CHAPTER 31

ESTIMATION OF WATER PARTICLE VELOCITY OF BREAKING WAVE

by

Tetsuo Sakai
Associate Professor

and
Yuichi Iwagaki
Professor
Department of Civil Engineering
Kyoto University, Kyoto, 606 Japan

ABSTRACT

A possibility to estimate the water particle velocity of breaking wave on beaches by giving only the values of the beach slope i and the deep-water wave steepness H_0/L_0 is investigated. The data of the water particle velocity and profile of breaking wave on the beaches measured by Iwagaki, Sakai, Tsukioka and Sawai(1974) are used for this purpose. A method of using empirical curves relating the coefficients of Dean's stream function to i and H_0/L_0 does not give good results. The relations between the asymmetric breaking wave profile, i and H_0/L_0 are examined. As well as the front face of the profile, the slope behind the crest above some level is found to become steep when H_0/L_0 becomes small. The breaking wave profiles are reproduced by using the empirical curves relating the parameters characterizing the breaking wave profile to i and H_0/L_0. The velocity of Dean's stream function calculated from the reproduced profile explains the measured velocity better than that of Stokes waves. It is therefore possible to estimate the water particle velocity of breaking wave on beaches by giving only the values of i and H_0/L_0.

INTRODUCTION

The water particle velocity of breaking wave on beaches is very important in discussing the wave forces on coastal structures and the sediment movement in surf zone. However, one can find few investigations on this subject(for example, Adeyemo(1970) and Thornton, Galvin, Bub and Richardson(1976)). This is because the measurement of the water particle velocity of breaking wave is very difficult.

Recently the horizontal water particle velocities of breaking waves on the beaches with the slopes of 1/10, 1/20 and 1/30 were measured in a wave tank with a high speed camera(Iwagaki, Sakai, Tsukioka and Sawai(1974)). It was found that the existing small and finite amplitude wave theories can not explain the measured water particle velocities. The velocities of small amplitude waves agree roughly with the measured ones in average, but the vertical distributions of small amplitude waves were more uniform than those of the measured ones. The vertical distributions of the horizontal velocity under the crest phase of the breaking wave were found to be classified by the breaker types.

The water particle velocities of Dean's stream function (Dean(1965)) were calculated by giving the simultaneously measured asymmetric breaking wave profiles(the wave record at the breaking point)(Iwagaki and Sakai(1976)). Although Dean's stream function is based on the assumption of permanent type waves on the horizontal bottom, the calculated velocities of Dean's stream function explained the measured velocities better than the existing theories of symmetric wave profile. It was concluded that the asymmetric breaking wave profile dominates the water particle velocity field of the breaking wave on beaches.

From the view point of practical use, it is desirable that the water particle velocities of breaking waves on beaches can be estimated by giving only the values of parameters such as the beach slope and the deep-water wave steepness or Iribarren number(Battjes(1974)). This investigation is one approach to this problem.

RELATIONS BETWEEN COEFFICIENTS OF DEAN'S STREAM FUNCTION, BEACH SLOPE AND DEEP-WATER WAVE STEEPNESS

Dean's stream function represents the parmanent waves on the horizontal bottom:

$$\psi = \frac{L}{T}z + \sum_{n=4,6,8,\cdots}^{N-1} \sinh\frac{(n-2)\pi(h+z)}{L} \times \left\{X_n\cos\frac{(n-2)\pi x}{L} + X_{n+1}\sin\frac{(n-2)\pi x}{L}\right\} \qquad (1)$$

in the reference x-z moving with the waves, where L is the wave length, T the wave period, x-axis in the direction of wave propagation, z-axis vertically upward from the still water level, h the water depth, Xn the unknown coefficients. The sine terms in Eq.(1) are introduced so as to represent the asymmetric wave profile. At the water surface $z = \eta$, the value of the stream function $\psi(x,\eta)$ becomes constant($= X_3$).

The wave profile η is therefore expressed as follows:

$$\eta = \frac{T}{L}X_3 - \frac{T}{L}\sum_{n=4,6,8,\cdots}^{N-1} \sinh\frac{(n-2)\pi(h+\eta)}{L} \times$$
$$\times\left(Xn\cos\frac{(n-2)\pi x}{L} + Xn{+}1\sin\frac{(n-2)\pi x}{L}\right). \qquad (2)$$

As seen from Eq.(2), the wave profile is determined by giving the unknown coefficients Xn. The relations between the coefficients and Iribarren number ξ_0 are examined at first.

The horizontal water particle velocity u is obtained by differentiating ψ with respect to z. The obtained horizontal water particle velocity in the fixed reference is expressed in non-dimensional form as follows:

$$\frac{u}{\sqrt{gh}} = -\frac{Y_2}{Y_1}\sum_{n=4,6,8,\cdots}^{N-1} (n-2)\pi\cosh\left((n-2)\pi Y_1\frac{h+z}{h}\right)\times$$
$$\times\left(Yn\cos\{(n-2)\pi\theta\} + Yn{+}1\sin\{(n-2)\pi\theta\}\right), \qquad (3)$$

where $Y_1 = h/L$, $Y_2 = H/L$, $Y_4 = X_4/LH\sqrt{g/h}$, ..., $Yn = Xn/LH\sqrt{g/h}$, ..., $\theta = x/L - t/T$. Table 1 shows the experimental conditions of 39 cases in which the breaking wave profiles were measured by Iwagaki, Sakai, Tsukioka and Sawai(1974). In this table hb is the breaking depth and Hb is the breaking wave height. In 20 cases among them the measured horizontal water particle velocities were compared with the theoretical velocities. Dean's stream functions were already calculated for all 39 cases. Fig.1 shows two examples of the relations between the non-dimensional coefficients Yn and Iribarren number ξ_0 for 39 cases. The quantity ξ_0 is defined as $i/\sqrt{H_0/L_0}$, where i is the beach slope and H_0/L_0 is the deep-water wave steepness.

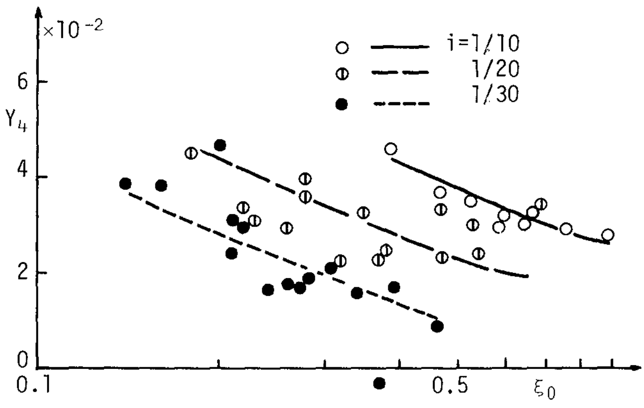
As seen from Fig.1, the data scatter considerably as well as in the relations between the breaking depth, the breaking wave height and the deep-water wave steepness. On the other hand, it is clear that the non-dimensional coefficients of Dean's stream function are not determined by Iribarren number only. The data for the different beach slope have a different trend. Empirical curves were drawn through the plotted points for each beach slope. Using these curves for all non-dimensional coefficients and Eq.(3), the horizontal water particle velocities at crest phase were calculated for the same 20 cases by giving the values of the beach slope and the deep-water wave steepness in each case. Fig.2,(1),(7) and (12) show three examples of the compari-

Table 1 Experimental conditions of analyzed data (Iwagaki, Sakai, Tsukioka and Sawai(1974))

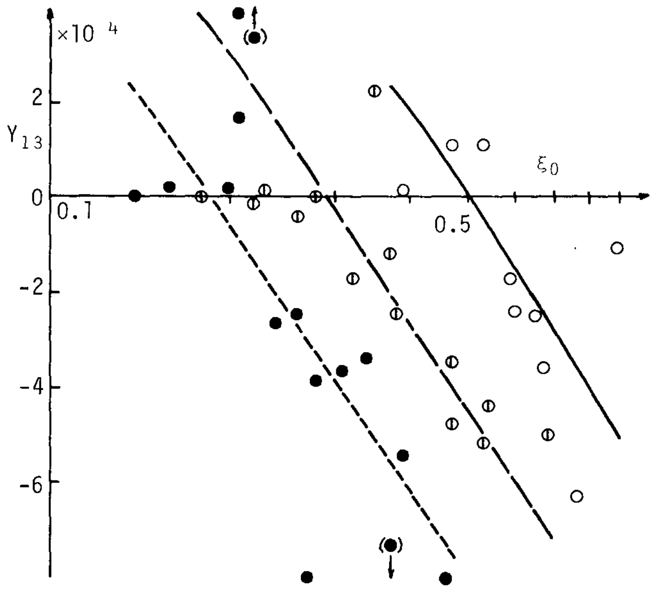
No.	i	T(sec)	H_0/L_0	h_b(cm)	$T\sqrt{g/h_b}$	H_b(cm)	H_b/h_b	**
* 1-1-1	1/10	1.00	0.066	11.2	9.4	9.7	0.87	1
* 2		0.96	0.046	7.4	11.0	6.7	0.91	2
* 3		1.01	0.029	6.2	12.7	5.3	0.85	3
2-1		1.26	0.035	10.5	12.2	10.3	0.98	
2		1.23	0.024	8.3	13.4	6.6	0.80	
3		1.27	0.017	6.5	15.6	6.4	0.98	
* 3-1		1.37	0.028	11.9	12.4	10.0	0.84	4
* 2		1.38	0.022	9.6	13.9	9.0	0.94	5
* 3		1.50	0.013	6.8	18.0	6.8	1.00	6
* 2-1-1	1/20	1.00	0.074	16.0	7.8	8.7	0.54	7
* 2		1.00	0.051	10.6	9.6	8.2	0.77	8
* 3		1.00	0.031	6.9	11.9	6.1	0.88	9
2-1		1.26	0.048	14.3	10.4	12.7	0.89	
2		1.24	0.038	11.0	11.7	9.4	0.85	
3		1.24	0.019	11.5	11.5	4.6	0.40	
* 3-1		1.49	0.032	14.7	12.2	12.5	0.85	10
* 2		1.48	0.020	10.4	14.4	8.3	0.80	11
* 3		1.46	0.012	6.2	18.4	5.7	0.92	12
4-1		1.64	0.025	14.5	13.5	11.5	0.79	
2		1.65	0.018	11.0	15.6	9.6	0.87	
3		1.65	0.009	6.8	19.8	5.8	0.85	
5-1		1.94	0.011	11.5	17.9	8.8	0.77	13
2		1.94	0.009	9.7	19.5	8.1	0.84	14
3		1.96	0.005	6.9	23.4	6.0	0.87	15
* 3-1-1	1/30	1.01	0.057	11.8	9.2	8.2	0.69	16
* 2		0.98	0.043	9.7	9.9	6.6	0.68	17
* 3		0.98	0.029	6.8	11.8	4.4	0.65	18
2-1		1.24	0.026	11.5	11.5	8.6	0.75	
2		1.23	0.024	8.5	13.2	7.0	0.82	
3		1.25	0.015	6.2	15.7	5.5	0.89	
* 3-1		1.49	0.025	12.8	13.6	10.9	0.85	19
* 2		1.50	0.016	10.1	14.8	7.6	0.75	20
3		1.46	0.009	6.3	18.2	4.5	0.71	
4-1		1.64	0.020	12.6	14.5	9.9	0.79	
2		1.60	0.014	9.7	16.1	7.6	0.78	
3		1.60	0.007	6.0	20.5	5.0	0.83	
* 5-1		1.92	0.012	12.2	17.2	9.6	0.79	21
* 2		1.98	0.008	9.9	19.7	8.1	0.82	22
* 3		1.92	0.005	6.6	23.4	5.9	0.89	23

* case in which the water particle velocity is dicussed

** No. used in Iwagaki, Sakai, Tsukioka and Sawai(1974) and Iwagaki and Sakai(1976)



(1)



(2)

Fig.1 Examples of relations between coefficients of Dean's stream function and Iribarren number ξ_0

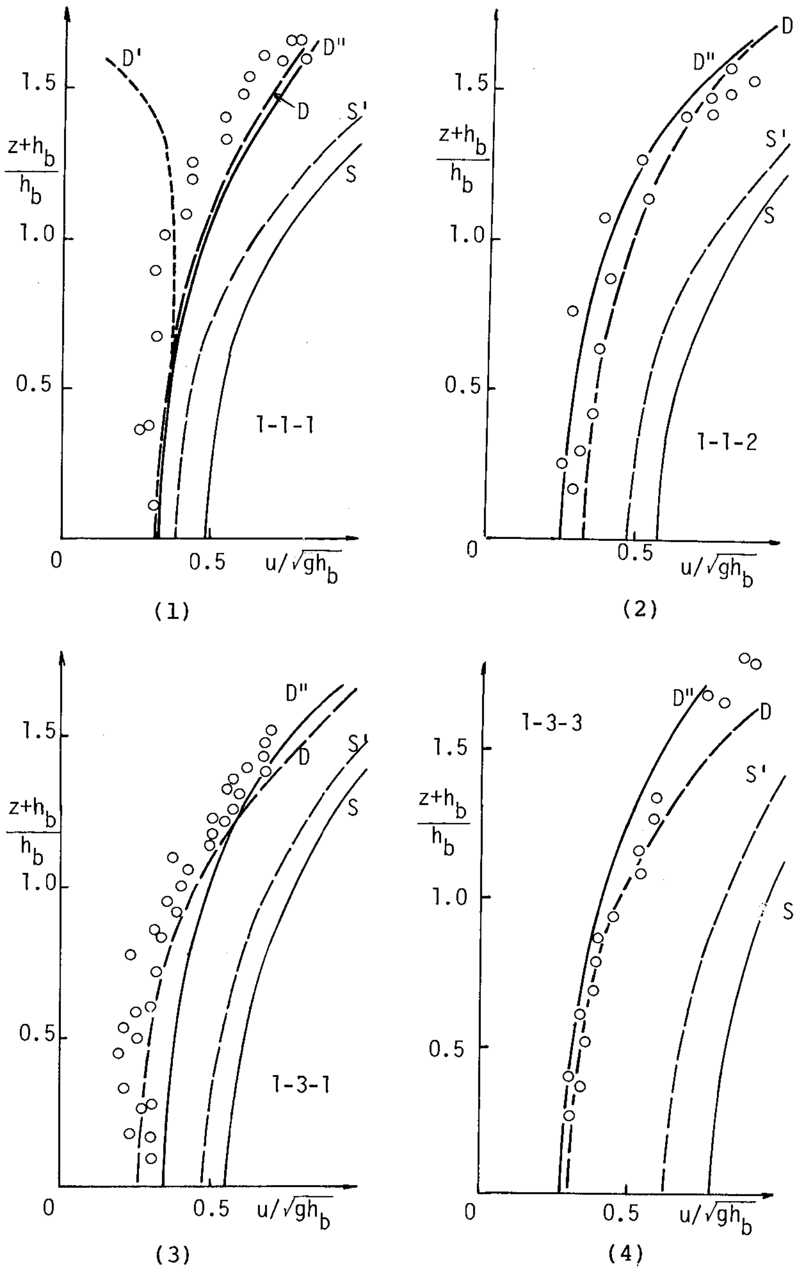


Fig.2,(1)∿(4) Examples of comparisons between calculated and measured water particle velocities

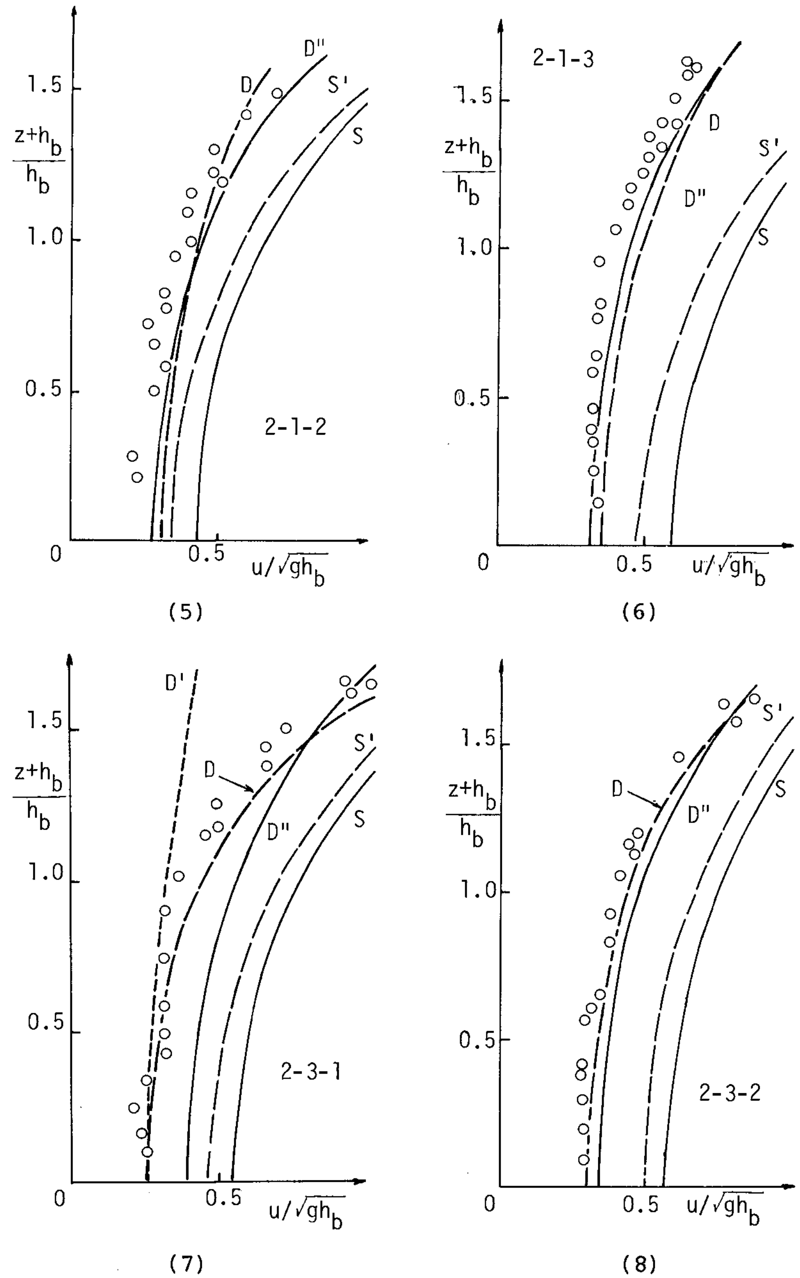


Fig.2,(5)∿(8) Examples of comparisons between calculated and measured water particle velocities

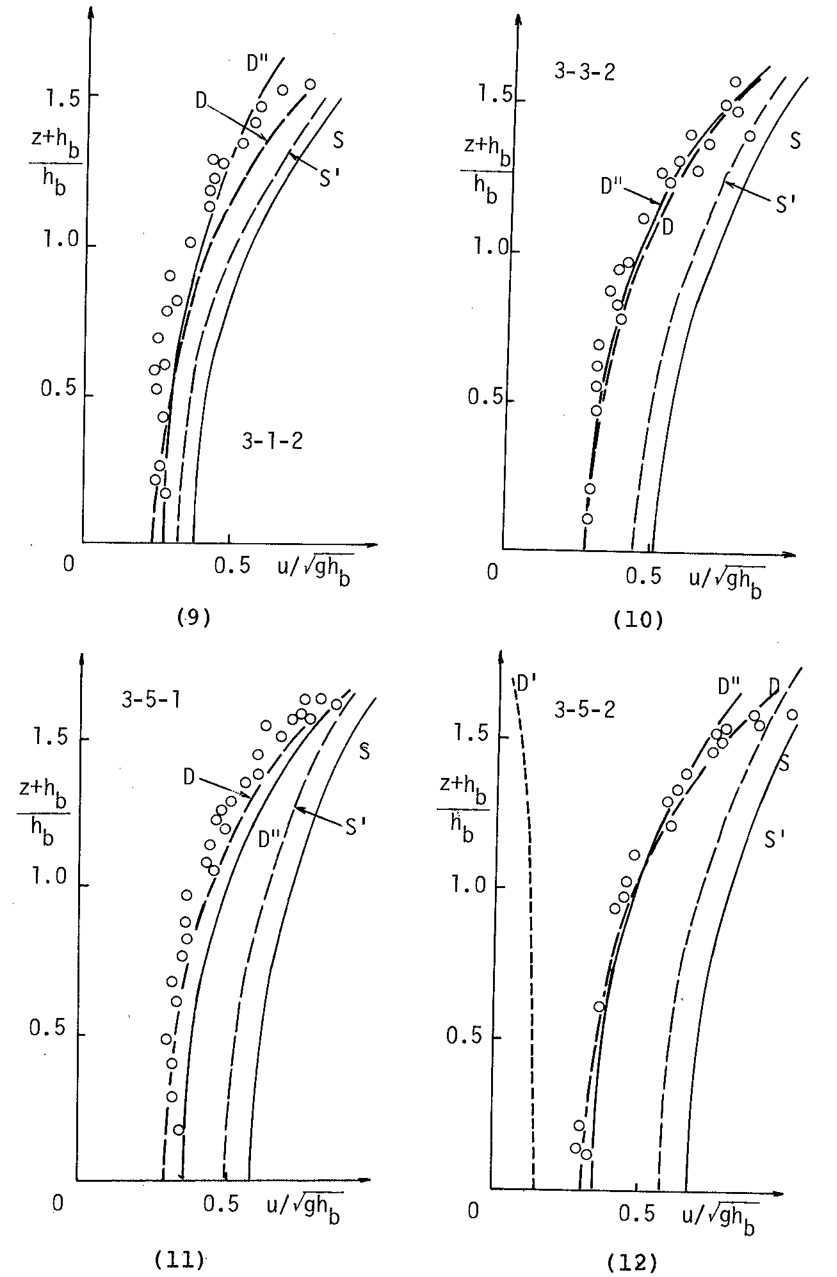


Fig.2,(9)∿(12) Examples of comparisons between calculated and measured water particle velocities

sons between the measured horizontal water particle velocities at crest phase and the calculated velocities denoted by D'. In these figures are also shown several theoretical velocities as follows(Iwagaki and Sakai(1976)):

- S : Stokes waves with first definition of wave celerity
- S' : Stokes waves with second definition of wave celerity(Tsuchiya and Yamaguchi(1972))
- D : Dean's stream function with measured wave profile
- D' : Dean's stream function with determined coefficients
- D" : Dean's stream function with reproduced wave profile(explained later)

Unfortunately the calculated velocities by using the empirical curves for the non-dimensional coefficients(D') are in general smaller than the measured ones. In Figs.2,(1) and (12), the velocity near the water surface is rather smaller than that near the bottom. One reason is of course the scatter of the values of the non-dimensional coefficients. Furthermore, the term in the summation of Eq.(3) contains $(n-2)\pi\cosh\{(n-2)\pi Y_1(h+z)/h\}$. Therefore, even if the value and the scatter of Y_n of large n are small, the error of estimating the term of large n(the higher order term) becomes large as n becomes large. The contribution of the higher order term to the water particle velocity is confined near the water surface. This is why the slope of the vertical distribution is reversed near the water surface.

RELATIONS BETWEEN BREAKING WAVE PROFILE, BEACH SLOPE AND DEEP-WATER WAVE STEEPNESS

Considering that the water particle velocity field of the breaking wave is dominated by the asymmetric breaking wave profile, it is tried to relate the measured breaking wave profiles(the wave record at the breaking point) to the beach slope and the deep-water wave steepness. Adeyemo(1970) proposed four parameters on the wave profile asymmetry. They were the relative positions of the crest during one wave period, the crest height and the slope of wave profile. In order to represent the asymmetric wave profile exactly, such a rough parameterization is not sufficient. 39 measured breaking wave profiles were characterized with 25 parameters shown in Fig.3. Two parameters t_a and η_a are respectively the phase and the level of the point on the profile behind the crest where the slope changes abruptly. Fig.4 shows the relations between the representative 9 parameters, the beach slope and the deep-water wave steepness. Although the scatter

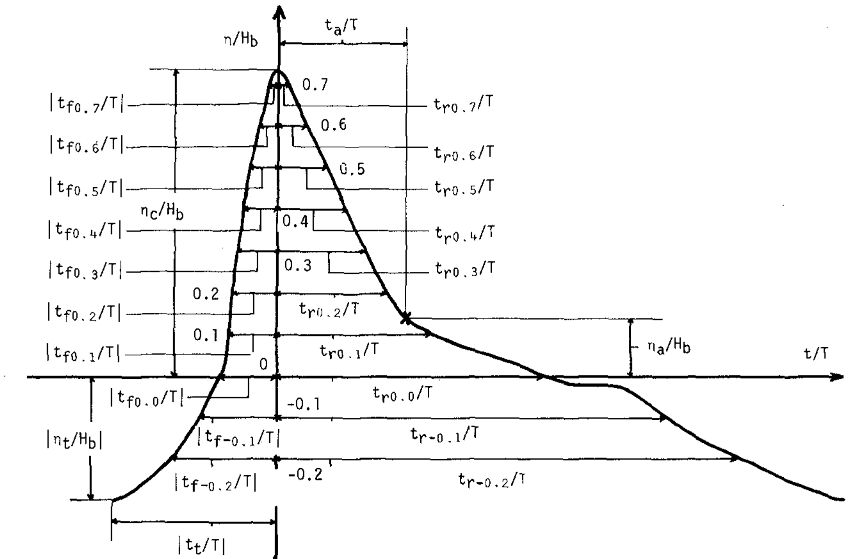
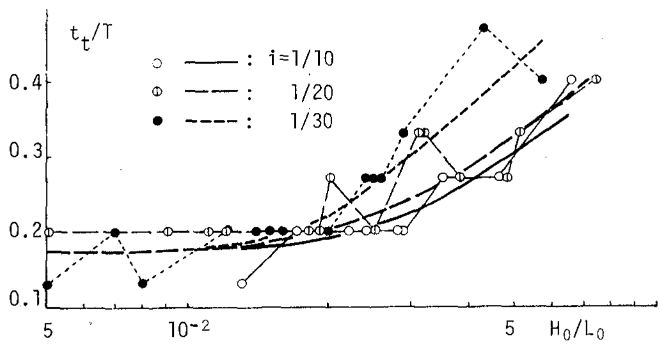
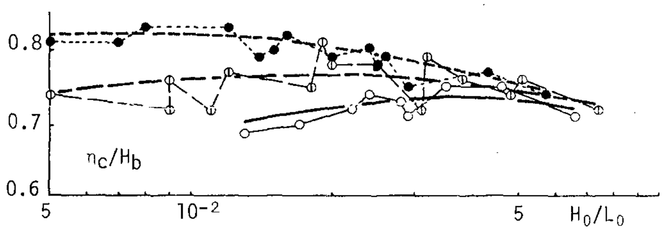


Fig.3 25 parameters characterizing breaking wave profile



(1)



(2)

Fig.4,(1),(2) Examples of relations between parameters of breaking wave profile, beach slope and deep-water wave steepness of the data in each figure is considerable, empirical curves were drawn through the plotted points for each beach slope.

It is well known, from the classification of the breaker types, that the front face of the breaking wave profile becomes steep and the slope behind the crest becomes gentle as the beach slope becomes large and the deep-water wave steepness becomes small. This trend is seen in Figs.4,(1)(the phase of the front trough $|t_t/T|$), (3)(the phase of the front face at $z/H_b = -0.1$ $|t_{f-0.1}/T|$) and (7)(the phase of the profile behind the crest at $z/H_b = 0.0$ $t_{r0.0}/T$). The same trend is seen in all phases of the front face and the slope behind the crest below the level of $z/H_b = 0.0$.

In Fig.4,(4), however, the experimental results of the phase of the slope behind the crest at $z/H_b = 0.3$ $t_{r0.3}/T$ show a different trend. The phase at $z/H_b = 0.3$ slightly decreases as the deep-water wave steepness decreases. Also as for other phases of the profile behind the crest above $z/H_b = 0.3$, the same trend is seen. This fact means that the slope behind the crest above $z/H_b = 0.3$ becomes steep as well as the front face when the deep-water wave steepness becomes small. On the other hand, the relations between the beach slope and the phases of the slope behind the crest above $z/H_b = 0.3$ are not clear.

At $z/H_b = 0.2$, as shown in Fig.4,(5), only in the case of 1/10 beach slope, the phase behind the crest becomes large as the deep-water wave steepness becomes small. At $z/H_b = 0.1$ (Fig.4,(6)), in two cases of $i = 1/10$ and $1/20$, the phase becomes large as the deep-water wave steepness becomes small. Thus, the beach slope has the effect on the level where the relation between the phase of the slope behind the crest and the deep-water wave steepness reverses. Fig.4,(9) on η_a/H_b explains the effects of the beach slope and the deep-water wave steepness on this level. The level(the value of η_a/H_b) becomes high as the beach slope becomes large and the deep-water wave steepness becomes small.

As seen from Fig.4,(8), the phase of the point of abrupt slope change t_a/T becomes small as the deep-water wave steepness becomes small. That is, the width of the base of the crest of the breaking wave profile becomes narrow as the deep-water wave steepness becomes small. The effect of the beach slope is not clear. The crest height η_c/H_b, as seen from Fig.4,(2), shows complicated trends.

REPRODUCTION OF BREAKING WAVE PROFILE

The empirical curves relating 25 parameters to the beach slope and the deep-water wave steepness in Fig.4 are used to

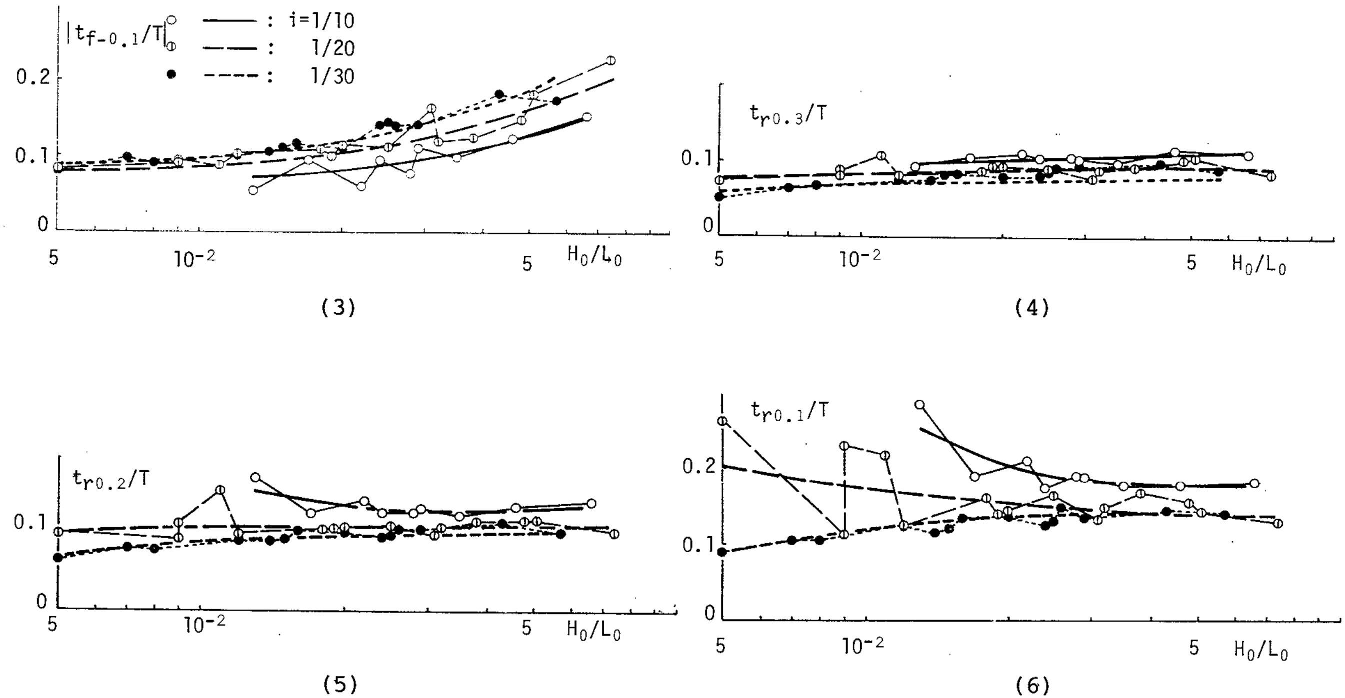


Fig.4,(3)~(6) Examples of relations between parameters of breaking wave profile, beach slope and deep-water wave steepness

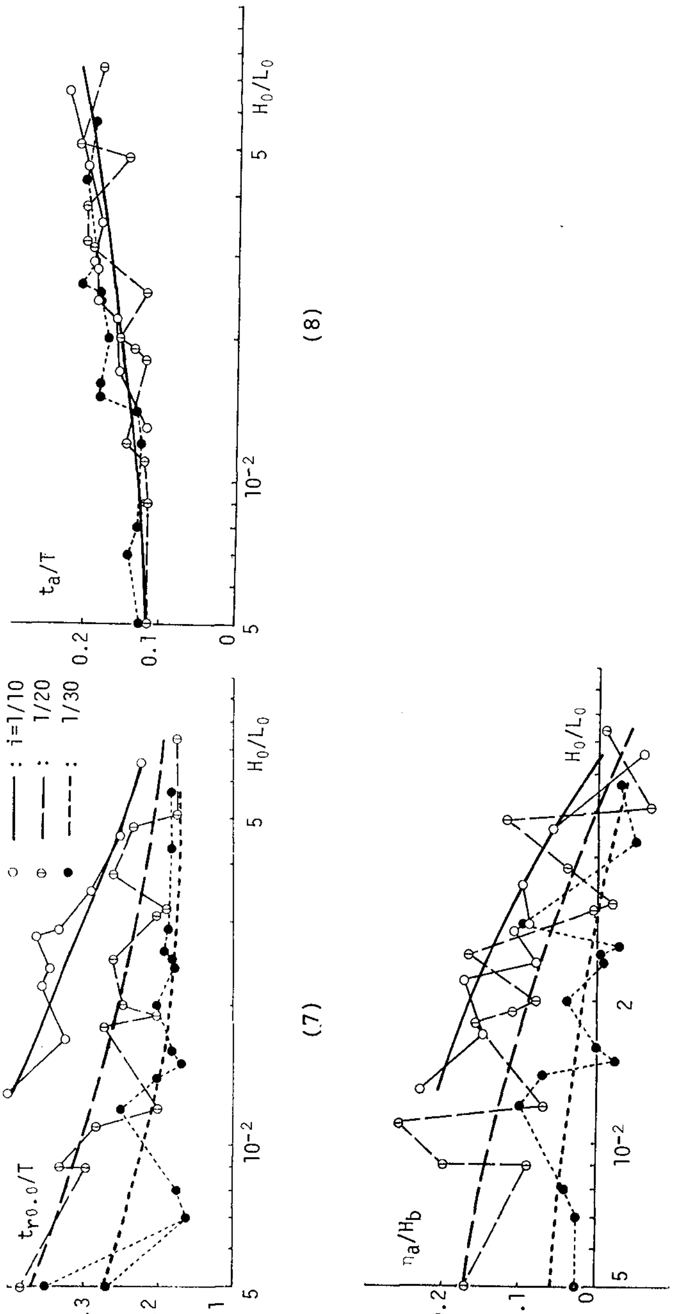


Fig.4,(7)~(9) Examples of relations between parameters of breaking wave profile, beach slope and deep-water wave steepness reproduce the breaking wave profiles by giving the values of the beach slope and the deep-water wave steepness. In some cases, the phases of the front and back faces at $z/Hb = 0.7$ $|tf0.7/T|$ and $tr0.7/T$ are given, while the crest height $\eta c/Hb$ is smaller than 0.7 because of using the empirical curves. In these cases the crest height is adopted. Also in some cases, the position of abrupt profile change(ta/T and $\eta a/Hb$) is far from the neighboring points determined by the values of tr/T. In these cases, only the value of ta/T is adopted.

Figs.5,(1)∿(3) show three examples of the comparisons between the reproduced and measured breaking wave profiles. In general, the reproduced breaking wave profiles coincide with the measured profiles except for the small deviations.

ESTIMATION OF WATER PARTICLE VELOCITY OF BREAKING WAVE ON BEACHES

The reproduced breaking wave profiles were used to calculate Dean's stream functions. The wave profile as the input to calculate the stream function is dimensional. The measured wave period and breaking wave height were used to convert the non-dimensional reproduced profiles into the dimensional ones. The water levels of the reproduced profiles were read at 15 points of equal distance on the time axis. The value of N was set equal to 13 which corresponds to Stokes waves of the fifth order. The calculations determining the unknown coefficients were 5 times repeated.

The curves denoted by D" in Figs.2,(1)∿(12) are the velocities obtained from the calculated Dean's stream functions with the reproduced breaking wave profiles. In these figures, are also shown the velocities obtained from the calculated Dean's stream functions with the measured profiles (D) and the velocities of Stokes waves of two different definitions of wave celerity(S and S'). The velocities of cnoidal waves are not shown because the vertical distributions are too steep compared with the measured ones. Also the velocities of small amplitude waves are not shown because the vertical distributions are more uniform than the measured ones.

As well as the velocities based on Dean's stream functions with the measured profiles, the velocities based on Dean's stream function with the reproduced profiles explain the measured velocities better than those of Stokes waves. Table 2 shows the relative errors of the calculated velocities at the still water level of 20 cases in which the water particle velocities were measured. u_E is the measured velocity at the still water level. After all, it is possible to calculate the

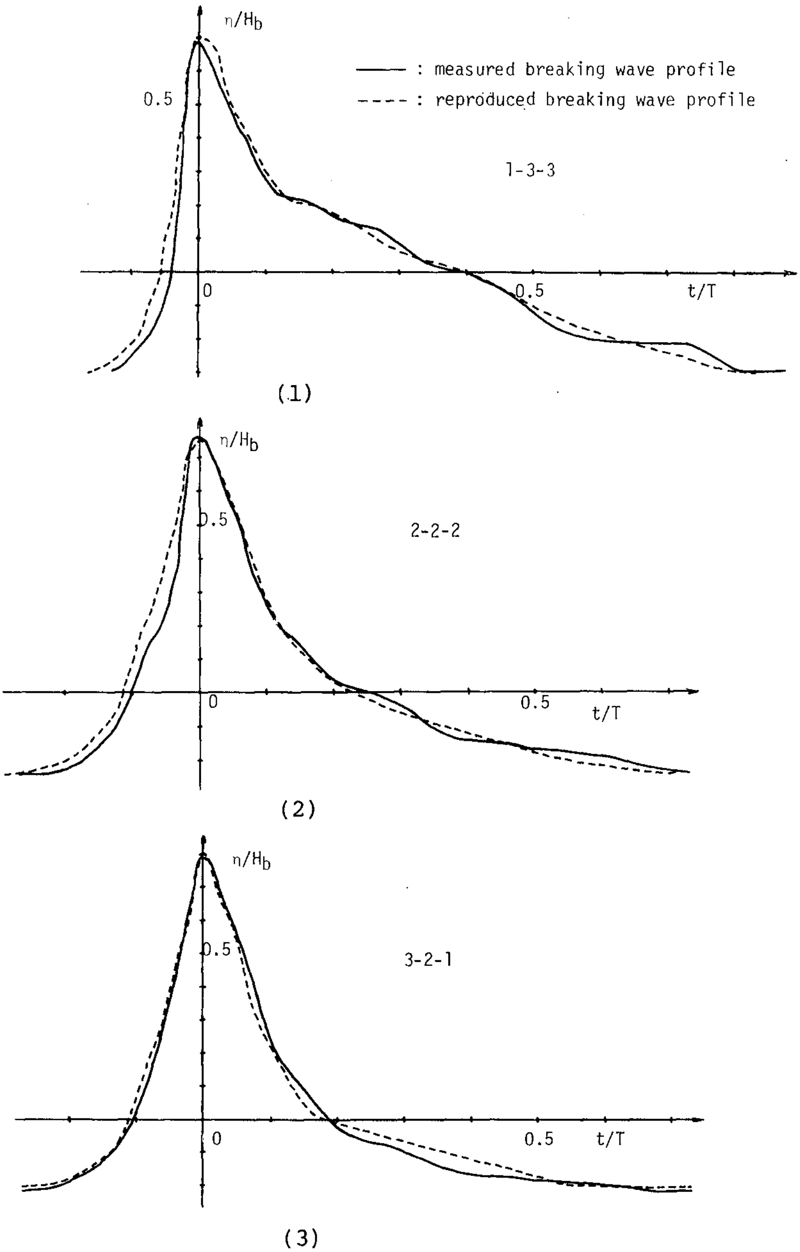


Fig.5 Examples of comparisons between reproduced and measured breaking wave profiles

Table 2 Relative errors of calculated velocities at still water level

No.	i	H_0/L_0	$(u_D-u_E)/u_E$	$(u_{D''}-u_E)/u_E$	$(u_{S'}-u_E)/u_E$
1-1-1	1/10	0.066	0.27	0.37	0.78
2		0.046	0.14	0.05	0.76
3		0.029	-0.06	-0.14	0.45
3-1		0.028	0.15	0.20	0.79
2		0.022	0.16	0.42	0.93
3		0.013	0.04	-0.15	0.74
2-1-1	1/20	0.074	0.27	0.71	0.53
2		0.051	0.08	0.13	0.51
3		0.031	0.26	0.20	0.90
3-1		0.032	0.15	0.51	0.79
2		0.020	0.02	0.23	0.60
3		0.012	-0.07	-0.26	0.42
3-1-1	1/30	0.057	-0.10	0.08	0.28
2		0.043	0.14	0.10	0.38
3		0.029		-0.22	
3-1		0.025	0.14	0.26	0.61
2		0.016	0.10	0.07	0.50
5-1		0.012	0.12	0.28	0.56
2		0.008	0.00	0.09	0.58
3		0.005	-0.02	-0.18	0.47
average of absolute values			0.12	0.23	0.61

horizontal water particle velocity at crest phase of the breaking wave on the beach more exactly by giving only the values of the beach slope and the deep-water wave steepness than with Stokes wave theory.

CONCLUSIONS

The possibility to estimate the horizontal water particle velocity at crest phase of breaking wave on beaches by giving only the values of the beach slope and the deep-water wave steepness was examined. The data of the velocities of breaking waves on the sloping beaches in the wave tank measured by Iwagaki, Sakai, Tsukioka and Sawai(1974) were used. The following conclusions are obtained:

1) The coefficients of Dean's stream function are not determined by Iribarren number only.

2) The velocity calculated by using the empirical curves relating the coefficients of Dean's stream function to the beach slope and the deep-water wave steepness can not explain the measured velocity.

3) As well as the front face, the slope behind the crest of the breaking wave profile above the level(z/H_b = 0.1∿0.3) which is determined by the beach slope and the deep-water wave steepness becomes steep when the deep-water wave steepness becomes small.

4) The breaking wave profile can be reproduced from the empirical curves for the parameters of the profile by giving only the values of the beach slope and the deep-water wave steepness.

5) The velocity based on Dean's stream function calculated from the reproduced breaking wave profile explains the measured velocity, as well as the velocity based on Dean's stream function calculated from the measured wave profile.

6) It is therefore possible to estimate the water particle velocity of breaking wave on sloping beaches by giving only the values of the beach slope and the deep-water wave steepness.

ACKNOWLEDGEMENTS

The authors wish to express their gratitudes to Mr. T. Asano, Mr. E. Otsuka and Mr. M. Tanaka for their assistances through the data analysis. This work was partly supported by Scientific Research Funds from the Ministry of Education.

REFERENCES

Adeyemo, M. D. : Velocity fields in breaker zone, Proc. 13th Conf. Coastal Eng., pp.435-460, 1970.

Battjes, J. A. : Surf similarity, Proc. 14th Conf. Coastal Eng., pp.466-480, 1974.

Dean, R. G. : Stream function representation of nonlinear ocean waves, Jour. Geophy. Res., Vol.70, No.18, pp.4561-4572, 1965.

Iwagaki, Y., T. Sakai, K. Tsukioka and N. Sawai : Relationship between vertical distribution of water particle velocity and type of breakers on beaches, Coastal Eng. in Japan, Vol.17, pp.51-58, 1974.

Iwagaki, Y. and T. Sakai : Representation of water particle velocity of breaking waves on beaches by Dean's stream function, Memo. Fac. Eng. Kyoto Univ., Vol.38, Part 1, pp.11-20, 1976.

Thornton, E. B., J. J. Galvin, F. L. Bub and D. P. Richardson : Kinematics of breaking waves, Proc. 15th Conf. Coastal Eng., pp.461-476, 1976.

Tsuchiya, Y. and M. Yamaguchi : Some considerations on water particle velocities of finite amplitude waves theories, Coastal Eng. in Japan, Vol.15, pp.43-58, 1972.