WAVE FORCING OF BEACH GROUNDWATER

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ABSTRACT

A field project was conducted to measure swash and surf characteristics and related processes on a beach with a high tide range and very low slope. Variables measured were beach waterlevels in the surf and swash zones, beach groundwater levels, and on-offshore and alongshore currents. A variety of deployment patterns and instrument combinations were used. Results show a nonlinear transfer of wave energy from high to low frequencies. This appears to have occurred because finite amplitude waves and bores caused an onshore mass flux which produced an increased local waterlevel. Periodically, this impounded water was released. Groundwater records indicate that low frequency waterlevel oscillations on the beach face were the dominant forcing function for oscillations of the beach watertable. A model is generated which adequately predicts some of the response characteristics of the groundwater table.

INTRODUCTION

As one of a series of studies conducted to examine in detail the physical mechanisms which are active in a dynamic beach system, the field project discussed in this paper was designed to examine the relationship between waves, swash, and groundwater on a very low sloping beach. The specific process to be examined in this paper is the response of the beach groundwater and water table to forcing mechanisms occurring at frequencies ranging from input wave frequencies to several minutes. It is apparent from these results that on such a low sloping beach the understanding of the groundwater flow exceeds the understanding of the complexly coupled forcing mechanisms.

METHODOLOGY

This experiment was conducted on Little Talbot Island, Florida in May 1973. Little Talbot Island is a barrier island in the St. Johns River estuary. A typical cross beach profile at the experimental site is given in Figure 1. The slope is very shallow with an average of approximately 1:70. This site was chosen because of its low slope, high tide range, and medium sand-sized sediment.

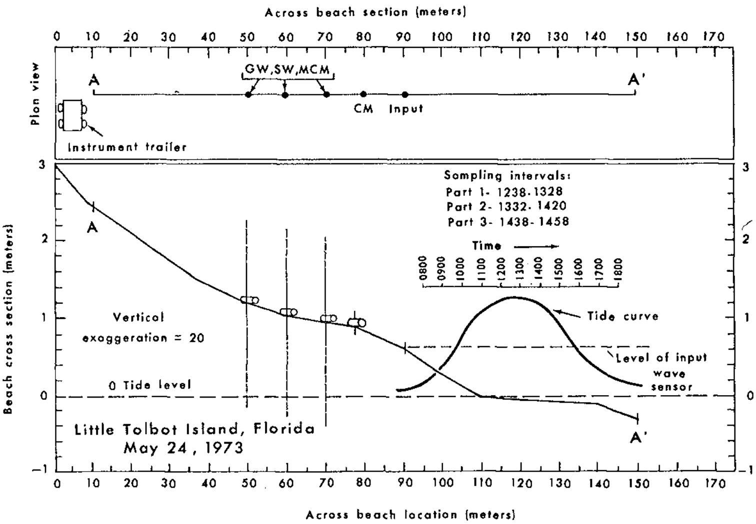

Figure 1. Beach profile, typical instrument deployment pattern, and representative tide curve. The plan view shows the cross beach locations of instruments where GW = groundwater well, SW = swash or beach waterlevel depth, input = input wave sensor. MCM and CM refer to current meters which are not discussed in this paper.

At the study site, the dominant semidiurnal tide was slightly tropic with a range of approximately 1.3 meters. This amplitude and period produced a beach face which rapidly migrated 120-150 meteres during half a tidal period. For this discussion, beach is any region which is affected by swash at some time during a tidal cycle. During high tide, most of the beach was inundated and waves shoaled from the seaward or outer limit of the beach at approximately x = 150 meters. Waves and bores moved directly onshore, i.e., wave orthogonals were perpendicular to the shoreline and no complex refraction patterns occurred.

The beach cross section shown in Figure 1 remained rather static and exhibited no fundamental changes during the several day experiment. Small scale sedimentary features such as ripples and low amplitude sand waves would appear and disappear during a tidal cycle.

The general cross beach profile was composed of five zones distinguished on the basis of local slope. These zones correspond closely to processes active on the beach during particular portions of a tidal cycle. Zone 1, which extends seaward of x = 140 is the nearshore, which is continually inundated during all tidal phases except under exceptional conditions. Zone 2 ($110 \le x \le 140$) is a low tide terrace which is occasionally exposed or has only very shallow water during low tide. Zone 3 ($75 \le x \le 110$) is the low tide swash slope and a region over which swash migrates rapidly during rising or falling tide. Zone 4 ($50 \le x \le 75$) is a high tide terrace. This is a region of very shallow water during high tide. It is in this zone that most instruments were deployed. Zone 5 ($20 \le x \le 50$) is the high tide swash slope. The exact location of Zone 5 varies with a result of diurnal and fortnightly variability of high tide level.

Measurements were made between x = 40 and x = 70 m with most water-level measurements being made between x = 50 meters and x = 70 meters. In this region the beach slope was approximately 1:70. Measurements of input parameters were made between 80 m and 90 m, usually at approximately 90 m. Thus, input statistics reflect wave phenomena just prior to their transiting the very shallow Zone 4. It is in this region, Zone 3, in which waves make a transition to solitary waves and bores. Just seaward of the input wave sensor, water depth decreases from over a meter to less than .3 m.

Instrumentation and Data Runs

Three types of instruments were deployed on the beach face. Due to limitations in the number of analog recorders, all instruments were not used during all data runs. The three instruments were: (1) capacitance type waterlevel sensors which were used to measure waterlevel on the beach and in groundwater wells; (2) bidirectional, ducted impellor, current meters; and (3) a pressure transducer used to measure input surf conditions. These instruments are described and discussed by Sonu and Pittigrew (1974).

Instruments were deployed in different locations and combinations during various data runs. An example of the instruments and their deployments are given in Figure 1. Included on this figure is a representative vertically exaggerated cross beach profile and a tide level record plotted relative to the profile datum. With this presentation water depths at any time and location can be readily determined.

Most data acquisition occurred during or near high tide. Conducting experiments during this period tended to minimize variations in local conditions that could cause a lack of stationarity in the processes active on the beach face.

Data was taken on several different days. For this discussion, primary instruments were swash probes and groundwater probes. Generally, a single instrument array was used each day. During a day, different data runs were conducted and these are referred to as Part 1, 2, or 3. A given data set is indexed by day and part. Thus "24-3" is part 3 on May 24th.

Analysis

Results of the measurement program were synchronous time series which represented variations in magnitude of the various phenomena measured. These records which have durations ranging from 10 minutes to over an hour were analyzed to give a frequency decomposition of the variance of each signal. In addition, physically related phenomena were analyzed for a frequency decomposition of the covariance of these processes. This technique permits isolation of major periodicities and associations which characterize the random processes being analyzed.

A standard method of spectral density analysis was used. In addition to spectral densities of individual time series, cross spectral densities were computed and the second order characteristics of coherence, transfer functions, and phase angle calculated. All data were demeanded and detrended during analysis to leave only periodic components of variability. Caution must be exercised when interpreting results of higher order spectral analysis. Many physical processes were related nonlinearly which can cause parameters such as coherence to be misleading.

RESULTS

Waterlevels

Design of this experiment required measurement of waterlevels in the region of the swash and surf zone. However, because of: (1) the desire to measure waterlevels during or immediately following high tide, (2) the diurnal variation in the elevation of high tide, (3) the necessity of putting instruments in place well in advance of high tide, and (4) the very low beach slope and hence very wide beach, it was not possible to have instruments in the same relative position during each data run. Therefore, during all data gathering periods, a given instrument or location does not measure or represent the same general environment. In fact, due to a lack of stationarity of still waterlevel, a given instrument may be sampling distinctly different environments in successive data runs; e.g., surf measurements in one run and swash measurements in the subsequent data run. As a result, specific values of variance density can not be examined; however, patterns can be examined and these should reflect the processes occurring on the beach.

In almost all data runs, a critical, although at times minor, spectral peak occurred between .0667 hz and .0769 hz. This peak was observed at all across beach stations and was the lowest frequency peak which was uniformly and consistently observed at each measurement station. The frequency of this spectral peak separates what will be referred to as high frequencies from low frequencies.

In these waterlevel spectra, Figure 2, two major features occurred: (1) higher frequency peaks apparently associated with input waves and bores, and (2) one or more low frequency peaks which are believed to have been locally generated by complex interaction of waves and topography. By local generation, it is meant that the process which produced this peak was not input to the beach system but rather was the result of local processes.

If time series of waterlevel at one location are considered as input and time series from a station further upslope are considered as output, then the relationship of the associated spectra can indicate whether spatial processes can be represented by a linear function in the frequency domain. If the spatial process is linear, then at any given frequency, the output is a linear multiple of the input; i.e.,

$$X_{out}(x_{out}, f) = |H(f)|^2_{x_1,\ x_2} X_{input}(x_{in}, f) \qquad (1)$$

where X(f, x) is the magnitude of the variance density at $x = X_{out}$ or x_{in} at frequency f, x = location on the beach relative to the arbitrary baseline in Figure 1 and H(f) = frequency response function.

If the spatial processes are linear, then variance can not be redistributed in the frequency domain as the waves move from x_{in} to x_{out}. In the present beach system, it is anticipated that the linear multiple should be less than or equal to unity since energy is being dissipated as waves move upslope.

These data, Figure 2, suggest that the systems function for the entire spatial shoaling process is nonlinear. This must be concluded because of the apparent local generation and behavior of the low frequency spectral peak. The upslope increase in low frequency peaks and simultaneous upslope decrease in high frequency peaks, strongly suggests that some nonlinear process is causing variance to be redistributed in the frequency domain (Waddell, 1976).

If high and low frequency variance are viewed independently, then high frequency processes appear to be well behaved and quasi linear since there is a gradual upslope variance attenuation. This is what might occur if there was no spatial gradient in mass flux and there was simple upslope energy dissipation resulting in a continual decrease of wave or bore height. Such a situation may occur if Airy waves are gradually dissipated.

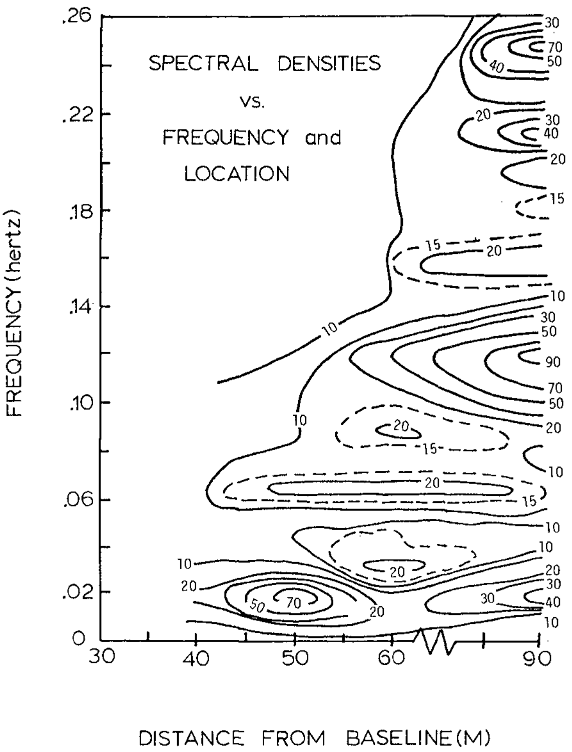

Figure 2. Contour plot of spectral density of beach waterlevels at each across beach location. The actual spectra at any location can be determined holding "distance from baseline" constant and examining changes in magnitude with frequency.

However, in the present situation, waves have definite nonlinear shape and there is most probably a spatial gradient in horizontal mass flux.

As indicated previously, for the environment being examined, local bore height depends on: (1) location, (2) initial wave energy, i.e., height, (3) rate of energy concentration, and (4) rate of energy dissipation. Therefore, it is expected that at some point on the beach face, total variance associated with the input bore spectral peaks will begin to decrease and will continue to decrease until completely attenuated. This complete attenuation may occur at the maximum limit of uprush or it may occur seaward of that point such that no wave induced uprush results (Le Mehaute, 1968).

An explanation of the cause and behavior of low frequency variance is much less straightforward. Possible sources for this variance are: (1) orderly variations in bore height causing variance at subharmonics of the input wave frequency; (2) a combined influence of radiation stress induced set-up in conjunction with arrival of wave groups, and (3) periodic impoundment and release of water moved onshore by shoaling waves and bores. One or more of these could occur simultaneously; however, in the present situation the last possibility seems the most probable.

If these low frequency peaks were the result of some orderly variation of the height of incident waves, then the low frequency peaks should occur at subharmonics of the dominant input periods. Additionally, since the dominant input spectral peak had a rather narrow bandwidth, such subharmonics should also be rather well defined. In the present data, there were no consistent patterns of low frequency variance at subharmonics of the input wave periods.

If orderly height variations caused the low frequency variance, then the magnitude of the low frequency peaks should be relatively small unless there were large differences between successive bore heights. If such were the case, there should be spatial changes in the period between successive bores because of the dependence of celerity on bore height, i.e.,

$$C = (g(h_o + H_B))^{\frac{1}{2}} \qquad (2)$$

where C = rate of change of location of bore front relative to the local water velocity, h_o = initial water depth and H_B = change in water level across the bore front. If there were large sequential variations in bore height in water where H_B was the same order of magnitude as h; i.e.,

$$O(H_B) \geq O(h) \qquad (3)$$

then the band width of the input wave spectral peak should increase upslope. No such pattern can be seen, in fact, the opposite generally occurs. As a result, it must be assumed that successive input waves did not display large sequential and orderly variations in height.

In these data some low frequency spectral peaks occurred at subharmonics of input wave spectral peaks, however, there was no consistent pattern and the magnitude of these spectral peaks was generally small.

It is expected that due to energy dissipation, a subharmonic peak should decrease in magnitude upslope unless secondary bores consistently rejuvenated primary bores. Even if this occurred, it is unlikely that such rejuvenation could account for low frequency spectral peaks having the magnitude of those observed.

If low frequency variance is associated with periodic set-up due to the arrival of wave groups, then the amplitude of the low frequency oscillation can be rather large. Examination of the original time series did not reveal clearly defined and repetitive wave groups, although there was some indication that small periodic variations of wave height might have been present at times. If such wave groups were the result of superposition of two deep water wave trains moving into shallow water, then it should be possible to identify the component waves in the input spectra and from these predict the approximate frequency of the low frequency oscillation. If there are two wave trains having similar amplitudes and angular frequencies ω_1 and ω_2, then amplitude of the carrier wave will vary at a frequency of $\frac{(\omega_1-\omega_2)}{2}$ and the variation in wave set-up will occur at $\omega_2-\omega_1$.

All possible pairs of high frequency spectral peaks were evaluated for the frequency of the envelope wave which would be produced if they were superimposed. In all these data runs only a few high frequency peaks could be combined to produce the observed low frequency peaks. There is serious doubt as to whether this association was any more than random chance. This doubt arises because the peaks were not consistently observed across the beach face.

By elimination, impoundment and release of water from the beach appears to be the most likely explanation for the low frequency waterlevel oscillations. Since the component highly nonlinear waves caused a shoreward mass flux, a mechanism for onshore flux is readily available; however, an adequate explanation for the periodicity of impoundment and release is difficult to provide. It is possible that due to the low beach slope, wide beach zone and existence of multiple random waves in the shallow portion of the surf zone, some natural periodicity is established.

Groundwater

Spectra of groundwater indicate that low frequency oscillations of waterlevel on the beach dominated the forced groundwater oscillations (Figure 3). This is to be expected and has been documented previously (Waddell, 1973). These low frequency oscillations dominate for two major reasons: (1) the beach matrix and well point are low pass filters, and (2) groundwater was usually measured on the shoreward side of the experimental site where lower frequencies generally dominated the waterlevel spectra.

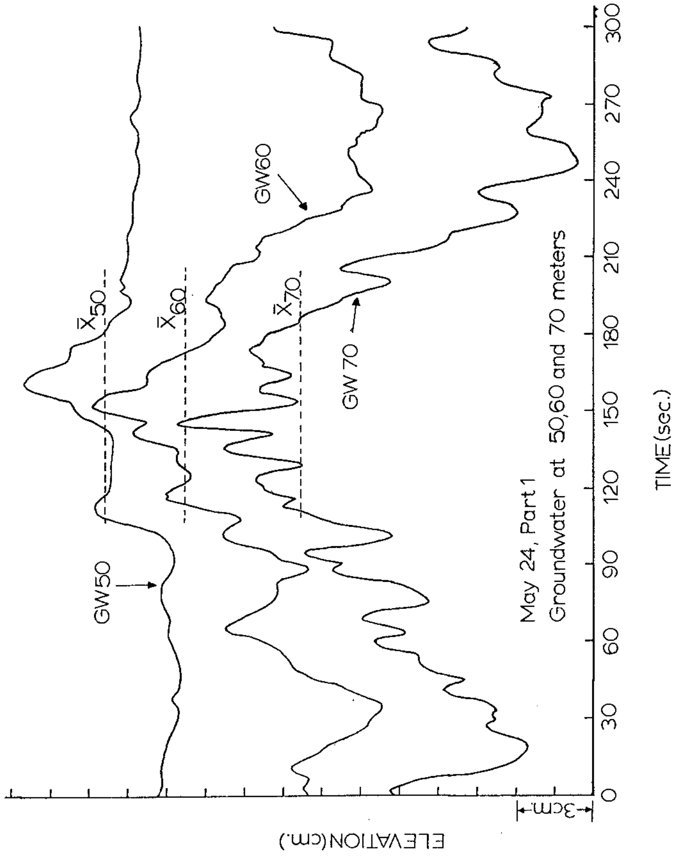

Figure 3. Groundwater oscillations at three across beach stations 10 meters apart (x = 50, 60, and 70 meters from baseline). The zero level or origin is shifted for each trace. Note the sequential upslope dominance of low frequency fluctuations. Also note the very slow fluctuation with a period of approximately 200 seconds.

As seen in Figure 4, examination of time-synchronized traces of beach waterlevel and well waterlevel shows that they correspond quite closely. This similarity suggests that possibly the local pressure gradient across the well wall may be the forcing function for the well waterlevel oscillations. Since velocities within the well are very small, especially at lower frequencies, pressure should be approximately hydrostatic. Also, pressure within a solitary wave is very close to hydrostatic (Ippen, 1962). Consequently, the time dependent hydrostatic spatial pressure gradient is probably the responsible forcing mechanism

A simplified model can be generated to explain the response of the beach waterlevel-well waterlevel system (Figure 5). Mass continuity dictates that flow through the well point is equal to the time rate of change of water within the well. Thus,

$$\rho A u = \frac{dQ}{dt} \qquad (4)$$

where A = area of openings in the well point, u = spatial mean velocity of flow across the well point, ρ = density of water, Q = storage of water within the well. Storage within the well is given by

$$Q = \rho \pi r^2 h_w \qquad (5)$$

where h_w = height of the waterlevel in the well and r = radius of the well and constant. Therefore,

$$Au = \pi r^2 \frac{dh_w}{dt} = \pi r^2 \frac{dh_1}{dt} \qquad (6)$$

where h_1 is waterlevel relative to an arbitrary datum since only the time rate of change is being considered and this is independent of datum level. Let

$$u \propto (h_0 - h_1) \qquad (7)$$

where h_0 = beach waterlevel and h_1 = well waterlevel, both being measured relative to the same arbitrary datum. Assuming a constant of proportionality K, then Eq. 7 is

$$u = K(h_0 - h_1) \qquad (8)$$

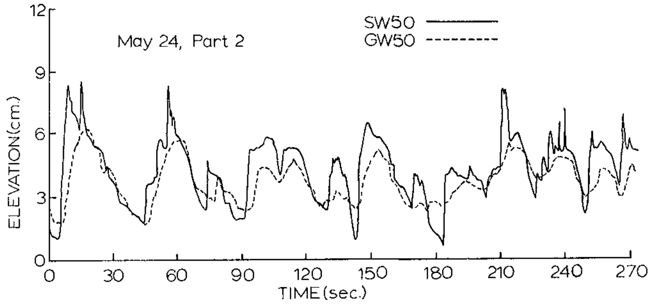

Figure 4. Synchronized beach waterlevel and ground waterlevel traces. The well waterlevels are attenuated relative to the swash fluctuations.

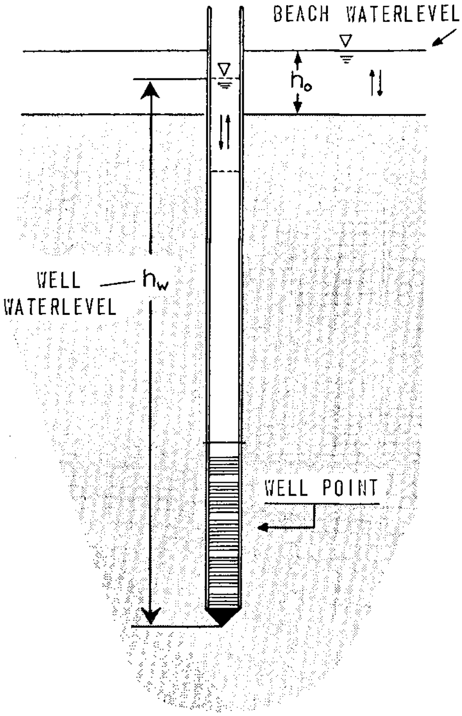

Figure 5. Schematic of groundwater well configuration and associated waterlevels.

Physically, $(h_0 - h_1)$ is the forcing function for the system and is a measure of the hydrostatic pressure gradient across the well point and can be expressed as

$$AK(h_0 - h_1) = \pi r^2 \frac{dh_1}{dt} \qquad (9)$$

Let $F = AK$, $D = \ r$, and let $h_0 = H_b \cos(wt)$ where w = angular frequency of beach waterlevel changes. Assigning values to h_0 establishes a datum level for h_1 and h_0. Eq. 9 can be rewritten as

$$\frac{dh_1}{dt} - \frac{F}{D} h_1 = \frac{F}{D} H_b \cos(wt) \qquad (10)$$

The solution to this equation is

$$h_1(t) = h_1(0) \exp\left(-\frac{F}{D}t\right) + \frac{F^2 H_b}{d^2w^2+F^2} \cos(wt) + \frac{FDwH_b}{D^2w^2+F^2} \sin(wt) \qquad (11)$$

where $h_1(0)$ = waterlevel in the well at $t = 0$. The first term on the right side of the above equation represents the initial response to excitation of the beach-well system. To examine a well waterlevel response relatively unaffected by the initial well waterlevel, assume $t=t_1$ where t_1 is large so that $h_1(0) \exp(-\frac{F}{D} t_1) \to 0$. In the time domian where $t \geq t_1$, Eq. 11 reduces to

$$h_1(t) \doteq \frac{F^2 H_b}{D^2w^2+F^2} \cos(wt) + \frac{F\ D\ w\ H_b}{D^2w^2+F^2} \sin(wt) \qquad (12)$$

The time derivative of Eq. 12 is evaluated for the first maxima, of $h_1(t)$, following a maximum in $h_0(t)$. This occurs at

$$t = \frac{1}{w} \tan^{-1} \left(\frac{Dw}{F}\right) \qquad (13)$$

where $t = \tau$ = lag of the well waterlevel maximum relative to the beach waterlevel maximum.

Given environmental parameters D and F, τ is frequency dependent. In this experiment D $\doteq$ 45.6. It remains to determine K, which can be viewed as incorporating a variety of effects generally associated with flow attenuation. For illustration it is assumed that K is a linear function of A, the surface area of the openings in the well point. Obviously, such a relationship would be dependent upon the geometry of the openings, but generally, given a geometry, as surface area increases for a given pressure gradient, the mean velocity would increase. This would be true for small openings where retarding forces due to boundaries of the openings and due to grain boundaries are significant.

If $$K = mA$$

then $$F = mA^2 \qquad (14)$$

For the present example we assume F = 4. Then,

$$\tau = \text{lag} = \frac{1}{w} \tan^{-1} (11.4w) \qquad (15)$$

This argument can be reversed to use measured values of τ to calculate an estimate of K which relates velocity to the pressure differential.

Since the driving force for changes in these two waterlevels appears to be a hydrostatic pressure gradient which is a linear operator, it is expected that the systems function (filter function relating these two) will be linear. This means there should be a high coherence between the two signals. This was generally observed, e.g. Figure 6.

Phase lags of surf depth and well waterlevel are anticipated (Figure 4). As seen above, these occur as a direct result of the frequency response function H(f), associated with the porous media and the well point, where

$$H(f) = |H(f)|e^{j\theta(f)} \qquad (16)$$

and $\theta(f)$ is the frequency response phase angle and is numerically identical to the cross spectral phase angle, $\alpha(f)$. Thus, for a linear system, τ is a direct result of the amplitude attenuation of the signal by the system.

Figure 7 contains a plot of "τf" as determined from Eq. 15. Additionally, on Figure 7 are plotted some observed values of $\tau(f)$, suggesting that for sinusoidal input signal, Eq. 15 provides a rather accurate prediction of well waterlevel oscillations. Considering: (1) that the observed values of $\alpha(f)$ represent cumulative influence of all input signals at a given frequency and (2) that the input signal in Eq. 10 was strictly sinusoidal, the agreement in Figure 7 is very good.

In some data runs the lack of close agreement between predicted and observed values at higher frequencies may in part result because the

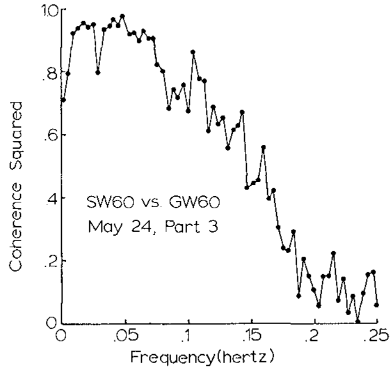

Figure 6. Coherence between waterlevels on the beach surface and in the groundwater wells. Note the coherence remains high at low frequencies ($f \leq .07$ hz) and decreases at higher frequencies.

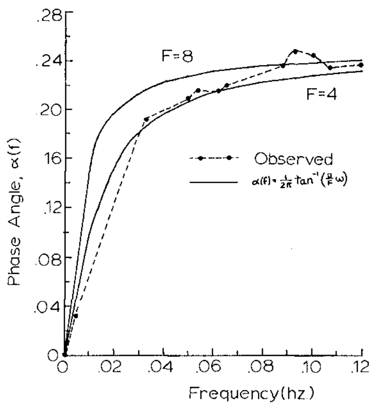

Figure 7. Phase angle or phase lag between the input signal (beach waterlevel) and the output signal (well waterlevel). Plotted are observed values and two examples of predicted phase lags from Eq. 15.

higher frequency signals were not sinusoidal but were more asymmetrical with a steep faced front and a more gradually sloping seaward side.

Specific frequency response functions between beach waterlevel and well waterlevel are, in part, dependent upon the depth of the well point below the beach surface. As suggested by the low pass characteristics of these functions, for a given beach, the deeper the well point, the lower the cut-off frequency of the filter function. Thus, in these experiments, if the well point had been closer to the surface, then the observed well waterlevels would have responded more vigorously to the input waves (i.e., 0.1 hertz).

Figures 8 and 9 are examples of the predicted response of well waterlevel to forcing functions of constant amplitude but having different frequencies. However, two identical wells placed in different locations can have different response characteristics as a result of differences in local sedimentary characteristics. Thus, to be useful in a predictive sense, each well must be "calibrated"; i.e., determine F in Eq. 13. If such a calibration is completed, nearshore wells could be used to identify slopes of oscillating nearshore waterlevels.

Since the flow parameter K is in reality dependent on the character and quantity of sediment above the well point, local changes in nearshore morphology can alter the frequency response function relating input and output signals.

High and low frequency pumping action may have very significant effects on local sediments. The pumping induced by the pressure gradient associated with the migrating bore front could cause dilation of surficial sediments. The depth of this layer would be limited by the frequency dependent amplitude attenuation which results from viscous flow through a porous media. However, low frequency pressure differentials could be transmitted to a much greater depth in the sediment deposit, thus causing flow and possible grain dilation to a much greater depth.

If spatial pressure gradients occur at high and low frequencies, it is possible that a rather thick sedimentary deposit could become fluidized. If this occurred, then such things as local shear strength of sediments could exhibit strong periodic components.

CONCLUSIONS

Waterlevel fluctuations on this low sloping, high tidal range beach exhibited a nonlinear transfer of energy from high to low frequencies. This spatial process appears to have resulted from the impoundment of water which was transferred onshore by bores and finite amplitude waves. Periodically this impounded water mass was released, resulting in low frequency waterlevel oscillations. These low frequency oscillations became increasingly dominant at the more shoreward stations. In some data runs near the mean shoreline these low frequency oscillations completely dominate almost to the exclusion of high frequency fluctuations. Thus, on the

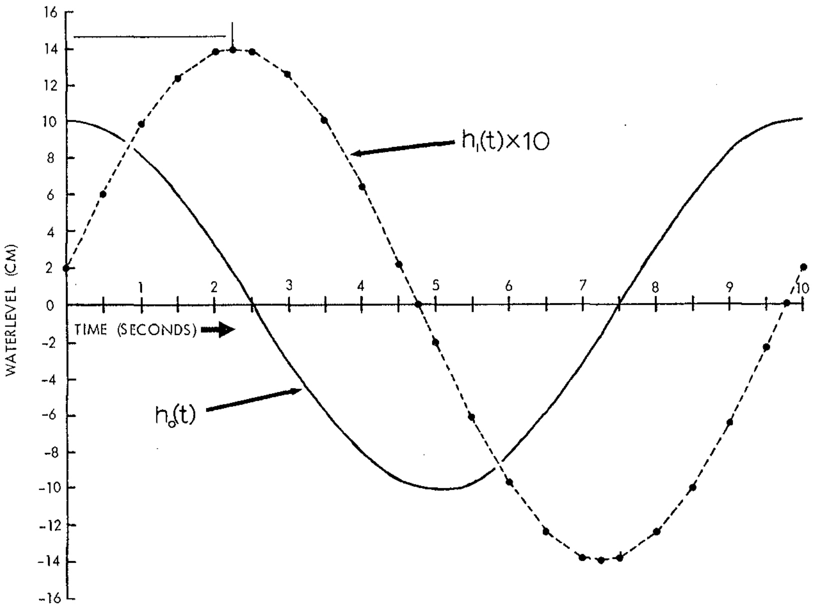

Figure 8. Input and output signals for high frequency (f=.1 hz) forcing. Note the low amplitude response to a 10 cm input amplitude.

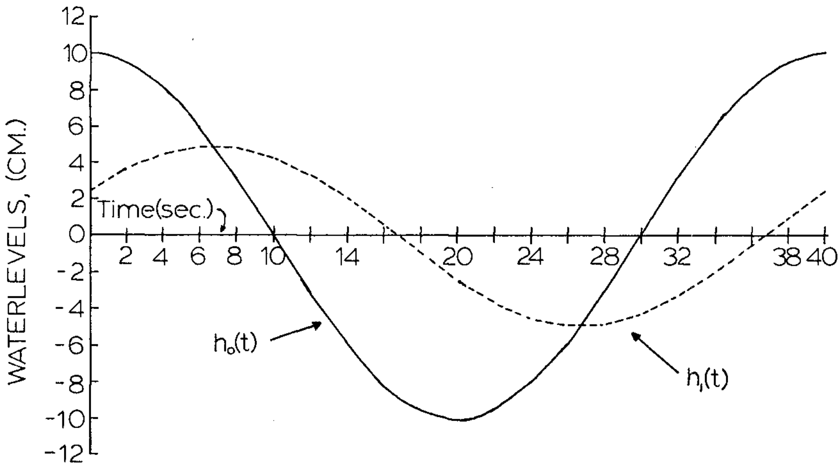

Figure 9. Low frequency response. Compare to Figure 8. Input signal has 10 cm amplitude but response is attenuated much less.

swash slope run-up due to individual bores may not even occur and periodic uprush and backwash may be associated with the low frequency waterlevel oscillations.

Watertable variance was dominated by lower frequency oscillations. This resulted because the beach acted as a low pass filter and the configuration of wells was such that high frequency fluctuations were almost completely attenuated at the depth of the well point. By assuming a form for the groundwater forcing function, it was possible to predict the lag of the groundwater maximum behind the beach waterlevel maximum. The influence of well placement and local sediment permeability and porosity are incorporated in parameters, F, D, which must be empirically determined.

ACKNOWLEDGMENTS

Field data used in this report was taken by personnel from the Coastal Studies Institute of Louisiana State University under the direction of Dr. C. J. Sonu, who is presently with Tekmarine, Inc. Use of these data are gratefully acknowledged as is the financial support of Science Applications, Inc.

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