CHAPTER 88

USES FOR A CALCULATED LIMIT DEPTH TO BEACH EROSION

by

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Abstract. A sediment entrainment parameter is used to calculate maximum water depth for intense bed agitation by shoaling linear waves of given height and period. Calculated limit depths agree with available laboratory measurements of water depth at an erosive wave cut into slopes of quartz and other fine sediments. Ignored variables have small effects on the agreement between calculations and laboratory measurements. On natural seasonal beaches, available measurements of seaward limit to appreciable sand level changes agree with limit depths calculated for extremely high waves expected 12 hours per year. The apparent accuracy and lack of scale effect in the calculated limit depth justify several applications in field and laboratory projects.

THE CALCULATED LIMIT DEPTH

In considering sediment transport by water waves on a beach, it is useful to divide the onshore-offshore profile into zones related to physical processes. The simplest division distinguishes two zones: a nearshore or littoral zone; and an offshore zone. In the offshore zone, wave shoaling is the dominant process and bed agitation remains relatively moderate. The littoral zone is characterized by increased bed stresses and sediment transport, caused by waves near breaking and induced fluid circulations.

The hypothetical boundary between these two zones is the seaward limit of intense bed agitation by shoaling wave action. Hallermeier (1977) proposed that the onset of intense bed agitation might be described as a critical value of a sediment entrainment parameter. This parameter has the form of a Froude number:

$$\Phi = U_b^2/\gamma' g\varepsilon d \tag{1}$$

where U_b is maximum horizontal water velocity near the bed, g is the acceleration of gravity, d is water depth, ε is a number less than unity, and γ' is the ratio of the density difference between sediment and fluid to fluid density. This Froude number is peak near-bottom wave energy per unit sediment grain volume, divided by the energy needed to raise an immersed grain a distance ($\varepsilon d/2$). Two order-of-magnitude choices, $\varepsilon = 0.03$ and $\Phi = 1$, are taken to describe the onset of intense wave agitation for fine sands (diameter, D, between 0.06 and 0.5 mm).

With these assumptions and linear theory for shoaling waves, equation 1 can be written as

$$\xi \sinh^2\xi \tanh^2\xi \left(1 + \frac{2\xi}{\sinh 2\xi}\right) = (329\ H_o{}^2/\gamma' L_o{}^2) \qquad (2)$$

where H_o and L_o are wave height and wavelength in deep water, and $\xi = (2\pi d_s/L_s)$ gives the limiting water depth, d_s, in a form normalized by local wavelength, L_s. For a certain γ' , H_o and $L_o = (gT^2/2\pi)$, where T is wave period, the ξ solving equation 2 can be conveniently computed using either a graph or an iterative root-finding procedure on a programmable calculator. The maximum water depth for intense bed agitation is $d_s = (\xi \tanh \xi)(L_o/2\pi)$.

The primary evidence for the usefulness of the calculated d_s is provided by laboratory tests with constant waves on an initially plane slope of sediment. The many published profiles developed in such tests provide a data base on sediment transport towards an equilibrium profile in controlled wave conditions. A common profile feature with fine sediment is a submarine cut by erosive waves into the initial slope, with sediment deposition offshore. The well-defined water depth at this wave cut, d_c, is the limit depth to the erosive action of the surface waves. The wave cut sometimes lies on the landward side of a bar, but more commonly is on a gently sloping terrace.

Figure 1 shows the good agreement of measured d_c with calculated d_s for 46 laboratory tests of profile development with an initially plane sediment slope. The filled points in Figure 1 denote tests with sediments having γ' appreciably different from quartz in water. The Appendix gives references and test conditions for the 46 profiles,

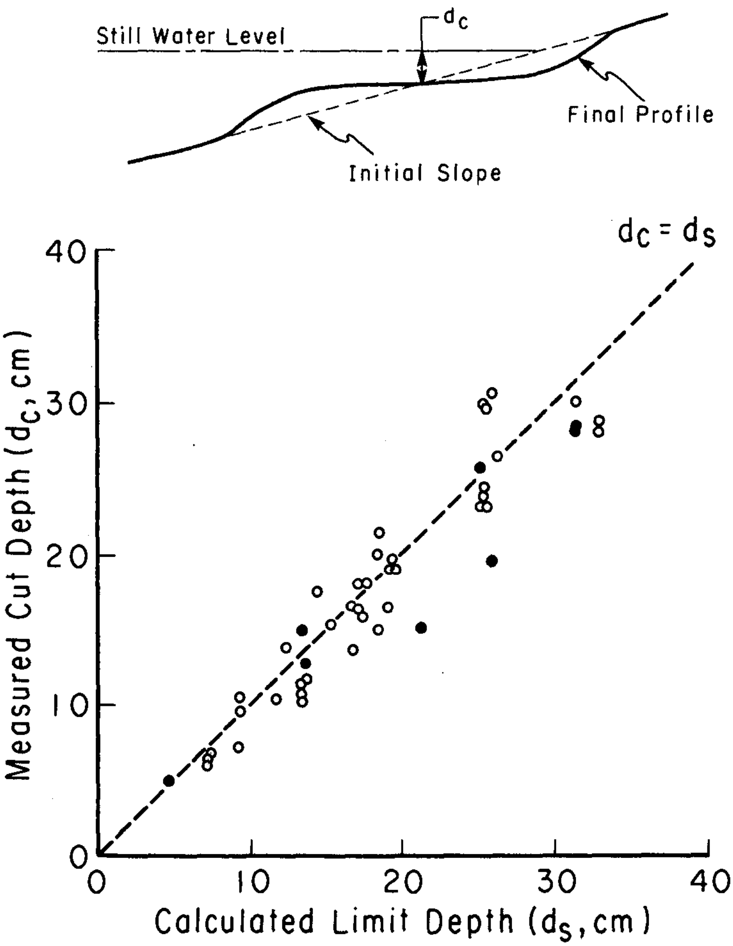

Figure 1. Measured profile cut depth, d_C, versus calculated limit depth, d_S. Conditions for 46 laboratory tests from 13 studies are presented in the Appendix.

selected as ideal cases of a wave cut. In each case, there is only sand deposition on the initial slope offshore of the cut depth.

Table 1 presents results of linear regressions on the Figure 1 points. For each regression line (including those of the data set halves discussed in the following section), the correlation coefficient, r, is large enough to reject the possibility of no linear relationship between d_c and d_s with at least 99.99% confidence, presuming the data points are random samples from a bivariate normal population (Freund, 1962). In each case, the intercept of the fit line is fairly near zero and the slope is near unity. The agreement between measurement and calculation is most ideal for the 38 tests with quartz sands. With the sediments of other densities (glass, coal, bakelite, maselite, and oolitic aragonite), agreement is still good, providing additional confidence that the calculated d_s is the limit depth to the erosive action of various waves causing offshore deposition of various fine sediments.

The footnote to Table 1 emphasizes the fact that d_s is approximately a linear multiple of incident wave height, for any given γ'. The measured d_c is well described by a linear dependence on H_o, but the fit is considerably better with the exact calculated d_s, which includes a slight effect of wave period.

It may be noted that a similar physical viewpoint is involved in the d_s calculated using the equation 1 consideration of sand entrainment energetics and in a limit depth arising in the empirical eroding-profile schematization of Swart (1974). That work considers an "equilibrium D-profile" geometry, with offshore water depth, d_1, at the lower limit stated to be related to the first occurrence of suspended-load transport. For the 6 tests considered in Figure 1 and in Swart (1974), $0.69d_1 < d_s < 0.78d_1$; this narrow range helps confirm a similar phenomenon is involved in d_s and d_1.

EFFECTS OF IGNORED VARIABLES

The laboratory data base was divided into approximate halves to isolate effects of larger or smaller: tank water depth; number of waves; initial slope; Stokes number, describing the nonlinearity of the

Table 1. Results of Linear Regressions: $d_c = \alpha + \beta d_s$.

Data Set	Number of Points	Correlation Coefficient, r	Intercept, α, cm	Slope, β
All Data	46	0.946	0.61	0.93
Quartz Sands*	38	0.950	-0.01	0.98
Other γ'	8	0.954	1.56	0.83
$(d_tL_s/d_sL_t) \leq 1.7$	23	0.923	1.53	0.88
$(d_tL_s/d_sL_t) > 1.7$	23	0.958	-0.72	1.04
		Quartz Sands*		
10K to 400K Waves	19	0.933	2.01	0.91
>400K Waves	19	0.969	-1.67	1.05
Slope $\geq$1/12	19	0.949	-0.80	1.07
Gentler Slopes	19	0.972	0.30	0.92
$11<S<21$	20	0.927	1.23	0.91
$21<S<119$	18	0.966	-0.84	1.03
$D \leq 0.21$ mm	18	0.956	-2.77	1.15
$D \geq 0.22$ mm	20	0.923	3.69	0.83
$125<(H_s/D)<375$	19	0.906	0.09	0.94
$415<(H_s/D)<710$	19	0.855	4.44	0.81
$7<\phi<24$	19	0.919	-0.47	1.00
$24<\phi<43$	19	0.858	2.90	0.87
$95<(a_s/D)<300$	19	0.928	-1.10	1.06
$300<(a_s/D)<950$	19	0.945	0.17	0.97

* Also, $d_c = 1.97$ cm $+ 1.70\ H_o$, with $r = 0.875$, for quartz sands

waves; and sand diameter. The separation criteria are given in Table 1 along with the linear regression results; data subsets of nearly equal size facilitate comparison of results. In Table 1, the relative tank depth is given in dimensionless form as $(d_t/L_t)/(d_s/L_s)$, where d_t is maximum water depth and L_t is linear wavelength in that depth for the given T; this form seems more appropriate than (d_t/d_s) for considering wave geometry effects. The Stokes number is $S = (H_s L_s^2/2d_s^3)$, where H_s is calculated wave height at d_s, according to linear theory for wave shoaling. Ignoring sand size is thought to be a significant assumption in the calculation procedure, so the influence of sand diameter was examined using four parameters: D directly; (H_s/D), suggested by Bagnold (1940) as a similarity parameter for beach profile development; and two useful parameters from dimensional analysis of oscillatory-flow interaction with a sediment bed (Lofquist, 1978), $\phi = (U_b^2/\gamma' gD) = (\varepsilon d_s/D)$ and $(a_s/D) = (H_s/2D \sinh \xi)$, where a_s is near-bottom horizontal fluid orbit amplitude.

There are differences in the fit lines and the correlation coefficients between each data subset of the eight pairs in Table 1. The quantity r^2 measures the amount of variance in d_c attributable to the derived linear dependence on d_s. This ranges between 73% and 95%, and differs somewhat between the two members of each of the eight pairs. In seven of the eight cases, the larger r occurs for the fit line in better agreement with the present model, i.e., α nearer zero and β nearer unity. (Results with gentler and steeper slopes show practically the same agreement with the model.)

For each of these eight cases, a nonparametric statistical test was applied to assess the parallelism of the two regression lines. This test (Hollander and Wolfe, 1973) compares two sets of slope estimators from pairs of points within each data base half, using the Wilcoxon signed rank test, with no assumptions about the distribution of deviations from linearity. Inferences from repeats of this test may differ slightly because a random pairing of slope estimators is involved.

The tank water depth was found to affect the slope at the 0.13 level of significance. This result implies there is a 0.13 probability that the parallelism of the fit lines has been rejected, when in fact they are parallel. With relatively large water depth, $(d_t L_s/d_s L_t) > 1.7$, the $(d_s\ d_c)$ data shows better agreement with the present model. Because 6 of 8 tests with non-quartz sands were done with relatively small water depth, these 8 points were deleted from the further analyses.

The number of waves was found to affect the slope at the 0.04 level of significance. More waves result in a better agreement between calculated and test results. For 25 of the 38 quartz-sand tests, the profile development in time was available, and d_c is found to oscillate somewhat but generally to increase slightly with running time. On the average, it is clear that d_s provides a useful estimate for the limit depth on an eroding profile approaching equilibrium.

The statistical test gave no conclusive indications of effects of initial slope and of Stokes parameter on the regression line slopes. Initial slope appears to have insignificant consequences for the present consideration, considering the large and nearly equal r for these two data subsets. However, the lack of a discernible effect of the Stokes parameter on the fit between d_c and d_s requires comment. Linear wave theory can be valid only if correction terms arising in second-order theory are negligible compared to the linear solution; an approximate form for this requirement is that the Stokes parameter be much less than about 50 (Madsen, 1976). The stream function results in Dean (1974) show that for $(HL^2/2d^3)$ greater than about 10, the calculated U_b is appreciably greater than that given by linear wave theory, although the difference is not monotonic with the Stokes parameter, clearly depending on (d/L). These results are for waves over an ideal immobile bed, and the present data base includes only $S > 11$. These facts might account for lack of a definite effect with larger Stokes parameter for the calculation procedure to underestimate U_b and thus d_s (equation 1), as the wave condition becomes increasingly nonlinear.

The examinations of sand size effects gave some interesting results. Splitting the data base directly according to reported D was found to affect the regression-line slope at the 0.18 level of significance; finer sands result in slightly better agreement between the model and measurements. When the data base is split using either (H_s/D), $(U_b{}^2/\gamma'gD)$ or (a_s/D), the parallelism of the two regression lines cannot be rejected even at the 0.4 level of significance.

The separation of the data base according to (H_s/D) appears to be the least constructive, because the fit line for each data subset shows relatively poor agreement with the present model. Bagnold (1940) suggested the change of bed material behavior from shingle to sand takes place for (H_s/D) on the order of 2,000, but the present consideration suggests the transition value is on the order of $(H_s/D) = 200$, if this is the proper descriptive parameter. A limited range of relatively fine sands are represented in the present data base. Bagnold (1940), Rector (1955), Popov (1960), and van Hijum (1974) have reported laboratory tests with waves acting on coarse quartz sediments: $3 \leq D \leq 7$ mm. A near-horizontal shelf or wave cut commonly occurred at water depth ranging from $(0.7\ H_o)$ to $(2\ H_o)$. These tests with coarse sediments all have (H/D) on the order of 40, but no clear conclusion is possible on the limit depth to the wave's erosive action. However, it seems clear that the calculated d_s should be an upper bound to the actual limit depth with erosive waves acting on coarser, less mobile sediments.

Although results are not too decisive when the data base is separated according to $(U_b^2/\gamma' gD)$ or (a_s/D), it is somewhat paradoxical that measurements appear in better agreement with the model for relatively small $(U_b^2/\gamma' gD)$ and for relatively large (a_s/D), because these two parameters are strongly and positively correlated in the present data base. Others of the ignored variables might cause this. $(U_b^2/\gamma' gD)$ roughly measures the intensity of sand motion, and the values of $(U_b^2/\gamma' gD)$ and (a_s/D) in the present data base indicate that $\Phi = 1$ may correspond to the onset of intense bed agitation as revealed by the decline of bed forms with increasing bed agitation (cf. Figure 32 in Lofquist, 1978).

In summary, the variables ignored in calculating the limit depth appear to have small or negligible effects. Emphasizing the non-negligible effects, d_s has been seen to agree better with the limit depth of equilibrium profile erosion for fine sands and unconstrained water depth. Each data subset gives less ideal agreement between d_c and d_s than the entire data base for quartz sands. The excellent agreement between d_c and d_s in the larger data base evidently results from an averaging of effects of ignored variables.

YEARLY LIMIT DEPTH TO PROFILE ACTIVITY ON SEASONAL BEACHES

Because d_s is calculated from a critical value of a Froude number, this limit depth should pertain to eroding natural beach profiles of fine sand, with comparable (H_o/L_o) and Φ, as well as to profiles developed in smaller-scale laboratory tests. This is partially confirmed by several profiles obtained by Saville (1957) in large-scale laboratory tests, as discussed in Hallermeier (1977). However, application of this calculated limit depth to natural beaches must consider complicating effects occurring in nature, including changing wave action.

For natural sand beaches, one limit depth useful in coastal engineering is the yearly limit to the very active nearshore profile, beyond which repetitive surveys reveal little sand level change throughout the seasonal wave climate changes. This profile close-out depth can be estimated using the cut depth calculated for a yearly extreme wave condition. Such high waves erode the nearshore and deposit sand offshore; the estimated yearly extreme cut depth should be a minimum limit to water depth for appreciable sand level changes. An appropriate extreme wave condition is proposed to be that exceeded for 12 hours per year (0.137%). This duration should permit moderate adjustment towards profile equilibrium and moderate quantities of sand deposited beyond the limit depth. Also, Maksimchuk (1976) stated the beach profile in varying wave action is dominated by a similar wave condition, that having a cumulative frequency of 0.2% (0.73 days per year).

To calculate limit depth for such extreme waves, an accurate approximation for equation 2 is convenient:

$$d_{se} = 2.28\, H_e - 68.5\, (H_e^2/gT_e^2) \qquad (3)$$

Here γ' has been taken as 1.6 (quartz sand in salt water), H_e and T_e are nearshore significant height and period of the extreme wave condition, and g is acceleration of gravity in appropriate units.

H_e is the dominant input in calculating d_{se}. Thompson and Harris (1972) reported measured nearshore wave heights for a complete yearly cycle occur according to a modified exponential distribution, and height of extreme waves is best estimated using the mean height, $\overline{H}$, and standard deviation of height, σ, defined by a full year of at-least-daily nearshore wave measurements. According to the exponential distribution, the 12-hour-per-year height is

$$H_e = \overline{H} + 5.6\,\sigma \quad . \tag{4}$$

T_e should be taken to be the typical period of measured high waves.

Any near-bed velocities above those caused by linear waves are ignored in calculating the limit depth in equation 3. Use of linear wave theory is warranted because of its simplicity and agreement with available field measurements of peak near-bed velocity (Grace (1976). However, any flows superposed on surface waves must increase the peak near-bed fluid kinetic energy, the numerator of equation 1, and thus increase the calculated limit depth in the denominator. To counteract this, it is proposed that the calculated d_{se} be used as a minimum estimate of profile close-out depth with respect to mean low(er) tide level.

Table 2 presents estimated profile close-out depths along with recorded close-out depths from published field studies including repetitive nearshore profile and wave measurements. The estimated water depths are calculated using equations 3 and 4, and measured depths are for profile superposition throughout a yearly cycle to within 1 foot, a typical resolution for nearshore surveys. In each case, the estimated close-out depth is quite close to and usually less than the measured depths. There is about the same agreement with measurement if limit depth is calculated using, rather than equation 3, the exact form of equation 2 ignoring shoaling wave height change (given in Hallermeier, 1977). Each field study in Table 2 was done on a different sea coast, with very different extreme wave conditions, increasing confidence in the usefulness of the calculated yearly limit depth. Agreement between calculated and measured depths is best for the most ideal data set, Torrey Pines Beach, where three profiles were surveyed monthly over a two-year interval.

On the other hand, Table 2 summarizes a very small amount of field investigation, and the present treatment has ignored several possibly important factors in considering only two-dimensional wave action.

Table 2. Comparison of estimated and measured profile close-out depths from field studies reporting repetitive nearshore profiles and wave measurements. Water depths are with respect to local mean low(er) tide level, and sand sizes are near estimated limit depths.

Locality; Reference	Description of Data Set: Profiles	Waves	Design Wave: H_e, Ft	T_e, Sec	Eq. 3 Limit Depth, Ft	Measured Close-out Depth*, Ft	Profile Line Name
Gold Coast, Australia, S. Pacific Ocean; Delft Hydr. Lab., 1970 (D = 0.2 mm)	5 lead-line surveys Jun 66-Dec 68	4/day records (1 year) from deep-water waveriders	14.5	8	26.1	28.5 (>25.5) 25.5 29.5 34.5 30.5	Letitia Greenmount Tugun Palm Broadbeach The Spit
Avondale, Fla., U.S., Gulf of Mexico; Balsillie, 1975; Poche, 1972 (D = 0.3 mm)	8 lead-line surveys, Jan-Aug 70	97% daily breaker observations (1 year)	7.9	5.4	13.5	14.8	Pier
Torrey Pines, Cal., U.S., N.Pacific Ocean; Nordstrom & Inman, 1975; Pawka et al., 1976 (D = 0.12 mm)	24 fathometer surveys, Jun 72-Apr 74	64% daily breaker observations (2 years); 4/day pressure records (16 months)	11	14	23.8	22.8 24.3 24.8	North Indian South

* Profiles superpose to within ± 0.5 ft; depth is average of two yearly cycles, except for Balsillie (1975).

Swart (1974) considered the effects on the onshore-offshore equilibrium beach profile of wave-induced longshore and rip currents; in erosive waves, each effect theoretically deepens the profile due to increased bed shear. Bijker (1967) reported the measured increase of bed shear in a uniform current due to wave action could be expressed as an empirical function of: angle between wave and current; the ratio of their peak near-bed speeds; and the bed roughness. These factors must be considered in predicting alongshore and onshore-offshore sediment transport within the nearshore beach zone.

However, the beach profile near d_{se} ($\approx 2\ H_e$) should be beyond the influence of intense wave-induced nearshore circulations. Wave breaking usually occurs at water depth on the order of wave height, i.e., in water appreciably shallower than d_{se}. Beyond wave breaking, the seaward extent of the induced currents seems a useful way to define the extent of the nearshore zone. Available theory and experiment on the wave-generated alongshore current give maximum current speed on planar fixed beds within the breakers, with monotonically decreasing current speed offshore of the maximum (Longuet-Higgins, 1970; James, 1974; Jonsson et al., 1974). The onshore-offshore profile of the longshore current evidently depends on wave condition and on the mechanisms of energy dissipation and lateral fluid mixing. There remains considerable uncertainty about the seaward extent of significant near-bed alongshore currents and rip currents, especially in extreme events, but the water depth d_{se} seems a reasonable estimate for the extreme extent of the nearshore zone, intensely active due to wave action.

At such water depth, tidal or wind-induced currents may significantly add to wave-induced velocities. Interaction of water waves and currents is a complex topic (Peregrine, 1976), and available results on bed agitation and suspended sediment provide no guidance on modifying equation 1 to account for the combined effect of wave action and a current. For this reason, the simple geometric correction has been proposed for the effect of tidal action: d_{se} is to be taken with respect to mean low(er) tide level for estimating minimum limit depth to the very active profile on natural seasonal beaches.

APPLICATIONS TO FIELD PROJECTS

The yearly limit depth from equations 3 and 4 has several applications in coastal engineering:

a. Any nearshore hydrographic survey should extend further seaward than the estimated limit depth, to ensure coverage of the very active zone of a sand beach. Minimum definition of conditions at a seasonal beach requires two such surveys, conducted when the beach shows maximum summer-wave accretion and winter-wave erosion.

b. For subaqueous beach nourishment, suitable material should be placed landward of the estimated limit depth to the active seasonal profile during the summer-wave accretionary beach phase. This guidance is consistent with field offshore nourishment attempts judged successful (Vera-Cruz, 1972; Mikkelsen, 1977; Schwartz and Musialowski, 1977) and unsuccessful (Harris, 1954; Hall and Watts, 1957; Wiegel, 1964). Nearshore wave measurements (Thompson, 1977) indicate average summer-wave height is about ($0.8\,\overline{H}$), and the estimated yearly limit depth is about half the maximum water depth for motion initiation with fine sands, summer waves, and the motion threshold criterion of Komar and Miller (1974).

c. For effective offshore disposal, material should be placed far seaward of the limit depth to the very active beach profile, so that it does not enter the nearshore system.

The estimated limit depth to the nearshore zone might also find applications in the design of coastal structures in sandy regions. It seems an offshore mound-type breakwater must be situated in water deeper than this limit for the nearby region, if it is to provide wave shelter while minimizing its effect as a littoral barrier. Also, it seems that dual jetties at a navigation channel extending to the limit depth may be expected to intercept almost all the littoral drift during a typical yearly cycle of waves. However, regions near navigation channels and structures are three-dimensional and have significant currents in addition to waves (Liu and Mei, 1975), factors not considered in the present treatment. Also, coastal structure design must consider the economics of construction and maintenance, as well as functional performance. Further research is needed before the estimated limit depth can be applied with confidence to structure design.

APPLICATION TO LABORATORY TEST DESIGN

Laboratory investigations of nearshore sediment transport can potentially provide valuable information for coastal engineering, since regulated experiments on sand response to controlled hydraulic forces are permitted in the laboratory. Interpretation of laboratory results requires consideration of scale effects in replicating prototype processes (Bijker, 1967; Kamphuis, 1975), and obtaining meaningful experimental results requires that laboratory effects, e.g., constrained geometry, be minimized (Chesnutt, 1978).

The previously discussed results (Table 1) on parameters influencing the cut depth by erosive waves clarify one requirement for unconstrained geometry in laboratory experiments on nearshore processes: water depth must be adequate to accommodate an equilibrium nearshore profile. The wave cut in laboratory sand slopes is a distinct geometrical feature with erosive waves, and the elevation of this feature on a two-dimensional profile approaching equilibrium is generally better explained by d_s from equation 2 with relatively large tank water depths: $(d_t L_s / d_s L_t) > 1.7$. Some profiles obtained by Monroe (1969) and by Masuda and Ito (1975) indicate untypical forms are associated with dimensionless water depths much less than this. Using typical laboratory values of $d_s = 2H$ (quartz sand) and $(L_t/L_s) = 1.5$, it seems water depth should be at least five times generated wave height for vertically unconstrained tests with fine quartz sand in water. For sediments of lower density, larger water depth is required (equation 1) for meaningful tests.

The laboratory requirements for a horizontally unconstrained beach profile remain somewhat uncertain, but appear harder to accommodate in general. Two-dimensional profiles developed by erosive waves commonly have a gently sloping terrace centered at the cut point in the initial slope. The profile developments in time reported by Sunamura and Horikawa (1974) and by Chesnutt (1978) show an approximately logarithmic increase in the length of this terrace with running time. The terrace slope is on the order of 1° (comparable to offshore slopes on natural beaches), and an equilibrium eroding profile must be very long, because the terrace should extend to a depth where sediment movement ceases, to include the entire active beach profile.

A practical alternative is to truncate the offshore profile, while including the entire nearshore zone in the laboratory situation. Kemp (1960) has noted that dynamic similarity between laboratory test and prototype is more likely within the turbulent breaker zone than off-shore. In designing the profile truncation, provision must be made for the offshore-zone function as a source or sink of nearshore sands, in various wave conditions. Also, the truncation cross-section should be shaped to minimize wave reflection, which has deleterious effects on laboratory beach studies (Chesnutt, 1978). Available laboratory results on perched beach designs (Chatham et al., 1973) provide a data base for further research on offshore profile truncation.

SUMMARY

The sediment entrainment parameter (equation 1) does not figure in any reported research results on wave propagation or interaction with sand beds. However, with linear wave theory, and two coupled order-of-magnitude assumptions, this parameter can be used to calculate a limiting water depth to the erosive action of waves causing offshore deposition of fine sands (equation 2). This calculated depth agrees well, over a wide range of conditions, with the elevation of the wave cut commonly occurring on eroding laboratory sand slopes (Figure 1). Also, for extremely high waves expected 12 hours per year, the estimated limit depth (equations 3 and 4) agrees well with recorded close-out depth to significant profile activity at seasonal beaches on several sea coasts (Table 2). The calculated limit depth has several types of application in field and laboratory projects, although further analysis and tests are clearly required.

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Appendix. Test conditions for profiles showing ideol wave cut.

Test I.D.	Initial Slope	D, mm	γ'	H_o, cm	T, sec	Total Waves	d_t/L_t	d_s/L_s	d_s, cm	d_c, cm	$\frac{H_s L_s^2}{2d_s^3}$	a_s/D
(Chesnutt, 1978)												
70X-06	1/10	0.23	1.65	11.88	1.9	398K	0.164	0.0886	25.25	23.2	31.3	484
70X-10	1/10	0.23	1.65	11.88	1.9	332K	0.164	0.0886	25.25	23.2	31.3	484
71Y-06	1/10	0.23	1.65	11.88	1.9	711K	0.164	0.0886	25.25	23.8	31.3	484
71Y-10	1/10	0.23	1.65	11.88	1.9	635K	0.164	0.0886	25.25	24.4	31.3	484
72A-06	1/10	0.22	1.65	8.56	3.75	130K	0.074	0.0439	25.90	30.5	118.3	946
72C-10	1/10	0.21	1.65	13.72	1.5	336K	0.227	0.1158	25.28	29.7	19.6	397
72D-06	1/10	0.22	1.65	11.88	1.9	341K	0.164	0.0886	25.25	29.8	31.3	477
(Collins and Chesnutt, 1976)												
14A	1/10	0.3	1.42	2.0	0.82	101K	0.320	0.0876	4.60	4.95	29.6	60
238	1/10	0.45	1.42	7.0	1.00	94K	0.224	0.1274	13.20	15.05	15.5	83
(Delft Hydraulics Laboratory; Swart, 1974)												
71-09A	1/20	0.22	1.65	11.4	1.15	423K	0.213	0.1353	19.28	19.5	15.2	254
71-098	1/20	0.16	1.65	11.4	1.15	423K	0.213	0.1353	19.28	19.0	15.2	349
71-11A	1/20	0.22	1.65	10.7	1.15	423K	0.213	0.1314	18.39	21.5	15.9	247
71-11B	1/20	0.16	1.65	11.2	1.15	423K	0.213	0.1342	19.03	19.0	15.4	349
71-12A	1/10	0.22	1.65	9.6	1.15	451K	0.213	0.1252	16.96	18.0	17.2	237
71-128	1/10	0.16	1.65	9.3	1.15	451K	0.213	0.1234	16.56	16.5	17.6	322
(Eogleson, Glenne and Dracup, 1961)												
2	1/30	0.37	1.67	9.57	1.15	523K	0.258	0.1247	16.84	16.4	17.4	143
3	1/20	0.37	1.67	9.91	1.15	429K	0.253	0.1266	17.29	16.0	17.0	143
(Horikawa, Sunamuro ond Kitoh, 1973)												
1	1/24	0.2	1.65	6.4	1.4	591K	0.168	0.0883	13.60	11.7	31.5	289
2	1/24	0.2	1.65	5.3	1.4	617K	0.168	0.0815	11.74	10.6	36.4	267
3	1/24	0.2	1.65	7.3	1.4	123K	0.168	0.0934	15.07	15.2	28.5	299
4	1/24	0.2	1.65	6.1	1.7	415K	0.132	0.0791	16.41	13.5	32.2	317
(Masuda and Ito, 1975)												
II-1	1/10	0.33	1.3	9.80	2.30	13K	0.099	0.0730	25.86	19.8	39.7	349

					(Monroe, 1969)							
1A	1/15	0.26	1.585	16.57	1.19	106K	0.231	0.1569	26.19	26.5	11.8	253
1B	1/15	0.27	1.81	16.57	1.19	106K	0.231	0.1522	24.97	25.8	13.2	255
2A	1/15	0.26	1.585	17.11	1.77	96K	0.133	0.1113	32.87	28.0	20.5	424
2B	1/15	0.27	1.81	17.11	1.77	96K	0.133	0.1081	31.24	28.3	23.1	426
8A	1/15	0.26	1.585	17.11	1.77	161K	0.133	0.1113	32.87	28.8	20.5	424
8B	1/15	0.27	1.81	17.11	1.77	161K	0.133	0.1081	31.24	28.0	23.1	426
					(Nicholson, 1968)							
79	1/10	0.42	1.61	8.23	1.00	50K	0.229	0.1332	14.21	17.5	15.4	98
					(Paul, Kamphuis and Brebner, 1972)							
5a	1/10	0.357	1.67	4.15	1.29	100K	0.183	0.0785	9.30	9.5	39.3	124
5e	1/10	0.357	1.67	10.33	1.29	100K	0.183	0.1165	18.87	16.7	19.5	175
6e	1/10	0.525	0.60	5.15	0.91	47K	0.182	0.1467	13.76	12.8	8.1	38
					(Raman and Earattupuzha, 1972)							
1	1/8	0.3	1.65	6.8	1.0	144K	0.246	0.1216	12.20	13.9	18.0	130
3	1/12	0.3	1.65	11.08	1.0	155K	0.246	0.1516	17.53	18.0	12.7	155
					(Shinohara, Tsubaki, Yoshitaka and Agemori, 1958)							
Fig. 2a	1/10	0.2	1.66	3.84	1.40	15K	0.154	0.0710	9.10	10.5	47.2	232
Fig. 2b	1/10	0.2	1.66	4.95	0.89	24K	0.299	0.1170	9.05	7.3	19.3	147
Fig. 2c	1/10	0.3	0.29	4.30	1.65	13K	0.126	0.0939	21.12	15.6	11.8	117
					(Sunamura and Horikawa, 1974)							
2	1/10	0.2	1.65	3.4	1.0	576K	> 0.200	0.0898	7.16	6.2	30.6	305
4	1/10	0.2	1.65	7.6	1.0	576K	> 0.274	0.1278	13.27	10.2	16.6	203
6	1/10	0.2	1.65	7.6	2.0	288K	0.116	0.0702	18.16	20.0	48.1	473
8	1/20	0.2	1.65	3.4	1.0	576K	> 0.178	0.0898	7.16	6.4	30.6	305
10	1/20	0.2	1.65	7.6	1.0	576K	> 0.241	0.1278	13.27	10.7	16.6	203
12	1/20	0.2	1.65	7.6	2.0	288K	> 0.114	0.0702	18.16	15.2	48.1	473
14	1/30	0.2	1.65	3.4	1.0	576K	> 0.143	0.0898	7.16	6.7	30.6	305
16	1/30	0.2	1.65	7.6	1.0	576K	> 0.200	0.1278	13.27	11.6	16.6	203
					(Watts and Dearduff, 1954)							
Fig. 7	1/20	0.22	1.65	15.24	2.0	108K	0.160	0.0943	31.30	29.9	28.0	567