CHAPTER 63

ROLE OF 2D AND 3D MODELS IN JONSDAP '76

by

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Abstract

This paper describes how a two-dimensional numerical model of the North Sea was used to determine optimum positions for the deployment of off-shore tide gauges during the JONSDAP '76 oceanographic exercise.

A three-dimensional model of the North West European Shelf is also described. Using this model the three-dimensional distribution of the M_2 tidal current over the shelf has been computed. This model has also been used to compute the wind induced circulation of the North Sea for the INOUT period of JONSDAP '76.

1. Introduction

In order to gain a more comprehensive understanding of the physical and biological oceanography of the North Sea, a large scale observational program took place in the northern North Sea during 1976, namely JONSDAP '76 (Joint North Sea Data Acquisition Program 1976), involving a number of European maritime countries.

This paper describes two hydrodynamic North Sea models. The application of one of these in determining optimum positions for off-shore tide gauges, prior to the JONSDAP '76 exercise is described. The use of these models in the interpretation of the physical data collected during the exercise is also considered. Employing a model to determine positions for off-shore tide gauges it is possible to take tidal measurements around its open boundaries at locations which have the greatest influence upon the tidal regime subsequently computed by it. Observational data collected at these points during the exercise constitutes an optimum open boundary data set for use with the model to simulate the motion of the sea for the period of the exercise.

The numerical model used to determine tide gauge sites, is two-dimensional, comprehensively non-linear, with a fine grid resolution of $1/9^{o}$ latitude by $1/6^{o}$ longitude; it covers the North Sea, the Skagerrak, the Kattegat and the eastern half of the English Channel (see Figure 1). The open boundaries correspond to those monitored during JONSDAP '76. Thus, the open boundary in the north runs along the line between the port of Wick and position $59^{o}20'$N, $0^{o}0'$W and then along latitude $59^{o}20'$N between this point and the Norwegian coast. The southern open boundary is the line of longitude crossing the English Channel at 2^{o}W. The southern end of the Kattegat is a very shallow region and has been considered closed.

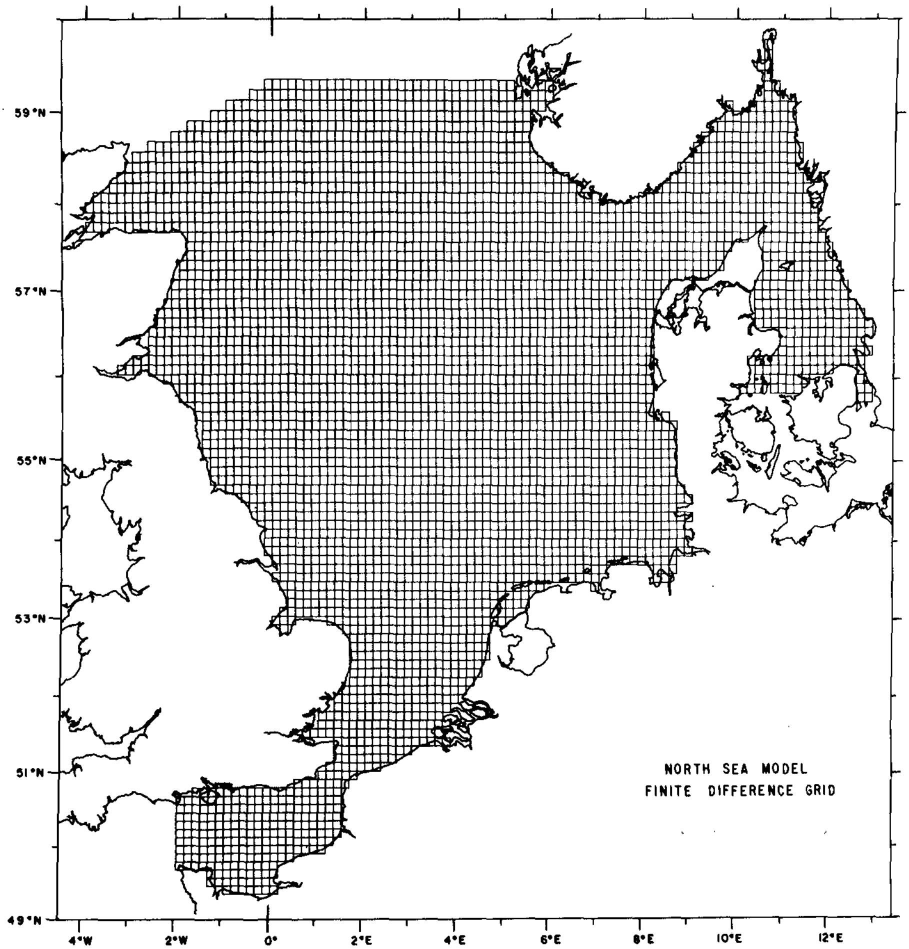

Figure 1. Two-dimensional North Sea model : finite difference grid.

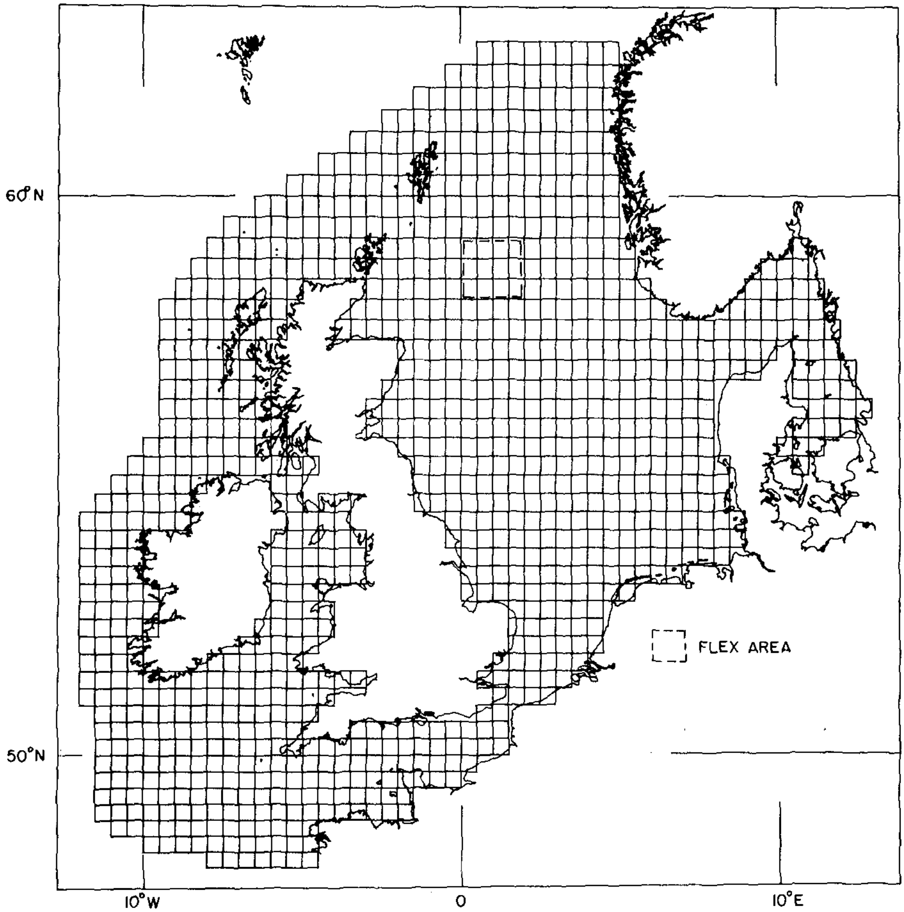

Figure 2. Three-dimensional shelf model : finite difference grid.

The second model employed is three-dimensional with a coarser grid resolution of $1/3°$ latitude by $1/2°$ longitude, covering the continental shelf as shown in Figure 2. The region in the northern North Sea, marked FLEX area on this diagram, denotes the sea area in which an intense biological measurement program took place concurrently with the physical measurements made during JONSDAP '76. The hydrodynamical equations include a quadratic bottom friction but are otherwise linear. Along the open boundaries, a radiation condition is employed to allow disturbances from the interior of the model to pass outwards.

This model is used initially to compute the distribution of the M_2 tide over the continental shelf. In this case boundary forcing is provided by M_2 tidal elevation alone. In a subsequent calculation the wind-induced circulation on the shelf for the period 15 March 1976 to 15 April 1976 (the INOUT period of JONSDAP '76) is computed. Boundary forcing for this case involved the M_2 tidal elevation and a meteorologically-induced elevation computed using the hydrostatic approximation. Meteorological forcing at the sea surface consisted of a time-average wind stress field and a time-averaged field of atmospheric pressure gradients. The time averaging was over the period cited above. The meteorologically induced circulation for the period was computed by subtracting the pure tidal motion from that computed with tide and meteorological forces.

2. Two-dimensional North Sea model (Figure 1)

The depth averaged equations of continuity and motion, including the non-linear advective terms and non-linear bottom friction may be written in polar coordinates as follows :

$$\frac{\partial \xi}{\partial t} + \frac{1}{R\cos\phi}\left\{\frac{\partial}{\partial \chi}(dU) + \frac{\partial}{\partial \phi}(dV\cos\phi)\right\} = 0 \qquad (1)$$

$$\frac{\partial U}{\partial t} + \frac{U}{R\cos\phi}\frac{\partial U}{\partial \chi} + \frac{V}{R\cos\phi}\frac{\partial}{\partial \phi}(U\cos\phi) - 2\omega\sin\phi\, V + \frac{kU(U^2+V^2)^{1/2}}{d} + \frac{g}{R\cos\phi}\frac{\partial \xi}{\partial \chi} = 0 \qquad (2)$$

$$\frac{\partial V}{\partial t} + \frac{U}{R\cos\phi}\frac{\partial V}{\partial \chi} + \frac{V}{R}\frac{\partial V}{\partial \phi} + \frac{U^2\tan\phi}{R} + 2\omega\sin\phi\, U + \frac{kV(U^2+V^2)^{1/2}}{d} + \frac{g}{R}\frac{\partial \xi}{\partial \phi} = 0 \qquad (3)$$

where the notation is :

χ, ϕ east-longitude and latitude, respectively,

t time,

ξ elevation of the sea surface above undisturbed level,

h undisturbed depth of water

$d = h + \xi$ total depth of water,

R radius of the Earth,

ω angular speed of the Earth's rotation,

k coefficient of bottom friction,

g acceleration due to gravity,

u, v components of current in the directions of increasing χ, ϕ respectively, at a depth z below the undisturbed sea surface,

U, V components of depth mean current given by

$$U = \frac{1}{h+\xi}\int_{-h}^{\xi} u(z)\,dz \quad , \quad V = \frac{1}{h+\xi}\int_{-h}^{\xi} v(z)\,dz \tag{4}$$

Equations (1) to (3) are approximated using finite difference methods : details are given by Davies and Flather [1978] and will not be presented here. An explicit finite difference technique is used to generate solutions through time and space over a staggered grid in which ξ, U and V are calculated at different mesh points. Solutions are generated from a state of zero displacement and motion, expressed by,

$$\xi = U = V = 0 \tag{5}$$

Along open and closed boundaries, appropriate dynamical conditions have to be satisfied. Thus, along a closed boundary the normal component of current is set to zero, i.e.

$$U \cos\psi + V \sin\psi = 0 \tag{6}$$

where ψ denotes the inclination of the normal to the direction of increasing χ. Along the open boundaries tidal elevation is specified according to the harmonic theory. Here we consider only the M_2 component of the tide, thus the tidal elevation on the open boundary is given by :

$$\zeta = H\cos(\sigma t - g), \tag{7}$$

where for this component, H denotes the amplitude, σ the speed and g the phase.

3. Calculation of the M_2 tide and determination of an optimum deployment pattern for tide gauges.

In principle in order to calculate the M_2 component of the tide throughout the region covered by the two-dimensional model (Figure 1) it is necessary to have a detailed knowledge of the tide along the open boundaries of the model. Obviously prior to the JONSDAP '76 exercise this data was not available, and thus it was necessary to take M_2 tidal input from a numerical model of the continental shelf (Flather [1976]). Input data around the open boundaries of the latter model was derived from the observational data taken by Cartwright [1976]. A distribution of the M_2 tide over the North Sea in good agreement with observation (Davies [1976]) was obtained from the two-dimensional model shown in Figure 1.

Having thus proved the model it was possible to determine the sensitivity of the distribution of the M_2 tide computed with the model to changes in amplitude and phase of the M_2 tidal input along various sections of the open boundary. To this end, the northern open boundary of the model was divided into seven sections. Section 1 spans the seven grid boxes between the Scottish coast and the point (58°40'N, 2°0'W), Section 2 the seven grid boxes between (58°40'N, 2°0'W) and (59°0'N, 0°50'W) and so on up to the seventh section which spans eight grid boxes between point (59°20'N, 3°50'E) and the Norwegian coast. An eighth section comprised the whole of the southern boundary. In order to test the response of the model, the amplitudes of the M_2 tide in the first section were increased by 10 cm; at the same time the phase in this section and the amplitudes and phases in the other sections were kept constant. The numerical model was then run until the tidal regime was established and differences in amplitude and phase, ΔH and ΔG, between the tides of the new distribution and those of the original one were compared at a number of ports throughout the region. In a similar experiment, the phase of the M_2 tidal input in Section 1 was changed by 10°, and the changes ΔH and ΔG at a number of ports were determined. Similar numerical experiments were performed for all seven northern boundary sections, and for the eighth boundary section across the English Channel.

This sensitivity analysis showed that the tides along the north-east coast of Britain from Aberdeen to Inner Dowsing were particularly sensitive to changes along the north-western part of the boundary. They were also sensitive to changes near the Norwegian coast. Changes in tidal amplitude along this section of boundary also affected the tide along the south-east coast of England and in the Channel.

Obviously it is not practical to measure the tide at each northern boundary point of the model. However based on the results of the sensitivity analysis, it was evident that the most important regions for tidal specification lay on the western and eastern sections of the Northern boundary and thus a distribution of five off-shore tide gauges that reflected this fact was used (Davies [1976]). Since the gauges were situated along the open boundaries of the model the tidal data measured during JONSDAP '76 can be readily incorporated in the model, and calculations using this data are presently in progress, aimed at determining the distribution of various tidal constituents over the North Sea.

4. Three-dimensional shelf model (Figure 2)

The equations of continuity and motion for homogeneous water, neglecting non-linear terms and shear in the horizontal, may be written in polar coordinates as :

$$\frac{\partial \xi}{\partial t} + \frac{1}{R\cos\phi}\left\{\frac{\partial}{\partial \chi}\int_0^h u\,dz + \frac{\partial}{\partial \phi}\int_0^h v\cos\phi\,dz\right\} = 0 \tag{8}$$

$$\frac{\partial u}{\partial t} - \gamma v = \frac{-g}{R\cos\phi}\frac{\partial \xi}{\partial \chi} - \frac{1}{\rho R\cos\phi}\frac{\partial P}{\partial \chi} + \frac{\partial}{\partial z}\left(N\frac{\partial u}{\partial z}\right) \tag{9}$$

$$\frac{\partial v}{\partial t} + \gamma u = \frac{-g}{R}\frac{\partial \xi}{\partial \phi} - \frac{1}{\rho R}\frac{\partial P}{\partial \phi} + \frac{\partial}{\partial z}\left(N\frac{\partial v}{\partial z}\right) \tag{10}$$

where we denote by :

P atmospheric pressure at the sea surface,

N the coefficient of vertical eddy viscosity, assumed to be a function of χ, ϕ, z and t. All other symbols are as defined previously.

In order to solve (8), (9) and (10) for ξ, u, v, boundary conditions have to be specified at the sea surface and at the sea bed. The surface conditions are

$$-\rho\left(N\frac{\partial u}{\partial z}\right)_0 = F_s \quad , \quad -\rho\left(N\frac{\partial v}{\partial z}\right)_0 = G_s \tag{11a,b}$$

where F_s, G_s denote the components of wind stress acting on the water surface in the χ and ϕ directions, suffix 0 denoting evaluation at $z=0$.

Similarly at the sea bed, $z=h$, assuming a slip condition and using a quadratic law of bottom friction, gives

$$-\left(N\frac{\partial u}{\partial z}\right)_h = k u_h (u_h^2+v_h^2)^{1/2}$$

$$-\left(N\frac{\partial v}{\partial z}\right)_h = k v_h (u_h^2+v_h^2)^{1/2} \tag{12a,b}$$

where k is the coefficient of bottom friction assumed constant.

We now seek a solution of equations (8), (9) and (10) for ξ, u, v subject to boundary conditions (11) and (12). Expanding the two components of velocity u, v in terms of m depth dependent functions $f_r(z)$ and horizontal-space and time-dependent coefficients $A_r(\chi,\phi,t)$ and $B_r(\chi,\phi,t)$ gives

$$u(\chi,\phi,z,t) = \sum_{r=1}^{m} A_r(\chi,\phi,t)\, f_r(z) \tag{13}$$

$$v(\chi,\phi,z,t) = \sum_{r=1}^{m} B_r(\chi,\phi,t)\, f_r(z) \tag{14}$$

Using the Galerkin method in the vertical space domain, equations (9) and (10) are multiplied by each of the basis functions f_k, and integrated with respect to z over the interval 0 to h. By integrating the term involving the vertical eddy viscosity, boundary conditions (11a,b) and (12a,b) can be included (Davies [in preparation]) giving

$$\int_0^h \frac{\partial u}{\partial t} f_k\,dz = \gamma\int_0^h v f_k\,dz - \frac{g}{R\cos\phi}\frac{\partial \xi}{\partial \chi}\int_0^h f_k\,dz - \frac{1}{\rho R\cos\phi}\frac{\partial P}{\partial \chi}\int_0^h f_k\,dz$$

$$+ f_k(0)\frac{F_s}{\rho} - f_k(h)\, k\, u_h(u_h^2+v_h^2)^{1/2} - \int_0^h N\frac{\partial u}{\partial z}\frac{\partial f_k}{\partial z}\,dz \tag{15}$$

and

$$\int_0^h \frac{\partial v}{\partial t} f_k\,dz = -\gamma\int_0^h u f_k\,dz - \frac{g}{R}\frac{\partial \xi}{\partial \phi}\int_0^h f_k\,dz - \frac{1}{\rho R}\frac{\partial P}{\partial \phi}\int_0^h f_k\,dz$$

$$+ f_k(0)\frac{G_s}{\rho} - f_k(h)\, k\, v_h(u_h^2+v_h^2)^{1/2} - \int_0^h N\frac{\partial v}{\partial z}\frac{\partial f_k}{\partial z}\,dz \tag{16}$$

for k = 1,2, ... , m

Consider now the choice of basis functions $f_\tau(z)$. Davies [in preparation] has demonstrated that an expansion of 10 cosine functions is sufficient to accurately reproduce the depth variation of current, and has successfully applied such an expansion to the computation of the wind induced circulation of the North West European Continental Shelf (Davies [1979]). In this paper we again use an expansion of 10 cosine functions, with $f_\tau(z)$ given by,

$$f_\tau(z) = \cos \alpha_\tau z/h \tag{17}$$

The choice of α_τ is quite arbitrary, provided $f_\tau(h)$ is non zero, and a suitable choice for α_τ is,

$$\alpha_\tau = (\tau - 1)\pi \qquad \text{for } \tau = 1, 2, \ldots, m \tag{18}$$

Substituting expansions (13) and (14) into equations (15), (16) and (8), gives a set of 2m + 1 equations, which when integrated forward through time subject to initial and boundary conditions, yields ξ and the set of coefficients A_τ, B_τ at progressive time steps. Using expansions (13) and (14), the U and V components of current at any depth can be computed from the set of A_τ and B_τ. Details of the method for the solution of the equations are given by Davies (1979) and will not be repeated here.

In order to solve equations (15) and (16) it is necessary to specify how N will vary with χ, ϕ and t. The horizontal variation of N over the North Sea for semi-diurnal tidal motion has been computed by Kraav (1969). He shows that N increases rapidly in the shallow coastal areas, particularly along the east coast of England, where the tidal currents are high, and to a first approximation it is evident that N increases with the square of the tidal current. On the basis of Kraav's results, an appropriate parameterization of vertical eddy viscosity is, using M.K.S. units,

$$N = 0{\cdot}2\,(\bar{U}^2 + \bar{V}^2)$$

where $\bar{U}$ and $\bar{V}$ are depth mean currents given in terms of the present theory by

$$\bar{U} = \sum_{\tau=1}^{m} A_\tau a_\tau \quad , \quad \bar{V} = \sum_{\tau=1}^{m} B_\tau a_\tau \tag{19}$$

$$a_\tau = \frac{1}{h}\int_0^h f_\tau(z)\,dz \tag{20}$$

This parameterization of vertical eddy viscosity was used in the tidal computations to be described in Section 5.

Alternatively Heaps (1972), from computations of the wind induced circulation of a simple rectangular basin, suggested that the term $k h/N$ should be constant and accordingly this term was fixed in the calculation of the wind induced circulation of the North Sea (see Section 6) in such a way that, in water of depth 65m, N had a value of 650 cm^2/s - a value suggested by Heaps (1972). In this calculation a coefficient of bottom friction k = 0.0025 was assumed over the whole shelf.

Solutions were generated from a state of zero displacement and motion, expressed by,

$$\zeta = A_r = B_r = 0 \quad \text{at } t = 0 \quad (r = 1,2, \ldots, m) \tag{21}$$

Along a closed boundary the normal component of current was set to zero, for all $t \geq 0$, thus,

$$A_r \cos\psi + B_r \sin\psi = 0 \quad (r = 1,2, \ldots, m) \tag{22}$$

where ψ denotes the inclination of the normal to the direction of increasing χ.

Consider now the open boundary conditions. For the computation of tidal motion alone over the continental shelf, M_2 tidal input determined from the two-dimensional model of Flather [1976] was used, together with a radiation condition. Details of this radiation condition can be found in Davies [1979]. For the computation of the tidal motion, the pressure gradient and wind stress terms in equations (15) and (16) were set to zero.

In computing the wind induced motion over the continental shelf the radiation condition was again used along the open boundaries of the model, with M_2 tidal input as described previously, together with meteorological input along the open boundaries, computed using the hydrostatic approximation.

The meteorological forcing functions, namely the two components of wind stress F_s and G_s, and the pressure gradients $\partial P/\partial \chi$ and $\partial P/\partial \phi$ were calculated from data supplied by the British Meteorological Office. Hourly values of wind stress and pressure gradients were computed at the grid points of the model for the period 0000h 15 March to 0000h 15 April 1976 from the hourly geopotential height data. From this time series mean values of pressure gradients and wind stresses over the sea area for this period were computed.

The meteorologically induced circulation for the period was computed by subtracting the pure tidal motion (computed using identical values of k and N to those used with the meteorological input) from that computed with tide and meteorological forces.

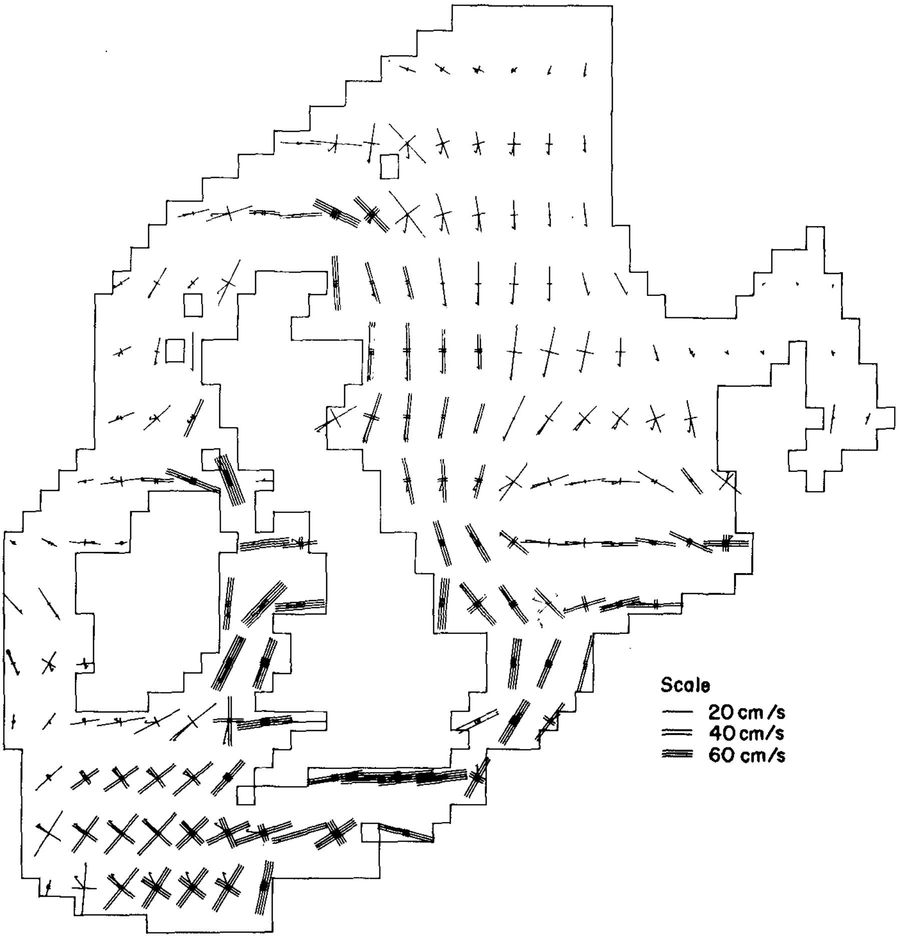

Figure 3a. Distribution over the continental shelf of M_2 current ellipses at sea surface.

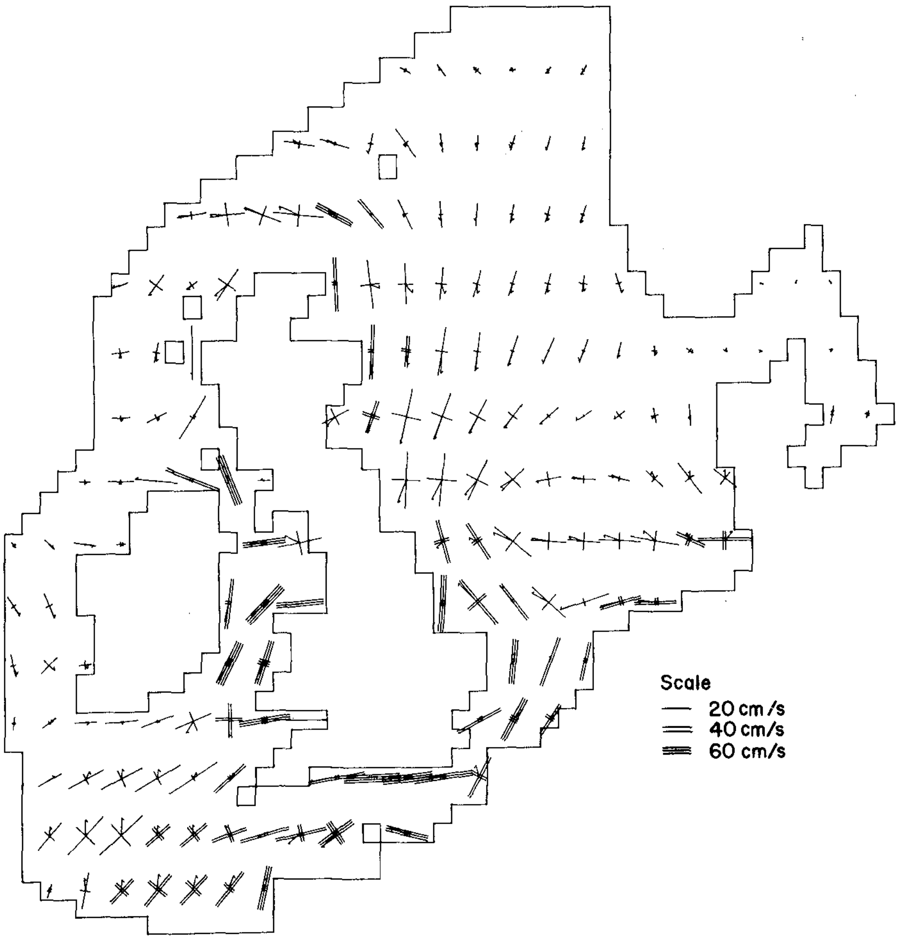

Figure 3b. Distribution over the continental shelf of M_2 current ellipses at sea bed.

5. Computed M_2 tidal currents

The distribution of the M_2 tidal currents over the continental shelf, and through depth were computed using the three-dimensional shelf model described previously. The variation of vertical eddy viscosity with tidal current described in Section 4 was used in the model.

In an initial series of calculations the computed amplitude of the M_2 tidal elevations and currents in shallow areas was found to be sensitive to the value of the coefficient of bottom friction k. Using a value of $k = 0.0025$ (an appropriate value for a two-dimensional model [Davies 1976]) the amplitude of the tidal velocity in shallow areas was higher than the observed value. However, increasing k to 0.0050 reduced the magnitude of these elevations and currents, although currents in the deeper areas were only slightly reduced.

Plots of the magnitude and direction of the major and minor axes of the M_2 tidal current and the sense of rotation of the ellipse computed using $k = 0.0050$, together with the time of lunar transit at Greenwich, at every third grid point of the model are given in Figures 3a,b. Figure 3a shows such a plot for the surface current, and the bottom current is presented in Figure 3b.

From Figures 3a,b it is evident that the maximum currents occur in the English Channel, between Cherbourg and the Isle of Wight, where the flow is nearly rectilinear. Tidal surface currents in excess of 100 cm/s occur in this area. Surface velocities of the order of 70-90 cm/s occur along the east coast of England, essentially flowing parallel to the coast. In the northern North Sea, surface tidal velocities are much lower, of order 5-15 cm/s with the orientation of the major axis of the ellipse in a north-south direction. However in the central North Sea, the orientation of the major axis is in a west-east direction, see Figures 3a,b, and the magnitude of the current is of order 25 cm/s.

Comparing Figures 3a and 3b it is evident that over most of the North Sea, both major and minor axes of the tidal current ellipse diminish with depth. However close to the Norwegian coast the computed ellipses do not exhibit this behaviour; the minor axis increases with depth close to the sea bed, and this increase is present in the M_2 current obtained from the harmonic analysis of the observations taken in this area during JONSDAP '76.

It is evident from Figures 3a and 3b that the current ellipses to the east of Aberdeen, rotate in the opposite directions at sea surface and sea bed. Computed current ellipses close to the Norwegian coast also have a different sense of rotation between surface and bottom, and these differences agree well with observational data taken in the area.

As more tidal data becomes available from the JONSDAP '76 program, a more rigorous comparison between computed and observed current ellipses will be possible.

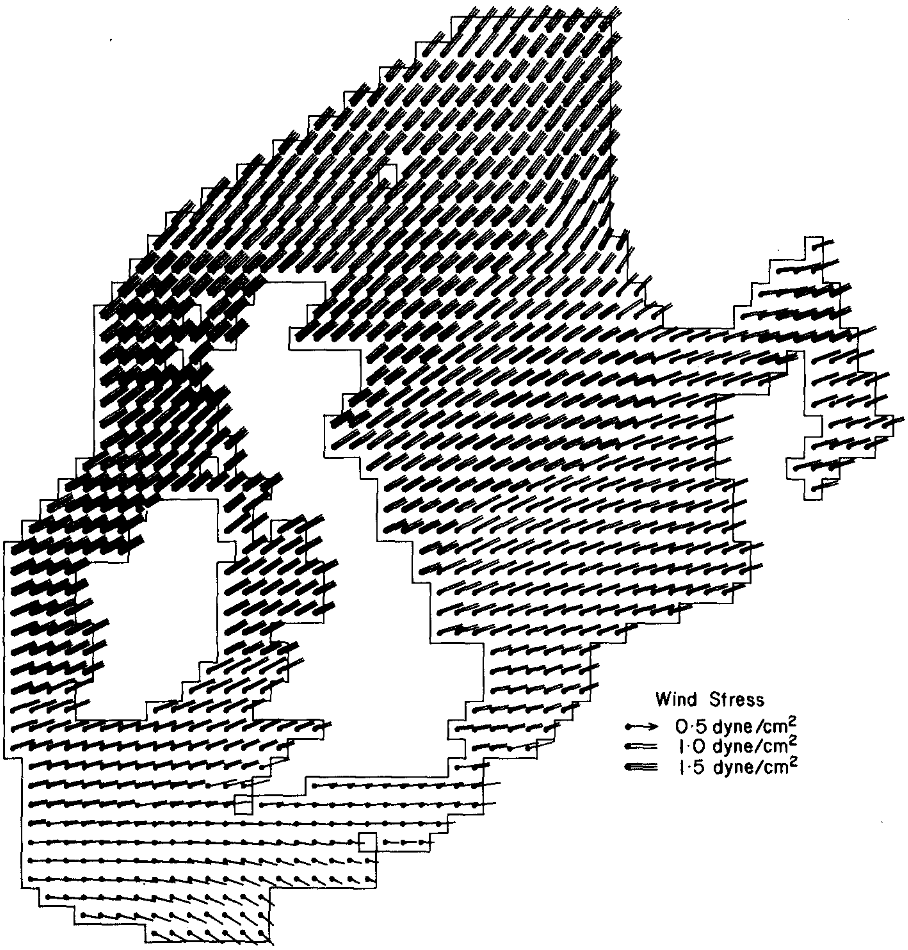

Figure 4. Distribution of mean-wind stress for the period 15 March to 15 April 1976 over the continental shelf.

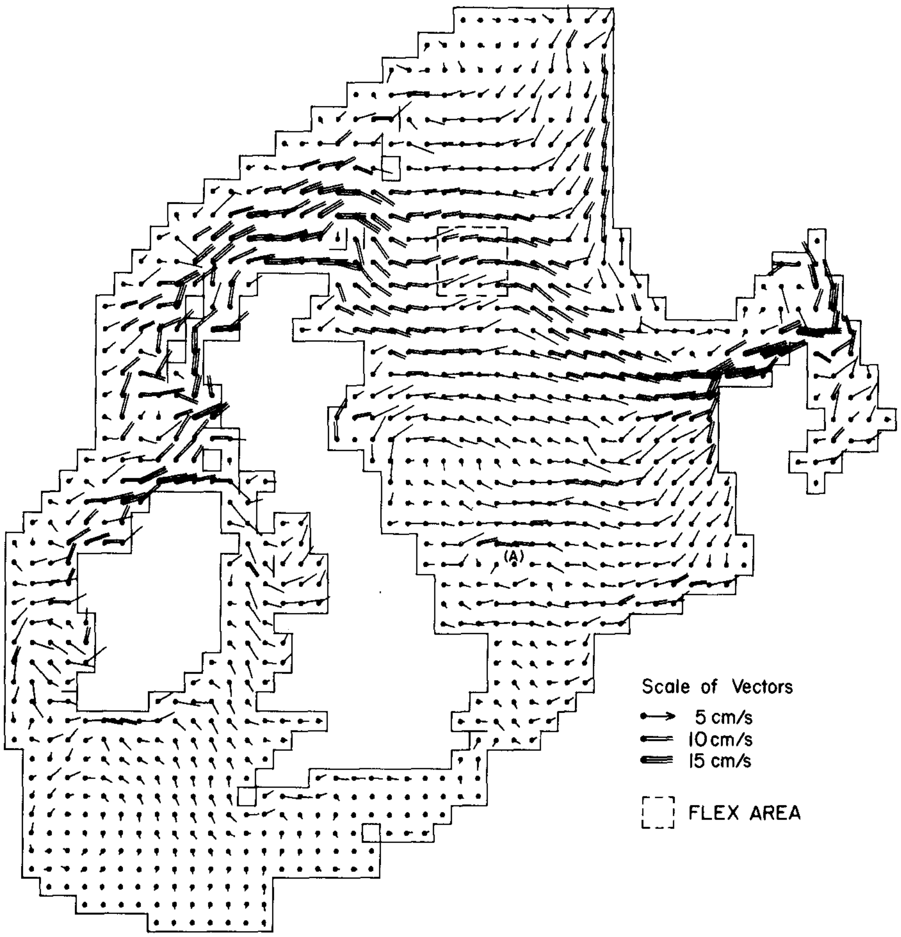

Figure 5a. Meteorologically-induced surface currents, means for the period 15 March to 15 April 1976, computed with the three-dimensional shelf model.

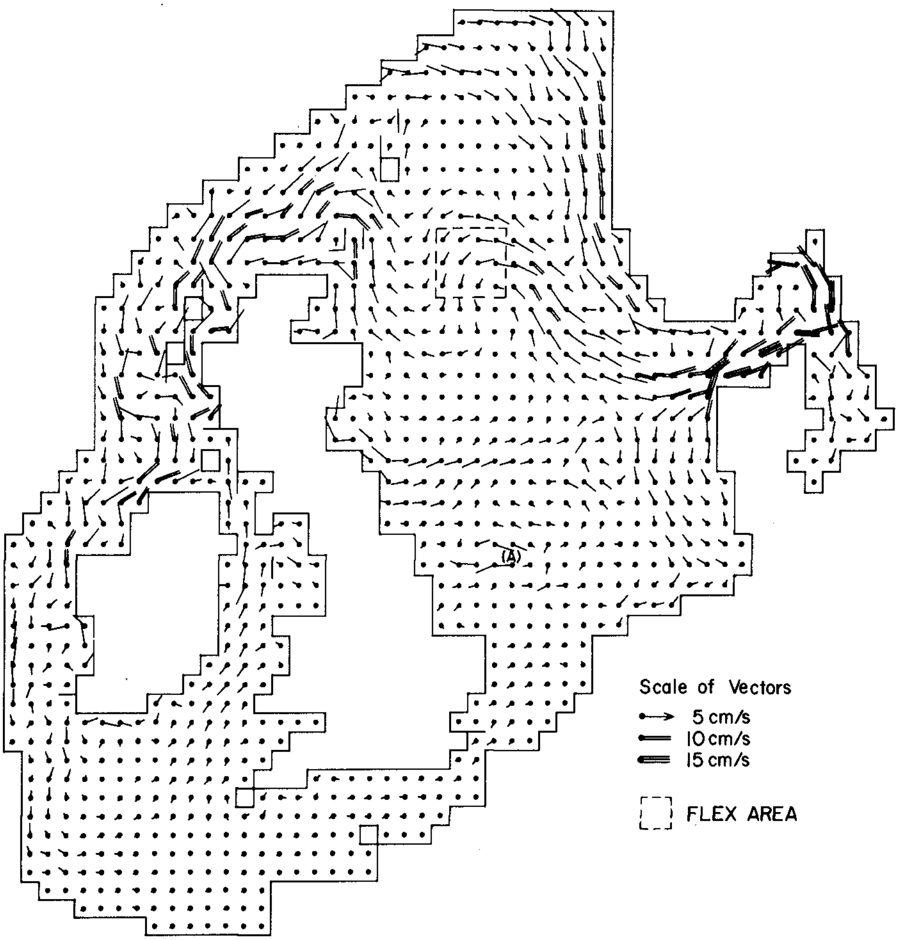

Figure 5b. Meteorologically-induced bottom currents, means for the period 15 March to 15 April 1976, computed with the three-dimensional shelf model.

6. Wind induced circulation for the period 15 March to 15 April, 1976.

Initial conditions corresponding to a state of rest were assumed, and the three-dimensional motion three days after the imposition of the meteorological forcing terms was calculated by integrating equations (8), (15) and (16) forward through time.

The distribution of the mean wind stress over the shelf is given in Figure 4.

The computed spatial distribution of meteorologically induced surface current over the shelf is shown in Figure 5a and comparing this with the computed distribution of bottom current in Figure 5b, it is evident that over most of the shelf the magnitude of the current decreases with depth, and that in certain areas significant differences in current direction, up to 180°, occur between surface and bottom currents.

The horizontal variation of surface current in the region of the FLEX box, to a first approximation is fairly uniform, reflecting the uniform wind stress field, being essentially an eastward flow, at about 45° to the right of the wind field, as would be expected from Ekman theory. However, the bottom flow (Figure 5b) shows a large horizontal variation, being directed southward down the Scottish coast in the sea area to the east of Wick, then flowing eastward at the latitude of Aberdeen, and subsequently turning northward at a point about 100 km to the south of the FLEX box. The flow through the FLEX box shows the current turning from a northward to an eastward flow, which then continues to the south east into the Skagerrak. This high horizontal spatial variation of bottom current in the northern North Sea presumably being due to the influence of bottom topography.

Differences in direction between surface and bottom currents in the German Bight are also evident from Figures 5a and 5b. The surface current is mainly to the north east, moving water towards the coast, whereas the bottom current is to the north west, taking water away from the coast.

A particularly interesting circulation pattern exists off the east coast of England in the region of the grid point marked (A) in Figures 5a and 5b. At grid point (A) the surface current is near zero, while the bottom current is the order of 3 or 4 cm/s towards the coast. The surface currents in the area surrounding point (A) are predominantly off shore, whereas at point (A), and due west of it, the bottom current is towards the coast. This type of circulation has been observed a number of times (Hill [1973]).

Further calculations are presently in progress to compute the daily residual circulation of the North Sea for the period considered here.

7. Concluding Remarks

The results described in Section 3 show that a numerical model can play an important role in the design of an oceanographic exercise. By making measurements at critical points along the open boundaries of the model, an optimum set of open boundary data can be derived and subsequently used with the model to simulate the motion of the sea during the period of the exercise.

Preliminary comparisons of the three-dimensional tidal currents, computed using the shelf model, with the observational data collected during JONSDAP '76 suggest that the model is reproducing the observed distribution of the tide over the North Sea. As more data becomes available a more rigorous comparison will be possible.

The three-dimensional meteorologically induced circulation of the North Sea, computed with the model, clearly shows the importance of bottom topography and coastal features upon the meteorologically induced circulation of the North Sea.

Further calculations to determine the daily residual flow of the North Sea are in progress and a comparison between these and observed residual flows measured during JONSDAP '76 will provide a rigorous test of the model.

Acknowledgements

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