CHAPTER 15

An Heuristic Model of Wave Height Distribution in Surf Zone

Masaru MIZUGUCHI
Department of Civil Engineering
Chuo University
Kasuga, Bunkyo-ku
Tokyo, JAPAN

INTRODUCTION

Until now, almost every study on coastal processes has considered the basic type of two-dimensional beach profile as being of constant slope. However, as our knowledge on this problem advances, we realize the importance of the influences of the bottom configuration on the hydrodynamic phenomena in a given area.

Figure 1 shows a recent experimental result on the longshore current profile on a step type beach. (Here the step type beaches are defined as those which have a step in the bottom profile, whether the beaches are of accretion type or not.) The wave height and mean water

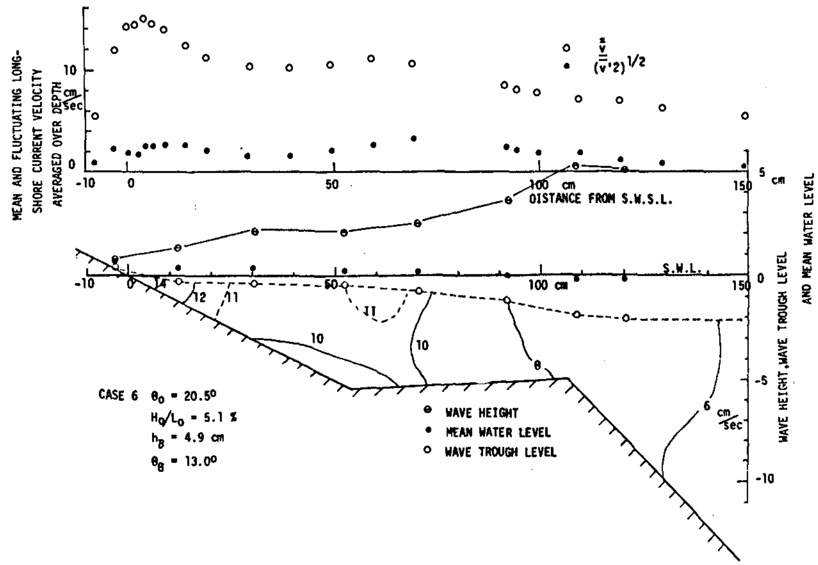

Figure 1 Longshore current on a step type beach [Mizuguchi & Horikawa (1978)]

level distribution are also plotted in this figure. The current velocity profile exhibits a different pattern from the one expected on a uniformly sloping beach. The difference is mainly due to the wave height distribution in the surf zone, which contributes to the radiation stress gradient and to genarating the current. On uniformly sloping beaches, the wave height in the surf zone is almost proportional to the water depth, and thus the external force to generate the current is a monotonous function of the onshore or offshore direction. The wave height distribution after breaking at the step in Fig. 1 is not proportional to the local water depth but, rather, is unique to the step type beach, showing exponential-like decay after the first breaking, recovery at the uniform depth area, second breaking on the inner slope and linear decay on the uniformly sloping beach. The longshore current velocity tends to increase shoreward where the wave height decreases, being modified by the turbulent viscosity. Therefore it is absolutely essential to be able to calculate the wave height distribution in a surf zone with a non-uniform bottom profile. And the beach profile usually observed in field experiments is not a uniformly sloping one, but a much more complex profile.

Outside the surf zone the energy equation, combined with a theory of permanent wave, is capable of giving the wave height change with good approximation, if the beach slope is not very steep. Inside the surf zone, turbulence induced by wave breaking plays an important role. At present, we have both experimental results and semi-emperical theories based on the energy conservation equation to evaluate the wave height (or energy) decay on a constant depth beach [for example, Horikawa & Kuo (1966)]. The most sophiscated but still gross treatment of that problem originates with the work of LeMehaute (1962), where the energy dissipation process is assumed to be similar to that in the hydraulic jump. Following this line, Battjes & Janssen (1978) treated the change of the r.m.s. value of the irregular wave height in the surf zone and succeeded in adjusting their theory to the experimental results, which included the case of a non-uniformly sloping beach.

In the above mentioned theories the energy dissipation depends on the wave height itself. This means that the wave height decays until reaching zero. However, suppose that the water depth after the wave breaking is constant; the broken wave will recover somewhere and start to propagate as a reformed wave in the constant depth area, if the energy dissipation due to the bottom friction is negligible.

In this paper an heuristic model is developed by employing the simplest formulation, that is, an eddy viscosity assumption. In order to construct the model, two limiting conditions were taken into account, namely, 1) wave breaking on a constant depth beach and 2) wave breaking on a uniformly sloping beach. These will be shown to be well described by the model.

DERIVATION OF THE MODEL

Four major assumptions are employed to derive the model as are now listed.

1) The following wave energy equation can be applied in the surf zone.

$$\frac{d}{dx}(Ec_g) = -\varepsilon \tag{1}$$

with expressions

$$E = (1/2)\rho g a^2 \tag{2}$$

and

$$c_g = \sqrt{gd} \tag{3}$$

where E: wave energy per unit area, c_g: energy propagation velocity, ρ: fluid density, g: gravity acceleration, a: wave amplitude, ε: energy dissipation rate, d: water depth. In a strict sence, these expressions are inadequate to describe wave motion in the surf zone. The validity of Eq. (1) depends on the dissipation rate ε. Equation (2) can be used if the ratio of the wave height squared to the wave energy is constant through the surf zone. The unknown constant ratio will be included in the dissipation rate. In the surf zone the wave energy propagation velocity may be the same as the phase velocity, and is well approximated by that of the solitary wave theory [Mizuguchi (1979)].

$$c_g = \sqrt{g(d+2a)} \tag{4}$$

Equation (3) can be considered to be a first approximation to Eq. (4).

2) When applying the idea of turbulent eddy viscosity, ν_e, the rate of energy dissipation is given by

$$\varepsilon = 2\rho g \nu_e (ka)^2 \tag{5}$$

where k: wave number. Here the energy dissipation due to the bottom friction is neglected. Equation (5) is obtained by replacing the molecular viscosity ν by the turbulent eddy viscosity ν_e in the final expression of the wave energy dissipation in irrotational flow. Ijima, Takahashi & Nakamura (1956) introduced Eqs. (1) and (5) to treat wave height decay in a surf zone of uniform depth with constant eddy viscosity. It should be noted that the eddy viscosity is not equivalent to that proposed by Battjes (1975), which is introduced through a discussion of energy dissipation due to (isotropic) turbulence induced in wave breaking.

3) Then the eddy viscosity is modeled in the following way. First, when the depth in the surf zone is constant, the broken wave should recover (or reform) as mentioned before. This suggests that the energy dissipation rate or the eddy viscosity should be a function of the difference between the real wave height and the reformed wave height. Secondly, the dissipation rate may also be related to the ratio of the real wave height to the possible maximum wave height at any location. On uniformly sloping beaches the waves are considered to be continously breaking and the state of the maximum wave height is always realized. Then the possible maximum wave height may be expressed by $\gamma'd$, where

γ' is the constant wave amplitude to water depth ratio in the surf zone on a uniformly sloping beach. These hypotheses result in the following expression for ν_e:

$$\nu_e = \nu_{eB}[\ (a-c'd)/\gamma'd\ ]^m \tag{6}$$

where c' is the ratio between the wave amplitude and the water depth in the wave recovery zone on a constant depth beach, and m is a constant to be determined later. The subscript B denotes quantities evaluated at the breaker line.

Then the nondimensional equation describing the wave height distribution in the surf zone is

$$\frac{d}{dX}(\ A^2D^{1/2}\ ) = -N_B(AD^{-1}-c_0)^m A^2 D^{-1} \tag{7}$$

where

$$N_B \doteqdot 4\nu_{eB}\sigma/(gd_B) \tag{8}$$

and $c_0 = c'/\gamma'$, $A = a/a_B$, $D = d/d_B$, $X = k_B x$ and σ: angular frequency. In Eq. (7) the constant m should be less than 1.0, since the broken wave should recover in a finite distance in a constant depth surf zone. Here we take m as 0.5, for the advantage of easy handling.

4) The nondimensional eddy viscosity N_B can be calculated as follows from the experimental fact that the ratio γ' is constant on uniformly sloping beaches.

$$N_B = 5(1-c_0)^{-1/2}s_B/(2k_Bd_B) \tag{9}$$

Here s_B is a representative beach slope (including the expected wave set-up) outside the breaker line, so that

$$s_B = s_B'/(1+3\gamma'^2/2) \tag{10}$$

where s_B' is a representative beach slope for wave breaking. Actually Eq. (9) is derived by equating the wave height decay ratio at the breaking point with the mean decay ratio a_B/x_B, where x_B is the surf width.

The wave set-up $\bar{\eta}$ in the surf zone is usually given by the following nondimensional equation,

$$\frac{dH}{dX} = -(3/2)\gamma'^2 AD^{-1}\frac{dA}{dX} \tag{11}$$

where the radiation S_{xx} is assumed to be given by 3E/2 with Eq. (2), and $H = \bar{\eta}/d_B$. Combining Eqs. (7) and (11), we can calculate both the wave height and the wave set-up distribution in the surf zone for arbitrary bottom profile d(x). The flowchart to solve Eqs. (7) and (11) numerically is shown in Fig. 2, for the case of no second wave recovery. The differential equations are solved numerically by the Runge-Kutta-Gill method. The calculation converges within two or three

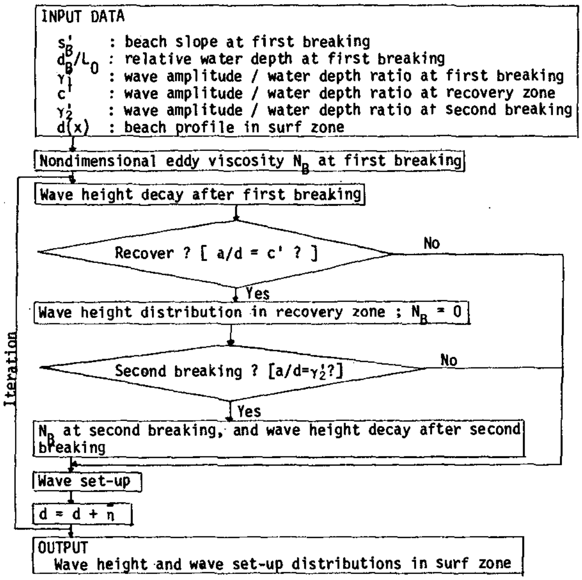

Figure 2 Flowchart to calculate wave height and wave set-up in the surf zone (no second wave recovery)

iterations as described in the flowchart.

As shown in Fig. 2, the unknowns are the breaker depth, the breaker height, the wave amplitude to water depth ratio, c', in the recovery zone, and the bottom topography d(x). The second wave breaking criterion γ_2' may be chosen as 0.4, which is an approximate value for breaking solitary wave, because once-broken waves may be better expressed by solitary wave theory. Any kind of breaker index can be used to predict the breaker depth and the breaker height (or the ratio γ_1') for uniformly sloping beaches [for example Goda (1970)]. The next simple empirical relation was given by Sunamura & Horikawa (1974):

$$a_B/H_0 = 0.5 s_B'^{0.2} (H_0/L_0)^{-0.25} \tag{12}$$

$$0.01 < s_B' < 0.1$$

$$0.003 < H_0/L_0 < 0.07$$

Equation (12) was obtained by using the collectd (and also corrected) data in Goda (1970). The following relation may also be derived from the same data:

$$a_B/d_B \ (= \gamma_1') = 0.75 s_B'^{0.3} (d_B/L_0)^{-0.1} \qquad (13)$$

$$0.03 < s_B' < 0.1$$

$$0.01 < d_B/L_0 < 0.05$$

In Eqs. (12) and (13), H_0 and L_0 denote deepwater wave height and deepwater wave length respectively. These equations provide full information for a wave to break. The ratio γ_1' has two limiting conditions. It tends to a constant as the slope s_B' goes to zero when the relative water depth is small, and the wave steepness should be limited when the relative water depth is large. Then Eq. (13) can hold only in the indicated range. The ratio c' is expected to be determined by the intensity of wave breaking. The values obtained in laboratory experiments [Mizuguchi,Tsujioka & Horikawa (1978) and Nagatoh & Ohishi (1979)] centered on the range from 0.21 to 0.28, slightly decreasing with increase of wave steepness.

If wave set-up is neglected, Eq. (7) can be solved in closed form for the case of a constant depth surf zone.

$$A = c_0 \sec^2 (-\sqrt{c_0} N_B X/4 + \arctan\sqrt{(1-c_0)/c_0}) \qquad (14)$$

Figure 3 show calculated curves for various values of N_B. The distance X_R to the recovery point where $A = c_0$ is given by

$$X_R = 4\arctan\sqrt{(1-c_0)/c_0}/(N_B\sqrt{c_0}) . \qquad (15)$$

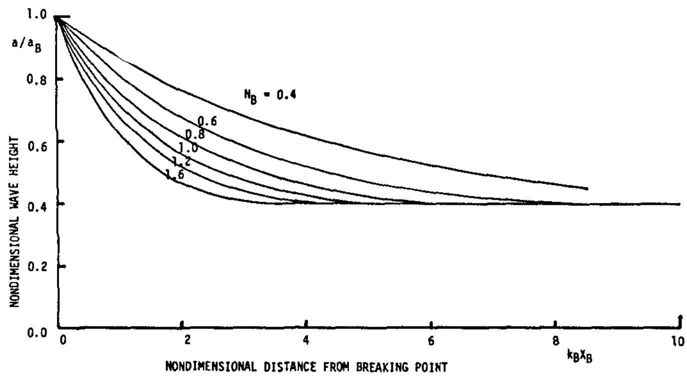

Figure 3 Wave height decay in a constant depth surf zone

COMPARISON WITH EXPERIMENTS

Figures 4 - 7 show the comparisons of the predicted results (solid lines) with the experimental results. The broken lines will be explained later. In the calculation the constant c' is roughly chosen as noted in the figures to give the best fitted curve in the recovery or almost recovery zone. In the case of uniformly sloping beaches as in Fig. 5, the value of c' does not affect the results.

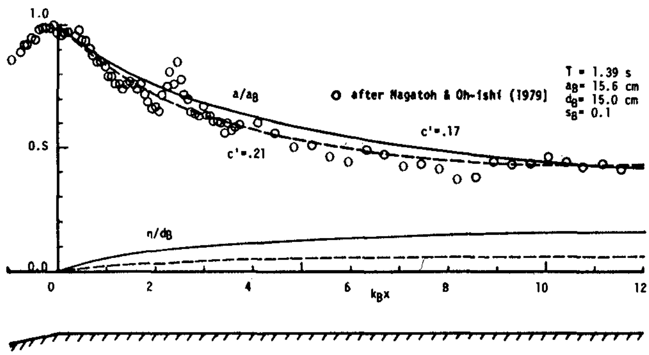

Figure 4 Comparison with experiment (constant depth beach)

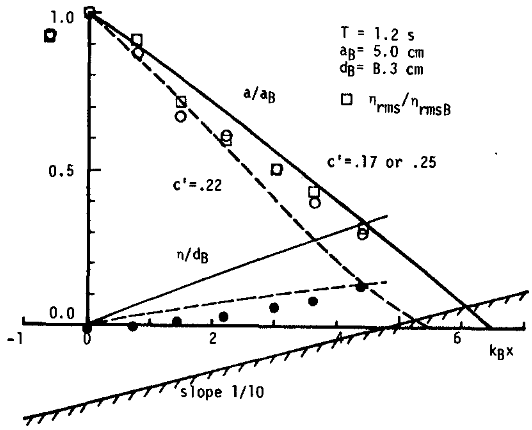

Figure 5 Comparison with experiment (uniformly sloping beach)

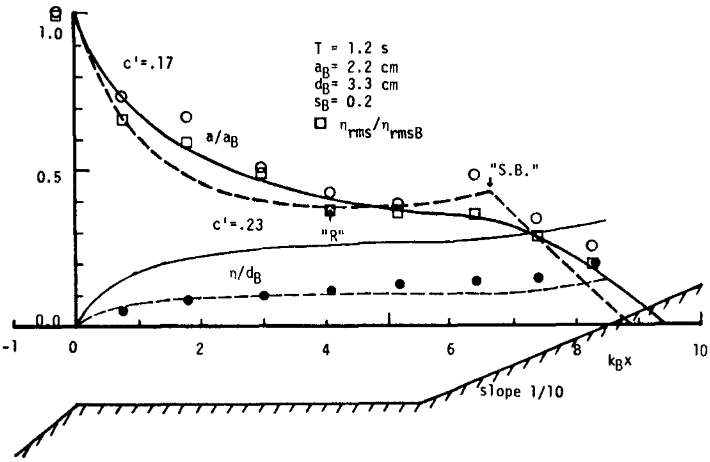

Figure 6 Comparison with experiment (step type beach)

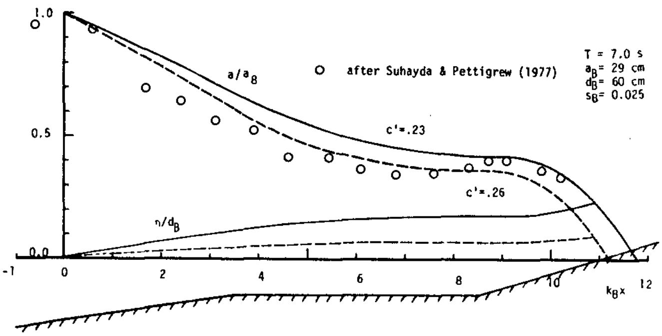

Figure 7 Comparison with field experiment

Generally the agreement is good enough. However, the model shows a tendency to underestimate the wave height decay near the first breaking, except in Fig. 6. In figs. 5 and 6, it is also seen that the wave set-up is also overestimated. The overestimated wave set-up increases the water depth, so that the wave height decay is relaxed. These are mainly caused by the fact that the wave energy is overestimated in the form given by Eq. (2). The delay of the wave set-up starting point from the wave breaking point, which is usually defined by the point of the maximum wave height, is also responsible for the results in Fig. 5.

Figure 8 shows the relation of the wave set-up $\bar{\eta}_R$ in the recovery zone to the wave breaking height on step type beaches. The solid line is from the next relation.

$$H_R \ (= \bar{\eta}_R/d_B) = 0.75(\gamma'^2 - c'^2) \tag{16}$$

which follows immediately from Eq. (11). The fitted line is

$$H_R = 0.25(\gamma'^2 - c'^2) \tag{17}$$

and indicates that the wave energy (or radiation stress) should be estimated by the following relation:

$$E = (1/6)\rho g a^2 \tag{18}$$

Here it is worth noting that the data plotted in Fig. 8 were odtained by changing the wave height while keeping the water depth fixed in the step region. Figure 8 also suggests that the value of c' is approximately 0.25.

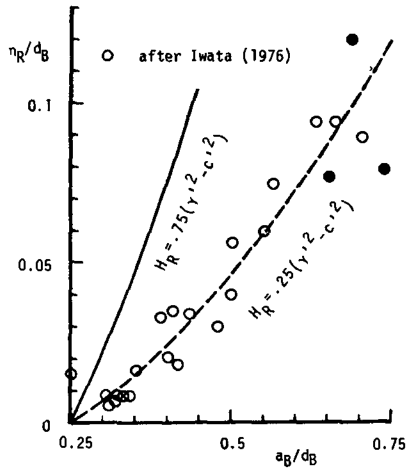

Figure 8 Wave set-up in recovery zone (constant depth or step type beach)

In Figs. 5 and 6, the nondimensional r.m.s. values of the water surface fluctuation η_{rms}/η_{rmsB} are also plotted. Taking the long wave or small amplitude wave assumption, we have the next relation.

$$E = \rho g \eta_{rms}^2 \tag{19}$$

Therefore, in a sense, the ratio of the wave height to η_{rms} denotes the unknown factor mentioned in the derivation of the model. The difference between η_{rms}/η_{rmsB} and a/a_B is within the error of the allowance for surf zone phenomena. Then the relation (18) is expected to hold through the surf zone and should be taken into consideration in evaluating the wave energy from the wave height. Actually the ratio of a^2 to η^2_{rms} is calculated from the experimental data for the cases in Figs. 5 and 6. In and near the surf zone the values are about 3 to 4, which is much larger than the value 2 for small amplitude waves but almost half of that expected from Eq. (18).

The broken lines in Figs. 4 - 7 are the results obtained by using the empirical relation (18). Then Eq. (10) is also modified. They agree better with the experimental results. A typical wave height transformation over a step type beach is shown in Fig. 6; decay after the first breaking, recovery at the uniform depth area (marked by "R"), increase due to the decrease in water depth, second wave breaking ("S.B.") after which the wave height linearly decreases to zero at the mean water shoreline. However, this time, the wave set-up is a little underestimated. It reflects the above-mentioned experimental results that the ratio a^2 to η^2_{rms} should be smaller than 6. In addition, the broken line shows the overestimation of the wave height decay just after the first breaking in Fig. 6. This is connected with the determination of the representative beach slope for the wave breaking. The step part of the bottom profile in the case of Fig. 6 was made of bent plate, which was placed on the uniformly sloping (1/10) beach. The slope just before the wave breaking was about 1/5, which was used to calculate the predicted curve. The slope might be steeper than the value which really governs the process of the wave breaking and gives too rapid decay of the wave height after breaking.

DISCUSSION

The model developed herein is an expedient, although it predicts rather well the experimental results over a wide range of conditions. There are two points which one should treat carefully when applying this model. The first point is how to choose the representative beach slope s'_B before wave breaking. In laboratory experiments, it should be pointed out that the beach slope s'_B is not equal to a real slope, unless the uniformly sloping beach section extends to a sufficient deep region. When the water is not so deep in the uniform depth region, waves may deform while travelling in that region and easily break. Then the breaking phenomena cannot be considered to depend only on the beach slope just before wave breaking. We suggest that Eq. (13) is to be applied to determine the beach slope s'_B, once the wave amplitude - water depth ratio γ'_1 at the breaking point is given independently. This ratio may be considered to indicate how the wave is deformed before breaking, representing the wave's history. The second point is how to give the value of c'. In this respect our knowledge on wave breaking phenomena is still unsatisfactory. Well-controlled experiments are required to give the proper value of the c' in this model.

Finally it should be pointed out that the eddy viscosity assumption in the energy equation is physically obscure, although it could be more easily modeled as in Eq. (6) than that proposed by Battjes (1975). This model reflects the present state of the study on wave properties in the surf zone and reveals some problems to be investigated in the future. For example, on uniformly sloping beaches where the waves are considered to be continously breaking, why is the wave height always proportional to the water depth through the surf zone and why does the wave set-up delay its start from the point of the maximum wave height? How is the turbulence generated and how does it affect the wave height decay? How should the constant c' be determined? What is the best expression for breaking or broken waves in and near the surf zone? The investigation to answer such questions will lead to a more complete modeling of the wave breaking process. The work by Sawaragi & Iwata (1974) and Svendsen, Madsen & Hansen (1978) are along these lines, but still further progress must be made before one can reach a firm understanding of the wave transformation in the surf zone.

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