WAVE FORCES ON AN INCLINED CIRCULAR CYLINDRICAL PILE

by

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ABSTRACT

This paper describes experimental results on the in-line and lift forces acting on inclined circular piles which are placed in two different planes: (1) a vertical plane parallel to the direction of wave propagation; and (2) a vertical plane parallel to the wave crest.

The in-line and lift force formulas for an inclined pile are formulated by referring to the conventional Morison and lift force formulas, respectively. Stokes third order wave theory is used for the estimation of flow kinematics induced around a pile. Based on these formulas, the time-independent and time-dependent values of the drag, mass and lift coefficients are determined by using several methods. Further, the time-dependent coefficients are expanded into Fourier series which consist of several significant components.

Reliability of these coefficient values are studied by examining the relative deviation of the predicted wave forces based on these coefficient values from the measured ones. The analysis finds that relative deviations of the in-line and lift forces exceed in many cases 15% and 100%, respectively, when the time-independent coefficients are used for the prediction of wave forces, but that they are reduced to 5% and 15%, respectively, when the time-dependent coefficients are used for it.

INTRODUCTION

It is commonly recognized that two types of wave forces are induced on a slender cylindrical pile subject to the motion of unbroken waves. One is called in-line force which acts on the pile to the direction of wave propagation and the other is called lift force which acts on the pile transversely to the direction of wave propagation. For the present, the in-line force is calculated by the Morison formula which evaluates the in-line force as a linear sum of the drag force and the inertia force. The lift force is calculated by the lift force formula which is same as that in a steady flow. However, when one attempts to predict these wave forces by the above formulas, one has to get reliable information on: (1) the flow kinematics induced around an inclined pile; and (2) the values of the hydrodynamic coefficients [C_D] and [C_M] included in the Morison formula and of the lift coefficient [C_L] included in the lift force formula. It is currently common practice to estimate the flow kinematics by using appropriate wave theories. Airy's wave theory has been widely used because of its simplicity although Stokes wave theory of higher order and Dean's stream function theory have also been used recently for the estimation of the flow kinematics. Hence, determination of C_D, C_M and C_L values experimentally has become one of the major themes in the present research of the wave forces.

Since Morison(1950) developed his unique method of determining C_D and C_M values experimentally, many distinguished works have been conducted on wave forces and coefficient values have been determined by several methods. Table 1 summarizes some of the representative works in which unique methods have been either developed or used to determine the coefficient values. Through a brief review of these previous works, the following facts can be noted:

1. Coefficient values for either vertical or horizontal piles have been determined mainly by the methods shown in Table 1.
2. There has not been sufficient study conducted to determine which method will provide the most reliable coefficient values for the prediction of wave forces.
3. Inasmuch as only a few works have been done on the wave forces acting on inclined piles, there is a limited amout of information available on the coefficient values for inclined piles.

It should be noted, however, that jecket-type offshore structures generally consist of many steel piles which are inclined in various planes at different angles. Thus, it is quite doubtful whether the coefficient values determined for either vertical or horizontal piles can be used directly for the evaluation of wave forces exerted on inclined piles.

Based on this, a basic experiment is performed in this study to investigate the characteristics of in-line and lift forces which act on inclined piles placed in two different planes: (1) a vertical plane parallel to the direction of wave propagation; and (2) a vertical plane parallel to the wave crest. The conventional Morison and lift force formulas are modified, respectively, to predict the in-line and lift forces acting on the inclined piles which are placed in each of both vertical planes. Further, Stokes third order wave theory is used to estimate the flow kinematics induced around the inclined piles. Based on these formulas, the time-independent and time-dependent values of C_D, C_M and C_L are determined by using either the methods shown in Table 1 or the methods which are derived by modifying the previous ones.

The analysis is focussed to determine: (1) whether the inclinations of piles affect the coefficient values; (2) which method will provide the most reliable coefficient values for the prediction of respective wave forces; and (3) how the inclinations of piles affect the ratio of the maximum lift force to the maximum in-line force exerted of an inclined pile over a wave cycle.

Table 1. Representative works on the wave forces acting on the piles.

Authors	Contents of investigations	Methods used for determining coefficient values
Morison (1950)	Circular and H-shaped piles and flat plates were placed vertically in progressive waves, and total in-line forces on them were measured.	C_D and C_M values were determined at two specific phases within a wave cycle where either the drag or inertia term became zero, respectively. This method determines time-independent values of each coefficient, and is called hereafter the "two-point method".
Keulegan and Carpenter (1958)	Circular piles and flat plates were placed horizontally at the nodal points of standing waves, and total in-line force on them were measured. Flow patterns were also observed photographically.	C_D and C_M values were determined by Fourier analysis:(1) by the primary coefficient of Fourier series; and (2) by the Fourier series of fully expanded. The former method determines time-independent values, and does the latter time-dependent values of each coefficient. The former method is called hereafter the "Fourier-averaged method".
Al-Kazily (1972)	Circular piles were placed horizontally in progressive waves, and total in-line, transverse and uplift forces on them were measured. Some experiments were also made on wave forces exerted on inclined piles.	C_D and C_M values were determined by the two-point method. These values were also determined by solving Morison formulas provided at successive phases, t and (t+Δt) simultaneously where Δt is an incremental phase. This method determines the time-dependent values of each coefficient, and is called hereafter the "Al-Kazily's method".
Sarpkaya (1975)	Circular piles and spheres were placed horizontally in oscillatory flows generated in the U-shaped tunnel, and total in-line and lift forces on them were measured.	C_D and C_M values were determined by the Fourier-averaged method and C_L values were determined at a specific phase where the lift force became maximum. This method also determines time-independent values of C_L, and is called hereafter the "one-point method".
Chakrabarti et al (1976)	Circular piles were placed vertically in progressive waves, and total and sectional in-line and lift forces on them were measured.	C_D and C_M values were determined by the least square method, which also determines time-independent values of them. This method is called hereafter "the least square method". C_L values were determined by the Fourier analysis.

THEORETICAL CONSIDERATION

This study employs an essential assumption that the in-line and lift forces exerted on any inclined piles can be evaluated by the conventional Morison and lift force formulas if certain modifications are made on them. This assumption is the basis of this research.

Wave Force Formulas for an Inclined Pile

Figure 1 shows a sketch of a circular pile with an outer diameter, D, placed in a vertical plane parallel to the direction of wave propagation at an inclining angle, θ, and the coordinate system used for the analysis. For the convenience of later discussion, this vertical plane will be called hereafter the "Plane A".

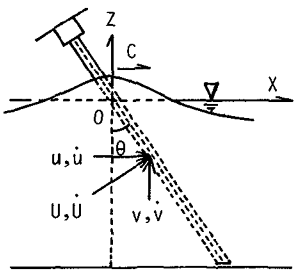


Figure 1. Schematic drawing of an inclined pile placed in the Plane A.

In this figure, u and v represent the horizontal and vertical velocity components of an induced flow at an arbitrary position, (x,z) on the circular pile, at an arbitrary phase, t. Similarly, $\dot{u}$ and $\dot{v}$ represent the acceleration components of u and v, respectively. Here assume that the velocity component normal to the pile axis, U and its accerelation, $\dot{U}$ can be composed into the following equations:

$$U = u\cos\theta + v\sin\theta \quad ....(1)$$

$$\dot{U} = \dot{u}\cos\theta + \dot{v}\sin\theta \quad ....(2)$$

Then, the total in-line force, $F_{txA}(t)$ which acts on the inclined pile at an arbitrary phase, t, will be evaluated by the following equation:

$$F_{txA}(t) = \int_{-h}^{\eta} C_{DU}\frac{1}{2}\rho D\,U|U|\sec\theta\,dz + \int_{-h}^{\eta} C_{M\dot{U}}\frac{1}{4}\rho\pi D^2\,\dot{U}\sec\theta\,dz + \frac{1}{4}\rho g\pi D^2\eta\tan\theta \quad ....(3)$$

where C_{DU} and $C_{M\dot{U}}$ represent the drag and mass coefficients of the inclined pile, and the last term represents the buoyancy caused by the fluctuation of the surface elevation, η.

Similarly, the total lift force, $F_{tLA}(t)$ which acts on the inclined pile at the same phase, t will be evaluated by the following equation:

$$F_{tLA}(t) = \int_{-h}^{\eta} C_{LU} \frac{1}{2}\rho D\, U^2 \sec\theta \quad \text{.......................} \quad (4)$$

where C_{LU} is the lift coefficient of the inclined pile.

Figure 2 shows a sketch of a circular pile with an outer diameter, D, placed in a vertical plane parallel to the wave crest at an arbitrary inclining angle, θ, and the coordinate system used for the analysis. For the convenience of later discussion, this vertical plane will be called hereafter the "Plane B".

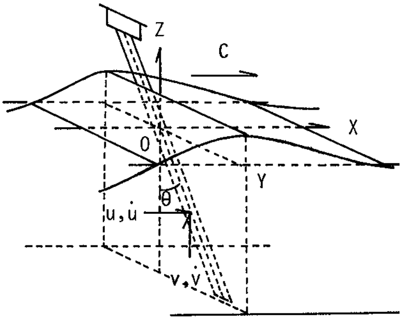


Figure 2. Schematic drwing of an inclined pile placed in Plane B.

In this figure, u and v represent the horizontal and vertical velocity components of an induced flow at an arbitrary position, (y,z) on the pile at an arbitrary phase, t. Similarly, $\dot{u}$ and $\dot{v}$ represent the acceleration components of u and v, respectively.

Here assume that the in-line force which acts on the inclined pile sonsists of the in-line force caused by u and $\dot{u}$, and the lift force caused by v. Similarly, assume that the lift force which acts on the inclined pile consists of the in-line force caused by v and $\dot{v}$, and the lift force caused by u.

Then, the total in-line force, $F_{txB}(t)$ which acts on the inclined pile at an arbitrary phase, t, will be evaluated by the following equation:

$$F_{txB}(t) = \int_{-h}^{\eta} C_{Du} \frac{1}{2}\rho D\, u|u| \sec\theta\, dz + \int_{-h}^{\eta} C_{M\dot{u}} \frac{1}{4}\rho \pi D^2 \dot{u} \sec\theta\, dz$$

$$+ \int_{-h}^{\eta} C_{Lv} \frac{1}{2}\rho (v \sin\theta)^2 \sec\theta\, dz \quad \text{...............} (5)$$

where the first two terms in the right-hand side of Eq.(5) represent the in-line force caused by u and $\dot{u}$, and the last term represents the lift force caused by v.

Similarly, the total lift force, $F_{tLB}(t)$ which acts on the inclined pile at an arbitrary phase, t, will be evaluated by the following equation:

$$F_{tLB}(t) = \int_{-h}^{\eta} C_{Lu} \frac{1}{2}\rho D\, u^2 \sec\theta\, dz + \int_{-h}^{\eta} C_{Dv} \frac{1}{2}\rho D\, v|v| \sin\theta \tan\theta\, dz$$
$$+ \int_{-h}^{\eta} C_{M\dot{v}} \frac{1}{4}\rho \pi D^2\, \dot{v} \tan\theta\, dz + \frac{1}{4}\rho g \pi D^2 \eta \tan\theta \quad \text{..........(6)}$$

where the first term in the right-hand side of Eq.(6) represents the lift force caused by u and the second and third terms represent the in-line force eaused by v and $\dot{v}$. Eqs. (3), (4), (5) and (6) were all derived by referring to the Morison and lift force formulas.

Determination of Coefficient Values

It is known that the coefficients in each of the above equations are functions of certain variables associated with the characteristics of the flow kinematics induced around a pile and of the pile itself. Thus, these coefficients may have different values from one point to another over the submerged portion of the pile.

In this study, however, the representative values of each coefficient over the submerged portion will be determined. Thus, the coefficients in each integral term of Eqs.(3), (4), (5) and (6) can be taken out of each integral sign. Consequently, Eqs.(3) and (4) can be rewritten as follows: rewritten as follows:

$$F_{txA}(t) = C_{DU} F_{DU}(t) + C_{M\dot{U}} F_{I\dot{U}} + F_B(t) \quad \text{...............(7)}$$

$$F_{tLA}(t) = C_{LU} F_{LU}(t) \quad \text{...............................(8)}$$

where $F_{DU}(t)$ and $F_{I\dot{U}}(t)$ in Eq.(7) represent the corresponding integral terms in Eq.(3), and $F_B(t)$ represents the buoyancy term in Eq.(3).

Similarly, Eqs.(5) and (6) can be rewritten as follows:

$$F_{txB}(t) = C_{Du} F_{Du}(t) + C_{M\dot{u}} F_{I\dot{u}} + C_{Lv} F_{Lv}(t) \quad \text{..........(9)}$$

$$F_{tLB}(t) = C_{Lu} F_{Lu}(t) + C_{Dv} F_{Dv}(t) + C_{M\dot{v}} F_{I\dot{v}}(t) + F_B(t) \quad \text{...(10)}$$

where $F_{Du}(t)$, $F_{I\dot{u}}(t)$ and $F_{Lv}(t)$ in Eq.(9) represent the corresponding integral terms in Eq.(5), and $F_{Lu}(t)$, $F_{Dv}(t)$ and $F_{I\dot{v}}(t)$ in Eq.(10) represent the corresponding integral terms in Eq.(6).

As it can be noted from these equations, both time-independent and time-dependent values of each coefficient can be determined once the flow kinematics around an inclined pile and measured values of the wave forces are known. In advance of determining these coefficient values, all integral terms in Eqs.(7), (8), (9) and (10) are calculated at every incremental phase of 0.01 second over a wave cycle, based on the information of flow kinematics estimated by the Stokes third order wave theory.

Further, recorded data of incident waves and wave forces are quantized at the corresponding phases and punched on input cards automatically by a special A-D converter.

1. Coefficient Values for Inclined Piles Placed in Plane A

Based on the data provided previously, the time-independent values of C_{DU} and $C_{M\dot{U}}$ in Eq.(7) are determined in this study by the two-point method and by the least square method which are shown in Table 1. Similarly, the time-independent values of C_{LU} in Eq.(8) are determined by the one-point method and by the least square method.

The time-dependent values of C_{DU} and $C_{M\dot{U}}$ at every incremental phase are also determined in this study by Al-Kazily's method which is shown in Table 1. Similarly, the time-dependent values of C_{LU} are determined by solving Eq.(8) directly at the same incremental phases. The time-dependent coefficients are all expanded into Fourier series in which several significant components are involved. The details of this method has been shown in Shigemura and Nishimura's paper(1979).

2. Coefficient Values for Inclined Piles Placed in Plane B

Eqs.(9) and (10) include three unknown coefficients in themselves. Thus, data should be provided at least at three successive phase for each equation to determine these coefficient values.

The time-independent values of these coefficients are determined by the following procedures:

1. Divide a wave cycle into four equal divisions. Further, determine three phases in each division by dividing it into four equal subdivisions.
2. At each division, establish three equations for both of Eqs.(9) and (10), based on the data provided at the three phases mentioned above.
3. Solve each pair of three equations simultaneously to determine the respective coefficient values in each division.
4. Calculate the arithmetic means of the respective coefficient values obtained in the four divisions.

This method will be called hereafter the "three-point method". The least square method is also used to determine the time-independent values of these coefficients.

The time-dependent values of each coefficient in Eqs.(9) and (10) are also determined by a method similar to the Al-Kazily's method. Namely, three equations are provided first for each of Eqs.(9) and (10), based on the data provided at three successive phases, $(t-\Delta t)$, t, and $(t+\Delta t)$ where Δt is an incremental phase. Two pairs of these three equations are then solved simultaneously under the assumption that each coefficient keeps constant value at the three successive phases. The coefficient values determined by this method are also expanded into Fourier series which consist of several significant components.

Reliability of Coefficient Values

Based on the coefficient values determined, theoretical values of each wave force are calculated at every incremental phase of 0.01 second over a wave cycle by Eqs.(7), (8), (9) and (10), respectively. To determine the reliability of each coefficient value quantitatively, relative deviations of theoretical wave forces from the measured ones are calculated by the following equation:

$$\delta F(\%) = \frac{\sqrt{\sum^{N} [F_m(t) - F_t(t)]^2/N}}{[F_t]_{max} - [F_t]_{min}} \times 100 \qquad \text{.................}(11)$$

In this equation, $F_m(t)$ and $F_t(t)$ represent the measured and theoretical values, respectively, of a wave force at an arbitrary phase, t. Further, $[F_t]_{max}$ and $[F_t]_{min}$ represent the maximum and minimum values, respectively of the theoretical wave force over a wave cycle, and N is sample number used for the calculation of δF. Note that δF also measures the propriety of formulation of each wave force formula.

EXPERIMENTAL SETUP AND PRECEDURES

A 4.5 meters wide, 1.2 meters deep and 12.0 meters long wave channel was used for this experimental study. This wave channel has a flap-type wave generator at the back of the channel. A self-driven truck sits astride the channel so that it can run from the wave generating paddle to the wave absorber installed at the front end of the channel. This truck was locked at a position of 6 meters from the wave generating paddle.

At a point below the main beam of this truck, a chuck was fixed in such a way that it could rotate around an axis parallel to the direction of wave propagation and around an axis parallel to the wave crest. This chuck allows placing a force meter on the beam. A commercialized force meter, or a three-component loadcell was clamped firmly by this chuck. This loadcell is designed to detect the electrical signals produced by two components of forces, F_x and F_y which are perpendicular to each other, and by a component of the bending moment caused by either F_x or F_y, simultaneously. This force meter was connected to an oscillograph through proper amplifiers.

A total of twelve model piles used in this experiment were made of aluminum and acrylic acid resin tubes having circular cross sections. The upper portion of each pile was fabricated so that it could be connected to the force meter, and its lower portion was cut off so that the cut face would be parallel to the bottom of the channel when the pile was connected to the force meter. The cut face of each pile was also shielded by vinyl sheet to avoid edge effects.

It was decided to give seven different angles of 0°, ±10°, ±20° and ±30° to each model pile placed in an allotted plane. These angles are those measured against an axis normal to the still water surface. Here, the plus sign in front of each angle indicates that a pile is inclined away from the direction of wave propagation and the minus sign means that a model pile is inclined towards the direction of wave propagation. This is the case when a model pile is placed in Plane A. In the case of model pile placed in Plane B, the plus sign indicates that a model pile is inclined towards the left side of the wave channel when one views the channel from the side of wave absorber, and the minus sign means that the pile is inclined towards the right side of wave channel. It was also decided to perform all tests at a constant water depth of 0.8 meters and seven waves were chosen as the experimental waves. Table 2 summarizes experimental conditions used in this study.

Table 2. Experimental conditions used in this study.

Characteristics of Model Piles				
Materials of Model Pile		Aluminum Tube	Acrilic Resin Tubes	
Outer Oia. of Model Pile(cm)		2.2	3.0	4.0
Inner Oia. of Model Pile(cm)		1.9	2.2	3.2
Placement of Model Piles				
Planes for Placing Model Pile		Plane A and Plane B		
Inclining Angles of Model Pile		0°, ±10°, ±20° and ±30°		
Characteristics of Experimental Waves				
Wave No.	Wave Period T(sec)	Wave Hight H(cm)	H/L	h/L
1	0.86	2.8	0.024	0.693
2	0.94	4.4	0.032	0.576
3	1.02	5.9	0.036	0.489
4	1.14	6.7	0.033	0.395
5	1.27	8.3	0.034	0.325
6	1.46	8.3	0.027	0.257
7	1.69	9.2	0.024	0.208

A total of 273 tests were conducted. In each test, both in-line and lift forces ex erted on the model pile were recorded on the oscillograph. Characteristics of the incident waves were also measured simultaneously by a capacitance-type wave gage which was placed at a position 15 cm from the right side of the model pile by aligning the gage front carefully with the intersection of model pile and still water surface, and were recorded on the same oscillogragh.

RESULTS AND DISCUSSIONS

Values of Drag, Mass and Lift Coefficients for Inclined Piles

Time-independent and time-dependent values of drag, mass and lift coefficients were determined for inclined piles placed in both Plane A and Plane B, by using the methods described in the previous Chapter, and characteristics of these coefficient values were studied.

1. Time-Independent Values of Drag, Mass and Lift Coefficients for Inclined Piles Placed in Plane A

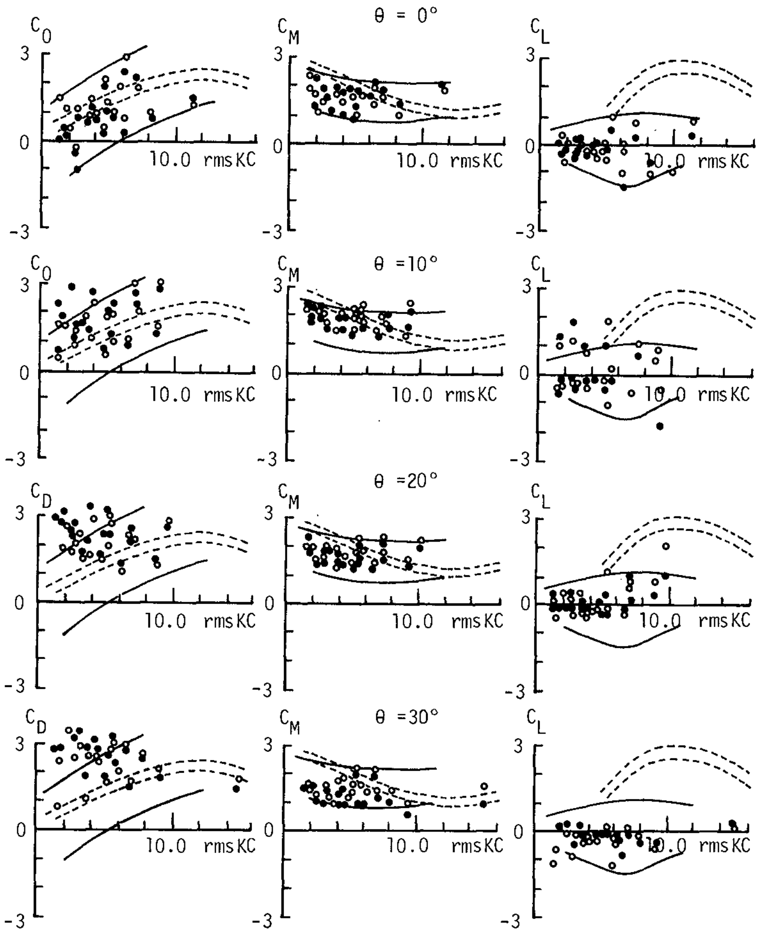
Time-independent values of the drag and mass coefficients in Eq.(7) were determined by the two-point method and by the least square method, respectively. Similarly, those of the lift coefficient in Eq.(8) were determined by the one-point method and by the least square method, respectively. For the convenience of later discussion, the following notations were given to the values of these coefficients:

C_{DU2}:value of the drag coefficient determined by the two-point method.
$C_{DU\ell}$:value of the drag coefficient determined by the least square method.
$C_{M\dot{U}2}$:value of the mass coefficient determined by the two-point method.
$C_{M\dot{U}\ell}$:value of the mass coefficient determined by the least square method.
C_{LU1}:value of the lift coefficient determined by the one-point method.
$C_{LU\ell}$:value of the lift coefficient determined by the least square method.

As mentioned previously, it has been pointed out by many researchers that the time-independent values of drag, mass and lift coefficients for either vertical or horizontal piles have some functional relationships with the period parameter or the Keulegan-Carpenter number. To determine whether this is true in the case of the coefficient values for inclined piles, coefficient values determined were plotted against the value of the rmsKC number, at every inclination of the pile. Here, the rmsKC number is the Keulegan-Carpenter number evaluated by $\sqrt{\bar{U}^2}\ T/D$ where $\sqrt{\bar{U}^2}$ is the root mean square of the velocity component, U, T is a wave period and D is a pile diameter.

Figure 3 shows the plots of the coefficient values determined for the piles inclined away from the direction of wave propagation. In this figure, the dotted lines show the Sarpkaya's results(1975) which were obtained for the piles placed horizontally in oscillatory flows, and the solid lines represent the variation ranges of each coefficient obtained in this study for the vertical piles. From this figure, the following facts were noted:

(1). Both C_{DU2} and $C_{DU\ell}$ increase their values first as the rmsKC number



o:variation curves of each coefficient determined by either the one-point or the two-point method.
•:variation curve of eavh coefficient determined by the least square method.
-----:variation curves found by Sarpkaya for horizontal piles.
——:variation curves found by the writer for vertical piles(θ=0°).

Figure 3. Variations of the coefficient values versus rmsKC number for circularpiles placed in Plane A at plus inclination.

increases its value up to approximately 10.0, then start decreasing their values as the rmsKC number increases its value farther, although values of both coefficients scatter considerably.

(2). Variations of C_{DU2} and C_{DUL} values versus rmsKC number agree considerably well to that found by Sarpkaya except for some data obtained for the piles inclined at 20° and 30°. It should be noted that these coefficients tend to decrease their values as the rmsKC number increases its value up to approximately 6.0.

(3). Both $C_{M\dot{U}2}$ and $C_{M\dot{U}L}$ decrease their values first as the rmsKC number increases its value up to approximately 10.0, then start increasing their values as the rmsKC number increases its value farther. In this case, both coefficient values do not scatter as greatly as the values of drag coefficient.

(4). Variations of $C_{M\dot{U}2}$ and $C_{M\dot{U}L}$ values versus rmsKC number agree considerably well to that found by Sarpkaya although inflection points of each variation curve appear at smaller value of the rmsKC number than the value of the KC number at which inflection point appears in the variation curve found by Sarpkaya. Further, distinct effect of pile inclination is not found on the variations of these coefficient values versus rmsKC number.

(5). Many of C_{LU1} and C_{LUL} take minus values over the whole range of the rmsKC number. Further, the values of both coefficients scatter considerably over the whole range of the rmsKC number.

(6). Variations of C_{LU1} and C_{LUL} values versus rmsKC number do not show any agreement with the variation curve found by Sarpkaya, although distinct effect of the pile inclination is not found on the variations of these coefficient values versus rmsKC number.

Similar plots were also made for the coefficient values of the piles inclined towards the direction of wave propagation. These plots found that variations of each coefficient value versus rmsKC number were quite similar to those found for the coefficient values of the piles inclined away from the direction of wave propagation except for the fact that many of the drag coefficient took minus values in the range where the value of the rmsKC number was smaller than approximately 6.0.

These facts found above may be caused partially by the following reasons:

(1). Coefficient values determined here are a sort of representative values averaged over the submerged portions of inclined piles. Thus, effects of the water depth for these coefficient values are not evaluated properly.

(2). The rmsKC number is used for the analysis of these coefficient values instead of the conventional period parameter.

(3). Experiments have been conducted in the range of relatively small values of rmsKC number where the inertia force is predominant in comparision to the drag force.

2. Time-Independent Values of Drag, Mass and Lift Coefficients for Inclined Piles placed in Plane B

Time-independent values of drag, mass and lift coefficients in Eqs.(9) and (10) were determined by the three-point method and by the least square method, respectively. Many of these coefficients, however, took incredibly large values. This might be caused by the defect of the determination methods which require each force term in Eqs.(9) and (10) to share equal weight for the determination of each coefficient value regardless to its magnitude. Then, the following ratios were calculated to find some information which help minimize this defect:

$$RX1(t) = F_{Du}(t)/F_{mxB}(t) \times 100(\%)$$
$$RX2(t) = F_{I\dot{u}}(t)/F_{mxB}(t) \times 100(\%)$$
$$RX3(T) = F_{Lv}(t)/F_{mxB}(t) \times 100(\%)$$
$$RL1(t) = F_{Lu}(t)/F_{mLB}(t) \times 100(\%)$$
$$RL2(t) = F_{Dv}(t)/F_{mLB}(t) \times 100(\%)$$
$$RL3(t) = F_{I\dot{v}}(t)/F_{mLB}(t) \times 100(\%)$$

In the above equations, $F_{mxB}(t)$ represents measured value of the in-line force exerted on a pile placed in Plane B at an arbitrary phase, t, and $F_{mLB}(t)$ represents measured value of the lift force on the same pile at the same phase. Hence, these six ratios measure the relative magnitude or the contribution rate of each force term in Eqs.(9) and (10) to the corresponding wave forces. These calculations found that values of RX3(t) were always quite small comparing to the values of the other ratios. Namely, the maxumum values of RX3(t) within a wave cycle were always smaller than 5% when piles were inclined at an inclination less than or equal to 20°. Further, some of RX3(t) surely exceeded 5% when piles were inclined at 30°, but they did not exceed 10% in these cases.

Based on this finding, it was assumed that contribution rate of $F_{Lv}(t)$ in eq.(9) to its total in-line force, $F_{txB}(t)$ is negligibly small in the cases of runs where the maximum value of RX3(t) is less than 5%, and the values of only C_{Du} and $C_{M\dot{u}}$ in Eq.(9) were determined in these cases again by the two-point method and by the least square method, respectively. For determining the coefficient values in Eq.(10), it was further assumed that the values of $C_{M\dot{v}}$ in Eq.(10) are approximately equal to the values of $C_{M\dot{u}}$ determined previously since values of $C_{M\dot{u}}$ were almost constant over the range of rmsKC number experienced in this study. Based on this assumption, values of only C_{Lu} and C_{Dv} in Eq.(10) were determined again by the two-point method and by the least square method, respectively.

Coefficient values determined by these procedures were also plotted similarly against the values of the rmsKC number. As a result, it was found that the values of thses coefficients also showed quite similar variations to those of the corresponding values of the coefficients found for the inclined piles placed in Plane A.

3. Time-Dependent Values of Drag, Mass and Lift Coefficients for Inclined Piles

Time-dependent values of drag, mass and lift coefficients for inclined piles placed in Plane A were determined by the same methods as described in the previous Chapter. Namely, time-dependent values of drag and mass coefficients in Eq.(9) were determined by the Al-Kazily's method, and those of the lift coefficient in Eq.(8) were determined by solving Eq.(8) at every incremental phase of 0.01 second over a wave cycle.

In the determination of time-dependent values of each coefficient for the piles placed in Plane B, however, coefficient values were determined by using the same procedures as developed previously for the determination of their time-independent values. Namely, the time-dependent values of each coefficient in Eqs.(9) and (10) were determined by the Al-Kazily's method when the contribution rate of each force term was less than 5%, although coefficient values were determined by the three-point method described in the previous Chapter when the contribution rates of all force terms were greater than 5%.

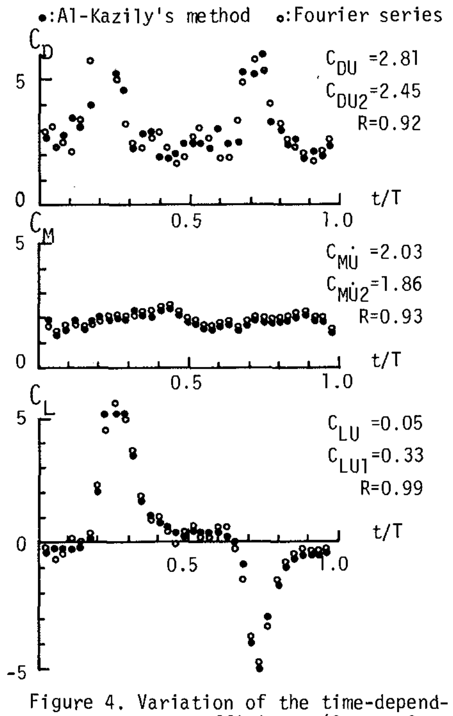


Figure 4. Variation of the time-dependent coefficients (θ = +20°, D = 4.0cm, rmsKC No. = 2.80)

Coefficient values determined by these methods were expanded into Fourier series which consist of six significant components whose frequencies are the multiples of the frequency of incident wave. As a result, it was found that correlation coefficients between determined values of each coefficient and the values of the corresponding coefficient estimated by the Fourier series reached approximately 0.8 in average.

Fiture 4 shows some comparisons of the time-dependent values of each coefficient determined by the Al-Kazily's method, with those estimated by the Fourier series mentioned above. It can be noted clearly from this figure that coefficient values are not constant but vary considerably within a wave cycle, and that Fourier series of significant components can trace determined values of each coefficient quite satisfactorily.

Reliability of the Coefficient Values

To check the reliability of the time-independent and time-dependent values of each coefficient determined above, quantitatively, relative deviations of the calculated wave forces from the measured ones were computed by Eq.(11) which was shown previously. This computation was made for both in-line and lift forces exerted on all piles placed in Plane A and Plane B. To distinguish these relative deviations, two suffixes were further added to the notations indicating the relative deviations of the in-line force, δF_X and that of the lift force, δF_L, respectively. Namely, the first suffix of the two indicates the plane where piles were placed: suffix [A] represents Plane A; and suffix [B] represents Plane B. On the other hand, the second suffix indicates the coefficients used for the calculation of wave force: suffix [1] represents the coefficeint determined by the one-point method; suffix [2] represents the coefficients determined by the two-point method; suffix [L] represents the coefficients determined by the least square method; and suffix [F] represents the coefficients determined by the Fourier series of six significant components. These values were plotted against the rmsKC number at every inclination of a pile.

Figure 5 shows the relative deviations of the in-line force exerted on the piles placed in Plane A and Plane B at plus inclinations, respectively. Namely, the left half of this figure shows the variations of relative deviations versus rmsKC number in the cases when the time-independent coefficients were used for the calculation of in-line force, and the right half of it shows the variations of the relative deviations versus rmsKC number in the cases when the time-dependent coefficients determined by the Fourier series were used for the calculation of in-line force. Form this figure, the following facts were noted:

(1). Relative deviations calculated basing on the time-independent coefficients are less than 10% mostly, regardless of the placing plane or the inclination of the piles, although some of them exceed 10% in the case of vertical pile($\theta = 0°$). It is further noted that these relative deviations tend to increase slightly as the rmsKC number increases although some scatters are found among them in the range of the rmsKC number smaller than approximately 4.0.

(2). Relative deviations calculated basing on the coefficients determined by the least square method are always few percents smaller than those calculated basing on the coefficients determined by the two-point method, regardless of the placing plane or the inclination of the piles.

(3). Relative deviations calculated basing on the time-dependent coefficients determined by the Fourier series are less than 5% mostly over the whole range of the rmsKC number, regardless of the placing plane or the inclination of the piles.

Similar facts were also found in the cases when piles were placed at minus inclinations. These facts found above may indicate that the time-independent coefficients can be used for the prediction of the in-line force exerted on the inclined piles although usage of the time-dependent coefficients is more desirable for that purpose. These facts may also

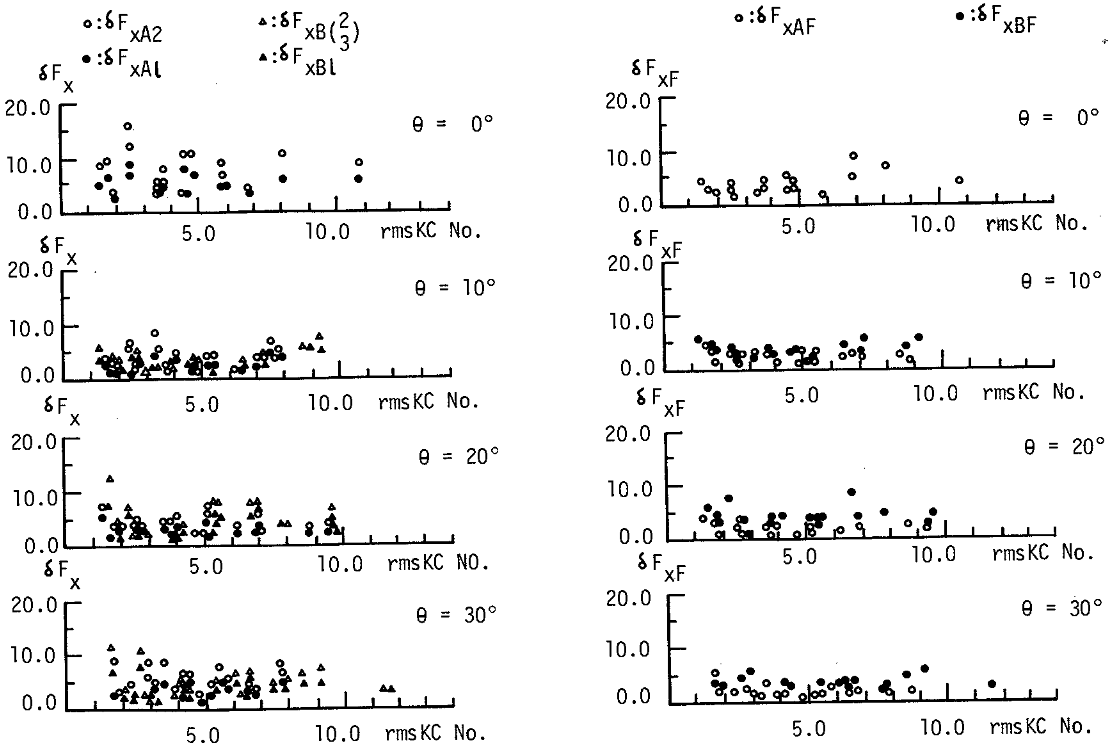


Figure 5. Relative deviation of in-line force plotted against rmsKC number.

endorse the propriety of wave force formulas formulated in this study for the inclined piles.

Figure 6 shows the relative deviations of the lift force exerted on the piles placed in Plane A and Plane B at plus inclinations, respectively. Namely, the left half of this figure summarizes variations of the relative deviations versus rmsKC number in the cases when the time-independent coefficients were used for the calculation of the lift force, and the right half of it summarizes those in the cases when the time-dependent coefficients determined by the Fourier series were used. From this figure, the following facts were noted:

(1). Relative deviations calculated basing on the time-independent coefficients scatter considerably over the whole range of the rmsKC number, regardless of the placing plane or the inclination of the piles, and some of them exceed 200%.

(2). Relative deviations calculated basing on the time-dependent coefficients are decreased significantly, irrespective of the placing plane or the inclination of the piles. Further, they tend to decrease as the rmsKC number increases, and most of them become approximately 10% when the rmsKC number reach 10.0.

Similar facts were also found in the cases when piles were placed at minus inclinations. The above facts may indicate that considerable amount of error would be induced if the time-independent coefficients are used for the prediction of lift force acting on the inclined piles, and that it is desirable to use some sorts of time-dependent coefficients such as the ones determined by the Fourier series for the prediction of lift force acting on the inclined piles.

Ratio of the Maximum Lift Force to the Maximum In-Line Force

Bidde(1971) conducted laboratory tests on the lift force acting on vertical circular piles and found that ratio of the lift force to the in-line force increased steadily as KC number increased, and that the ratio reached about 0.6 at the KC number of approximately 15 although it stopped increasing at the range of KC number greater than 15.

Sarpkaya(1975) also measured the lift force acting on horizontal circular piles and found that the ratio of the maximum lift force to the maximum in-line force varied with two humps while KC number increased up to about 30 although it kept almost constant value of about 0.8 at the range of KC number greater than 30. Further, he found that the ratio reached the maximum value of about 1.3 at one of the humps which appeared at the KC number of about 18.

In this study, ratio of the maximum lift force to the maximum in-line force recorded within a wave cycle were checked for all test cases and plotted at every inclination against the rmsKC number. Figure 7 shows these ratios found in the cases of the tests where piles were placed at plus inclination. In this figure, solid lines show the variation range of the ratio found by Bidde(1971). From this figure, the following facts were noted:

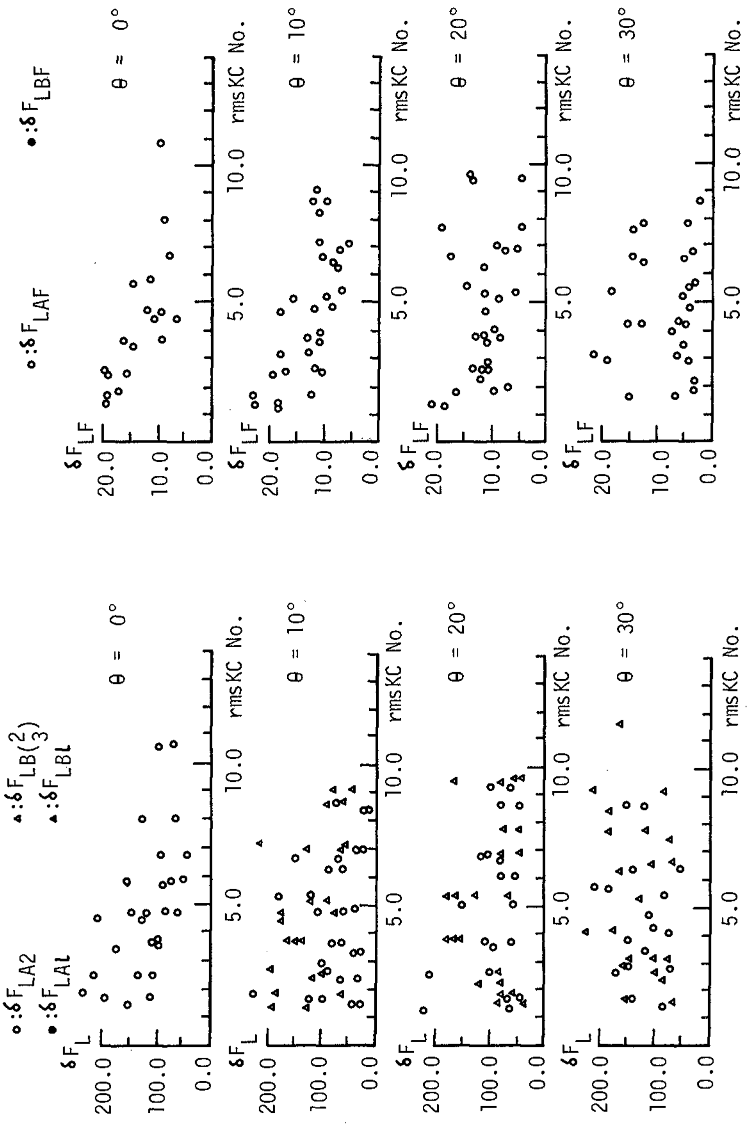


Figure 6. Relative deviation of lift force plotted against rmsKC number.

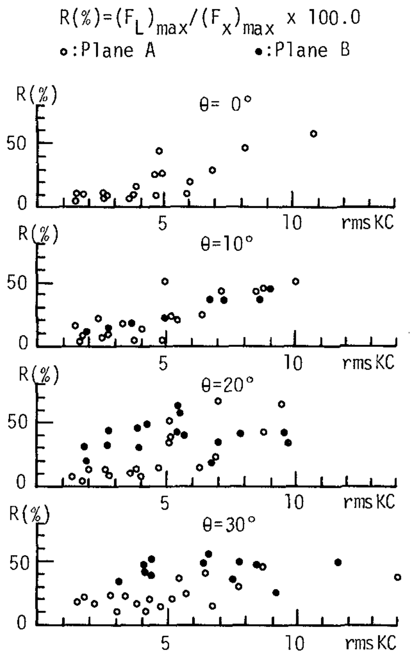


Figure 7. Ratio of the maximum lift force to the maximum in-line force versus rmsKC number(in cases of plus inclinations).

(1). Ratio of the maximum lift force to the maximum in-line force tend to increase generally as the rmsKC number increases. Further, variations of the ratios versus rmsKC number agree quite well to the upper envelope of the variations found by Bidde, when piles are inclined at the inclination equal to or less than 10°.

(2). When piles are inclined at the inclination equal to or greater than 20°, some scatters appear among the values of the ratios for the piles placed in Plane B. Namely, in this case, the ratios become almost constant ranging from approximately 0.3 to 0.6.

Similar facts were also found in the cases where piles were inclined at minus inclination.

The fact described in (2) may be caused partially by the effect of vertical velocity component of the induced flow. In any rate, readers should recognize the fact that considerably large lift force may possibly appear even at the small value of rmsKC number when piles are inclined more than 20°.

CONCLUSIONS

Both time-independent and time-dependent values of drag, mass and lift coefficients for inclined piles were determined by several methods, based on the respective wave force formulas which were derived by referring to the conventional Morison and lift force formulas. Reliability of these coefficient values were examined by checking the relative deviations of the calculated wave forces from the measured ones. Further, ratio of the maximum lift force to the maximum in-line force was also studied. As a result, the following conclusions were drawn:

(1). Time-independent values of drag and mass coefficients can be used for the prediction of in-line force acting on the inclined piles. Relative deviation calculated basing on these coefficients increases as the rmsKC number increases, but it will be 10% at most if the rmsKC number is smaller tha about 12.0.

(2). Time-independent values of lift coefficient can not be used for the prediction of lift force acting on the inclined piles. If they are used for prediction, relative deviation is about 100% in average over the whole range of rmsKC number. Lift coefficient determined by the Fourier series of six significant components will be a good one to be used for the prediction of lift force. If this coefficient is used for prediction, relative deviation is 20% at most. Further, it will decrease as the rmsKC number increases.

(3). Ratio of the maximum lift force to the maximum in-line force increases slightly as the rmsKC number increases, and variation of the ratio versus rmsKC number agrees quite well to the upper envelope of the variation of the ratio versus KC number found by Bidde, as long as the pile inclination is equal to or less than 10°. However, considerably large value of the ratio is found at the small value of rmsKC number if the piles are inclined more than 20°.

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