CHAPTER 52

# EXTREME SEA LEVELS FROM TIDE AND SURGE PROBABILITY

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## ABSTRACT

This paper considers a method of deriving the probability of occurrence of extreme sea levels by combining the probability functions of surge and tide. As a result the quantity of data required is less than with traditional methods. The philosophy of the method is discussed and the conditions under which the theory can be applied are examined. Finally the technique is applied to a number of ports in the United Kingdom and the results are compared with known estimates of extreme levels.

## 1. INTRODUCTION

Realistic estimates of the probability, or inversely the 'return period', of extremely high sea levels are necessary, for example, for the design of harbours and coastal defences. Estimates of the probability of extreme low levels, although not so widely used, are needed by the nuclear power industry for example to site cooling water intakes. Other applications came from the harbour authorities and the shipping industry for dredging approaches to ports or for towing deep draught oil rigs from their moorings across shallow banks to the open sea.

There are many ports particularly in northern Britain from which a few years of sea level data are available but not the several decades of data that one associates with traditional studies of extreme levels. It was our intention to make use of some of this data. Traditional estimates of extremes are made by considering the distribution of annual maximum and minimum sea levels (for example Lennon 1963 or Suthons 1963). In general curves are fitted to the distributions to produce a linear function and the sea level corresponding to a particular return period is estimated by linear extrapolation.

The technique developed in this paper enables estimates to be made from a few years of data by considering separately the probability distributions of the tidal and the surge (non-tidal residual) components of sea level. By extracting the surge component a continuous record of all surges that occur during the period is made available and these can be used to reinforce the statistics that otherwise would be generated from a single annual extreme value. For example large positive surges

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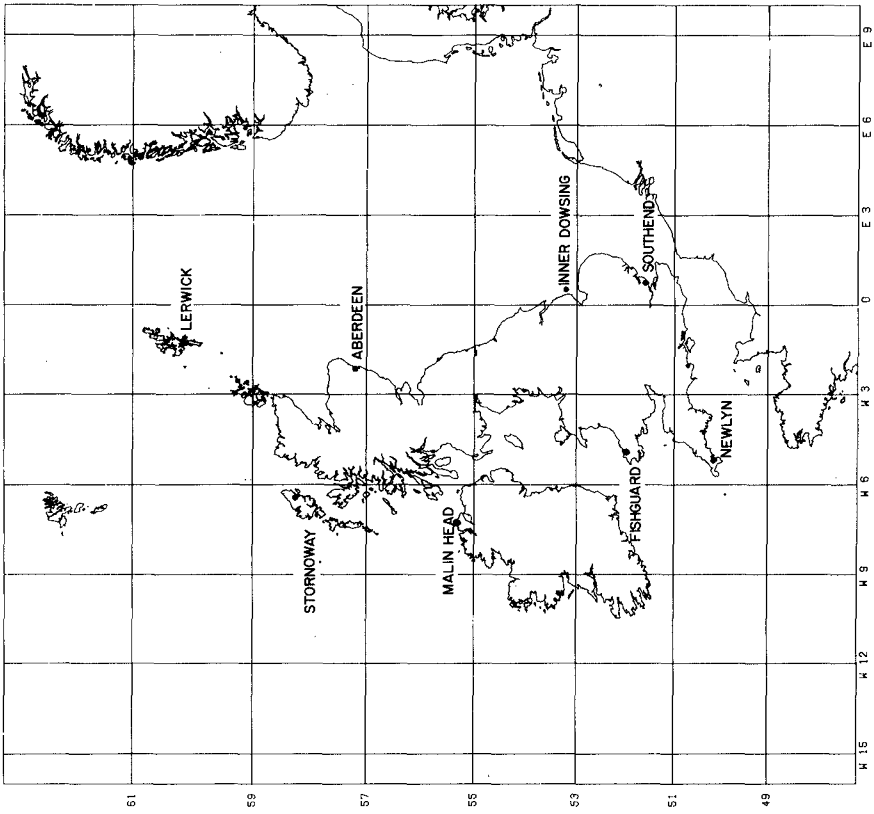


LERWICK
ABERDEEN
STORNOWAY
MALIN HEAD
INNER DOWSING
SOUTHEND
FISHGUARD
NEWLYN
N 61
N 59
N 57
N 55
N 53
N 51
N 49
E 9
E 6
E 3
0
W 3
W 6
W 9
W 12
W 15


FIG. 1.

that occur at low tide, which do not produce extreme total levels, are included in our analyses. Ackers and Ruxton (1974) used a similar technique for the Essex coastline, but only for surges that occurred at high water, in which they added the surge distribution at high water to the predicted tide at high water.

Our study had two main aims:

a) To produce estimates of extreme sea levels from data whose duration was much shorter than the return periods by examining tide and surge probability functions separately and combining them in the appropriate way.

b) To apply the method to a group of ports that would give a reasonable coverage of conditions round the United Kingdom.

The ports for which the technique was applied and the amount of data that was available were Newlyn (19 years), Fishguard (9), Malin Head (12), Stornoway (15), Lerwick (14), Aberdeen (10) and Southend (19). The location of each port is shown in figure 1. Southend was included to study the effect of the strong interaction between tides and surges which exists in the southern North Sea. At the other ports our investigation showed that there was little or no interaction and so tide and surge were treated as independent variables.

One advantage of the technique developed here was that mean sea level was eliminated from the computations. Of course it was defined relative to a local benchmark which in most cases had been levelled to Ordnance Datum Newlyn. Trends in the mean level had little effect because of the short lengths of data used. Rossiter (1966) and more recently Thompson (in preparation) have studied mean sea level trends and these may be easily incorporated into the extremal statistics on the assumption that they are statistically independent of the extremes.

## 2. Philosophy of the Method

At any time (t) the observed sea level ($\zeta$) can be considered as the sum of a tidal component (x), a surge component (y) and a mean level ($Z_o$)

$$\zeta(t) = x(t) + y(t) + Z_o \qquad 2.1$$

This is illustrated diagrammatically in figure 2.

The tidal component is the coherent part of the sea level that responds directly or indirectly to astronomical forcing. It was removed from the observed sea level records as the finite sum of harmonic constants which had the form:

$$x(t) = \sum_{n=1}^{N} H_n \cos(\sigma_n t + V_n - g_n) \qquad 2.2$$

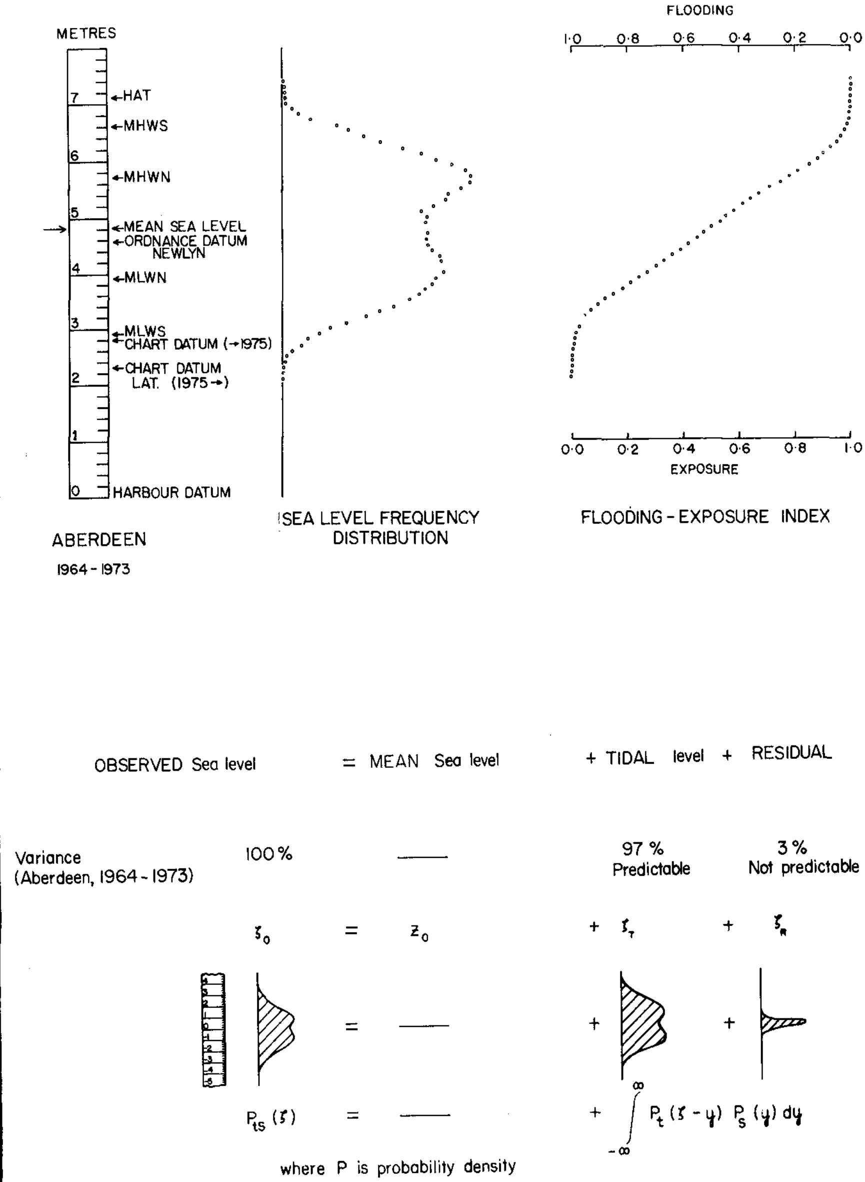

METRES
HAT
MHWS
MHWN
MEAN SEA LEVEL
ORDNANCE DATUM NEWLYN
MLWN
MLWS
CHART DATUM (→1975)
CHART DATUM LAT. (1975→)
HARBOUR DATUM
ABERDEEN
1964 - 1973
SEA LEVEL FREQUENCY DISTRIBUTION
FLOODING
1·0 0·8 0·6 0·4 0·2 0·0
0·0 0·2 0·4 0·6 0·8 1·0
EXPOSURE
FLOODING - EXPOSURE INDEX
OBSERVED Sea level = MEAN Sea level + TIDAL level + RESIDUAL
Variance (Aberdeen, 1964 - 1973)
100 %
97 % Predictable
3 % Not predictable
$\zeta_0 = z_0 + \zeta_T + \zeta_R$
$P_{ts}(\zeta) = \text{——} + \int_{-\infty}^{\infty} P_t(\zeta - y)\, P_s(y)\, dy$
where P is probability density

FIG. 2.

where $H_n$ = amplitude of each constituent

$\sigma_n$ = speed "

$V_n$ = equilibrium phase

$g_n$ = phase lag of constituent on the equilibrium tide

Removal of the tide did not require an excessive amount of data as a satisfactory tidal analysis was obtained from 1 year of observations.

The surge or non-tidal component is that part of the sea level which remains once the tide and mean level have been removed.

Over a long enough period the surge is a random variable. Obviously over a short period of say one month very few surges are likely to occur and any that do will not produce a random phase. Under these circumstances some of the surge energy is likely to appear in the harmonic constants and be depleted from the surge component. With longer data sets of say one year or more the effect is minimal and the surge is well separated from the tide. Only a few percent of the variance of sea level movements is attributable to the surge for typical shelf measurements. The remainder is tidal. Of course considerable damage is caused if the surge coincides with a high tidal level.

Unlike traditional methods the method of tide surge separation is not one of extrapolation. In this method it is intrinsically assumed that the available surge record is a representative sample of the population of all possible surges and that the surges could occur in a different configuration such that, for example, the largest surge could coincide with equinoxial spring tide. Of course the probability of these two coinciding is very small and should show as a correspondingly low probability in the final statistics.

Whether the above assumption is true is considered later but if it is, which is akin to saying that tide and surge are independent, the tidal and surge probability density functions (p.d.f.) can be combined as the sum of two independent variables (see for example Cramer 1971) to give the p.d.f. for total sea level.

If $p_t(x)$ = p.d.f. for tidal component

$p_s(y)$ = p.d.f. for surge component

and $p(\zeta)$ = p.d.f. for total sea level

then $$p(\zeta) = \int_{-\infty}^{\infty} p_t(\zeta - y) \, . \, p_s(y) \, . dy \qquad 2.3$$

In simple terms equation 2.3 states that the probability of sea level reaching say 10 metres is the sum of all surge and tidal probabilities that combine to produce a level of 10 m. For example a surge of 2 m and a tide of 8 m, or a surge of 1 m and a tide of 9 m.

The integral is taken over an interval which includes all possible tidal and surge levels.

From $p(\xi)$ the probability of exceedance of a particular level can be evaluated by generating the cumulative distribution function $F(\eta)$. where

$$F(\eta) = \int_{\eta}^{\infty} p(\xi) \cdot d\xi \qquad 2.4$$

The 'return period' for the level $\eta$ is given by the reciprocal of the function $F(\eta)$ provided that the time series representing the surge is not auto-correlated.

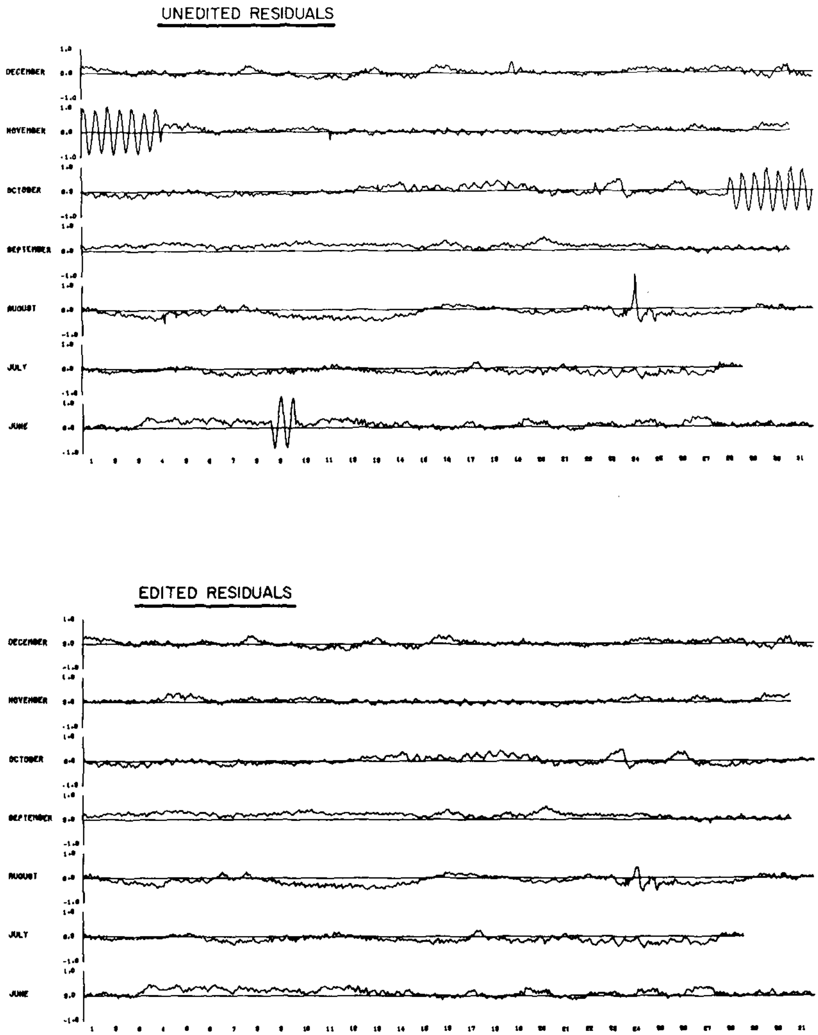
In a similar fashion the probability of exposure of a low level is given by

$$F(\eta) = \int_{-\infty}^{\eta} p(\xi) \cdot d\xi \qquad 2.5$$

Essentially the method consisted of a tidal analysis of the data for the harmonic constants and a prediction of the tide from these harmonic constants. From this the function $p_t$ was generated and from the difference between sea level and the tidal time series the function $p_s$ was generated. These were combined according to equation 2.3 and finally equations 2.4 and 2.5 were used to generate the cumulative distributions from which the return period and corresponding sea level were extracted.

## 3. THE DATA

The data consisted of hourly values of sea level digitised from tide gauge charts in the normal way. Each year comprised therefore on average 8766 values of hourly heights of sea level measured relative to the local tide gauge benchmark. The length of the records varied from a minimum of 9 years for Fishguard to 19 years for Newlyn and Southend. For these latter ports an extended record covering 50 years was available but only part of it was chosen for the present study. A summary of the data used is given in Table 1.




NON-TIDAL RESIDUALS

THE PLOT SHOWS SEVEN MONTHS OF HOURLY NON-TIDAL RESIDUALS GENERATED BY REMOVING THE TIDE FROM SEA LEVEL OBSERVATIONS. THE UNEDITED RESIDUALS SHOW ERRORS THAT ARE TYPICAL OF DIGITISED TIDE GAUGE RECORDS. THE MONTHS OF JUNE, OCTOBER AND NOVEMBER ILLUSTRATE TIMING ERRORS. IN AUGUST THERE IS A SINGLE MISPUNCHED VALUE. THESE ERRORS FALSELY SHOW UP AS A SIGNIFICANT SURGE.

FIG. 3

TABLE 1

| | | | |
|---|---|---|---|
| Newlyn | 19 years | 1951-69 | continuous |
| Fishguard | 9 | 1963-71 | a few gaps |
| Malin Head | 12 | 1960-71 | sections of the record dubious |
| Stornoway | 15 | 1957-72 | a few gaps |
| Lerwick | 14 | 1959-72 | a few gaps |
| Aberdeen | 10 | 1964-73 | many gaps |
| Southend | 19 | 1951-69 | continuous |

Most of the records were fairly complete except for short periods when the tide gauges malfunctioned. Malin Head gauge had two breakdowns and three sections lasting several months where the record was dubious for tidal analysis but adequate for surge statistics. The record from Aberdeen had many gaps but the data from the periods when the gauge was working was satisfactory.

The fact that gaps did exist in the data was of no consequence to the analysis procedure which was designed to deal with this. It might have had some effect on the surge statistics if the gaps had been systematic and had occurred only at particular times of the year. For example, if records had been available only from summer months the statistics would have been biased downwards because the surges were more frequent in the winter.

A typical surge record is shown in figure 3. This was produced by subtracting the tidal signal from the sea level record and there was therefore a need for careful editing of the data. This is not so critical in the annual extremes method which deals only with absolute levels.

When the charts were removed from the tide gauge drum they usually contained seven or fourteen days data and were such that every day occupied the same portion of the chart. Occasionally it happened that the chart was digitised erroneously by following the wrong part of the trace. Another fairly common type of error was due to timing, the chart was placed on the drum offset from the timing zero.

In the annual extremes method neither of the above errors would have had any effect as the absolute levels would not have been changed. However when the tide was removed both types of error produced a large tidally varying residual giving the false impression of a surge of significant height. Errors in reading the absolute height of the sea level curve would have affected the annual extreme and surge methods equally.

When the original tide gauge records were digitised they were at that time subjected to error checking routines. A Lagrangian interpolation curve was fitted to adjacent observations and a check was made for discontinuities between the smooth curve and the digitised level. This was found inadequate for detecting all the possible errors to the accuracy required.

The method chosen to examine the sea level record consisted fundamentally of plotting the surge as a function of time and of examining the plotted values by eye for irregularities. Errors were then corrected by referring to the original tide gauge chart.

The method of data editing is summarised below.

1. Pilot tidal analysis from 1 year of data.

2. Generate non-tidal residuals and plot as in figure 3.

3. Identify major extremes and areas of dubious record.

4. Refer to original tide gauge charts to correct errors.

5. Check weather records during periods of extremes.

6. Edit original sea level record.

7. Tidal analysis of the full period of data.

8. Reproduce non-tidal residuals.

This method was found to be very powerful for detecting errors and less time consuming than orthodox methods in cases where the record was consistently bad. Timing errors of a few minutes were easily detectable.

If there was some area of doubt such as a large surge when weather conditions were calm the data concerned was omitted from the analysis, but this rarely happened.

## 4. THE ASTRONOMICAL TIDES

The tidal part of the record was represented by the harmonic constants given in equation 2.2. From these constants a tidal time series was generated using the same equation and incrementing t from zero in units of one hour. The time series extended over a period of 19 years which is the time of regression of the moon's nodes, a fundamental period in the orbital motion of the Earth-Moon system. 19 years was necessary to generate the correct probability density function (p.d.f.) for the tides as the modulation in tidal elevation due to nodal motion could have been as much as 4%. The resulting p.d.f. for each port is shown in figure 4. These were generated numerically from the tidal time series in class intervals of 0.1 metre.

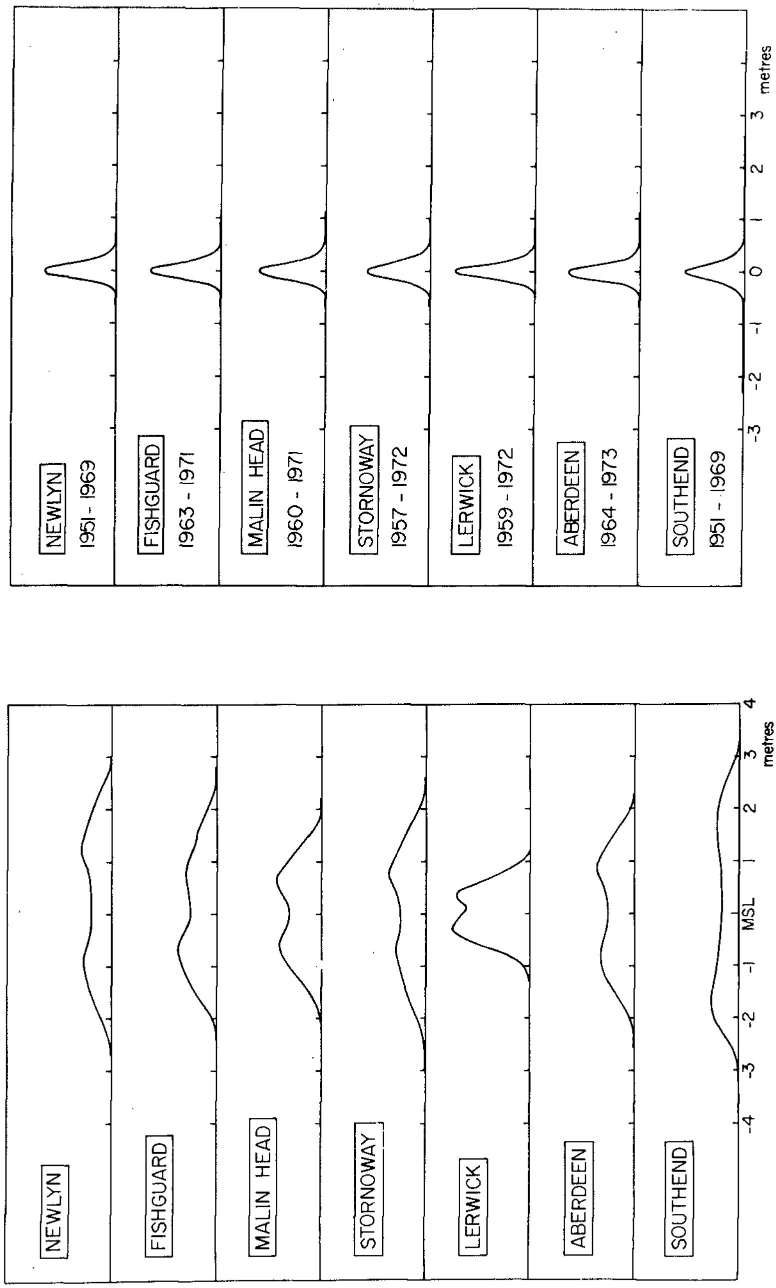



**FIG. 4.** Numerical computations of TIDAL frequency distribution at selected British gauges (based on 18 years of hourly predictions)

Observed SURGE frequency distribution for selected British gauges (based on hourly levels)

It was possible to get some way towards an analytical expression for the tidal probability distribution from the harmonic constants using the sum of independent sinusoids but all such attempts produced a symmetric distribution whereas almost all the measured tidal distributions were asymmetric.

The measured distribution was always bimodal, the two peaks, or modes, occurring near mean high water neaps and mean low water neaps, but with a considerable tail above and below these levels. The tails did not of course extend to infinity, they terminated at Highest and Lowest Astronomical Tide (HAT and LAT) which in the general case were not evenly distributed about mean sea level. The values of HAT and LAT were extracted from the tidal time series and their values are presented in Table 2 relative to the mean level at each port.

TABLE 2

HAT and LAT relative to mean sea level

| | HAT(m) | LAT(m) |
|---|---|---|
| Newlyn | 2.914 | -2.928 |
| Fishguard | 2.791 | -2.429 |
| Malin Head | 2.177 | -2.128 |
| Stornoway | 2.626 | -2.904 |
| Lerwick | 1.188 | -1.272 |
| Aberdeen | 2.281 | -2.576 |
| Southend | 3.311 | -3.131 |

One interesting feature of the tidal distribution was the asymmetry of the modes, i.e. they were of different heights. This has been positively related to the shallow water tides $M_4$ and $S_4$ which are at exactly twice the frequency of the main lunar and solar tides $M_2$ and $S_2$. It can be shown that the asymmetry is a maximum when the fourth-diurnal tides are in quadrative phase to the semi-diurnal tides. The asymmetry was noticeable even at Lerwick where the non-linear tides were fairly small in relation to the main tide.

## 5. THE SURGE OR NON-TIDAL RESIDUAL

The mean surge level for all ports was necessarily zero because of the manner in which the tides were removed from the observations of sea level. The time series (figure 3) had the appearance of a background noise continuum with occasional large surges protruding above the noise level. As with the tides the p.d.f. for the surges was generated numerically from the time series. Figure 4 contains examples of the surge p.d.f's.

The p.d.f's had a Gaussian appearance, but with some asymmetry, which varied in width depending on location. In some cases, particularly Southend, the tail of the distributions were very wide indicating large surges but of infrequent occurrence.

TABLE 3

| | maximum | minimum | standard deviation | Skewness | Kurtosis |
|---|---|---|---|---|---|
| Newlyn | 0.877m | -0.639m | 0.149m | 0.0016 | 0.0019 |
| Fishguard | 0.962 | -0.734 | 0.156 | 0.0019 | 0.0024 |
| Malin Head | 1.123 | -0.682 | 0.169 | 0.0023 | 0.0034 |
| Stornoway | 0.957 | -0.739 | 0.169 | 0.0016 | 0.0030 |
| Lerwick | 0.756 | -0.524 | 0.136 | 0.0010 | 0.0012 |
| Aberdeen | 1.054 | -0.743 | 0.155 | 0.0024 | 0.0026 |
| Southend | 2.638 | -2.307 | 0.230 | 0.0057 | 0.0247 |

Statistics of surge distributions based on hourly values

Table 3 contains the principal moments of the surge distribution for each port in the form of the standard deviation, the coefficients of Skewness and Kurtosis and the maximum and minimum levels reached during the period. The coefficients were defined as follows:

rth moment $= \mu_r = \frac{1}{N} \sum_{i=1}^{N} (y_i)^r$ where $y_i$ = ith surge level

standard deviation $= \sigma = (\mu_2)^{\frac{1}{2}}$ N = total number of observations of surge level.

coefficient of skewness $= \mu_3/\sigma^3$

coeffieient of kurtosis $= \mu_4/\sigma^4 - 3$

The standard deviation increased down the east coast from 13.6m at Lerwick to 23.0 cm at Southend. On the west coast the variation was found to be smaller lying between 14.9 cm and 16.9 cm. The distributions in all cases showed a positive skewness indicative that extreme high levels (positive surges) were more probable than low levels. This was reflected in the maximumand minimum levels which in all cases showed extremes on the positive rather than negative side. The kurtosis which is a measure of the flattening of a distribution relative to a normal distribution was always positive, more so where the standard deviation was large, but in general was small.

No allowance was necessary in the moments for Sheppard's corrections due to grouping. These were insignificant in comparison to the moments themselves because of the small class interval used in computing each p.d.f.

Tests were carried out on the variation of the p.d.f. over subsets of a year. These showed a remarkable stability in the main parameters of the p.d.f. except for the very extremes of the distribution with which of course this work was primarily concerned. This was expected because the extremes occurred infrequently during the period of even the longest data set and did not occur during every year.

## 6. ADDITION OF SURGE TO HAT AND LAT

This section is included in the paper not because is is part of the main theme but because it gave upper bounds to the extreme statistics. Also it was found to be a useful technique in cases where safe design criteria were essential. The method consisted of adding the surge extremes to Highest Astronomical Tide instead of combining surge and tide in the manner discussed in section 8. The procedure is illustrated in figure 5 where Fishguard is used as an example.

To estimate the extremes of surge p.d.f. the upper part of the cumulative distribution, shown in the lower part of the diagram, was linearised by fitting a logarithmic curve. This curve proved adequate even although the uppermost surges, which were only three in number, deviated slightly from it. An extrapolation of the curve gave the surge levels associated with return periods of between 1 year and 1000 years.

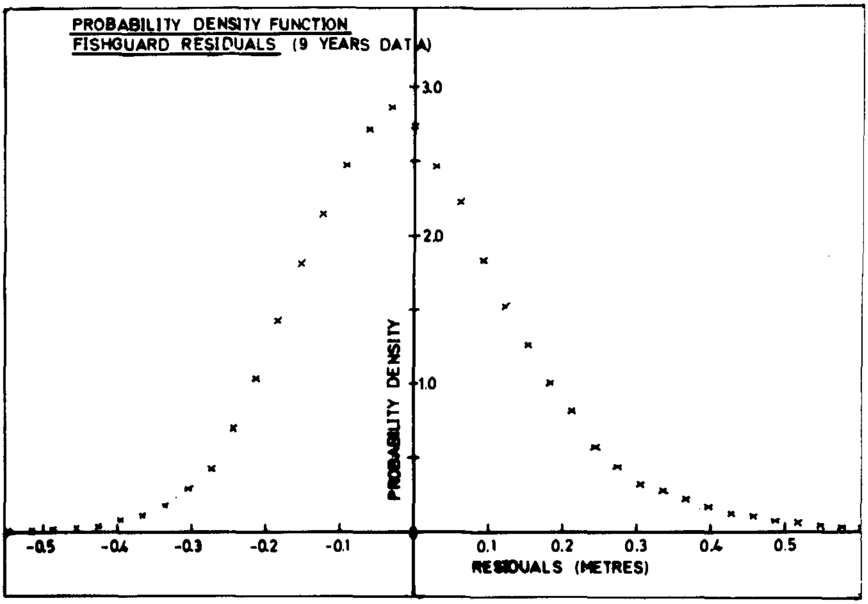
The total sea level appropriate to each return period was then derived by adding the surge statistics to HAT.

This technique of estimating extremes tended to overestimate the probability values and therefore gave, as expected, pessimistic results for the levels associated with each return period. The results are discussed in section 9.

## 7. INDEPENDENCE OF TIDE AND SURGE

The equation for the combination of tidal and surge distributions (2.3) assumes independence between the two and it is important to consider whether or not they can be treated as statistically independent variables.

One form of possible dependence stems from the fact that surges are more prevalent in the winter than at other times of the year and maximum



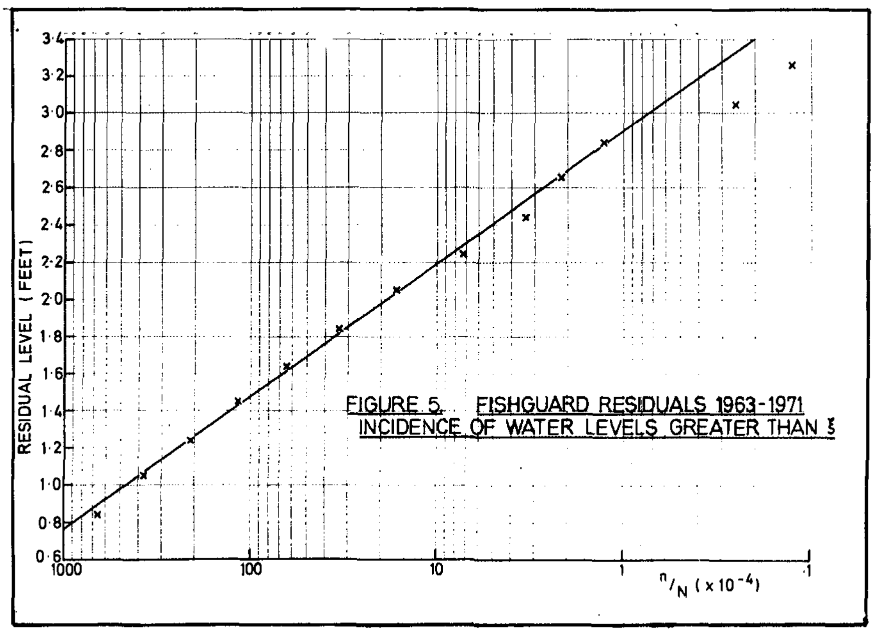




FIGURE 5. FISHGUARD RESIDUALS 1963-1971
INCIDENCE OF WATER LEVELS GREATER THAN ξ

tides occur at the equinoxes. However this is liable to be of secondary importance because the levels of the tides at the equinoxes are reached by spring tides at other times of the year to within a few tens of centimetres. Also the extreme surges with which we are primarily concerned do occur at times of the year other than winter. For example, one of the highest surges at Avonmouth occurred close to the spring equinox in March 1947. Fortunately it occurred very near low water or it would have caused extensive damage.

The main cause of dependence that has to be considered is surge/tide interaction such as occurs in the southern region of the North Sea. It is well known that at Southend the peak of the surge occurs on the rising tide and is somehow prevented from coinciding with high water.

To find if interaction was a problem at the six other ports a study was made of the moments of the surge distributions as a function of tidal level. Any interaction would have caused a variation of the moments with the height of the tide. Table 4 reproduces the results for two stations, Aberdeen and Southend.

TABLE 4a

INTERACTION STATISTICS FOR ABERDEEN

The moments of the surge distribution are shown as a function of tidal height.

| TIDAL LEVEL (metres) to m.s.l. | SURGE MOMENTS MEAN m | VARIANCE $m^2$ | SKEWNESS | KURTOSIS |
|---|---|---|---|---|
| -2.4 | 0.011 | 0.024 | 0.001 | 0.002 |
| -2.0 | 0.007 | 0.023 | 0.003 | 0.003 |
| -1.6 | 0.0 | 0.022 | 0.002 | 0.002 |
| -1.2 | -0.002 | 0.024 | 0.003 | 0.003 |
| -0.8 | -0.003 | 0.025 | 0.003 | 0.003 |
| -0.4 | -0.002 | 0.026 | 0.003 | 0.003 |
| 0.0 | 0.0 | 0.025 | 0.002 | 0.003 |
| 0.4 | 0.002 | 0.024 | 0.002 | 0.003 |
| 0.8 | 0.005 | 0.024 | 0.002 | 0.002 |
| 1.2 | 0.001 | 0.023 | 0.002 | 0.002 |
| 1.6 | -0.001 | 0.022 | 0.002 | 0.002 |
| 2.0 | -0.001 | 0.025 | 0.001 | 0.002 |
| 2.4 | -0.025 | 0.020 | -0.001 | 0.001 |

For Aberdeen there was little variation in any of the first four moments. The mean level was of course small but should have been zero. The second moment which is the most important as a test of interaction because it represents the square of the amplitude of the surge, was remarkably constant at 0.024 $m^2$ with an increase of only 2 $mm^2$ near the mid-tide level.

Southend on the other hand showed a marked increase in the second, third and fourth moments towards the mid-tide level and a decrease in the mean surge level. The second moment for example increased by a factor of two between the top and middle of the tidal range.

The other ports, although not reproduced here, were very similar to Aberdeen. It was concluded that, except for Southend, any interaction effects were of second order and that tide and surge could be treated as statistically independent variables.

TABLE 4b

INTERACTION STATISTICS FOR SOUTHEND

| TIDAL LEVEL (metres) to m.s.l. | SURGE MOMENTS MEAN m | VARIANCE $m^2$ | SKEWNESS | KURTOSIS |
|---|---|---|---|---|
| -3.2 | 0.121 | 0.043 | -0.007 | 0.008 |
| -2.8 | 0.052 | 0.040 | 0.002 | 0.011 |
| -2.4 | 0.014 | 0.039 | 0.004 | 0.011 |
| -2.0 | -0.005 | 0.047 | 0.009 | 0.022 |
| -1.6 | -0.006 | 0.054 | 0.010 | 0.025 |
| -1.2 | -0.013 | 0.061 | 0.012 | 0.036 |
| -0.8 | -0.011 | 0.064 | 0.009 | 0.034 |
| -0.4 | -0.013 | 0.072 | 0.009 | 0.043 |
| 0.0 | -0.005 | 0.069 | 0.008 | 0.040 |
| 0.4 | -0.008 | 0.066 | 0.004 | 0.030 |
| 0.8 | -0.001 | 0.066 | 0.004 | 0.033 |
| 1.2 | 0.019 | 0.057 | 0.002 | 0.020 |
| 1.6 | 0.021 | 0.052 | -0.002 | 0.018 |
| 2.0 | 0.012 | 0.047 | 0.001 | 0.017 |
| 2.4 | 0.006 | 0.041 | 0.0 | 0.010 |
| 2.8 | -0.020 | 0.039 | -0.001 | 0.010 |
| 3.2 | -0.086 | 0.035 | -0.007 | 0.012 |

## 8. COMBINED TIDAL P.D.F. AND SURGE P.D.F.

The sea level probability distribution was derived from the combined tide and surge distributions according to equation 2.3. In practice a discrete form of the integral was used because of the need to generate the individual distributions in discrete bands or class intervals.

$$p(\xi) = \sum_{i=-N}^{N} p_t(\xi - ih).p_s(ih) \qquad 8.1$$

where h = class interval = 0.1m.

Summation was over a number of bands 2N+1 which was wide enough to include all possible tidal and surge levels.

The cumulative distributions $F(\eta)$ were also discrete and were calculated as a discrete form of equations (2.4) and (2.5). For the high levels, these distributions gave the probability that a particular level would be exceeded which of course decreased as the level was increased. Table 5 shows the cumulative distribution function for each port but here the probability value is expressed as the return period.

Return period in years = 1/(exceedance probability x m)
where m is the number of hourly values in a year = 8766

TABLE 5

Return periods in years for exceedance of specified sea levels by combination of tidal and surge distributions

| Level (m) above msl | NEWLYN | FISHGUARD | MALIN HEAD | STORNOWAY | LERWICK* | ABERDEEN |
|---|---|---|---|---|---|---|
| 3.6 | 1670 | | | | | |
| 3.5 | 311 | 926 | | | | |
| 3.4 | 74 | 261 | | | | |
| 3.3 | 20 | 81 | | 762 | | |
| 3.2 | 5.9 | 26 | | 182 | | |
| 3.1 | 1.9 | 8.7 | | 50 | | 863 |
| 3.0 | 0.7 | 3.1 | 1520 | 15 | | 248 |
| 2.9 | | 1.1 | 392 | 4.9 | | 80 |
| 2.8 | | | 109 | 1.7 | 1520 | 27 |
| 2.7 | | | 35 | 0.6 | 199 | 9.8 |
| 2.6 | | | 12 | | 27 | 3.6 |
| 2.5 | | | 4.4 | | 4.8 | 1.4 |
| 2.4 | | | 1.7 | | 1.0 | • |

* Lerwick levels are related to Mean sea level + 1 metre

TABLE 6

Return periods in years for exposure of specified sea levels by combination of tidal and surge distributions

| Level (m) below msl | NEWLYN | FISHGUARD | MALIN HEAD | STORNOWAY | LERWICK* | ABERDEEN |
|---|---|---|---|---|---|---|
| -2.3 | | | 1.3 | | • | |
| -2.4 | | | 5.6 | | 1.1 | |
| -2.5 | | 0.8 | 28 | | 7.2 | |
| -2.6 | | 3.4 | 178 | | 56 | 0.7 |
| -2.7 | | 18 | 1655 | | 641 | 2.6 |
| -2.8 | | 101 | | | | 12.4 |
| -2.9 | | 584 | | 0.6 | | 64 |
| -3.0 | 1.3 | | | 2.0 | | 316 |
| -3.1 | 6.6 | | | 7.0 | | 1590 |
| -3.2 | 42 | | | 28 | | |
| -3.3 | 335 | | | 120 | | |
| -3.4 | | | | 560 | | |
| -3.5 | | | | | | |
| -3.6 | | | | | | |

* Lerwick levels are related to Mean sea level - 1 metre

Table 6 gives the equivalent values for the exposure of low levels again expressed as a return period. In both tables Lerwick has been offset relative to the mean level by 1 metre to fit in the table. The extremes at Lerwick were very much smaller than those at other ports because the tides had a relatively small range.

The return periods in some cases exceeded 1000 years. This might seem surprising because only a few years of data was used but in fact occurred because the combined probability was the product of two probabilities which near the limits of the distributions were themselves small. The limit of the tidal p.d.f. corresponded to the occurrence of equinoxial tide which lasted only a few hours each year (say $p_t = 0.001$). The surge probability was even smaller than that of the tide as it corresponded to the extreme surge in say 10 years ($p_s = 0.00001$). This gave a product of $(10)^{-8}$ and an equivalent return period of 10000 years. Therefore return periods of 1000 years contained at least 10 samples of surge in the upper levels of the distribution.

It is common practice in extreme levels work to quote return periods of 50 years and 100 years. The combined distribution did not produce these levels automatically. It was necessary to interpolate between height intervals to determine a particular return period. This was best done from the probability function because this was a smoother function of level than the return period. The total levels with a return period of 50 years and 100 years are shown in Table 7.

TABLE 7

Sea levels corresponding to 50 year and 100 year return periods. Levels are in metres relative to mean sea level.

| | 50 years | | 100 years | |
|---|---|---|---|---|
| | Exceedance Level | Exposure Level | Exceedance Level | Exposure Level |
| Newlyn | 3.38 | -3.32 | 3.43 | -3.37 |
| Fishguard | 3.27 | -2.78 | 3.33 | -2.80 |
| Malin Head | 2.75 | -2.55 | 2.80 | -2.59 |
| Stornoway | 3.10 | -3.34 | 3.18 | -3.31 |
| Lerwick | 1.65 | -1.60 | 1.68 | -1.65 |
| Aberdeen | 2.87 | -2.89 | 2.93 | -2.94 |

## 9. DISCUSSION

One reason for the inclusion of Newlyn in this work was that estimates were already available for high levels using the annual extremes method (Blackman and Graff, 1978). These were useful for comparison of the results from the two methods. The calculated levels which had a return period of 50 years and 100 years are reproduced in Table 8 from the annual extremes method, from the combined surge-tide method and from the addition of the surge distribution to Highest Astronomical Tide.

The values derived from each method can only be estimates of the probable extremes and should not be considered as absolute. Indeed, it is debatable whether absolute or correct values exist. The maximum observed level during the 62 years from 1915 to 1976 was 3.18m which was close to the 50 year level derived from annual extremes. The 100 year level by the same method was a slight underestimate when compared to the overall maximum.

The combined surge-tide probability produced values for the 50 year and 100 year return periods which were higher than those from annual extremes by 24 cm and 27 cm respectively. The agreement is good when the very different ways in which the data have been treated is considered. The 3.18 m level during the 62 years suggests that the method of annual extremes may give slightly low estimates. For engineering applications slight overestimates are preferable.

Our method will tend to produce overestimates because the surges are auto-correlated in that successive hourly samples of the time series are not independent of one another. Surges persist for more than one hour. If this grouping of hourly values had been taken into account then the estimated extreme levels would have been reduced. This problem is being investigated.

Another problem is the interaction between surge and tide. For this reason the results for Southend are notpresented in this paper. Estimates produced for Southend were greater than the values calculated from annual extremes by as much as 60 cm which requires further consideration. The surge probability distribution for Southend is a function of tidal height because of surge tide interaction (Prandle and Wolf, 1978) and it is intended to model this function so that surge and tide may be combined correctly.

TABLE 8

Comparison of return levels computed for NEWLYN
(above 1951-1969 mean sea level)

| | 50 years | 100 years |
|---|---|---|
| 62 years of annual maxima (1915-1976) | 3.14m | 3.16m |
| Combined tide-surge probability (1951-1969 observations) | 3.38m | 3.43m |
| Highest astronomical tide + surge (1951-1969 observations) | 3.90m | 3.93m |
| Maximum observed value 1915-1976 | 3.18m | |

## 10. CONCLUSIONS

We have developed a method for estimating extreme sea levels by studying the probability distributions of surge and of tide separately and then recombining them in a manner which assumes that they are statistically independent.

The method produces realistic estimates for extreme sea levels by comparison with traditional methods, but from much shorter lengths of data. This is important because few coastal sites have long sea level records. The assumption of independence of surge and tidal levels is proved to be valid for all the ports considered, with the exception of Southend.

The estimates are slightly biased on the high side for the upper extremes by comparison with traditional estimates. Some overestimation is inevitable because of the grouping of hourly surge values. The effects of this auto-correlation are being examined in an attempt to reduce the bias.

Our method also enables estimates of extreme low level probabilities. However, figures are not available from the annual extremes method for comparison.

The problem of surge-tide interaction is also being examined so that the method may be applied to ports such as Southend which are in shallow water and where, in consequence, surge and tide cannot be considered as statistically independent.

## ACKNOWLEDGEMENT

The work described in this paper was funded by the Consortium consisting of the Natural Environment Research Council, the Ministry of Agriculture Fisheries and Food and the Departments of Energy, Environment and Industry.

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