AN ELECTROMAGNETIC ANALOGY FOR LONG WATER WAVES

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1. Introduction

Seiching must be considered when designing mooring, berthing or navigational facilities in semi enclosed basins and harbours. The considerable oscillatory currents which may be generated along nodal lines of a seiche can result in serious surging of a vessel moored in such areas. Hydraulic modelling of the long gravity waves which form the seiche poses particular problems for the hydraulic modeller. In the prototype situation, a balance is achieved between the long wave energy entering a basin, the energy dissipation within the basin, and the wave energy which is radiated back into the ocean. The presence of the wave generator in the model may result in much of the radiated energy being reflected back into the basin, thereby distorting the seiche observed in the model.

The relatively low Reynolds numbers present in the model leads to exaggerated frictional damping of the waves and possible suppression of certain resonant modes. Scaling of long wave amplitudes in resonance situations cannot be determined from simple Froude laws but must be based on equivalent dissipation rates in the model and the prototype (Ippen, 1966).

Alternative methods of physically reproducing long wave behaviour have been studied, such as acoustic modelling (Nakamura, 1977). However, these models also have their limitations and are largely confined to constant depth situations.

It is suggested that electromagnetic radiation in the microwave range may be used to model long gravity waves and that this method has many advantages over the various techniques which have previously been used. It can be rigorously shown that the laws which govern the propagation, reflection and dissipation of electromagnetic radiation (Maxwell's equations) are identical to the linearised equations describing the motion of long gravity waves, (Jackson and McKee, 1980). The linear wave equation gives an accurate description of long wave oscillation in basins as the wave length is of the same order as the basin dimensions and non-linear effects do not have time to develop.

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The microwave model consists of a flat box or cavity whose plan outline is geometrically similar to the boundaries of the harbour basin under investigation. The entrance to the basin is connected to a source of micro wave energy, a klystron, via a length of wave guide. As the klystron generates microwaves which propagate in all directions, the vertical dimension of the wave guide and the microwave cavity is made less than half a wave length of the microwave. This eliminates the vertical component of the microwave radiation. Microwaves in the frequency range from 1 to 10 G.Hertz are most suitable for these models and the corresponding wave lengths are 30 to 3 cm. Thus the model size is typically 10 to 20 cm in plan by 1 cm deep.

The properties of the wave field are determined by measuring the voltage induced on small antennae probes which are inserted into measurement ports in the cavity wall.

The unwanted secondary interactions between the seiche and the wave generator, which poses a major problem in hydraulic models, can be eliminated in the microwave model by installing an isolator between the energy source and the microwave cavity. The isolator will pass the incident wave but totally dissipate the reflected wave travelling in the opposite direction. Microwave models are not restricted to constant depth situations as are acoustic models. The parameters equivalent to water depth in the microwave model are the electric and magnetic permeabilities of the cavity interior. The former can be readily varied by using materials with different dielectric constants (air, perspex, teflon etc.) to represent different contour intervals in the model.

Microwaves are potentially a relatively inexpensive, convenient and accurate means of modelling long wave motions in semi enclosed basins and harbours. In this paper the performance of microwave analogue is examined for some simple geometries whose prototype behaviour is well known.

Formulation

It is well known that many similarities of behaviour exist between water waves and electromagnetic radiation. Penny and Price (1953) adopted the equations describing the diffraction of light to describe the diffraction of water waves around breakwaters. Both types of waves are known to obey similar laws of reflection and refraction.

At first sight Maxwell's equations, which describe the propagation and characteristics of electromagnetic radiation, bear little resemblance to the equations describing the motion of long gravity waves (compare columns (i) and (ii) respectively in Table 1). However, if a vector potential $\tilde{W}$ is defined as

$$\tilde{W} = - \hat{i}hv + \hat{j}hu$$

where $\hat{i}$ and $\hat{j}$ are the horizontal unit vectors in the x and y directions

u and v are the x and y velocity components

and h is the local depth

The conventional linearised long wave equations transform to a set which are identical to Maxwell's equations. The transformation is somewhat involved and is described in detail by Jackson and McKee (1980).

For a plane wave the vector potential $\tilde{W}$ has a magnitude which is equal to the instantaneous flow per unit wave front but has a direction which is orthogonal to the direction of wave propagation (i.e. along the crest).

Table 1 gives equivalent expressions for electromagnetic radiation and long gravity waves.

Table 1: Field Equations

(i) Maxwell's Equations for Electromagnetic Waves	(ii) Long Water Waves (constant depth) Conventional Form	(iii) In terms of the Vector Function	
Faraday's Law $\nabla \times \tilde{E} = -\frac{\partial \tilde{B}}{\partial t}$	Continuity Equation $\frac{1}{h}\frac{\partial \eta}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$	$\nabla \times \tilde{W} = -\frac{\partial \tilde{\xi}}{\partial t}$	(1)
$\nabla \times \tilde{B} = \varepsilon \mu_0 \frac{\partial E}{\partial t}$	Linearised Bernoulli Equation $\frac{\partial u}{\partial t} + g\frac{\partial \eta}{\partial x} = 0$ $\frac{\partial v}{\partial t} + g\frac{\partial \eta}{\partial y} = 0$	$\nabla \times \tilde{\xi} = \frac{1}{gh}\frac{\partial \tilde{W}}{\partial t}$	(2)
$\nabla . \tilde{E} = 0$	Irrotational Flow $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$	$\nabla . \tilde{W} = 0$	(3)
$\nabla . \tilde{B} = 0$	Horizontal Wave Field $\frac{\partial \eta}{\partial z} = 0$	$\nabla . \tilde{\xi} = 0$	(4)
where $\tilde{E}$ = electric field vector $\tilde{B}$ = magnetic flux density vector ε = dielectric constant μ_0 = permeability (constant)	where η = local elevation of water surface above the mean level h = mean water depth u and v = horizontal components of water particle velocity g = gravitational acceleration see Proudman (1953) page 310	where $\tilde{W} = -\hat{i}v + \hat{j}u$ $\tilde{\xi} = (0, 0, \eta)$	

Boundary Conditions

The tangential component of $\tilde{E}$ is continuous	: The tangential component of $\tilde{W}$ is continuous
The normal component of $\varepsilon \tilde{E}$ is continuous	: The normal component of $\frac{\tilde{W}}{h}$ is continuous
The tangential and normal components of $\tilde{B}$ are continuous	: The tangential and normal components of $\tilde{\xi}$ are continuous

The equivalent parameters in the two systems are

$\tilde{E}$ and $\tilde{W}$

$\tilde{B}$ and $\tilde{\xi}$

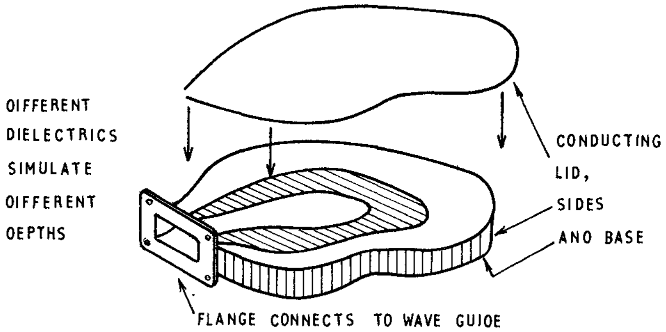
ε and $\frac{1}{h}$

μ_o and $\frac{1}{g}$

Furthermore frictional effects can be simulated by having the interior of the microwave cavity partially conducting. It can be shown that this has an effect equivalent to that of introducing a linearised friction term into Bernoulli's equation (Jackson and McKee, 1980)

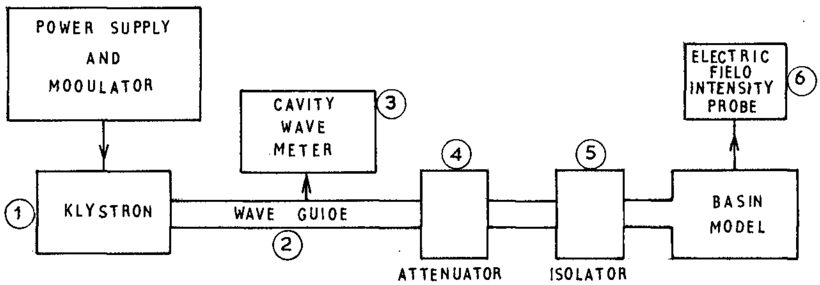
The Microwave Modelling Technique

Although in principle any frequency electromagnetic radiation could be used to model long gravity waves, radiation in the microwave range is of a convenient wave length for model construction. Since vector potential $\tilde{W}$ vanishes normal to a reflecting boundary its electromagnetic equivalent the electric field vector $\tilde{E}$ must be made to do the same. This is achieved if the boundary is electrically conducting so that the boundaries of the microwave cavity are constructed of highly conductive material such as copper sheet. Various materials such as air, glass and various plastics have dielectric constants which vary over a range of five or more and by using these materials between the various contour intervals, varied bottom topographies can be simulated by a series of step changes in depth as shown in Figure 1.



EXPLODED VIEW OF MICROWAVE CAVITY

FIGURE 1.



SCHEMATIC DIAGRAM OF THE MICROWAVE SET-UP

FIGURE 2.

Figure 2 shows a schematic diagram of typical microwave setup. It might be noted that similar equipment is readily available in most electrical engineering departments of tertiary institutions and the only irregular item would be the cavity model itself. The various items of equipment are briefly described.

1. The Microwave Source generates microwaves with a wavelength of 2.42 to 3.66 cm. Because of the high frequency of these waves (typically 10 GHz) it is impossible to detect them directly, hence they are modulated by a 1 KHz square wave.

2. The wave guide is simply a hollow metal tube of rectangular cross section which conveys the microwaves from the source to the model.

3. The cavity wave meter is a device for accurately determining the wave length of the microwaves using an adjustable resonator cavity. It is a passive device and has negligible effect on the microwave in the wave guide.

4. The attenuator reduces the microwave amplitude to any desired level.

5. The isolator permits microwaves to pass in one direction but dissipates the energy of microwaves travelling in the reverse direction. Therefore microwaves radiating back out the cavity are totally attenuated and do not affect readings within the harbour cavity. The major problem encountered with hydraulic models is thereby eliminated.

6. A probe, consisting of a fine needle approximately 0.3mm diameter and 3mm long, is inserted into the model. A voltage is induced onto the probe which is proportional to the intensity of the electric field at that location. This is equivalent to the instantaneous volume flux of the waves.

The microwave system used for the experiments described in this paper operated in the X band and generated microwaves with a frequency of 9 to 11 G.Hertz and a wave length of approximately 3 cm. This is too small for modelling complex basins but was adequate for the relatively simple geometries examined in the experiments described here. Systems are available for generating microwaves with wave lengths of up to 1m. However, a wave length of 30cm would be adequate for most model studies.

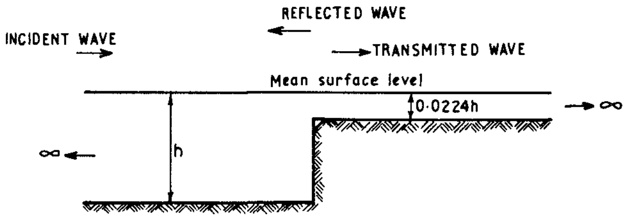
Two further points must be considered when designing a microwave model.

1. The microwave source is an essential point source of radiation; the waves do not propagate down the waveguide as plane waves but rather "bounce" from wall to wall. This has the effect of limiting the wave length that can be used with each size wave guide and the maximum wave length which will propagate down an X band wave guide which is 4.57 cm. (Lance 1964, p.87).

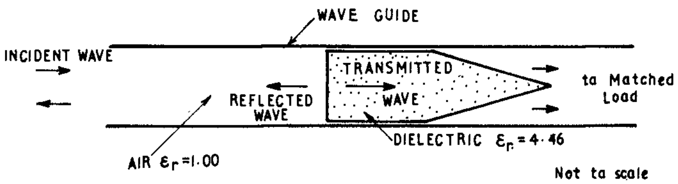
2. It is far simpler to measure the electric field intensity $\tilde{E}$ than the magnetic flux density $\tilde{B}$. The hydraulic equivalents are the vector potential $\tilde{W}$ and the water surface elevation vector $\tilde{\xi}$. However η can easily be calculated once $\tilde{W}$ and the wave period are determined.

Experiments

The interactions of long waves with two simple geometries were examined using the microwave analogy and compared with well known theoretical solutions for the same geometries.



HYDRAULIC BOUNDARY CONFIGURATION



MICROWAVE EQUIVALENT OF THE ABOVE GEOMETRY

FIGURE 3. – EXPERIMENT 1.

Experiment 1

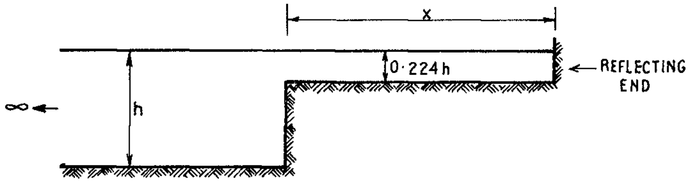
The reflection and transmission characteristics of a long wave normally incident to a step change in water depth were studied. The physical geometry and its microwave equivalent are shown in Figure 3. The water is assumed to be of infinite extent on either side of the step so that reflected waves are only generated at the step. The depth on the shallow side of the step was taken as 0.224 of the depth on the deeper side. Since this is basically a one dimensional problem it was modelled in a length of wave guide. The shallower region was simulated by inserting an acrylic plastic with a relative permitivity of $(0.224)^{-1} = 4.46$ (at 9.0 G.Hertz) into the wave guide, while air was the dielectric on the deeper side of the step. In order to avoid reflections from the end of the acrylic (which must be of finite length) it was tapered to a knife edge over a distance of several wave lengths.

The reflection coefficient for the reflected wave was found to be 0.357 compared with theoretical value of 0.365 (Lamb, 1932) while the transmission coefficient was found to be 1.34 compared with the theoretical value of 1.35. Wave heights determined from the microwave experiment are in close agreement with the theoretical values for long water waves.

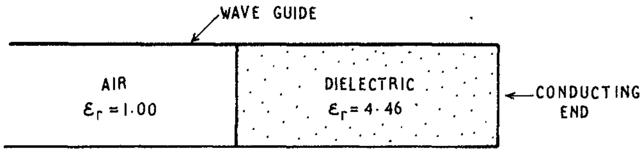
Experiment 2

This experiment was similar to the previous experiment except that the shallow region was truncated at a vertical reflective wall as shown in Figure 4. In this case a resonant condition is possible on the shelf if its length is an odd multiple of the quarter wave length of waves travelling in the shallower depth.

The microwave setup is similar to that shown previously except that the shelf region, which is again simulated by the acrylic dielectric is terminated by a conducting end which is normal to the wave guide. A probe was inserted adjacent to the step on the shallow side, and wave heights at the reflecting end wall were determined from measurements of the field intensity (equivalent to the volume flow) at the step. The wave height at the end wall (H_{max}) expressed as a fraction of the incident wave height (H_o) is shown in Figure 5 for a range of wave lengths (L) normalised with respect to the length of the shelf (X). Resonance occurs when $\frac{X}{L} = \frac{1}{4}$ and again agreement between the theoretical wave heights and those deduced from the microwave experiment are in close agreement. Near resonance dissipative effects become significant both in the prototype and the model and the data points depart from the non-dissipative solution which is graphed in Figure 5.

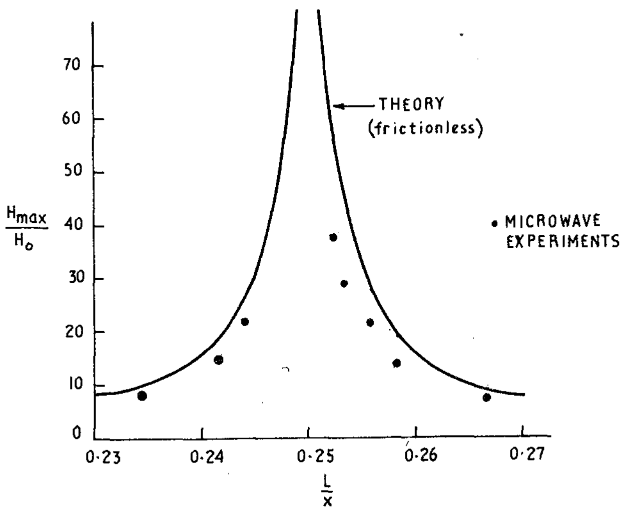


HYDRAULIC BOUNDARY CONFIGURATION



MICROWAVE EQUIVALENT OF THE ABOVE GEOMETRY

FIGURE 4. – EXPERIMENT 2.



WAVE HEIGHT AS A FUNCTION OF WAVE LENGTH FOR THE GEOMETRY SHOWN IN FIGURE 4

FIGURE 5.

Conclusions

Microwave cavities promise to be a relatively simple and effective means of studying long water waves in semi enclosed basins. Many of the inaccuracies invariably present in conventional hydraulic models can be eliminated in a microwave analogue. The vector potential (the equivalent of the instantaneous wave induced flow) is the most readily measured parameter in a microwave study and in many cases it is this parameter which is of most interest in the prototype. The analogy has been verified for some simple one dimensional geometries, whose behaviour is well known, and the work is being extended to look at more realistic geometries.

If a partially conducting dielectric is used in the model, some microwave energy is dissipated and the effect is exactly equivalent to that of introducing a linear friction term into the equation of motion for the water waves. It should therefore be possible to incorporate frictional effects into the microwave model.

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