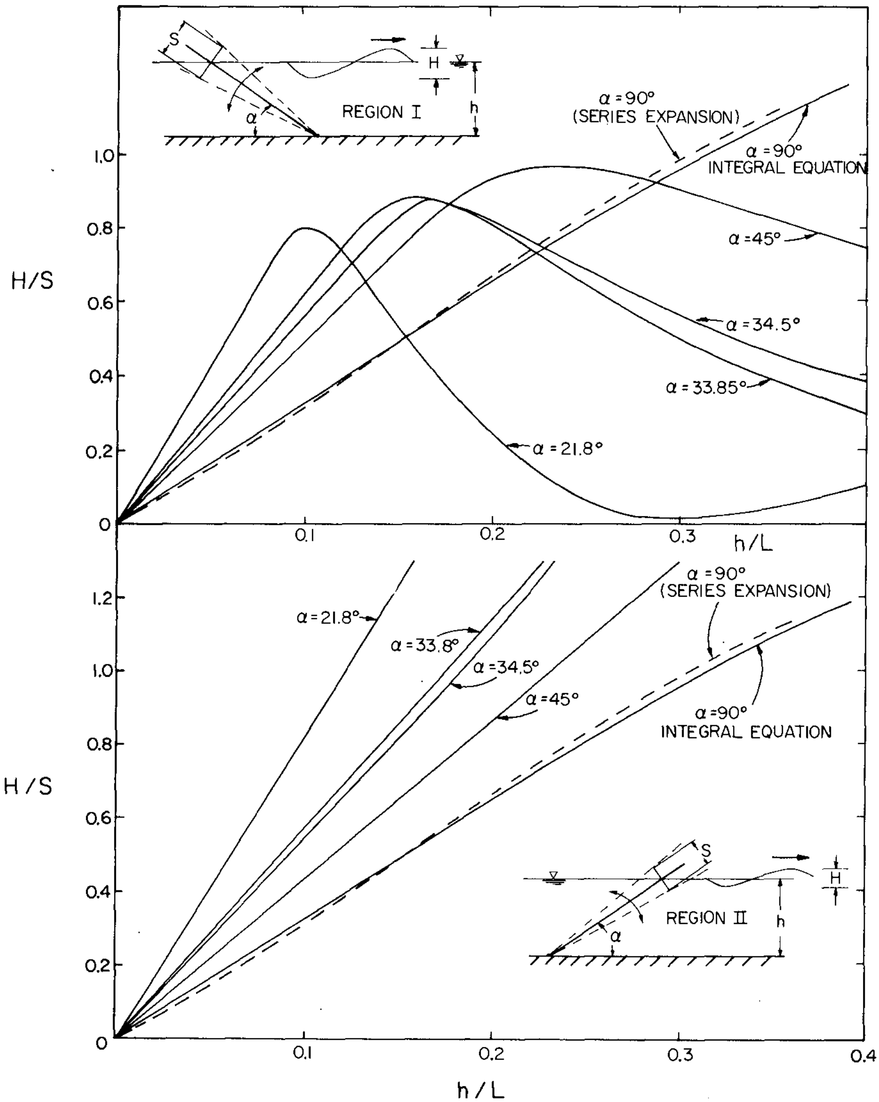

Figure 2 Theoretical Results for the Waves Generated by an Oscillating Inclined-Plate.

theoretical results are presented for the wave-maker theory described previously in the two regions: Region I where the region is bounded by the wave machine making an obtuse angle with the bottom, and Region II whose upstream limit is where the wave machine makes an acute angle with the bottom. (These two regions are shown in the insets in Figure 2.) The ordinate is the ratio of the wave height to the total stroke of the wave machine and the abscissa is the ratio of the depth to the wave length in the constant depth region. The stroke is the maximum excursion of the plate measured perpendicular to the plate at the plate-water-surface intersection with the generator in its at-rest-position. The wave length used is determined from small amplitude water wave theory for the given depth in the constant depth region and the measured wave period.

Attention is first directed to the lower portion of Figure 2 where the generation curves are shown for Region II for five different angles of inclination of the wave machine. A comparison is shown between the results using a series expansion for a perpendicular wave machine, i.e., $\alpha = 90°$, and the results from the integral equation technique developed in this study. (The results of the analysis using a series expansion can be found in several references, e.g., Biesel and Suquet (1951)). Reasonable agreement between the two theories is seen with the disagreement attributed to certain approximations which are inherent in the present numerical approach.

It is noted that only a limited portion of the theoretical curves are shown, although the range in the ratio of depth to wave length extends from shallow water wave to nearly deep water wave conditions. Figure 2 shows, for a given ratio of depth to wave length, as the angle α decreases, the ratio of wave height to stroke increases, and the increase is approximately proportional to $(1/\sin\alpha)$. This is because the wetted length of the wave maker relative to the perpendicular plate increases in that ratio. It is recalled, the value of H/S for a perpendicular generator for large h/L is two; hence, for the inclined cases one would expect the height-to-stroke ratio to approach a limit of $(2/\sin\alpha)$.

The wave generation characteristics of the inclined plate in Region I are quite different from those just shown in Region II. Theoretical curves of the variation of the ratio of wave height to wave machine stroke with the ratio of depth-to-wave-length are shown in the upper portion of Figure 2 for plates with five angles of inclination. Again, for reference, the theoretical curves are presented for the hinged wave generator which is perpendicular to the water surface in its at-rest position ($\alpha = 90°$) for both theories mentioned. The appearance of the other curves is interesting in that for a given wave stroke as the ratio of the depth to the wave length increases the wave height reaches a maximum and then decreases. (For small values of h/L the curves shown are nearly the same as the corresponding curves in Region II.) For the smallest angle investigated ($\alpha = 21.8°$) the maximum wave height generated for a given stroke is about 0.8 and occurs at a ratio of depth-to-wave length of approximately 0.1; a minimum of nearly zero is realized for a depth-to-wave length ratio of about 0.3.

The reason for this variation is that waves are generated along this sloping face and propagate to the constant depth region. As waves propagate past the hinge point into Region I, a portion is reflected back into the triangular region above the wave plate. For certain ranges of h/L the reflection is more than for others. This action essentially describes a resonant condition which can lead to maximum and minimum efficiencies of wave generation. This does not take place when waves are generated in Region II, since there is not a free surface above the wave generator for that case.

In Figure 3 experimental results are presented in addition to the theory for wave generators with four different angles: $\alpha = 21.8°$, 33.85°, 45°, and 90°. In general, it appears that the theory agrees reasonably well with the experimental results. There is some disagreement which can be noted, and the reasons for this are: the effect of dissipation on the wave heights measured 24 to 38 depths from the wave generator, energy leakage around the sides of the wave generator, and, of course, the approximate nature of numerical theories. With the leakage less than 1% of the wetted area one would not expect as large a difference as observed for the case of $\alpha = 45°$ (see Madsen, 1970).

CONCLUSIONS

In general, it can be concluded the numerical method developed provides a reasonable means of predicting the characteristics of the inclined plate wave generator. Two regions of generation are apparent with the response of the generator in these two regions distinctly different except for small ratios of depth-to-wave length. For certain at-rest angles of inclination and ratios of depth-to-wave length the generator could be described as being totally inefficient.

ACKNOWLEDGMENT

This study was supported in part by Grants ENV77-20499 and ENV77-01599 of the National Science Foundation and from support of the U.S. Navy Civil Engineering Laboratory. Certain of the experiments were conducted by Ella Wong.

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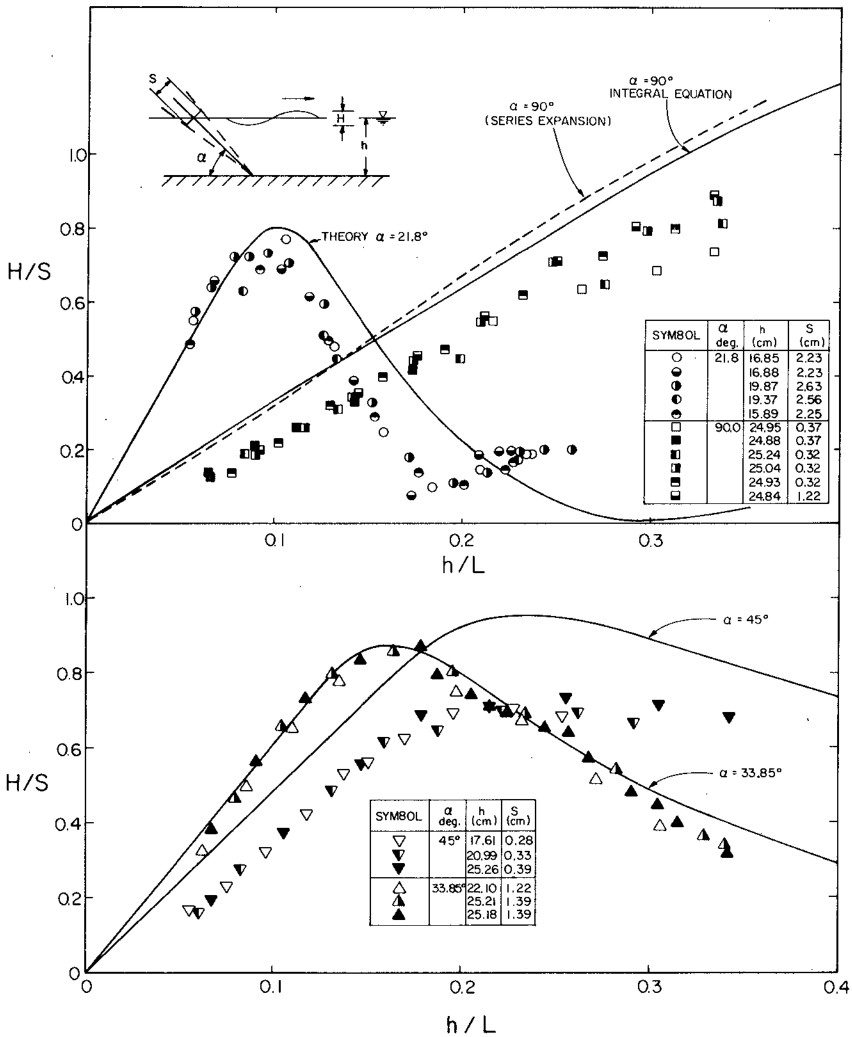

Figure 3 A Comparison of Experimental and Theoretical Results for the Waves Generated by an Oscillating Inclined-Plate.

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