LONGSHORE WATER AND SEDIMENT MOVEMENT

by

D.H. Swart* and C.A. Fleming**

ABSTRACT

The mean sediment transport rate obtained by using six known longshore transport formulae, for which the input variables are determined consistently, is used as best estimate of the transport. A good comparison is obtained when this package deal approach is compared with a prototype situation where the transport rates are inferred from quarterly bathymetric surveys over two years. The accuracy of the input variables is reviewed theoretically and the results are used to perform a sensitivity analysis.

1. INTRODUCTION

The Inman/Bagnold/Komar formula (referred to below as the SPM formula) for the prediction of total longshore transport rates is suggested in the Shore Protection Manual (SPM, 1973) for use in the coastal environment. In the design of large coastal structures, such as the breakwaters for a harbour which can accommodate oil tankers and ore carriers, the length of the breakwaters is determined largely by navigational requirements and the longshore transport is important only for the determination of the method of sediment by-passing, if that should be required. In such instances the output as given by the SPM formula, that is, a total annual transport rate without any indication about the area within which this transport takes place, is mostly sufficient. However, the present-day trend is for more and more smaller coastal structures to be built, such as small fishing harbours or small-craft harbours. Navigational requirements only demand a water depth of, say, 7 m at the entrance to such harbours. This depth could quite easily fall within the active coastal transport zone, which implies that the sediment transport determines the entrance depth to the harbour. Therefore is is important to have both the longshore transport rate, as predicted by the SPM formula, and the distribution normal to shore of this transport. For this reason it is imperative that the longshore transport distribution normal to shore is predicted in terms of the local hydraulic parameters. Such formulae, which were called detail predictors by Swart (1976a), use the wave characteristics, hydraulic bed roughness and local longshore current velocity to predict instantaneous longshore transport rates. The accuracy of predicted sediment transport rates for given hydraulic input parameters is, even under steady-flow conditions, strongly dependent on the accuracy of the input variables. Even a small inaccuracy in the input velocity can, for example, cause substantial errors in the predicted transport rates. The input variables used for the prediction of longshore transport rates are usually also predicted, either because of difficulties in measuring the required parameters in the coastal environment, or because of the impracticability of measuring over a long period of time, say one year, to deduce seasonal trends from the input data. If the techniques used to predict the input variables, which are in

* Coastal Engineering and Hydraulics Division, National Research Institute for Oceanology, Stellenbosch, South Africa.

** Sir William Halcrow and Partners, Consulting Engineers, Swindon, U.K.

turn to be used to predict sediment transport rates, are not quite accurate, the computed longshore transport rates could be in error by an appreciable margin. For this reason it seems a sound practice to use as many reliable predictors for the longshore transport rate as are available, and to infer the best estimate of the longshore transport rate from the results of all these formulae. In addition, it is imperative to always use the same method of predicting the input variables, to eliminate the possibility of differences in predicted transport rates because of the use of different techniques to predict the input variables. This paper describes the elements of such a *'package deal'* approach. The following *input variables* are required to allow the application of the package deal approach.

(1) *Wave data* are needed in either intermediate or deep water over the period for which transport rates are required (mostly one year to allow an evaluation of seasonal effects), that is,

- significant wave height $(H_s)_I$
- peak energy wave period T_p
- angle of incidence Θ_I
- fraction of the time that every wave condition occurs f

Subscript I signifies 'input'.

(2) The following details of the *bed topography* are required:

- the topography of the whole area to allow the construction of wave refraction diagrams;
- beach profiles at the locations where transport computations are to be performed, to decide on the depths d_i in which transport rates are needed and to find the representative widths of the transport zones thus defined;
- the mean bed slope in the breaker zone at the locations where longshore current velocities are to be calculated.

(3) The *characteristics of the sediment* (bed material) influence the longshore sediment transport rates, namely,

- the relative apparent density $\Delta_s = (\rho_s - \rho_w)/\rho_w$ where ρ = mass density, subscript s refers to 'sediment' and subscript w to 'water';
- the distribution of grain sizes; D_{10}, D_{16}, D_{35}, D_{50}, D_{65}, D_{90}.

With the aid of the above input variables a comprehensive set of output variables is produced.

(1) The standard output of a *refraction study* is obtained, namely,

- significant wave height H_s
- wave length λ
- angle of incidence Θ

in all water depths d_i, as chosen above in the area between the water line and a depth equal to four times the maximum depth in which wave breaking can occur. This area is referred to below as the *'test area'*.

(2) A wide range of *wave characteristics* is calculated for water depths d_i in the test area. Included are, for example, the horizontal orbital velocity at the bed u_{bc} and the orbital amplitude a_o at the bed. All computations regarding wave characteristics are at present performed with linear wave theory.

(3) *Bed-form characteristics* are computed at depths d_i in the test area by using the technique outlined by Swart (1976a) and are then used to compute the following:

- the hydraulic bed roughness $r = 25\Delta_r^2/\lambda_r$, see Swart (1976a) where Δ_r and λ_r are the bed-form height and length respectively;
- the wave friction factor f_w, found empirically by Swart and Loubser (1980) from 640 data sets for turbulent flow in the boundary layer, namely,

For $a_o/r < 7$

$$f_w = \frac{0.30}{1+0.28(\frac{a_o}{r})^{1.1}}$$

For $7 \leq a_o/r \leq 160$

$$f_w = 0.0066 + 0.483(\frac{a_o}{r})^{-0.91}$$

For $160 < a_o/r$

$$f_w = 0.0146(\frac{a_o}{r})^{-0.157} + 0.483(\frac{a_o}{r})^{-0.91} \qquad \ldots (1.1)$$

- the chezy roughness coefficient C_h, and
- the friction factor C_{LH} as used by Longuet-Higgins (1970) in his theory for the prediction of longshore current velocity.

(4) *Wave-induced properties* which are also *functions of the bed roughness* r are computed at depths d_i in the test area, namely,

- bed shear stresses, and
- wave power.

(5) *Longshore current velocities* v_i are computed for depths d_i in the test area, with the technique outlined in Section 2.

(6) At the same depths *longshore transport rates* are computed for five different detail longshore transport formulae, namely, the formulae of

- Bÿker (1971)
- Fleming (1976, 1977)
- adapted Engelund-Hansen (Swart, 1976b)
- Nielsen (1978, 1979)
- adapted Ackers-White (Swart and Lenhoff, to be published in 1980).

In addition the total longshore transport, as predicted by the SPM formula, is also obtained.

(7) *Statistical properties* of the computed transport rates are calculated, namely,

- the mean total transport μ obtained from the six formulae listed under (6), as well as the standard deviation σ around the mean,
- the mean transport/unit width normal to shore obtained from the five detail predictors listed under (6), as well as the standard deviation around the mean.

Similar means and standard deviations are also calculated for the gross transport and the total upcoast and downcoast transport.

The individual items of the package deal approach outlined above have mostly been reported on by various researchers in literature and are therefore not repeated in detail in this paper. The only exceptions are the prediction of longshore current and the adaptation of the Ackers-White formula for the prediction of sediment transport under steady-state flow conditions. These two aspects are therefore discussed in more detail in Sections 2 and 3 below before the package deal approach is applied to a prototype example.

2. LONGSHORE CURRENT VELOCITY

The momentum balance in the longshore direction determines the strength and distribution of the longshore current, that is

$$\frac{\partial R_{yx}}{\partial y} - (B_x + \frac{\partial D_y}{\partial y}) = 0 \qquad \ldots(2.1)$$

where the x- and y-axes are assumed to be parallel and normal to the (straight) coastline, R_{yx} is the flux of longshore momentum across a line parallel to the shoreline, B_x is the bed shear in the direction of the longshore current and $(\partial D/\partial y)$ is the exchange of momentum due to horizontal turbulent eddies. The radiation stress term represents the driving force whereas the bed shear and the lateral mixing are the dissipative forces.

Longuet-Higgins (1970) formulated equation (2.1) and found a solution to this equation for regular waves breaking as spilling breakers on a gently sloping beach. The lateral mixing, which is the key factor in the determination of the shape of the velocity profile, that is, in the distribution of velocity across the breaker zone and beyond the breaker line, is difficult to determine quantitatively. Longuet-Higgins assumed the lateral mixing to increase with the distance offshore to the power 1,5, even outside the breaker zone. The lateral mixing depends to some degree on the rate of change of wave energy dissipation as the wave approaches the shore, that is, on the type of breaker that occurs, or stated differently, on the shape of the velocity profile itself. The area within which the wave energy is dissipated is greater in the case of irregular waves with a spectrum of wave heights (extends further offshore) than is the case for regular waves and in addition the local rate of energy dissipation is also lower for irregular waves than for regular waves. It therefore seems probable that the effect of lateral mixing on the velocity profile will be less in the case of irregular waves. Furthermore it is truer to nature since, although the assumption of spilling breakers will still be required, a spectrum of wave heights is assumed. With this in mind Swart (CSIR, 1978) developed an explicit formula for the prediction of longshore current which is valid for long-crested irregular waves. The main points of this derivation are outlined below.

The following *assumptions* are made:

(1) Linear wave theory is used for all wave properties.
(2) Random long-crested waves with a Rayleigh distribution of wave heights are assumed.
(3) All waves in the spectrum break as spilling breakers with a constant breaker index γ = ratio of wave height at breaking to mean water depth.
(4) Each wave is assumed to retain a height equal to γ times the local water depth as it approaches the shoreline, that is, its wave height decreases at the same rate as the water depth in the breaker zone. In this manner a truncated wave height spectrum is formed, as assumed by Battjes (1974) in his treatment of random breaking waves.

(5) Wave set-up is neglected.
(6) The bed slope α in the breaker zone is assumed to be constant.
(7) The friction coefficient is assumed constant over the breaker zone.
(8) Lateral mixing is neglected.

Except for assumption (8), the other assumptions are the same as those made by Longuet-Higgins (1970). The resulting form of the three components in the momentum balance equation, based on these assumptions, is given below.

Radiation stress term ($\partial R_{yx}/\partial y$)

Longuet-Higgins (1971) showed that the radiation stress R_{yx} is given to the second order of approximation by

$$R_{yx} = En \sin \Theta \cos \Theta \qquad \ldots(2.2)$$

where E is the wave energy per unit surface area, n is the ratio group velocity to wave celerity and Θ is the angle of wave incidence. Battjes (1974) showed that the wave energy in a breaking wave spectrum is reduced to a fraction q_b of the value it would have attained if shoaling and refraction had taken place uninterrupted, that is, if no breaking had taken place, when a fraction $(1-q_b)$ of the waves are breaking. It follows from the definition of the Rayleigh distribution that

$$q_b = 1 - \exp(-H_b^2/H_{frms}^2) \qquad \ldots(2.3)$$

where H_b is the local breaker height ($= \gamma d$) and H_{frms} is the root-mean-square wave height that would have been attained in the absence of wave breaking. Since the wave height cannot exceed a value (γd) this wave height is fictitious.

It therefore follows from equation (2.2) that the radiation stress R_{yx} is reduced to a fraction q_b of its fictitious value R_{yxf} in the absence of wave breaking, that is,

$$R_{yx} = q_b R_{yxf} \qquad \ldots(2.4)$$

where subscript f refers to 'fictitious'.
Since it is assumed that the fictitious waves do not break, R_{yxf} is independent of the distance offshore. Therefore

$$\frac{\partial R_{yx}}{\partial y} = R_{yxf} \frac{\partial q_b}{\partial y} \qquad \ldots(2.5)$$

With the aid of equations (2.2) to (2.5) it can be shown an analogous manner to Longuet-Higgins' derivation for regular waves that

$$\frac{\partial R_{yx}}{\partial y} = \frac{5}{16} \rho\gamma^2 gd \exp\left(\frac{-\gamma^2 d^2}{H_{frms}^2}\right) \sin \Theta \cos \Theta \tan \alpha \qquad \ldots(2.6)$$

Bed shear term B_x

Longuet-Higgins (1970) showed that, provided that the longshore transport velocity v is much less than the maximum horizontal orbital velocity u_{bc} at the bed, the bed shear B_x can be approximated by

$$B_x = \frac{2}{\pi} C_{LH} \rho u_{bc} v \qquad \ldots(2.7)$$

For shallow-water waves the orbital velocity u_{bc} reduces to

$$u_{bc} = \frac{1}{2} \left(\frac{\overline{H}}{d}\right) (gd)^{\frac{1}{2}} \qquad \text{...(2.8)}$$

and therefore $B_x = \frac{1}{\pi} \rho \, C_{LH} \left(\frac{\overline{H}}{d}\right) (gd)^{\frac{1}{2}} \, v$ ...(2.9)

where $\overline{H}$ is the mean wave height of the breaking wave spectrum in a water depth of d.

Solution

Since the lateral mixing is neglected, it follows from equations (2.1), (2.6) and (2.9) that the longshore current velocity v equals:

$$v = \frac{5\pi}{16} \frac{\gamma^2 (gd)^{\frac{1}{2}}}{C_{LH}} \left(\frac{\overline{H}}{d}\right)^{-1} \exp \left\{\frac{-\gamma^2 d^2}{H_{frms}^2}\right\} \sin \Theta \cos \Theta \tan \alpha \qquad \text{...(2.10)}$$

An example of the velocity profile as predicted by equation (2.10) is given in Figure 1. Both axes are non-dimensionalized, the vertical axis by dividing by a scaling velocity v_{or} and the horizontal axis by dividing by the distance from the shoreline to the location in the profile where the significant wave height will start breaking, that is, where 13.5 per cent of the waves in the spectrum will have started to break. Also shown in this figure is the solution for regular waves of Longuet-Higgins, applied with a lateral mixing which can be shown to be the average of the possible mixings. The agreement between the two solutions is very good. This indicates that the assumption to neglect lateral mixing is justified for irregular waves and therefore equation (2.10) is used for the prediction of longshore current velocity in the package deal approach.

Longshore velocity friction factor C_{LH}

The magnitude of the friction factor C_{LH} must be known before equation (2.10) can be used to determine longshore current velocities. Galvin and Nelson (1967) compiled all available longshore current data. Each of these data sets can be used to determine a value for C_{LH}. Longuet-Higgins (1970) evaluated this information and concluded that C_{LH} is 'of order 0.01'. It is suggested in the Shore Protection Manual (SPM, 1973) that C_{LH} is actually less than 0.01. A value of C_{LH} = 0.0071 is suggested as the best estimate of C_{LH}. Using the same data, Komar and Inman (1970) found that

$$C_{LH} = 0.15 \tan \alpha \qquad \text{...(2.11)}$$

Swart suggested that the friction factor should also be a function of the bed roughness r (CSIR, 1978). He showed theoretically that

$$C_{LH} = \Phi \left(\frac{f_{wb} g}{2C_{hb}^2}\right)^{\frac{1}{2}} \qquad \text{...(2.12)}$$

where Φ is an unknown coefficient and subscript b refers to the significant breaker line.

Galvin and Nelson's data were then used to find a value for Φ, namely,

$$\Phi = 25 (\tan \alpha)^{0.85} \qquad \text{...(2.13)}$$

that is,

$$C_{LH} = 25 \left(\frac{f_{wb} g}{2C_{hb}^2}\right)^{\frac{1}{2}} (\tan \alpha)^{0.85} \qquad \text{...(2.14)}$$

Galvin and Nelson's data are plotted in Figure 2 for each of the above suggestions for the friction factor C_{LH}. From the four one-to-one plots of velocity listed in Galvin and Nelson's paper to predicted longshore current velocity it is quite apparent that the last alternative for C_{LH}, that is, equation (2.14), as given by Swart and Fleming (in CSIR, 1978) yields by far the best agreement between data and prediction, followed by Komar and Inman's equation (equation (2.11)). The value of C_{LH} as predicted from equation (2.14) is therefore used in the package deal approach.

3. PREDICTION OF LONGSHORE TRANSPORT RATES

As stated in Section 1, there are two types of formulae for the prediction of longshore transport rates, namely, overall predictors and detail predictors. Of the one overall predictor and five detail predictors listed in Section 1 only the adapted Ackers-White formula (Swart and Lenhoff, to be published in 1980) has not been reported on in the international literature. Therefore only a short summary is given of the first five transport equations, whereafter the adapted Ackers-White formula is discussed in more detail.

SPM formula (SPM, 1973; Swart, 1976a)
This formula is based on an empirical relationship between the longshore component of the energy flux due to wave action and the total longshore transport rate. No distribution is given of transport across the breaker zone. The formula for the total transport rate for each wave condition i is:

$$S_{SPM} = K(D)\ f_i\ T_p\ H_{os}^2\ K_{rb}^2\ \sin 2\Theta_b \quad \text{(in m}^3\text{/year)} \qquad \ldots(3.1)$$

where H_{os} is the deep-water significant wave height, K_{rb} is the refraction coefficient at the significant breaker line and Θ_b is the angle of incidence at the significant breaker line. Swart (1976a) showed that the proportionality factor K(D) is not a constant but is a function of the median grain size D_{50} of the bed material, namely,

$$K(D) = 91 \times 10^4 \log_{10} \left\{\frac{0.00146}{D_{50}}\right\} \qquad \ldots(3.2)$$

Bÿker formula (Bÿker, 1971)
Bÿker's formula, developed in 1967, was the first detail predictor and at the time constituted a major breakthrough in the prediction of longshore transport rates. It is built up of two components, namely, a bed load component and a suspended load component. The bed load formula was adapted from the Frÿlink formula for sediment transport rates under riverine conditions by adapting the shear stress terms:

$$S_{bB} = 5\ D_{50}\ \frac{v}{C_h}\ g^{\frac{1}{2}}\ \exp\left\{\frac{-0.27\ \Delta_s\ g\ D_{50}}{\mu(\tau_{wc}/\rho)}\right\} \quad \text{(in m}^3\text{/m/s)} \qquad \ldots(3.3)$$

where μ is a ripple factor defined by Bÿker and the bed shear stress due to waves and current τ_{wc} is:

$$\tau_{wc} = \rho\ \frac{v^2 g}{C_h^2}\ (1 + \tfrac{1}{2}\ (\frac{\xi u_o}{v})^2 \qquad \ldots(3.4)$$

Swart (1976b) subsequently showed that $\xi = C_h\ (f_w/(2g))^{\frac{1}{2}}$. The suspended load was found from the Rouse-Einstein distribution of suspended material from bed to free surface and integrating the product instantaneous velocity times concentration of bed material with depth, resulting in:

$$S_{sB} = 1.83\ S_{bB}\ \{I_1\ \ell n\ (\frac{33d}{r}) + I_2\} \qquad \ldots(3.5)$$

where I_1 and I_2 are elliptic integrals, for which Bÿker determined quantitative values by numerical integration. The total transport per metre in any given water depth d_i is then

$$S_B = S_{bB} + S_{sB} \qquad \text{(in } m^3/m/s) \qquad \ldots(3.6)$$

Although the formula contains no incipient motion criterion, the transport rates predicted for velocities below the threshold velocity are very low.

Fleming formula (Fleming and Hunt, 1976; Fleming, 1977)
Fleming developed a transport formula by which the total load (bed plus suspended load) for wave action can be predicted. He defined a reference concentration C_e close to the bed (a small distance e above the bed). He used the force balance of bed particles to derive a theoretical expression for C_e, which contains an incipient motion criterion. Fleming assumed that the concentration at the bed cannot exceed 0.52 and that the eddy diffusivity is constant over the whole water mass above the elevation z = e. A simple one-seventh power rule was assumed for the variation with distance from the bed of the longshore current velocity. The following equations must be integrated numerically to find the total longshore transport at any water depth d_i:

For $0 \leq z \leq e$

$$C(z) = 0.52\ (\frac{C_e}{0.52})^{(z/e)} \qquad \ldots(3.7)$$

$$v(z) = \frac{8}{7}\ v\ (\frac{z}{d_i})^{1/7} \qquad \ldots(3.8)$$

where z is the distance above the bed.

For $e < z \leq d_i$

$$C(z) = C_e\ \exp\ \{J_m\ (1-(^z/_e)^{0.75})\} \qquad \ldots(3.9)$$

The velocity v(z) is again determined from equation (3.8). Fleming presented equations for the finding of J_m and e. The resulting transport is given in $m^3/m/s$.

Adapted Engelund-Hansen formula (Swart, 1976b)
Swart adapted the original formula by Engelund and Hansen (1967) for the prediction of total sediment transport rates in any depth d_i under steady flow conditions in an analogous manner to the Bÿker formula. The resulting equation is:

$$S_{EH} = \frac{0.05\ vC_h\ (\tau_{wc}/\rho)^2}{g^{5/2}\ \Delta_s^2\ D_{50}} \qquad \text{(in } m^3/m/s) \qquad \ldots(3.10)$$

Obviously, equation (3.10) contains no incipient motion criterion. For longshore current velocities below the threshold velocity the formula predicts longshore transport rates which are higher than those predicted by the Bÿker formula.

Nielsen formula (Nielsen et al., 1978)
Nielsen determined the distribution of suspended sediment with distance from the bed for breaking (spilling breakers) and non-breaking wave conditions in the laboratory. He used the data to determine empirically quantitative predictors for the eddy diffusitivity ε, which he found is constant with distance from the bed for non-breaking waves and increases strongly with distance from the bed for breaking waves, and the concentration of suspended material at the top of the bed forms C_o, which contains an incipient motion criterion. The effect of the spilling breakers was to increase the eddy diffusitivity at the bed by two orders of magnitude. The variation of concentration of suspended material with distance from the bed for a nonuniform material is given by

$$C(z) = C_o \left(\frac{1}{1+\sigma V}\right)^{1/V} \qquad \ldots(3.11)$$

where $V = \text{var}(w)/w_{50}^2$, with w = fall velocity of bed material; and σ = parameter including the eddy diffusitivity. In the package deal approach the product of the concentration in equation (3.11) and the velocity in equation (3.8) is integrated numerically with respect to z to find the total load in any given depth d_i. The appropriate value of the eddy diffusitivity (with or without wave breaking) is used inside and outside the breaker line.

Adapted Ackers-White formula (Swart and Lenhoff, to be published, 1980)
The original formula for the prediction of sediment transport rates under steady-state conditions was as follows (Ackers and White, 1971):

$$S = 1.45\, v\, D_{35}\, C \underbrace{\left(\frac{P_{fg}/\rho}{v_{*fg}^3}\right)^n \left(\frac{P_{cg}/\rho}{v_{*cg}^2 v_r}\right)^{1-n}}_{\text{efficiency term}} \left(\frac{F_{gr}}{A} - 1\right)^m \qquad \ldots(3.12)$$

where C, n, m and A are grain size dependent parameters, for which empirical relationships are given by Ackers and White; F_{gr} is the sediment mobility = $(v_{*fg}^n\, v_{*cg}^{1-n})/(\Delta_s g D_{35})^{\frac{1}{2}}$; P is the stream power; v_r the resultant velocity (= v for steady state) and v_* is the shear velocity = $(\tau/\rho)^{\frac{1}{2}}$; subscripts 'fg' and 'cg' respectively denote the 'fine grain' and 'coarse grain' versions of the properties. The value for fine-grained sediment is obtained by using the actual bed roughness r, as described before, whereas the value for coarse-grained sediment is obtained by using the grain size D_{35} instead of r in the appropriate equations. For steady-state conditions the efficiency term reduces to $(v/v_{*fg})^n$. At present there are four adapted versions of equation (3.12) which are used in coastal engineering application, each containing different assumptions.

(1) SWANBY-version (Swart, 1976b)

Only the fine grain component of the shear terms was adapted to include the effect of wave action, which has now been found to be incorrect. Both the fine grain and the coarse grain components should be adapted.

(2) Willis-version (Willis, 1978)

Willis concluded that the critical (incipient motion) value A of the mobility number is different for combined wave and current action from the value for current action alone. To compensate for this difference he multiplied the wave-induced shear stresses by an empirical coefficient w_c^2, where w_c^2 is a function of the grain size. Willis adapted both the fine grain and the coarse grain components of the shear stress to include the effect of wave action, but he used an erroneous equation for wave power {$c_g \tau_{wc}$ (where c_g = group velocity of the waves) and the power was treated as a scalar instead of finding the mean value of the product (instantaneous resultant velocity at the bed times instantaneous bed shear stress)}, which yielded too high values for the power. The effect of this mistake on the predicted transport rates is masked because w_c^2 was computed by using the equation for longshore sediment transport rate with the erroneous expression for the wave power.

(3) V.d. Graaff and V. Overeem-version (V.d. Graaff and V. Overeem, 1979)

V.d. Graaff and V. Overeem added the effect of waves on the shear stresses to both the fine- and coarse-grain components, but used the same critical mobility number as for steady-state conditions and also used the steady-state version of the efficiency term instead of the actual wave power.

(4) Swart and Lenhoff-version (to be published, 1980)

All three versions discussed above are therefore erroneous in one way or another. Swart and Lenhoff therefore defined a fourth version, in which all previous shortcomings are eliminated. Three points are important.

a. The *instantaneous sediment mobility* $F_{gr}(t)$ for waves and currents is given by:

$$F_{gf}(t) = \frac{v_{*fg}(t)^n \; v_{*cg}(t)^{1-n}}{(\Delta_s \; g \; D_{35})^{\frac{1}{2}}} \qquad \ldots(3.13)$$

The 't' denotes time variation.

Instead of adapting the fine and coarse grain components of the shear stress individually by integrating each separately with respect to time, it is more logical to compute the average effect of the inclusion of waves on the mean mobility number by integrating the instantaneous mobility number with respect to time, that is

$$F_{wc} = \overline{F_{gr}(t)} = \frac{1}{T} \int_0^T F_{gr}(t) \; dt \qquad \ldots(3.14)$$

where F_{wc} is the mean sediment mobility for combined current and wave action.

b. Similarly, the *instantaneous* value $E_f(t)$ of the *efficiency term*, as given in equation (3.12), is averaged to obtain the mean value E_{fwc} of the efficiency term for combined current and wave action, namely,

$$E_{fwc} = \overline{E_f(t)} = \frac{1}{T} \int_0^T E_f(t) \; dt \qquad \ldots(3.15)$$

$$\text{where } E_f(t) = \left(\frac{P_{fg}(t)/\rho}{v_{*fg}(t)^3}\right)^n \left(\frac{P_{cg}(t)/\rho}{v_{*cg}(t)^2 \; v_r(t)}\right)^{1-n} \qquad \ldots(3.16)$$

$$P(t) = v_r(t) \; \tau(t) \qquad \ldots(3.17)$$

Values for the instantaneous resultant velocity at the bed $v_r(t)$ and the instantaneous shear stress at the bed $\tau(t)$ are found by vector addition of the contributions by the waves and the currents.

When applying the formula for sediment transport rates (equation (3.12)), values for F_{wc} and E_{fwc} are found by numerical integration.

c. A new empirical relationship for the *critical mobility number A* was determined from more than 800 data sets of four different types, namely:

- observed incipient motion data on a flat bed for waves only;
- observed incipient motion data on a rippled bed for waves only;
- observed incipient motion data on a flat bed for combined current and wave action;
- sediment load data over rippled beds for waves only as well as combined wave and current action.

In the case of the first three types the mobility number could be determined directly from the data. For the fourth type, however, all variables in equation (3.12) were known, except the critical mobility A, which could then be easily computed. The results are presented in Figure 3 in the form of a Reynolds number R_* (= $(\overline{\tau}_{cr}/\rho)^{\frac{1}{2}} D_{50}/v$, where $\overline{\tau}_{cr}$ is the mean shear stress at incipient motion and v is the kinematic viscosity) versus a dimensionless grain size D_* (= $(g\Delta_s/v^2)^{1/3} D_{50}$). This figure indicates that all four types of data follow the same relationship between R_* and D_*, namely,

$$\log_{10}(R_*) = 0.092\,(\log_{10} D_*)^2 + 1.158 \log_{10} D_* - 0.367 \qquad \ldots(3.18)$$

The critical mobility is now simply

$$A = R_* D_*^{-1\frac{1}{2}} \qquad \ldots(3.19)$$

This implies that equation (3.12) is in fact universally applicable to all types of flow, namely, to waves only, current only and combined waves and currents. Since the incipient motion data and sediment load data are completely independent and have been treated completely differently, it also implies that this fourth adapted version of the original Ackers-White formula is indeed correct. It is consequently also the only adapted version of Ackers-White which is included in the package deal approach.

4. PROTOTYPE APPLICATION

An example is now given to illustrate the results obtained with the package deal approach for a prototype application. The input variables are summarized in Figure 4. It can be seen that the location under consideration is situated on a high-energy coastline. Quarterly bathymetric surveys over a two-year period indicated that the beach under consideration was in overall equilibrium (see Figure 5) although a substantial seasonal variation took place in the total volume of sediment in the control area. Fleming (1976) showed that the offshore-directed transport through the seaward boundary of the control area was negligible and that the volumetric changes depicted in Figure 5 are associated with a distinct sloshing of the material in the bay. During the summer months with persistent southerly winds and wave conditions, accretion took place in the northern half of the control area and (less) erosion occurred in the southern half of the control area. The opposite process took place during the winter months when northerly winds and wave conditions are more predominant. The data therefore indicate a clear wave-driven sloshing in the bay, with a corresponding average transport residual per year maintained over two years of $1.45 \times 10^6 m^3/yr$ (see Figure 5). The results obtained from the package deal approach are summarized in Figure 6. The data indicate that the volume of material in the control area will increase during periods of northbound transport and will decrease during periods of southbound transport. This result agrees with the prototype data. The average value of the difference in transport capacity at the up- and down-coast boundaries of the control area is predicted to be $1.65 \times 10^6 m^3/yr$ plus or minus about 20 per cent of this mean value, which agrees well with the observed value of $1.45 \times 10^6 m^3/yr$. Furthermore, the theoretical results also indicate that the volume of material in the control area will return approximately to its initial value after each year ($+ 1.7 \times 10^6 - 1.6 \times 10^6$), which also agrees with the prototype data. Since the wave climate has a marked seasonal component, the package deal approach therefore indicates a wave-driven sloshing mode of the same magnitude as observed in prototype. Figure 7 indicates that the prediction of the distribution of longshore transport with distance offshore is very consistent for the five detail predictors. The 95 per cent confidence band of the mean predicted longshore transport rate is approximately twice the standard deviation, which means that it is possible to predict the mean annual transport rate from the six predictors with 95 per cent certainty to within plus or minus 40 per cent. *It can therefore be concluded that the six formulae used to predict annual longshore transport rates are consistent with the given input variables.*

5. POSSIBLE SOURCES OF INACCURACY IN THE INPUT VARIABLES

In order to investigate whether this consistent answer obtained by means of the package deal approach is necessarily the correct one, a few possible sources of inaccuracies in the input variables that have to be specified for the application of the package deal approach are discussed briefly below.

Wave characteristics

The wave height H, which is a very significant parameter in the determination of every wave-induced process, could have been obtained from measurements in either shallow or deep water. In both instances the obtained wave height could be inaccurate.

For waves *measured in shallow water* the method of analysis of the records determines the extent of the possible inaccuracies. The actual waves are non-linear, that is, they are not sinusoidal anymore. For that reason a random wave train in shallow water consists of a number of non-linear components. By assuming that all these non-linear components can be represented by Vocoidal waves (see Swart and Loubser, 1978) it can be shown that a normal Fourier analysis, that is, an analysis in the frequency domain, will underestimate both the significant wave height H_s and peak wave period T_p of wave records measured in shallow water (see Swart, 1980). This is the result of the decomposition into their higher-frequency components of the non-linear waves in the spectrum. An example of this behaviour is given in Figure 8a for a random shallow-water wave record, simulated by the random superposition of Vocoidal component waves. The input values for the example shown are the root-mean-square wave height $H_{rms} = 0.711$ m and the peak wave period $T_p = 14.621$ s. The results of the normal Fourier analysis are denoted by a subscript '1' and are $H_{rms1} = 0.49$ m and $T_{p1} = 5.686$ s. Also shown in this figure is the result of a higher-order analysis, developed by Swart (1980), in which the Vocoidal components (instead of the sinusoidal components) are extracted from the original wave record. In the normal Fourier analysis the wave height can be underpredicted by up to 50 per cent, whereas the wave period can be underpredicted by up to 70 per cent. On the other hand, a Draper-analysis, in the time domain, of the same simulated record also underpredicts both height and period to the same extent as a normal Fourier analysis, in this instance as a result of the fact that the component waves are not symmetrical around the mean water level but do in fact have more pronounced crests. For the specific example shown in Figure 8a the Draper results are HSIGD = 0.581 m and TZ (zero-crossing period) = 3.617 s.

Waves *measured in deep water* can be analysed properly, because all component waves are either sinusoidal or closely resemble sinusoidal waves. In this case it is the method of transfer (via shoaling/refraction) of the wave from deep to shallow water which determines the extent of the inaccuracies. Figure 8b shows the error, at the location where H/d = 0.6, in the wave height obtained from linear shoaling and refraction of a wave with a deep water angle of incidence of 20°, when compared to Vocoidal theory. This is a valid way of determining the errors, since Swart and Loubser (1979) have shown that Vocoidal theory shows both a good agreement with measured wave data and adheres closely to the theoretical boundary conditions. The wave height is consistently underpredicted by linear (Airy) wave theory. For the whole range of values of $T\,(g/d)^{\frac{1}{2}}$ given in Figure 8b the error in the angle of incidence at H/d = 0.6 is about -0.13 for the example under consideration, again as compared to Vocoidal theory. The corresponding errors in the radiation stress components R_{yx} (used in the determination of longshore current velocity) and R_{xx} (which determines the mean water level but is not as yet part of the package deal approach) are given in Figures 8d and 8c respectively. It is shown that linear theory consistently overpredicts the radiation stresses. The error in R_{yx} does not necessarily reflect the possible error in the longshore current velocity v, since the friction coefficient C_{LH} is determined empirically. Since it is, however, possible that the formula for longshore current is applied outside the area of empirical determination of the friction factor C_{LH}, the extent of errors in R_{yx} point to a very possible source of inaccuracy in the current velocity v.

Bed roughness

The bed roughness is determined by the geometry of the bed forms, which can consist of wave-induced ripples and completely three-dimensional dune and bar patterns. In the package deal approach only the wave-induced ripples are used for this purpose, because they are the only predictable bed forms. Although this approach has to date given quite satisfactory results, it is obvious that it is an obvious source of error in the determination of the roughness, which in turn affects the friction factors C_{LH} and f_w and therefore also the longshore current velocity and ultimately the predicted longshore transport rate.

Breaking waves

The package deal approach is applied throughout the whole sediment transport zone, that is, also in the breaker zone. This is for a few reasons not correct:

(1) The wave characteristics inside the breaker zone, such as orbital velocities and wave profile, are definately non-sinusoidal.

(2) The expressions for the determination of bed roughness and wave-induced bed forms, derived for non-breaking waves, are not necessarily valid inside the breaker zone.

(3) The amount of sediment is suspension is determined, not only by the bed-related turbulence structure, but also by the turbulence induced by wave breaking. Figure 8e contains an example of the sediment load in the breaker zone, as predicted by the technique of Nielsen et al. (1978) by alternatively assuming breaking and no breaking for the same wave conditions. Although for the example shown the case with breaking leads to a total load which is 173 per cent more than in the case without breaking, the average percentage underprediction due to the fact that the effect of breaking waves on the turbulence was not taken into account would have amounted to 33 per cent for the annual transport in the example in Section 4. This is significant, since the Nielsen formula is the only one of the five detail predictors which includes the effect of wave breaking in his formula.

(4) The longshore current profile across the breaker zone is determined to a large extent by the type of wave breaking which occurs. At present the computation performed in the package deal approach assumes, as is done by everybody else, that the waves break in the spilling mode.

Longshore current velocity
The effect of the bed roughness, friction factor C_{LH} and breaker type on the longshore current velocity have already been touched upon in the above discussion. The beach profile itself also affects the longshore current. At present a representative constant beach slope is assumed for the longshore current determination. In reality the beach profile is often of the bar-type, which contains a well-developed trough landwards of the breaker bar, in which the longshore current will tend to concentrate. Recent research by Dette (1980) is the first step towards understanding this phenomenon.

It is therefore apparent that all input variables could be subject to possible inaccuracies, which could affect all six longshore transport predictors used in the package deal approach.

6. SENSITIVITY ANALYSIS

For this reason a sensitivity analysis was carried out to establish the extent of the influence of inaccuracies in the prime input variables (that is, wave height H longshore current velocity v and bed roughness r) on the average annual longshore transport rate, as predicted for the situation outlined in Section 4 with the five detail predictors. These three input variables were allowed to vary in the following ranges, namely, $0.5 \leq r_r \leq 5.0$; $0.2 \leq r_v \leq 1.0$; and $0.5 \leq r_H \leq 2.0$. The definitions for r_r, r_v and r_H are given in Figure 9, which also contains the results of the sensitivity analysis. Two graphs are given in Figure 9 for each input variable (r, v, H). In each case the left-hand graph indicates the variation of S/Spm with the ratio r_r (or r_v or r_H), where S is the mean computed transport for the five detail predictors and Spm is the transport given by the SPM formula in the original package deal approach. The right-hand graph represents the relative standard deviation σ/μ for the five detail predictors in terms of the ratio r_r (or r_V or r_H), where μ is the mean transport (equal to S in the left-hand graph) and σ is the standard deviation of the five elements of which μ is made up. Figure 9 indicates a few interesting things:

(1) The package deal approach, that is, $r_r = r_V = r_H = 1$, seems to yield the best correspondence with the SPM results of all ratios (of r_r, r_V and r_H) tested.

(2) The relative standard deviation σ/μ reaches a minimum value for $r_r = r_V = r_H = 1$, that is, for the package deal approach.

It can therefore be concluded that the package deal approach for the detail predictors yields the same annual transport rate as the SPM formula, which in a way contains empirically all the physical influences on the input parameters mentioned in Section 5, that is, the bed form and bed roughness, wave breaking and bar profiles. It is therefore reasonable to assume that the results obtained via the package deal approach at present represent the best estimate of annual longshore transport rates. Since both the overall and the detail predictors depend on the predicted wave characteristics, which have been shown to be possibly in error, the package deal results could be in error. The extent of the error will depend on the empirical relationships for the longshore friction factor C_{LH} and for the parameter K(D) in the SPM formula, if both of these are determined from available data by using a good non-linear wave theory, for example, Vocoidal theory, for the prediction of shallow water wave characteristics instead of linear Airy wave theory.

7. CONCLUDING REMARKS

We have therefore at present got a *package deal* approach that yields consistent transport rates and distributions across the breaker zone, but we should do further research to improve our input variables, namely, (1) wave characteristics; (2) bed forms and bed roughness; (3) longshore current velocity and distribution across the breaker zone for bar-type beaches; (4) breaking wave phenomena in general; and (5) the effect on sediment suspension of increased turbulence due to breaking.

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The figures are given in the following sequence:

3; 9; 1; 2; 4; 5; 6; 7; 8.

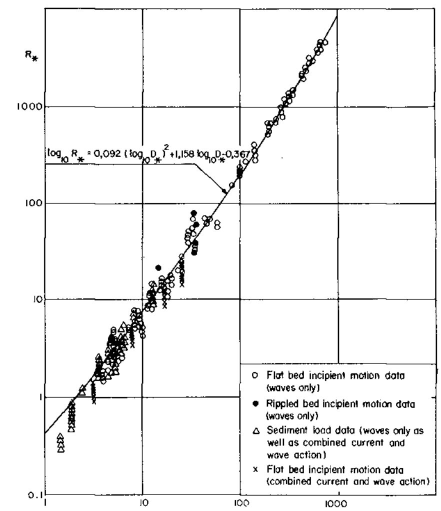

FIGURE 3
INCIPIENT MOTION CRITERION FOR COMBINED CURRENT AND WAVE ACTION

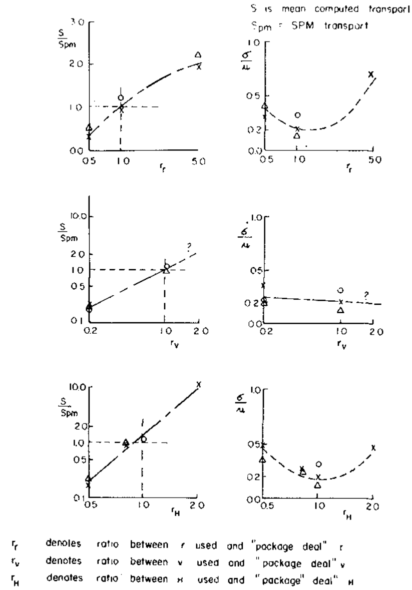

FIGURE 9 SENSITIVITY ANALYSIS ON ANNUAL LONGSHORE TRANSPORT RATE

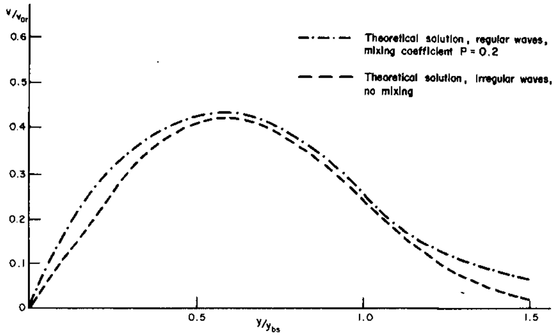

FIGURE 1
COMPARISON BETWEEN THEORETICAL SOLUTIONS FOR LONGSHORE CURRENTS GENERATED BY REGULAR AND IRREGULAR WAVES

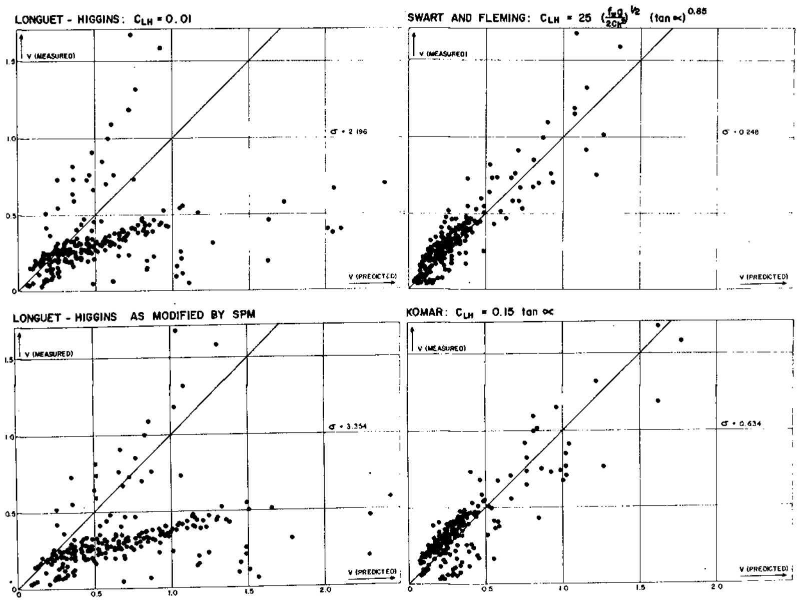

FIGURE 2
PREDICTED VERSUS MEASURED LONGSHORE CURRENT VELOCITIES BY FOUR DIFFERENT METHODS

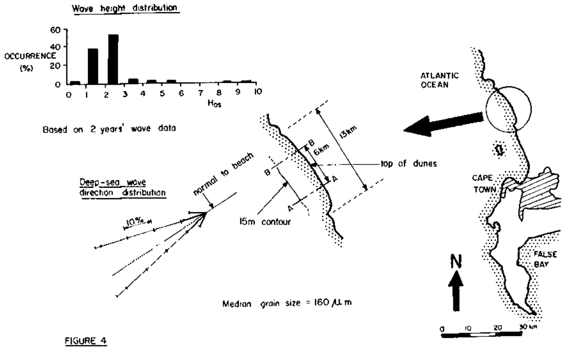

FIGURE 4
APPLICATION TO PROTOTYPE SITUATION

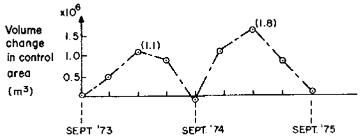

FIGURE 5
PROTOTYPE DATA OVER TWO YEAR PERIOD

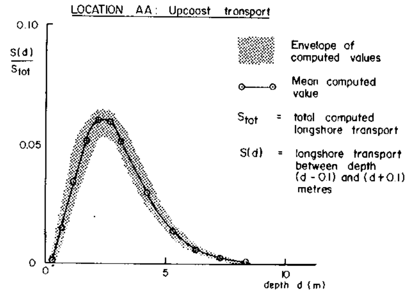

FIGURE 7
DISTRIBUTION NORMAL TO SHORE OF LONGSHORE SEDIMENT TRANSPORT

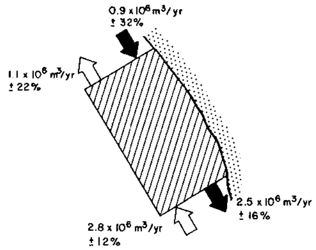

FIGURE 6
PACKAGE DEAL RESULTS FOR ONE YEAR

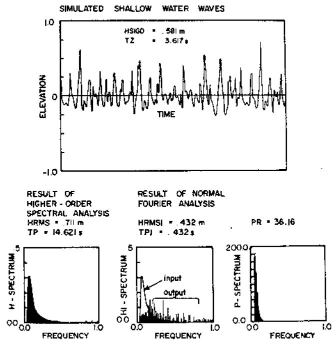

FIGURE 8a

HIGHER - ORDER WAVE ANALYSIS FOR H_{rms} = .711 d = 1.0 T_p = 14.621

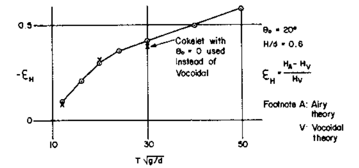

FIGURE 8b

ERROR ε_H IN WAVE HEIGHT AS OBTAINED AFTER REFRACTION WITH LINEAR AIRY THEORY

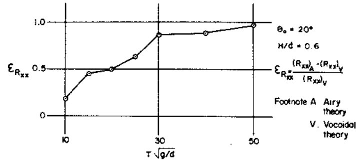

FIGURE 8c

ERROR $\varepsilon_{R_{xx}}$ IN PRINCIPAL RADIATION STRESS R_{xx} AS OBTAINED AFTER REFRACTION WITH LINEAR (AIRY) WAVE THEORY

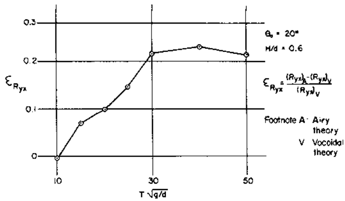

FIGURE 8d

ERROR $\varepsilon_{R_{yx}}$ IN RADIATION STRESS R_{yx} AS OBTAINED AFTER REFRACTION WITH LINEAR (AIRY) WAVE THEORY

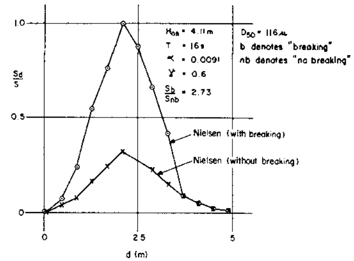

FIGURE 8e

EFFECT OF INCREASED TURBULENCE DUE TO BREAKING ON LONGSHORE TRANSPORT