CHAPTER 107

ON THE GEOMETRY OF RIPPLES DUE TO WAVES

by

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ABSTRACT

The present paper is an attempt to determine a single curve for the prediction of the length of ripples forming due to wind waves in shallow waters. The curve is revealed by normalising the field and laboratory data supplied by various authors. The concept of the unified plot embodied by the single curve mentioned is developed by using dimensional methods and by considering the fact that the specific weight and the density of the cohesionless bed material do not affect the length of ripples in a detectable manner. It is shown that the present formulation of the length of ripples due to waves satisfies the requirement of transition into the corresponding formulation of the unidirectional flow ripples when the period and the amplitude of the oscillatory motion increase indefinitely, while their ratio (implying "the velocity") remains finite.

INTRODUCTION

The occurrence of ripples alters the roughness of the movable bed and thus exerts a considerable influence on the mechanical structure of the flow and on sediment transport. Hence it is not surprising that the formation of ripples has attracted the attention of many researchers for many years. In spite of this, however, our knowledge of the origin of ripples and our methods of predicting their dimensions is far from complete, and therefore some further contributions on the topic cannot be regarded as superfluous. The present paper concerns the prediction of the length (Λ) of ripples generated by short waves (wind waves) in shallow waters. It is assumed that the initial surface of the horizontal movable bed is flat, that the granular material is cohesionless, and that the oscillatory motion of the two phases (transporting fluid and transported sediment) is two-dimensional. Furthermore, it is assumed that this oscillatory motion is completely symmetrical, i.e. that no "not drift" is present.

DIMENSIONLESS FORMULATION OF RIPPLE LENGTH

If the internal geometry of a granular material (i.e. the shape of its grains and the shape of its sieve curve) is specified, then, under the conditions stated above, the oscillatory two-phase

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motion in the vicinity of the bed is determined by the following dimensionless variables (Refs. [1] and [2])

$$X = \frac{\gamma_s D^3}{\rho \nu^2} \qquad Y = \frac{\rho D}{\gamma_s T^2} \qquad Z = \frac{a}{D} \qquad W = \frac{\rho_s}{\rho} \tag{1}$$

Here	ρ =	fluid density	fluid
	ν =	kinematic viscosity	
	ρ_s =	density of grains	granular material
	γ_s =	specific weight of grains in fluid	
	D =	typical grain size (usually D_{50})	
	T =	period of the oscillatory fluid motion	fluid kinematics
	a =	orbit length of fluid motion at the boundary layer level	

The dimensionless variables (1) ensure the possibility of determining <u>any</u> quantitative property of the oscillatory two-phase motion. This, however, does not mean that all four of these variables must necessarily be present in the expression of <u>every</u> property. The number of variables that must be present and the form of their appearance vary depending on the property under consideration.

The graphs produced to date for the prediction of ripple length Λ , or of its dimensionless version Λ/D , are in the form of a family of curves plotted versus the orbit length a or $Z = a/D$. (See e.g. Figs. 1 and 2 reproduced from Ref. [3] and [2] respectively.) Thus, contemporary practice is to treat Λ/D as a function of two variables, i.e. as

$$\frac{\Lambda}{D} = f(Z,\Theta) \tag{2}$$

say, and to plot it by using Z as the abscissa and Θ as a parameter differentiating individual curves (each curve corresponding to a constant value of Θ). There is general agreement that Z is the most important variable in the expression of Λ/D . There is, however, no agreement yet as to what is exactly the "parameter Θ ", i.e. how it should be expressed in terms of the remaining variables X , Y and W ; (see e.g. from Figs. 1 and 2 that in Ref. [2] the parameter Θ is identified with the variable Y , whereas in Ref. [3] the curves are classified simply by using the grain size D). The next section is an attempt to reveal the expression of Θ .

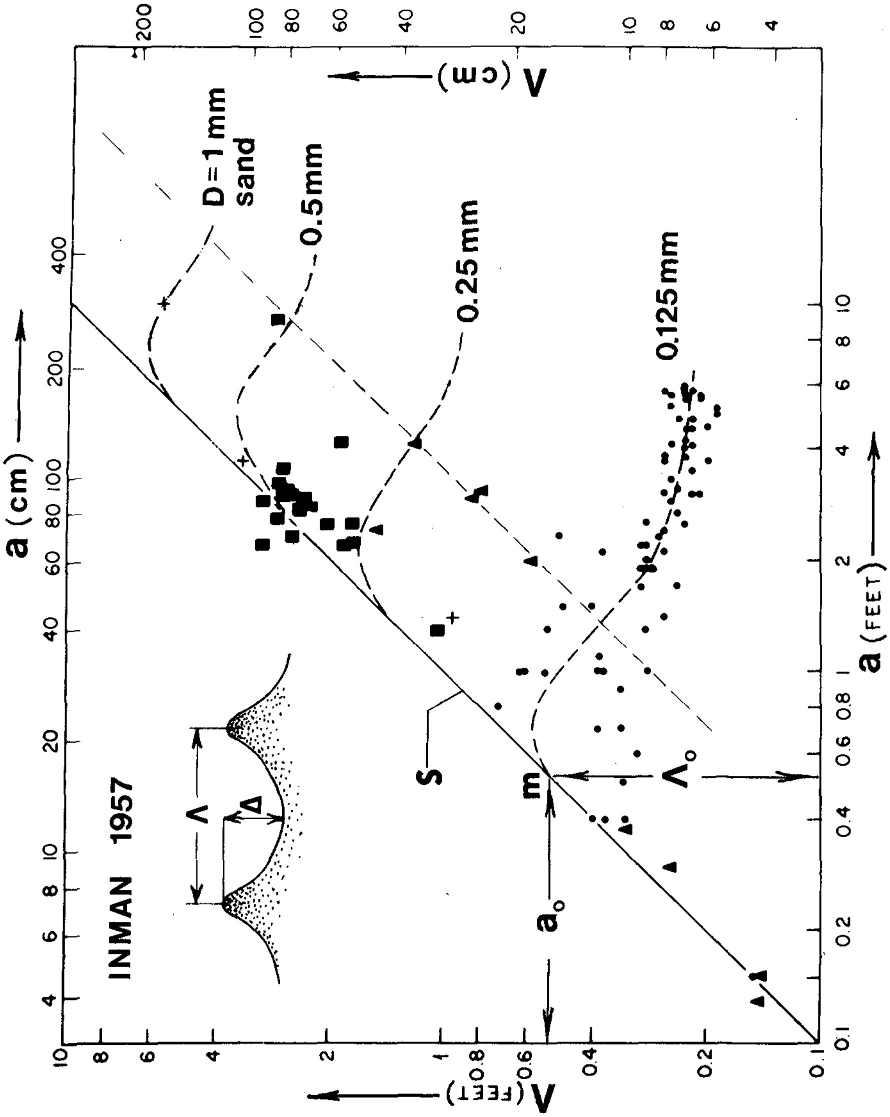
INMAN 1957
D=1 mm
sand
0.5mm
0.25mm
0.125mm
a (cm)
Λ (cm)
a (FEET)
Λ (FEET)
S
m
a₀
Λ₀
Λ
Δ

Fig. 1

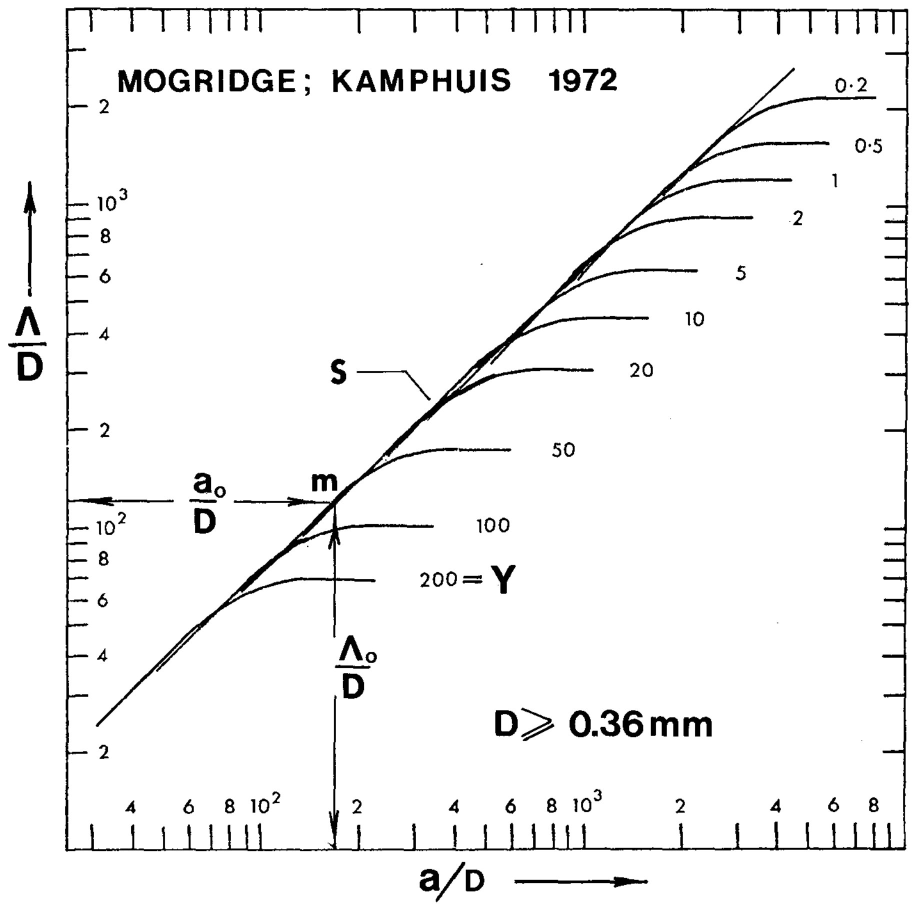
MOGRIDGE; KAMPHUIS 1972
Λ/D
a/D
S
m
a₀/D
Λ₀/D
200 = Y
100
50
20
10
5
2
1
0·5
0·2
D ⩾ 0.36 mm

Fig. 2

PARAMETER Θ

Consider the information supplied by experiment.

In Ref. [4] the measurements of the ripple length were carried out for three different granular materials: quartz $(\gamma_s/\gamma = 1.65)$; coal $(\gamma_s/\gamma = 0.50)$ and steel $(\gamma_s/\gamma = 6.6)$. No influence of γ_s or ρ_s was detected in the behaviour of the curves representing the variation of the ripple length Λ with the orbit length a . (See e.g. the three curves corresponding to the same grain size $D = 0.36$ mm in Fig. 1 of Ref. [4]).

Similarly from Fig. 3 of Ref. [1] it can be seen that the variation of Λ/D with Z for sand $(\gamma_s/\gamma = 1.65)$ and perspex $(\gamma_s/\gamma = 0.19)$ is almost the same if the period is the same $(T = 1.82_s)$ and if their grain size is comparable. (Compare e.g. the patterns of the points corresponding to $D = 0.52$ mm sand and $D = 0.48$ mm perspex in Fig. 3 of Ref. [1] (last two lines in the table on this graph)). These are only a few of the examples which indicate that the length Λ of ripples does not depend on the nature of the granular material, i.e. on its specific weight γ_s and density ρ_s . But the quantities γ_s and ρ_s can only disappear from the expression of Λ if W is excluded, while X and Y appear in the form of the product

$$X \cdot Y = \left(\frac{\gamma_s D^3}{\rho \nu^2}\right) \cdot \left(\frac{\rho D}{\gamma_s T}\right) = \frac{D^4}{\nu^2 T^2} \tag{3}$$

(or any power thereof). Accordingly, the relation determining Λ/D can be expressed in the form

$$\frac{\Lambda}{D} = f\left(Z , \frac{\nu T}{D^2} \right) \tag{4}$$

From the comparison of (4) and (2) it follows that

$$\Theta = \frac{\nu T}{D^2} = (X \cdot Y)^{-1/2} \tag{5}$$

DISCUSSION

(i) The parameter Θ which reflects the influence of T in conjunction with ν is, at the same time, an indicator which shows how the viscous effect (at the bed) compares with that of turbulence. Indeed, let δ_v and δ_t be the boundary layer thicknesses of viscous and fully turbulent flows respectively. If the flows are oscillatory and the bed is flat, then

$$\delta_v \sim \sqrt{\nu T} \tag{6}$$

and

$$\delta_t \sim K_s \sim D \tag{7}$$

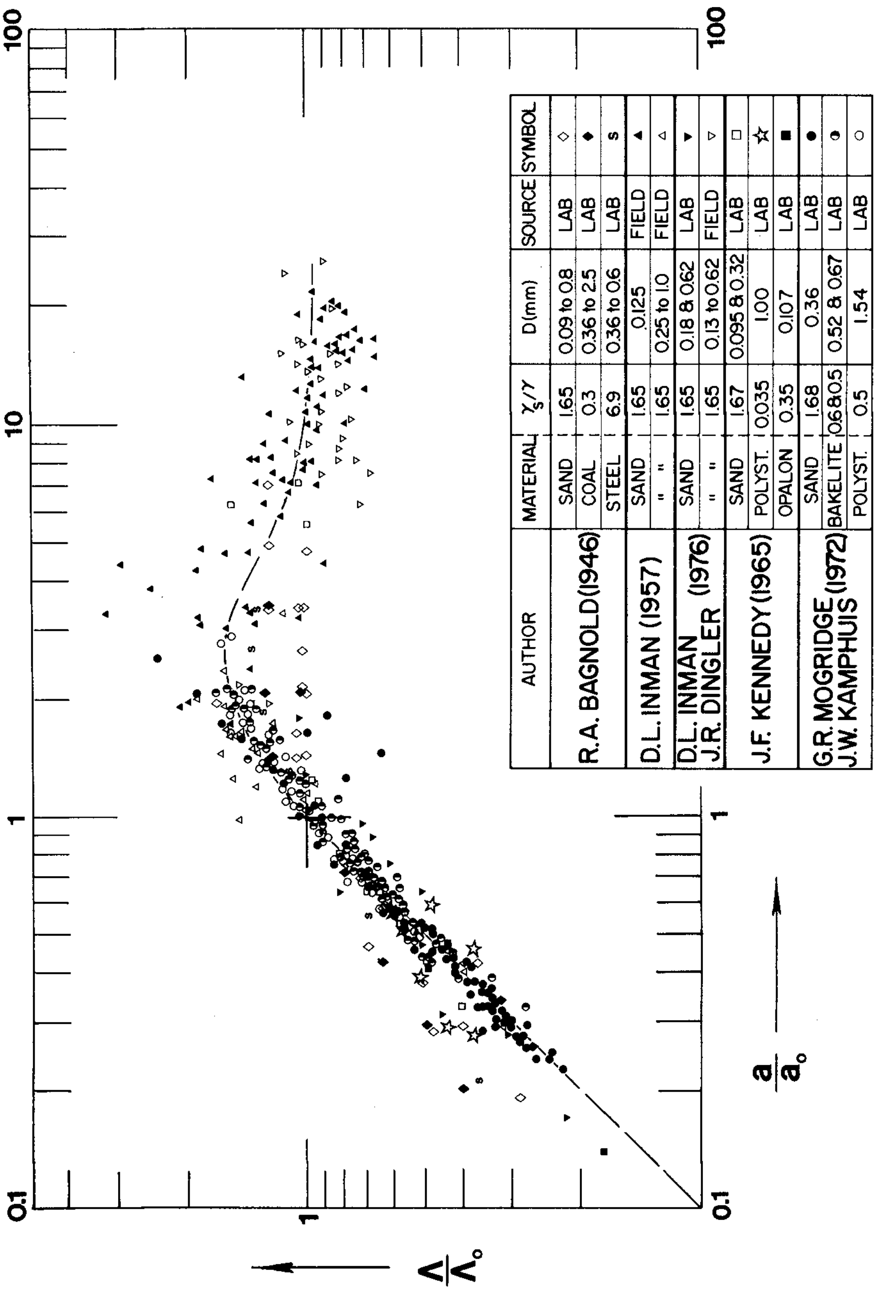
0.1
1
10
100
a/a₀
Λ/Λ₀
AUTHOR
MATERIAL
γs/γ
D(mm)
SOURCE
SYMBOL
R.A. BAGNOLD (1946)
SAND 1.65 0.09 to 0.8 LAB
COAL 0.3 0.36 to 2.5 LAB
STEEL 6.9 0.36 to 0.6 LAB
D.L. INMAN (1957)
SAND 1.65 0.125 FIELD
" " 1.65 0.25 to 1.0 FIELD
D.L. INMAN J.R. DINGLER (1976)
SAND 1.65 0.18 & 0.62 LAB
" " 1.65 0.13 to 0.62 FIELD
J.F. KENNEDY (1965)
SAND 1.67 0.095 & 0.32 LAB
POLYST. 0.035 1.00 LAB
OPALON 0.35 0.107 LAB
G.R. MOGRIDGE J.W. KAMPHUIS (1972)
SAND 1.68 0.36 LAB
BAKELITE 0.6805 0.52 & 0.67 LAB
POLYST. 0.5 1.54 LAB

Fig. 3

Thus
$$\Theta \sim \left(\frac{\delta_v}{\delta_t}\right)^2 \tag{8}$$

(If the "geometry" of the oscillatory motion over the flat (initial) bed is given (by $Z = a/D$) then the subsequent wave-like deformation of this bed appears to be dependent only on the comparative degree of turbulence.)

(ii) In the field of unidirectional flows it is well known that the length (Λ) of sand waves does not depend on γ_s or ρ_s. Indeed, none of the expressions produced to date for the length of ripples, dunes or antidunes contain γ_s or ρ_s; (these quantities affect only the speed of their development). In particular the length of ripples forming in a (two-dimensional) unidirectional flow is determined only by the parameters v_*, D and ν (Ref. [5]) :

$$\frac{\Lambda}{D} = \phi\left(\frac{v_* D}{\nu}\right) \quad . \tag{9}$$

Consider now an oscillatory flow having

$$(a \to \infty \ , \ T \to \infty) \quad \text{but} \quad (\frac{a}{T} \ \text{finite}) \tag{10}$$

Suppose that the (finite) mean orbital velocity $U = 2(a/T)$ of this flow is as sufficiently large as to induce the transport of sediment and consequently to generate ripples. Clearly the ripples generated by such an oscillatory flow must be virtually the same as those of a unidirectional flow. But if so, then the function (4) (of two variables) must in the limit (10) reduce into the function (9) (of one variable). In other words, the function (4) must possess the property

$$\lim_{\substack{a\to\infty \\ T\to\infty}} \Big[f(Z,\Theta) \Big] = \phi\left(\frac{v_* D}{\nu}\right) \tag{11}$$

If a and T are very large, then their individual influence vanishes and they affect the phenomenon in the form of the ratio a/T meaning "the velocity" ($u \sim a/T$). But the ratio a/T can only appear in the expression of Λ/D if two variables Z and Θ of the function (4) merge into a single variable Z/Θ, which means that

$$\lim_{\substack{a\to\infty \\ T\to\infty}} \Big[f(Z,\Theta) \Big] = \phi\left(\frac{Z}{\Theta}\right) \tag{12}$$

must also be valid. From comparison of (11) and (12) it follows that the necessary expectation in the limit (10) can only be satisfied if Z/Θ has the same meaning as the Reynolds number $v_* D/\nu$. Observe that this is indeed so

$$\frac{Z}{\Theta} = \frac{(a/D)}{(\nu T/D^2)} = \frac{(a/T)\cdot D}{\nu} \sim \frac{UD}{\nu} \sim \frac{v_*D}{\nu} \tag{13}$$

(An analogous transition into the Reynolds number v_*D/ν (which does not contain γ_s or ρ_s) cannot be achieved by any other expression of Θ (other than (5)), as any combination of X , Y and W (other than $(X\ Y)^n$) will inevitably contain γ_s and/or ρ_s).

NORMALISED PLOT

When the values of Λ/D corresponding to a constant value of Θ are plotted versus Z , then the experimental points first follow a common 45°-straight line S , then they diverge from it and form some curves (Figs. 1 and 2). These curves (point patterns) are rather similar to each other and it is very likely that each of them is in fact "the same curve shifted in the 45°-direction". Let m be the point (location) where an individual curve diverges from the common straight line S , and a_o and Λ_o be the coordinates of m (Fig. 1). If the individual curves are indeed the shifted versions of each other, then one can normalise the curve family (i.e. one can make all the curves collapse into a single unified curve) by plotting Λ/Λ_o versus a/a_o . Estimating the locations m from the data available (Refs. [2], [3], [4], [6], [7]) such a normalised plot was produced and it is shown in Fig. 3. The scatter is considerable (especially that due to the field data) and yet the points (symbols) do not tend to form some patterns of "their own". Thus the idea of a normalised plot appears to be feasible. It would certainly be desirable if some further (laboratory) measurements were carried out in the range

$$\simeq 2 < \frac{a}{a_o} < \simeq 20$$

(in order to reveal more clearly the position of the curve in this region).

The graph in Fig. 3 can only be used for the prediction of Λ if one knows the values of a_o and Λ_o . Since the divergence points m lie on the straight line S (Figs. 1 and 2) a_o and Λ_o are interrelated by the simple proportion

$$a_o = (\text{const})\cdot\Lambda_o \tag{14}$$

where $$\simeq 0.75 < (\text{const}) \leq \simeq 1 \tag{15}$$

(In fact it is only the curve family of Ref. [2] which yields (const) $\simeq 0.75$. The curve families of the rest of the authors quoted yield (const) $\simeq 1$, and therefore with regard to practical purposes it would be only reasonable to adopt simply $a_o \simeq \Lambda_o$).

Thus it remains to reveal the value of Λ_o . Since the divergence point m is displaced together with the curve (Figs. 1 and 2), and since the position of each curve is determined only by the parameter Θ, the value of Λ_o/D must also be determined by Θ only.

$$\frac{\Lambda_o}{D} = \psi (\Theta) \tag{16}$$

Plotting the ordinates Λ_o/D of the points m (estimated from the data available) versus the corresponding values of $\Theta = \nu T/D^2$ one arrives at the graph in Fig. 4. (Note from this graph that the points corresponding to different γ_s and ρ_s do not tend to form different patterns).

Hence, knowing D, ν and T (and consequently Θ) one determines from Fig. 4 the value of Λ_o, and then from Eqn. (14) the value of a_o. Knowing a (and thus a/a_o) one can predict Λ/Λ_o (and thus Λ) from Fig. 3.

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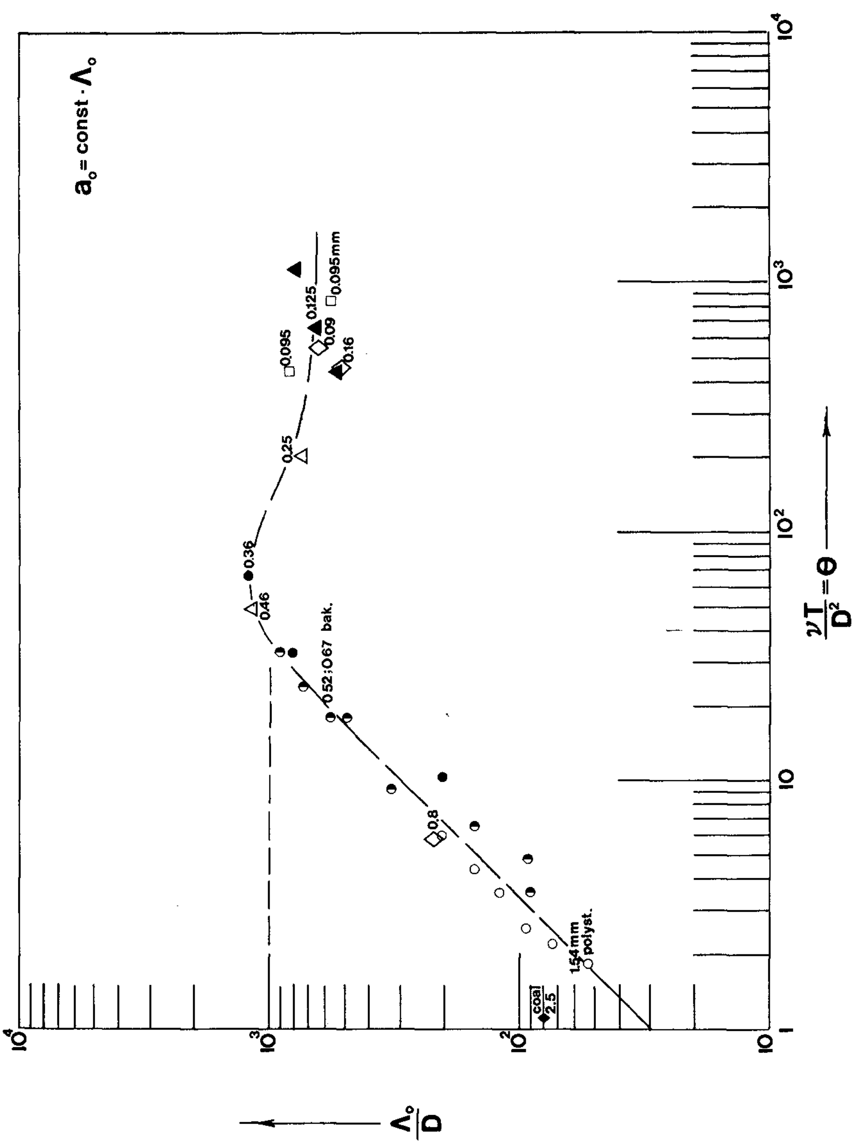

$a_o = \text{const} \cdot \Lambda_o$
$\frac{\Lambda_o}{D}$
$\frac{\nu T}{D^2} = \Theta$
0.095 mm
0.125
0.095
0.09
0.16
0.25
0.36
0.46
0.52; 0.67 bak.
0.8
1.54 mm polyst.
coal 2.5

Fig. 4

LIST OF SYMBOLS

ρ	fluid density
ν	kinematic viscosity
ρ_s	density of grains
γ_s	specific weight of grains in fluid
D	typical grain size (usually D_{50})
T	wave period
a	orbit length of the oscillatory fluid motion due to waves at the boundary layer level
$U = 2a/T$	mean orbital velocity at the boundary layer level
K_s	height of the granular roughness of the (initial) flat bed
v_*	shear velocity of the unidirectional open channel flow
Λ	ripple length
δ_v	boundary layer thickness of the viscous oscillatory flow
δ_t	boundary layer thickness of the fully turbulent oscillatory flow
X,Y,Z,W	dimensionless variables of the two dimensional oscillatory two-phase motion at the mobile bed, as defined by Eqns. (1)
$\Theta = \nu T/D^2$	dimensionless combination reflecting the influence of period (in conjunction with viscosity)
a_o and Λ_o	upper limits of the proportionality between a and Λ which correspond to a given Θ = const