TWO-DIMENSIONAL SURF BEAT

by

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ABSTRACT

Surface elevation and velocity measurements in shallow water on beaches show large fluctuations known as surf beat in the long period range from approximately 20s to 2000s (Munk, 1949; Tucker, 1950). A laboratory study was conducted to determine if two shoaling primary waves of nearly equal frequency would generate two-dimensional surf beat at their difference frequency. The experiments, carried out in the Scripps Institution of Oceanography Hydraulics Laboratory 30 m glass walled wave channel, show that the beat frequency motion in the channel consists of the sum of a forced progressive wave and two free standing waves. The progressive wave is forced by the local nonlinear interaction of the primary waves and grows sharply in shallow water. One of the free standing waves is generated directly by the wavemaker. The data is consistent with the hypotheses that the second, much larger, free wave is generated in shallow water as the reflection of the long, forced progressive wave, leading to the observed standing wave surf beat pattern.

Introduction

Surf beat was first observed by Munk (1949) and by Tucker (1950) who noted from field records of swell in shallow water that the envelope of wave amplitude was correlated with wave energy at the envelope, or beat, period. Longuet-Higgins and Stewart (1962) derived the result that the difference interaction of the incoming swell nonlinearly generated forced waves ("set-down" wave) at the difference frequency and wavenumber with some properties like those observed by Munk and Tucker. However, the nonlinear forcing hypotheses could not explain the observed

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lag between the envelope of the swell offshore and the arrival of the difference frequency wave. This discrepancy led to the idea that the nonlinearly forced wave was reflected from the shore line as a free wave (Tucker, 1950; Longuet-Higgins and Stewart, 1962), but a plausible mechanism for this was not proposed.

At about the same time, Munk et al (1964) showed that low frequency energy on the continental shelf was mostly in the form of trapped, three-dimensional edge waves. Furthermore, Gallagher (1971) was able to show that resonant growth of edge waves due to nonlinear difference interaction of incoming waves was theoretically possible and consistent with field data. Inman et al (1976) presented field data suggesting that beat period edge waves can be trapped and grow because of longshore discontinuities in topography such as headlands and submarine canyons. Recently, Bowen and Guza (1978) showed in the laboratory that difference interactions of obliquely incident waves caused resonant edge wave growth according to the Gallagher (1971) hypotheses.

With all the interest in edge waves, however, it still remained to show whether or not two-dimensional free long waves existed as a consequence of reflection or surf zone forcing. It was suggested by Tait and Inman (1969) that special surf zone widths and bars and other slope discontinuities could lead to enhancement effects in the surf zone which would amplify runup at special frequencies. Observations by Inman and Tait (unpublished manuscript) show that large runup amplitudes may be observed at certain frequencies. However, it was found that some of the runup peaks appeared at resonant frequencies of the channel, while some did not, thus still leaving the problem unresolved.

A long wave channel conveniently suppresses three-dimensional motions such as edge waves which have longshore dependence. Thus, on-offshore processes can be measured without interference. For example, the upper panel of Figure 1 shows two superposed spectra taken from two single frequency wave experiments at primary frequencies σ_1 and σ_2 run separately. Only the harmonics of each primary appear as secondary peaks. The lower panel of Figure 1 shows the spectrum resulting from measurements made when the two primary waves appear together. Note the energy present at sum and difference frequencies, particularly $\sigma_2 - \sigma_1$.

Wave channels introduce a completely different set of problems which complicate the interpretation of data, especially long wave measurements. These problems are essentially associated either with the wavemaker or with reflection and dissipation in the flume, or in some cases with both (Ursell, et al, 1960; Fontanet, 1961; Madsen, 1971; Hansen and Svendsen, 1974; Bowers, 1977; Flick and Guza, 1980).

It was the purpose of the experiments outlined in this paper to clarify the role of free, two-dimensional surf beat in light of reflection and wavemaker effects peculiar to wavechannels. The measurements were conducted in the Scripps Institution of Oceanography Hydraulics Laboratory. Figure 2 illustrates schematically the 30 m glass wall wave channel, the high-pressure servo-hydraulic wavemaker system and the high resolution resistance wire wavestaff system (Flick, et al, 1979) used in the study.

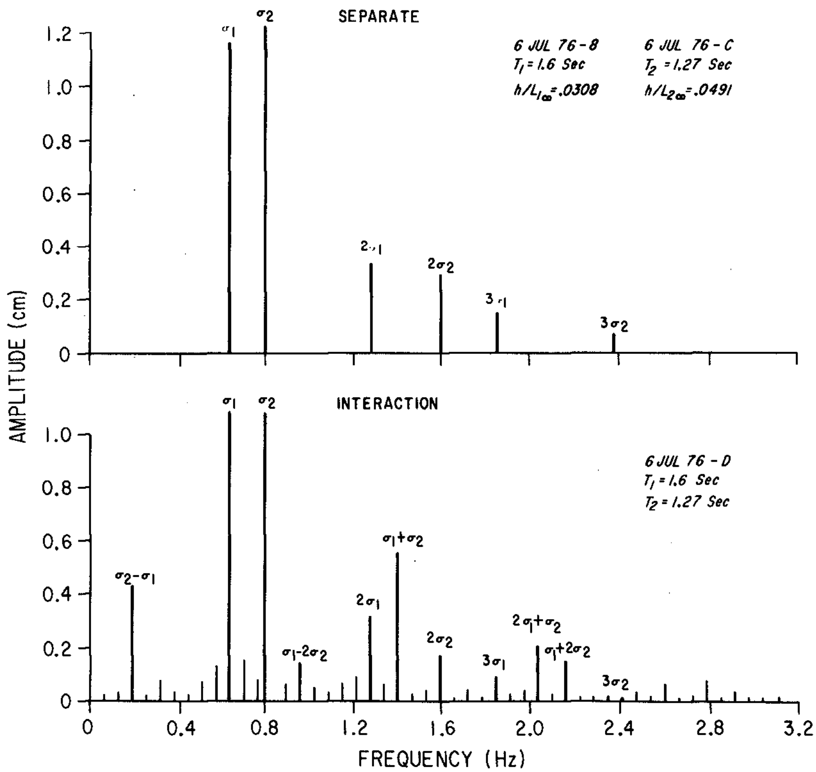

Figure 1. Upper panel shows superposed spectra of single frequency (σ_1 and σ_2) wave experiments. Note harmonics of each primary. Lower panel shows spectrum of run with both primaries present simultaneously. Note sum and difference frequencies, particularly $\sigma_2 - \sigma_1$ peak.

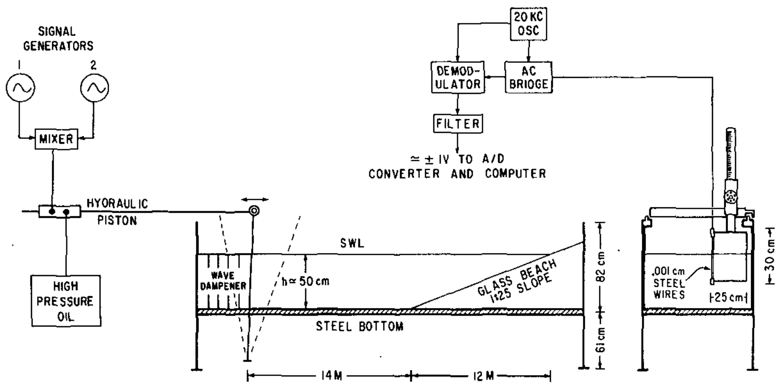

Figure 2. Schematic of Scripps Institution of Oceanography Hydraulics Laboratory 30 m glass wall wave channel, servo-hydraulic wavemaker and resistance wire wavestaff system.

It can be shown (Madsen, 1971) that free difference frequency waves always exist at some amplitude (perhaps small) when a wavemaker is used to generate two primary waves, unless paddle motion corrections are imposed (Bowers, 1977). These free waves were shown by Flick and Guza (1980) to theoretically be of the same order as the forced nonlinear "set-down" wave propagating with wave groups (Longuet-Higgins and Stewart, 1962). These free waves are generated at the wavemaker as progressive waves. After several wave periods, reflections produce a standing wave pattern. Assuming free, long waves are also generated in shallow water or in the surf zone, these also soon exist as standing waves. This makes it difficult to distinguish the source of free waves in wave channels.

The effect of dissipation, particularly in shallow water, may also be important. Long standing waves of sufficiently small amplitude usually exhibit downchannel profile changes too small to measure. However, swash dissipation on the beach face can be significant (Guza and Bowen, 1976) and can lead to progressive wave components in the standing wave profile which further complicate long wave data interpretation. On the other hand, strongly enhanced shallow water wave dissipation by means of a wave absorber was used in this study to eliminate the primary waves before the breakpoint without affecting the free long waves.

Results and Discussion

Flick and Guza (1980) have shown that to lowest order, the surface elevation of free standing waves in a channel with a flat section and a sloping beach (Figure 3) can be written

$$\eta \;=\; a_p f(x)\,\cos\left[\int_0^x k\,d\tilde{x} - \frac{\pi}{4}\right]\cos\sigma t \qquad \text{(a)}$$

where

$$f(x) = \begin{cases} 1 & , \; x_S < x < x_L \quad \text{(flat)} \\ \left(\frac{x_S}{x}\right)^{1/4} & , \; 0 < x < x_S \quad \text{(slope)} \end{cases} \qquad \text{(b)} \qquad (1)$$

$$\int_0^x k\,d\tilde{x} = \begin{cases} \left(\frac{\sigma^2}{g\beta x_S}\right)^{1/2}(x + x_S), & \text{(flat)} \\ \left(\frac{4\sigma^2 x}{g\beta}\right)^{1/2} & , \; \text{(slope)} \end{cases} \qquad \text{(c)}$$

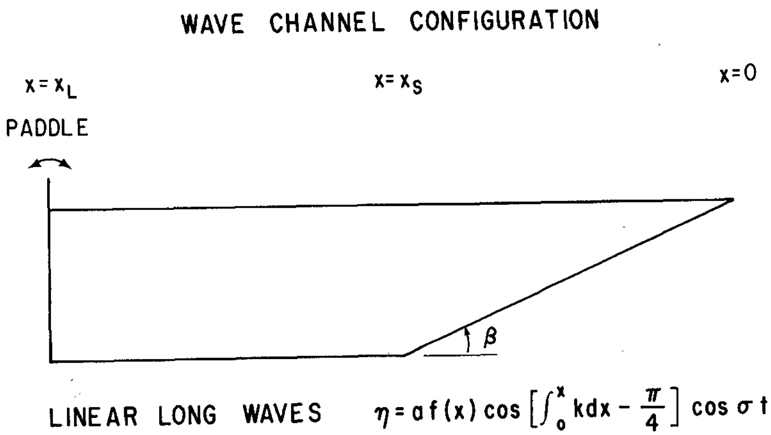

Figure 3. Definition sketch of wave channel with a beach showing location of wavemaker and the coordinate system used.

Here σ is the radian wave frequency, k the wavenumber, β the beach slope, g the acceleration of gravity and the coordinate system is defined in Figure 3. This form is valid except very near the shoreline.

The channel resonance frequencies can be computed by assuming that resonant standing waves will have an amplitude antinode at $x = x_L$. The resonance frequencies are

$$\sigma_n = (n + \tfrac{1}{4})\pi \frac{(g\beta x_S)^{\frac{1}{2}}}{x_L + x_S} \qquad n = 1, 2, \ldots \qquad (2)$$

Relations (1) and (2) follow from the asymptotic form for large arguments of the J_o Bessel function (Abramowitz and Stegun, 1965) which is the exact linear standing wave solution on a sloping beach, together with the required matching at the beach toe $x = x_S$.

Figure 4 shows the amplitude of surface displacement for two paddle generated standing waves in a channel with a beach. The triangles show wave elevation data taken with a short-wave absorbing barrier at the location indicated. The solid dots indicate data taken in the absence of the barrier. The location of the barrier in both experiments shown in Figure 4 was chosen so as not to change the location of nodes and antinodes of displacement of the paddle generated long waves. The theoretical wave amplitude normalized by the vertical runup amplitude, according to equation (1) is shown as a solid line.

The upper part of Figure 4 shows the displacement of a free wave with frequency between that of two adjacent channel modes (non-resonant). The wave length is determined by the period and depth since the wave is free. The condition that no flow can occur through the beach face means that the wave has an antinode at the beach. Therefore, the location of nodes and antinodes is determined. The phase of the wave at the paddle then depends only on the channel length. This phase for the non-resonant wave is very close to a node, while in the case of the wave at a channel resonance frequency (Figure 4, lower) the phase at the paddle corresponds to an antinode. In fact, the location of the antinode at the paddle is the resonance condition.

Since non-resonant standing waves generated directly by the wavemaker have antinodes of displacement at the beach face, it is a plausible assumption that standing waves generated in shallow water at the other end of the flume have antinodes at the paddle. The existence of antinodes at the paddle for non-resonant modes would therefore constitute evidence for shallow water long wave generation.

The results of eight beat wave experiments are summarized in Table 1. Four runs were at channel resonance frequencies and the other four were non-resonant. Each experiment consisted of driving the wavemaker simultaneously at two primary wave frequencies (periods T_1 and T_2) and measuring the amplitude of the difference frequency oscillation as a function of position in the channel. Each run consisted of two parts: one with the barrier to absorb the short primary waves and one without the barrier. The position of the barrier was chosen in each

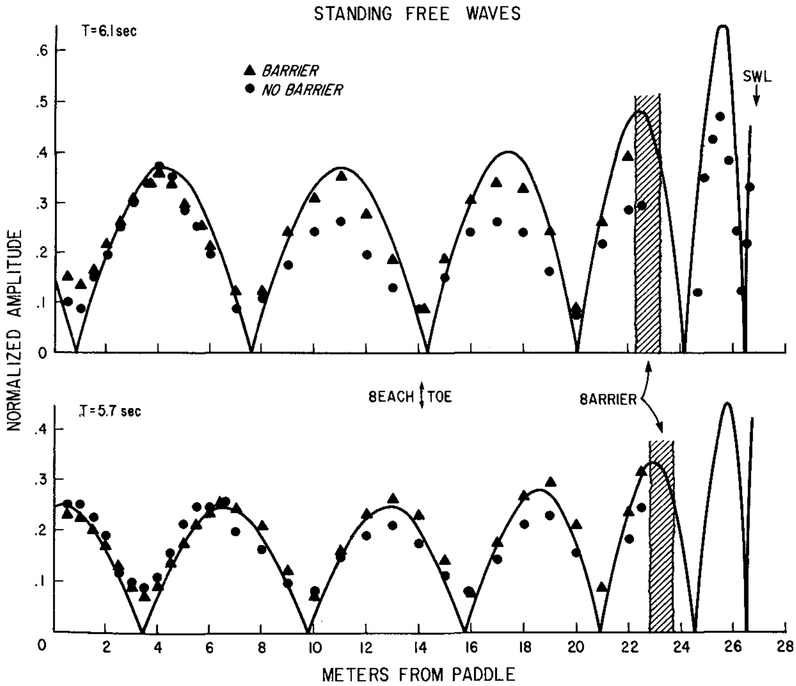

Figure 4. Amplitude of standing waves generated directly by the wavemaker. Location of short wave absorbing barrier shown by cross hatching. Triangles are data taken with the barrier installed, circles were taken without barrier. Solid line is theory from relation (1).

Table 1. Experimental Conditions for Beat Wave Experiments

RUN	BEAT PERIOD (sec)	T_1/ T_2 (sec)	TYPE	a_1/a_2 (cm)	a_Δ (cm)	a_B (cm)	a_P (cm)	c_r
1	6.1	1.60	No Barrier	1.2	.037	0.	.15	2.17
	Non res.	1.27	Barrier	1.2	.037	0.	.07	
2	6.1	0.78	No Barrier	4.0	.152	0.	.20	---
	Non res.	0.90	Barrier	4.0	.152	.03	0.	
3	6.1	1.60	No Barrier	2.0	.103	0.	.30	---
	Non res.	1.27	Barrier	2.0	.103	0.	.10	
4	7.47	1.60	No Barrier	1.5	.062	0.	.40	---
	Non res.	1.32	Barrier	1.5	.062	0.	.40	
5	5.7	1.60	No Barrier	1.0	.025	0.	.20	1.04
	Resonant	1.25	Barrier	1.0	.025	0.	.07	
6	5.7	1.60	No Barrier	1./1.	.025	0.	.32	1.86
	Resonant	1.25	Barrier	1.5/1.5	.100	0.	.20	
7	5.7	1.60	No Barrier	2.5/2.5	.156	0.	.20	2.64
	Resonant	1.25	Barrier	3./3.	.225	0.	.60	
8	6.73	1.30	No Barrier	2./2.	.072	0.	.40	---
	Resonant	1.09	Barrier	3./3.	.163	0.	.40	

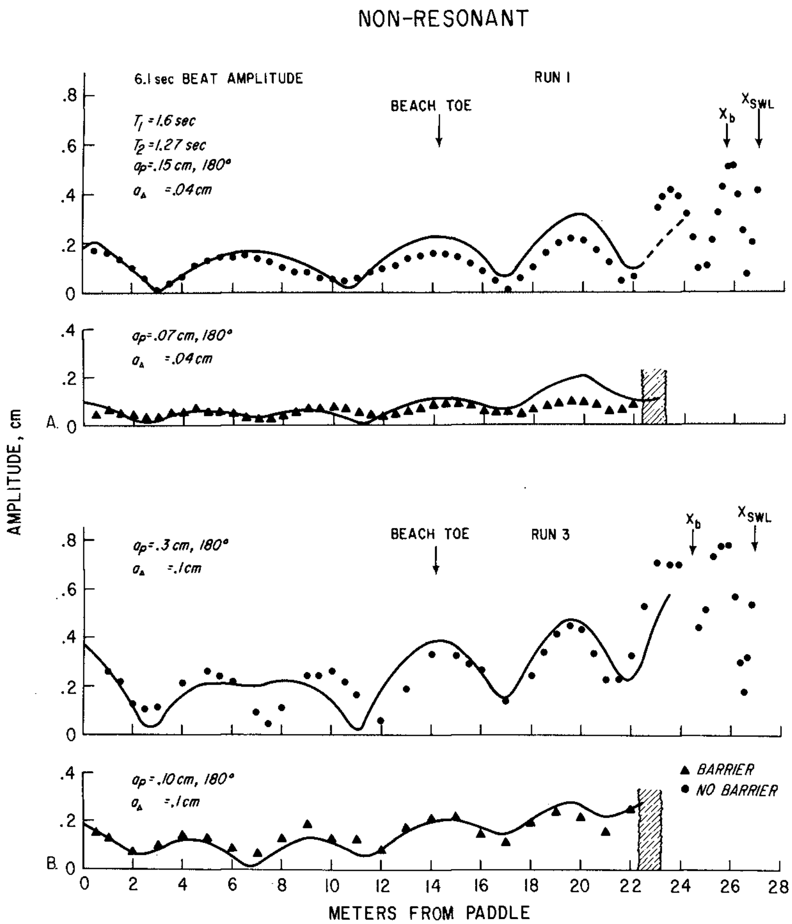

Figure 5. Detailed data of two non-resonant beat wave runs summarized in Table 1. Dots show beat frequency fluctuation without a barrier, triangles are taken with the barrier. Solid line is the best fit of an ad-hoc surf beat theory described in the text.

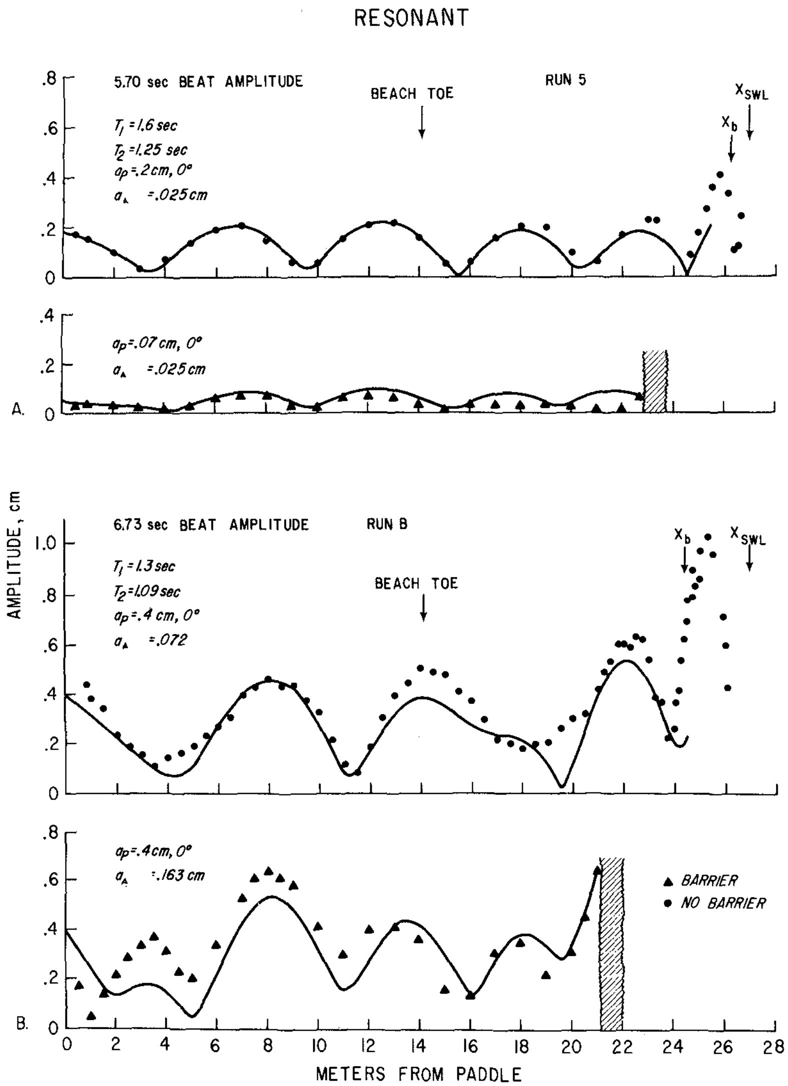

Figure 6. Same as Figure 5 for resonant beat waves.

run to correspond to the position found from earlier long wave measurements (Figure 4) not to affect the paddle generated beat frequency free wave of the same period. The purpose of the barrier was to eliminate the primary wave forcing and the surf zone and any associated long waves generated there. This scheme was supposed to leave only the set-down wave of size a_Δ and any free waves generated by the paddle. Further discussion of the effect of the barrier is given below.

Figures 5 and 6 show the results of four beat wave experiments in detail. Runs 1 and 3 (Figure 5) are non-resonant examples and 5 and 8 (Figure 6) are resonant. First, it is clear that the phase at the paddle corresponds more closely to an antinode than to a node in the non-resonant data both with and without the barrier. This is also true for the other non-resonant runs not shown in detail, but summarized in Table 1. Of course, there is also an antinode at the paddle for resonant runs, but this is expected and does not help distinguish the source of the free wave component.

The solid lines in Figures 5 and 6 represent the results of an <u>ad-hoc</u> surf beat theory consisting of the sum of two free standing waves and the set-down wave. One standing wave with amplitude a_B is given by equation (1) and has an antinode on the beach face. The second standing wave is similar in form, but has an antinode at the paddle and amplitude denoted a_p. The set-down wave is a forced, progressive correction and is given by Longuet-Higgins and Stewart (1962) in terms of the primary wave parameters. The amplitudes a_Δ, a_B and a_p have been adjusted to best fit the data and are listed for all runs in Table 1. The results show that a_B is negligible compared with a_p for all runs except run 2 where it is small. Therefore, only the progressive set-down wave and the free wave generated or reflected from shallow water seem to be important in the non-resonant runs 1-4.

The data shown are consistent with the hypotheses that the non-linearly generated set-down wave is reflected from the beach when there is no barrier, and from the barrier when it is present. In runs 1, 2, 3, 5 and 6 the standing wave amplitude without the barrier is always larger than with the barrier. For the same primary wave forcing, the amplitude of a_p is doubled by removing the barrier in both runs 1 and 5. In runs 3 and 6 removal of the barrier increased the response by about 50%.

The set-down wave amplitude increases sharply in shallow water. This fact accounts qualitatively for the increased standing wave amplitude in the absence of the barrier, since the beat wave is forced to larger amplitude before being reflected. In the presence of the barrier, the progressive wave is apparently reflected from the barrier, thus reaching relatively smaller amplitude.

The data suggest that the standing wave a_p is merely the result of reflection of the incoming wave a_Δ. No special enhancement was observed in the conditions tested. The absolute amplitude of the standing wave response depended on whether or not the frequency corresponded to a channel mode. The proportional increase in response caused by removing the barrier was the same for the resonant or non-resonant cases until saturation was reached. The saturation hypothesis advanced by Guza and Bowen (1976) is illustrated by the standing waves in runs 4, 7 and 8. Removal of the barrier caused no increase in response in runs 4 and B, and actually decreased response in run 7. In these cases, the reflection parameter c_r exceeds 2,

$$c_r = \frac{a_J \sigma^2}{g \tan^2 \beta}$$

where a_J is the standing wave amplitude at the beach face. This results in increased long wave dissipation in the shallow water region shoreward of the barrier (Guza and Bowen 1976). Run 4 corresponded to a non-resonant case, so that the free wave amplitude could not increase due to resonance with or without the barrier. Run 7, however, corresponded to a resonant case and the amplitude was saturated without the barrier. With the barrier, the dissipative region in shallow water was removed but the resonance condition was unaltered, so that the response was larger. The increase in primary amplitude while other conditions remain the same results in increased standing wave response until saturation is reached (see runs 1, 3, 5, 6 and 7).

In runs 6, 7 and 8 primary amplitudes seem to be larger in the presence of the barrier than without. Without contrary evidence, it seems reasonable to assume that the primary waves were partially reflected from the barrier in these runs and so were able to increase their amplitude. Why the primary waves in the analogous non-resonant experiments, runs 1, 3 and 4 were not so affected is not known.

Finally, Figure 7 shows a plot of the "forcing" a_Δ versus the beat wave response, a_p. The squares denote data with the barrier in place, the triangles are data taken without the barrier. Filled symbols were resonant runs, open symbols were non-resonant. The amplitudes plotted were chosen from the ad-hoc theory described above. When the barrier was in position, a_Δ at the barrier location was used. When the barrier was absent, the value of a_Δ at the breakpoint of the largest wave in the sets was used. The value of a_p is that on the flat portion of the channel. The data plotted in Figure 7 show that a line $a_p = a_\Delta$ separates resonant from non-resonant runs so that resonance occurs for $a_p \gtrsim a_\Delta$.

Conclusion

It has been shown that beat wave data in a 2-dimensional laboratory channel are consistent with the hypotheses that the set-down wave described by Longuet-Higgins and Stewart (1962) is reflected from the shore when the primary waves break. Surf beat experiments in wave channels are severely complicated by the fact that long waves reflect from the beach and from the paddle and thus exist as standing waves. This and the presence of free paddle generated waves make it difficult to distinguish the source of long waves.

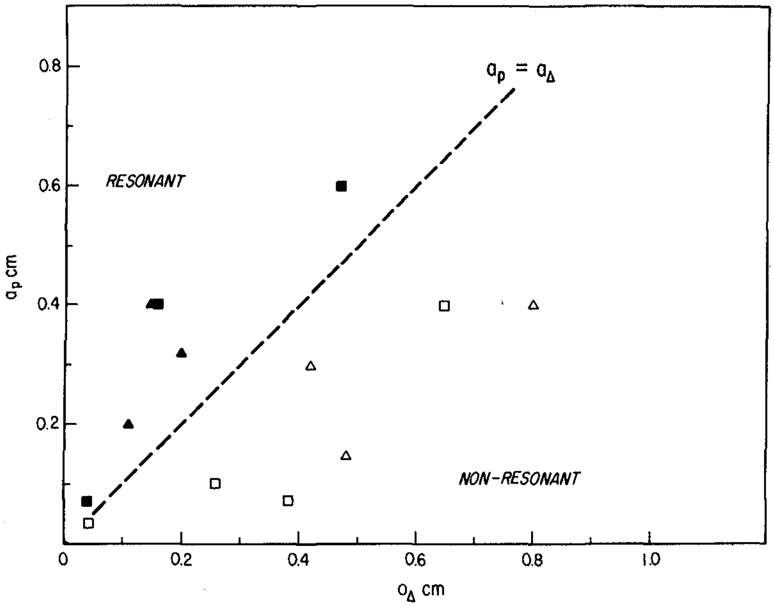

Figure 7. Standing beat wave "response" (a_p) as a function of set-down wave "forcing" (a_Δ). Amplitudes plotted are taken from ad-hoc surf beat theory as described in the text. Line $a_p = a_\Delta$ separates resonant from non-resonant runs.

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