CHAPTER 103

A NUMERICAL MODEL FOR SEDIMENT TRANSPORT

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ABSTRACT
We introduce here a numerical two dimensional model for sediment transport which permits to compute the impact of a coastal structure on the bottom evolution.
The introduction of current disturbance and some assumptions using difference of time scale between current and bottom evolutions permits to obtain a propagation equation driving the bottom evolution. The model has been calibrated in the case of the local scour around a jetty. At last, it has been applied to the bottom evolution in the vicinity of the new port of Dunkerque.

INTRODUCTION

One of the impacts of a large coastal structure is its effect on current pattern in the vicinity of the structure. These changes in current conditions will induce changes in the sediment transport pattern and may disturb an existing equilibrum thus causing large changes in bottom topography in the vicinity of the structure. To assess the severity and extend of topographical changes induced by the structure the interaction of the resulting fluid motion with the bottom evolution must be properly reproduced.
The study of sediment drifting and movable bed evolution is a difficult problem from a physical and mechanical point of view. But the sediment transport relationship admitted, the problem is reduced to the study of a conservative phenomena.
An other problem is the difference of time scale between current and bottom evolution. It is impossible (because of cost), to compute simultanously the bottom evolution and the current by the classical way. Nevertheless, the interaction between the two is fundamental for the bottom evolution.
This paper presents a two dimensional mathematical sediment transport model taking into account the influence of the bottom evolution upon the current pattern and shows how this particular aspect of the interaction drives the ripples propagation.

THEORETICAL ANALYSES

Bed continuity equation and sediment transport relationship

Let $\vec{T}$ be the sediment transport vector and ξ the bottom elevation ; the bed continuity equation may be expressed as

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$$\frac{\partial \xi}{\partial t} + \text{div}\,\bar{T} = 0$$

How express $\bar{T}$ as a function of the velocity ? That is a real problem. Many relations can be found taking into account waves or not. For ourselves we have used the Meyer-Peter relationship for the sediment transport vector $\bar{T}$ which is supposed in the direction of the current bottom shear stress which is evaluated using Chezy's relationship.

So the bed continuity equation cas be transformed into :

$$\frac{\partial \xi}{\partial t} + T_{Xu}\frac{\partial u}{\partial x} + T_{Xv}\frac{\partial v}{\partial x} + T_{Yu}\frac{\partial u}{\partial y} + T_{Yv}\frac{\partial v}{\partial Y} = 0$$

with

$$T_{Xu} = \frac{u}{W}\frac{\partial T}{\partial u} + T\frac{v^2}{W^3}, \qquad T_{Yu} = \frac{v}{W}\frac{\partial T}{\partial u} - T\frac{uv}{W^3}$$

$$T_{Xv} = \frac{u}{W}\frac{\partial T}{\partial v} - T\frac{uv}{W^3}, \qquad T_{Yv} = \frac{v}{W}\frac{\partial T}{\partial v} + T\frac{u^2}{W^3}$$

$$\left.\begin{aligned} T &= 8\sqrt{\frac{g}{\varpi}}\,\frac{1}{\varpi_S - \varpi}\,(\tau - \tau_C)^{3/2} && \text{if } \tau > \tau \\ T &= 0 && \text{if } \tau > \tau_C \end{aligned}\right]\ \text{sediment transport}$$

$$\tau_C = A\,(\varpi_S - \varpi)\,D_M \quad (0{,}02 < A < 0{,}06 \text{ Shields}). \text{ Critical bottom shear stress}$$

$$\tau = \varpi\frac{W^2}{C^2} \quad \text{bottom shear stress}$$

u, v are the two components of the depth averaged current
$W^2 = u^2 + v^2$

ϖ, ϖ_S specific weight of water and sediment

D_M mean diameter of sediment.

Influence of bottom evolution upon the current pattern

With the initial bottom shape ξ_o and the new geometric conditions the depth averaged flow pattern is (u_o, v_o). This current modifies the bottom shape which in turn modifies the current by $(u_1\ (t), v_1\ (t))$.
At time t, the current pattern is given by $(u_o + u_1\ (t), v_o + v_1\ (t))$ and the bottom level by $\xi(t)$ ($\xi_S = \xi - \xi_o$ is the bottom evolution).

The resulting disturbance (u_1, v_1) is assumed to be without effect upon the surface elevation z_o. This assumption is equivalent to neglect the characteristic response time of the surface wave propagation compared to the characteristic response time of the bottom evolution.
The resolution of the fluid continuity equation shows that the current disturbance (u_1, v_1) can be written in two different terms :
- the first one ($\bar{u}_1$, $\bar{v}_1$) comes directly from the bottom elevation ξ and expresses the flow conservation along the stream lines of the undisturbed field of currents (u_o v_o)

$$\bar{u}_1 = u_o \frac{\xi - \xi_o}{z_o - \xi} = u_o \frac{\xi_s}{h} \qquad \bar{v}_1 = v_o \frac{\xi - \xi_o}{z_o - \xi} = v_o \frac{\xi_s}{h}$$

- the second one ($\tilde{u}_1$, $\tilde{v}_1$) is a deviation of the flow due to the bottom slope. It is governed by :

$$\frac{\partial}{\partial x} \left[\tilde{u}_1 \ (z_o - \xi)\right] + \frac{\partial}{\partial y} \left[\tilde{v}_1 \ (z_o - \xi)\right] = 0$$

Bottom equation
These two terms are introduced in the bed continuity equation (1) which can be written :

$$\frac{\partial \xi_s}{\partial t} + C \left(\frac{u}{W} \frac{\partial \xi_s}{\partial x} + \frac{v}{W} \frac{\partial \xi_s}{\partial y}\right) = - T_{Xu} \left[\frac{\partial}{\partial x} (u_o + \tilde{u}_1) + \xi_s \frac{\partial}{\partial x} \left(\frac{u_o}{h_o}\right)\right]$$

$$- T_{Xv} \left[\frac{\partial}{\partial x} (v_o + \tilde{v}_1) + \xi_s \frac{\partial}{\partial x} \left(\frac{v_o}{h_o}\right)\right]$$

$$- T_{Yu} \left[\frac{\partial}{\partial y} (u_o + \tilde{u}_1) + \xi_s \frac{\partial}{\partial y} \left(\frac{u_o}{h_o}\right)\right] \qquad (2)$$

$$- T_{Yu} \left[\frac{\partial}{\partial y} (v_o + \tilde{v}_1) + \xi_s \frac{\partial}{\partial y} \left(\frac{v_o}{h_o}\right)\right]$$

with $C = \frac{1}{h} \left(u \frac{\partial T}{\partial u} + v \frac{\partial T}{\partial v}\right)$

Equation (2) governs a ripples propagation in the direction of the initial current pattern with the celerity C. This phenomena comes directly from the adaptation of current disturbance ($\bar{u}_1$, $\bar{v}_1$). By neglecting the disturbance it is impossible to reproduce the ripples propagation.
The second member can be divided in two different parts :
- contribution of the initial current pattern which is conserved at time t
- contribution of the deviation of the flow ($\tilde{u}_1$, $\tilde{v}_1$) which drives a ripple deformation.

Fluid equation

To determine the current disturbance (u_1, v_1) an other assumption is required : an irrotational current disturbance pattern ($\bar{u}_1 + \tilde{u}_1$, $\bar{v}_1 + \tilde{v}_1$) is assumed. So $\tilde{u}_1$ and $\tilde{v}_1$ are obtained from the three-dimensional stream function ψ, which yields a Poisson type equation (3).
So the actual current pattern is defined by :

$$u = u_o + u_o \frac{\xi_s}{h} + \overbrace{\frac{1}{h}\frac{\partial \psi}{\partial y}}^{\tilde{u}_1}$$

$$v = v_o + v_o \frac{\xi_s}{h} - \underbrace{\frac{1}{h}\frac{\partial \psi}{\partial x}}_{\tilde{v}_1}$$

$h = z_o - \xi$ actual depth and ψ obtained from

$$\Delta\psi = + h \frac{\partial}{\partial x} (v_o \frac{\xi_s}{h}) - \frac{\partial}{\partial h} (u_o \frac{\xi_s}{h}) + \tilde{u}_1 \frac{\partial h}{\partial y} - \tilde{v}_1 \frac{\partial h}{\partial x} \qquad (3)$$

NUMERICAL MODEL

A finite difference scheme is used to solve equations (2) and (3). The computational grids ψ and u, v, ξ are shifted. The initial conditions (u_o, v_o, z_o, ξ_o) are obtained with an other numerical model or recorded on a scale model.
Each time step involves two stages :

- computation of the bottom level ξ ; equation (2) is solved by the characteristic method. All functions are explicited but the scheme is stable.
- computation of the new velocities ; only $\tilde{u}_1$, $\tilde{v}_1$ have to be computed. Equation (3) is solved by an iterative process.

NUMERICAL EXAMPLES

Local scour around a jetty

Several numerical examples have been computed. In figures 1 and 2, the local scour around a jetty, and the flow pattern evolution are shown. The conditions are : flat initial bottom, far field mean velocity = 41 cm/s, water depth = 20 cm, width = 46 cm, ratio jetty length over flume width = 1/3 and particle diameter 4,5 mm. The initial current pattern has been computed with an other numerical model. In figure 3, comparison between computed and measured scour is shown.

Study of new port of Dunkerque

The Port Autonome of Dunkerque has built a new port able to receive 22 metters draught ships. Many studies have been

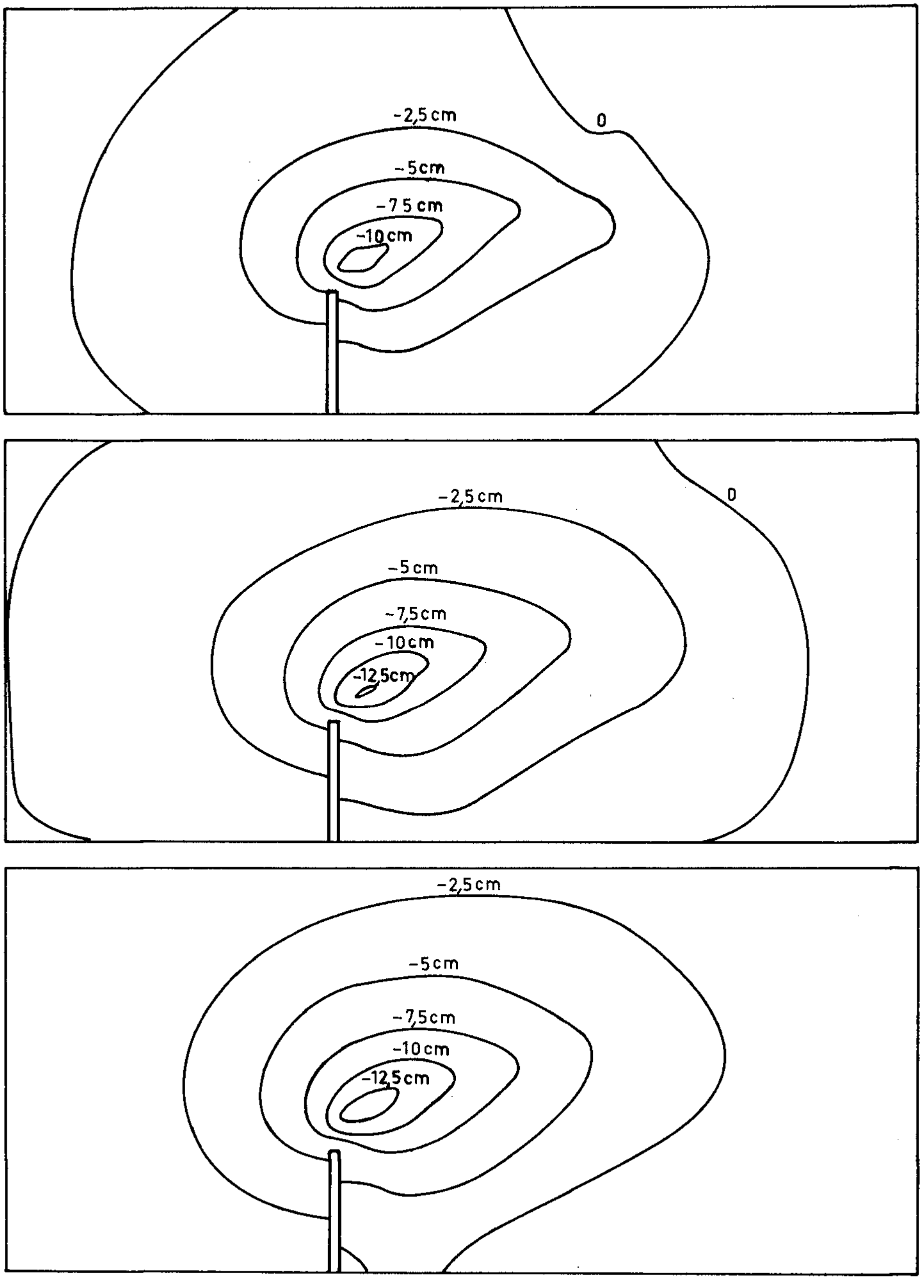

Fig.1_EROSIONS AFTER 1,2 AND 3 HOURS

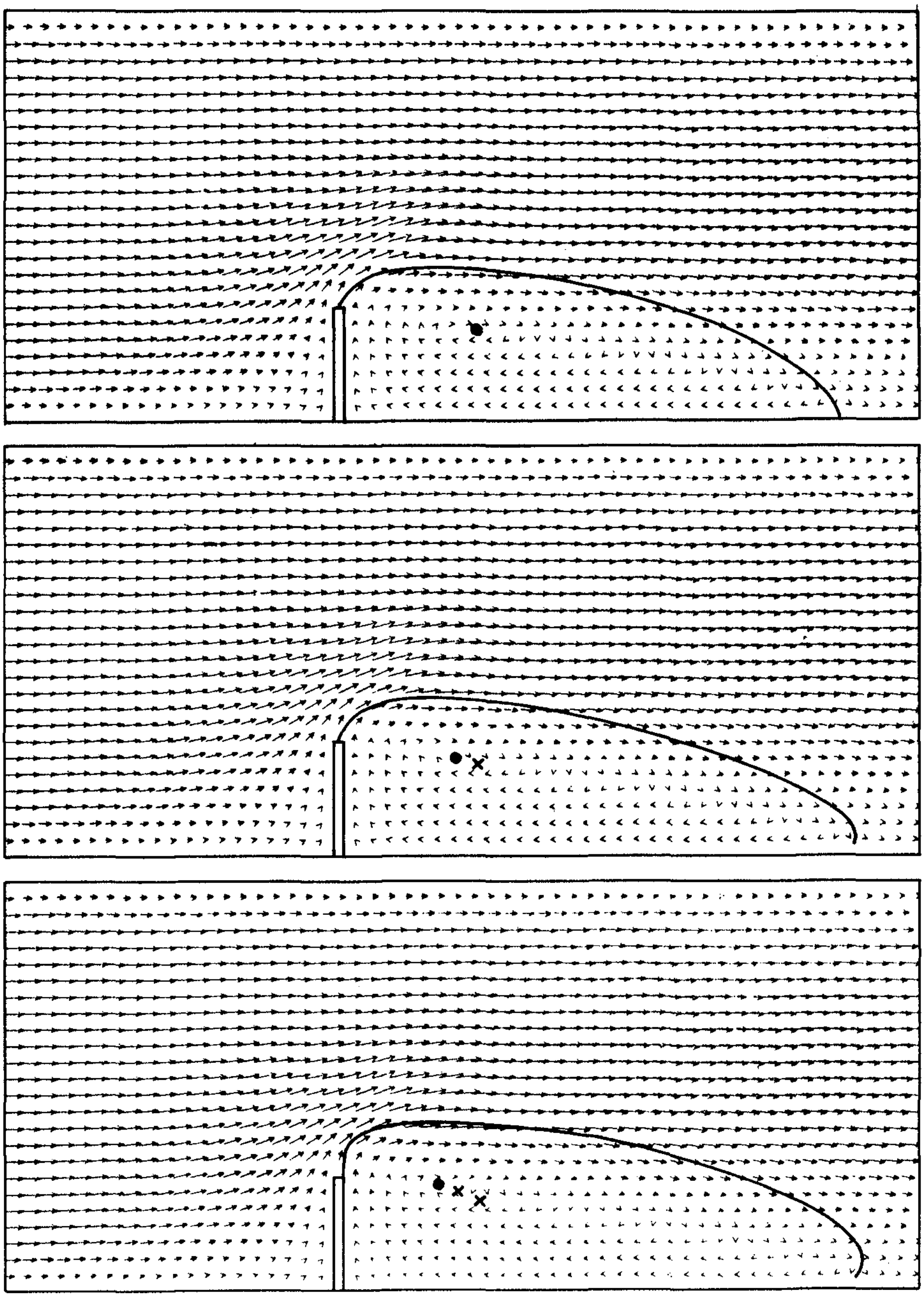

Fig.2 _ CURRENT PATTERN AFTER 1,2 AND 3 HOURS

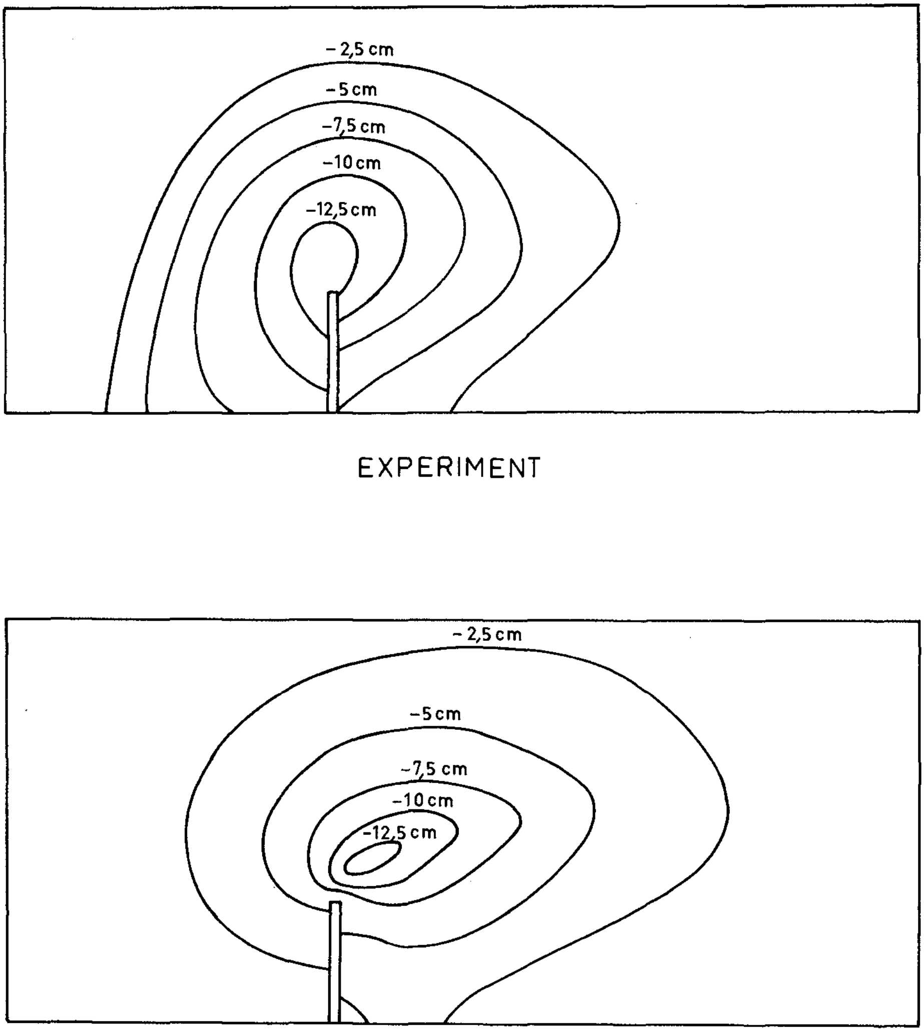

Fig. 3 _ COMPARISON BETWEEN MEASURED AND COMPUTED EROSIONS

carried on during ten years. Particularly, a movable bed model have been built to study the bottom evolution due to tidal currents near the new port.
The numerical model has been used in this particular case, but to decrease the cost of computation the second kind of disturbance has been neglected. Only equation (2) was solved. The initial current pattern used for the computation was recorded on the scale model.
The comparison between the computed and mesured erosions and accretions is presented on figures 4 and 5. The main difference takes place near the jetties and it probably comes from the initial current pattern which was not conservative because of the precision of measurements on the scale model.

CONCLUSION

A simple kinematical study of the sediment transport equation has shown how can the ripples propagation be obtained. It has also allowed a numerical integration on a computer. The characteristic response time of the surface wave propagation compared to the characteristic response time of the bottom evolution put a stop to any sort of computation of the disturbed current in the classical way. The introduction of current disturbance and several assumptions permit the computation of the bottom evolution during a long time.
This kinematical and mathematical aspect almost understood, studies are going on a more physical and dynamical point of view to determine the influence of the different parameters in transport relationship and to find a best dynamical approximation of the current disturbance. In the same time, a mean of averaging the tide in tidal problems is investigated.

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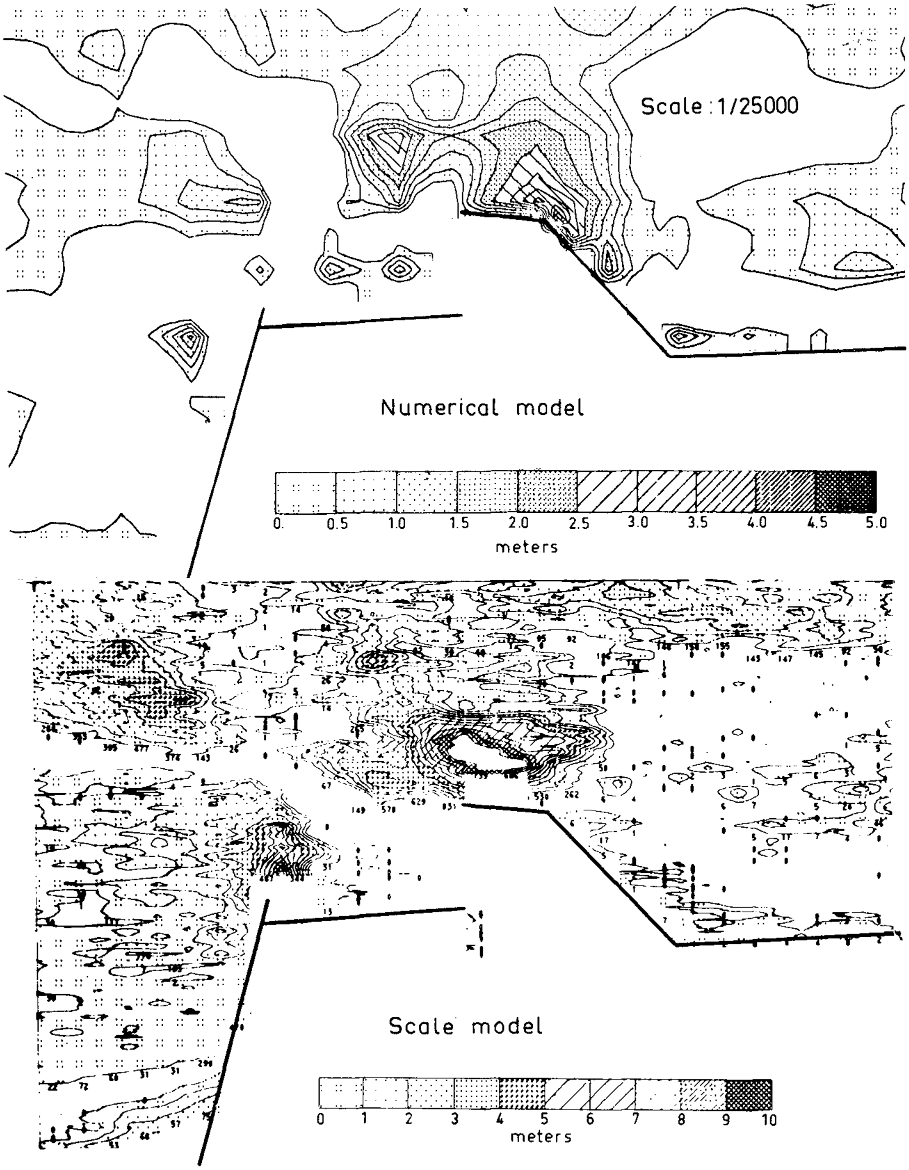

Fig.4 _ EROSIONS NEAR DUNKERQUE PORT

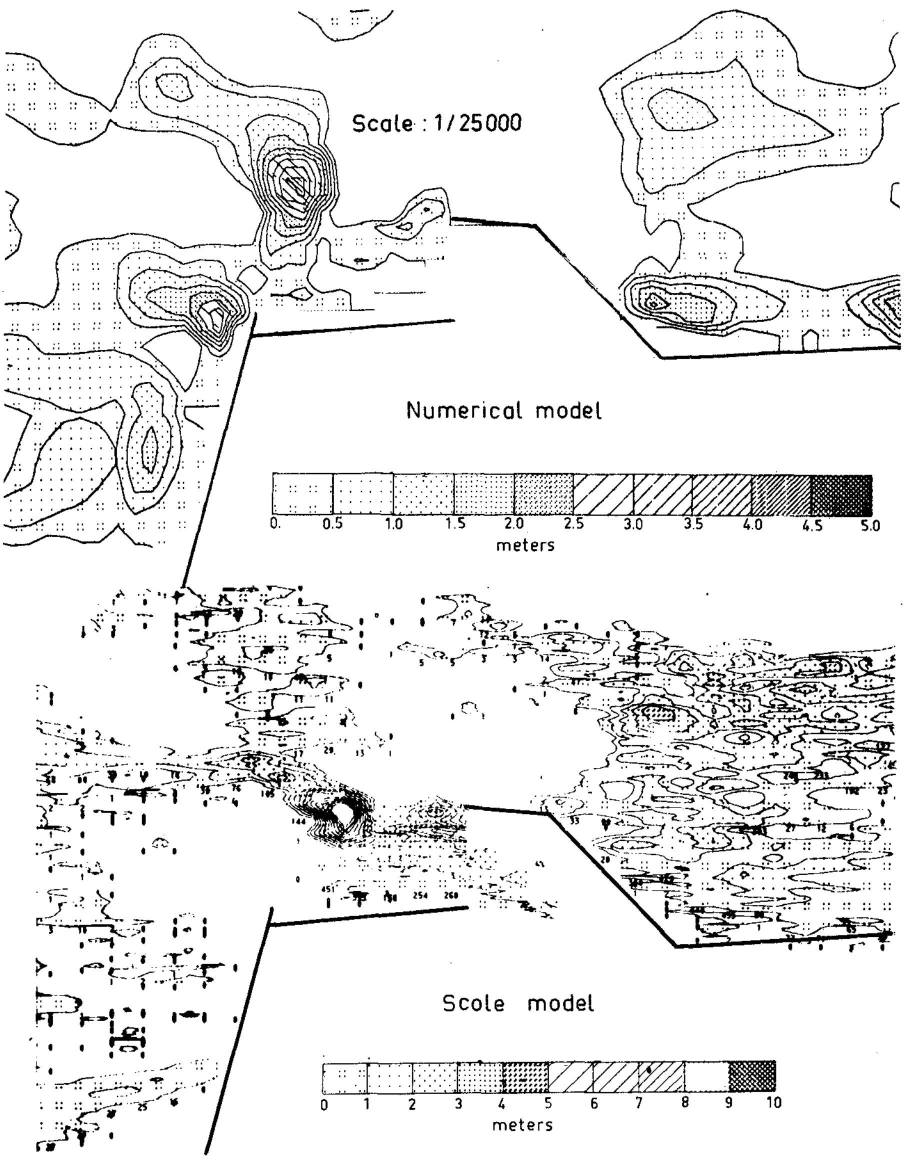

Fig. 5 _ ACCRETIONS NEAR DUNKERQUE PORT