CAN WE DEVELOP NEW BREAKWATER ARMOUR FORMULAE ?

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Introduction

The purpose of this paper is to draw attention to the limitations of present empirical methods for the design of rubble mound breakwaters, particularly those using concrete armour units. Although it is unlikely that analytical techniques could become an adequate substitute for model testing, they should contribute significantly to a major advance in our understanding of breakwater behaviour. This is now long overdue and the art of breakwater design should be capable of substantial development.

Hudson's Formula

$$W_r = \frac{\gamma_c . H^3}{K_D \left(\frac{\gamma_c}{\gamma_w} - 1\right)^3 \operatorname{Cot} \alpha} \qquad (1.)$$

During the last quarter of a century, most engineers and research workers have used Hudson's formula (ref: 1) for comparing different types of breakwater armour and for other purposes such as the design of simple breakwaters and the interpretation and correction of hydraulic model tests. Other formulae, such as Iribarren's formula have also been used to a lesser extent.

As new types of armour unit have been developed, their effectiveness has generally been measured by quoting the value of K_D in Hudson's formula in relation to their behaviour in model tests. However, Hudson's formula was originally developed to represent the behaviour of natural rock-type materials which retain their stability under wave action principally by their own weight and without any significant interlock with adjacent units.

In representing the behaviour of armour units of this basic type, Hudson's formula has been a very convenient tool even though in a single formula it over simplifies a very complex situation.

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The development of new types of artificial armour units, which to varying degrees do not behave in the same way as rock, has highlighted the limitations of Hudson's formula. The best-known such unit is the dolos which relies principally upon its interlock with a number of surrounding and underlying units to prevent displacement. For this reason dolos units may be less than a third of the weight of the natural rocks required to resist similar conditions. The unsuitability of the Hudson formula for dolos units has long been apparent (refs: 2, 3 & 4) e.g. it is necessary to quote different K_D values according to slope for these units. This has made it very difficult to interpret the results of different model tests.

It is also important to consider whether there are any other factors influencing stability which were not found to be of significance when Hudson's formula was developed for gravity-type armouring and which become of increased importance with units in which interlock is a major or predominant feature. There are indications from recent model testing (ref: 5 & 6) that contact friction between armour units could be one such important factor and that wave steepness may well be another.

What is now becoming very clear is that breakwater designers must appreciate the circumstances under which Hudson's and Iribarren's formulae were developed and the conditions for which they were designed. To use these formulae out of their original context is likely to be misleading and even dangerous. The application of Hudson K_D values to dolos and other interlocking units is convenient in comparing their weight-for-weight effectiveness against more traditional forms of breakwater armour; however, we must not be deceived into thinking that Hudson's formula therefore represents their behaviour or that such comparisons are valid.

Although most engineers would only advocate use of Hudson's formula for direct design purposes in the simplest cases, and would prefer to rely upon specific model tests for any important breakwaters, they would nevertheless in most cases have recourse to Hudson's formula for correcting hydraulic model test results for errors in modelling such as in water and material densities.

However, the fact that certain variables such as contact friction (or natural angle of repose) are not included in the formula does not imply that they are necessarily insignificant nor that they may be ignored with impunity when constructing and operating hydraulic models.

Before considering what other phenomena may be important to breakwater stability and then suggesting how breakwater design techniques might be developed, it is worth studying in some detail the development and derivation of Iribarren's formula.

Development and Derivation of Iribarren's Formula

The formula was first published in 1938 (ref: 7) in the form:

$$P = \frac{N A^3 d}{(f \cos \alpha - \sin \alpha)^3 (d - 1)^3} \qquad (2)$$

It followed an earlier formula by Castro in 1933 (ref: 9).

A number of modifications have been suggested to the original formula. Some of these are expressed below using symbols defined in the legend

$$Wr = \frac{K' \gamma_c \mu^3 H^3}{(\mu \cos \alpha - \sin \alpha)^3 (Sr - 1)^3} \qquad (3)$$

after Hudson (ref: 1)

$$Wr = \frac{\gamma_c \mu^3 H^2 Lp}{Ko \left(\frac{\gamma_c}{\gamma_w} - 1\right)^3 (\mu \cos \alpha - \sin \alpha)^3} \qquad (4)$$

after Gravesen et al (ref: 6)

The derivation of the term $(\mu \cos\alpha - \sin\alpha)$ shown in figures 1 and 2, is very simple but is worth mentioning to clarify thinking.

This derivation represents the stability of an isolated block tending to slide down a slope (α) with a coefficient of friction between the slope and block of $\mu = \tan \phi$ where ϕ is the angle of repose. Compared with $\cot\alpha$ in Hudson's formula it has the merit that $(\mu\cos\alpha - \sin\alpha)$ tends to zero as α approaches the angle of repose ϕ; the weight of armour required therefore tends to the infinite.

Let us now remind ourselves of the derivation of the other main components of Iribarren's formula.

Disturbing force

Water velocity $V \propto \sqrt{gH}$

Drag force $\propto$ M.V.Area

or $\gamma_w . V.V. \left(\frac{W}{\gamma_c}\right)^{2/3}$ or $\gamma_w g H \left(\frac{W}{\gamma_c}\right)^{2/3}$ (5)

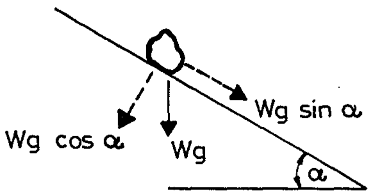
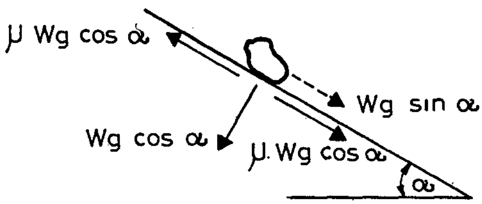


FIG. 1.



For simple shapes μ = coefficient of friction

FIG. 2.

Force required for movement

Sliding down - $Wg\,(\mu \cos \alpha - \sin \alpha)$

Sliding up - $Wg\,(\mu \cos \alpha + \sin \alpha)$

If submerged $\times \dfrac{(\gamma_c - \gamma_w)}{\gamma_c}$

Stabilising force (resisting down-rush)

$$\frac{Wg}{\gamma_c} (\gamma_c - \gamma_w) (\mu \cos \alpha - \sin \alpha) \quad (6)$$

Equating these and re-arranging

$$\gamma_w g H \left(\frac{W}{\gamma_c}\right)^{2/3} \propto \frac{Wg}{\gamma_c} (\gamma_c - \gamma_w)(\mu \cos\alpha - \sin\alpha) \quad (7)$$

$$W^{1/3} \propto \frac{\gamma_w H \gamma_c^{1/3}}{(\gamma_c - \gamma_w)(\mu \cos\alpha - \sin\alpha)}$$

or $$W \propto \frac{\gamma_c H^3}{\left(\frac{\gamma_c}{\gamma_w} - 1\right)^3 (\mu \cos\alpha - \sin\alpha)^3} \quad (8)$$

It will be seen that the derivation of the modified form of the formula clearly represents the stability of the slope in resisting drag forces caused by wave down-rush. The formula does not represent the effects of impact forces on the armour units, nor does it consider stability of units subject to uprush (Hedar drew attention to this in 1960).

Some further points may be worth mentioning.

It will be seen that the third power relationship between W and most of the other factors arises from the basic fact that the main stabilising forces must be proportional to the volume of the units whilst the main disturbing forces must be proportional to the exposed area.

If W were not proportional to H^3 it would not, of course, be possible to use hydraulic models for testing breakwaters. This involves an assumption that the pressures of water on the armour units are proportional to the height of approaching waves and that the velocity of water within the armour layer is proportional to $\sqrt{H}$. An examination of the limitations of this assumption would be a fruitful area for further research and with the great advances of recent years in mathematical analysis of waves, this may perhaps be an area for mathematical research.

In the case of wave uprush it should also be considered whether impact forces obey the same relationship and over what range of conditions.

If the function $\left(\frac{\gamma_c}{\gamma_w} - 1\right)^3$ is now considered it will be seen that it comes partly from the disturbing force and partly from the stabilising. The function does, however, appear to be fundamentally appropriate to the third power as in the Hudson and Iribarren formulae.

In 1972 Zwamborn (ref: 2 and 3) advanced the view that some other power may be more correct in the case of dolosse. One might rather question whether dolosse used in tests with different specific gravities may also have had some other differences such as different co-efficients of friction as this could provide an explanation of the effects reported by Zwamborn.

Factors which might affect Slope Stability

In considering the possibility of improving on present techniques of designing rubble mound slopes and the need to develop new tools for new types of armour unit and new applications, we must take a fundamental look at breakwater behaviour. Although the stabilising and disturbing forces must be related to the factors included in Hudson's and Iribarren's formulae, we can identify a number of other phenomena which may be of differing relative importance for different types of armour and in different circumstances but which do not appear in these formulae. These would include:-

(a) Wave uprush.

(b) Effect of surface slope angle and porosity on the wave behaviour.

(c) Effect of bed depth and slope on the wave behaviour.

(d) Incident wave steepness.

(e) Dynamic and static stresses in armour units.

Development of New Formulae

If one starts to break the problem into its constituent parts it soon becomes clear that the single K_D factor of Hudson or the comparable factor in any of the versions of Iribarren cannot be expected to behave as a constant even for a particular armour unit. If we separate K_D into a number of functions (not factors) related to each element of the problem, each of these functions is necessarily a complex combination of many different variables.

Figs: 3 and 4 illustrate the problem and indicate functions f_1, f_2 .... f_7. The different elements which can be expected to enter into each function are tabulated in Tables 1 and 2.

In addition to functions f_1 to f_7 which relate to different aspects of stabilising and disturbing forces, one more function, f_8 has been added to represent the damage condition at which the formula is being applied.

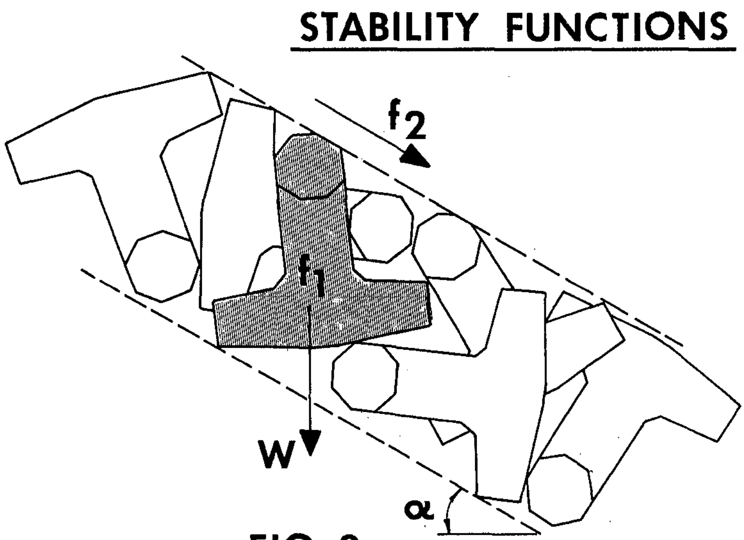

STABILITY FUNCTIONS
f2
f1
W
α

FIG. 3

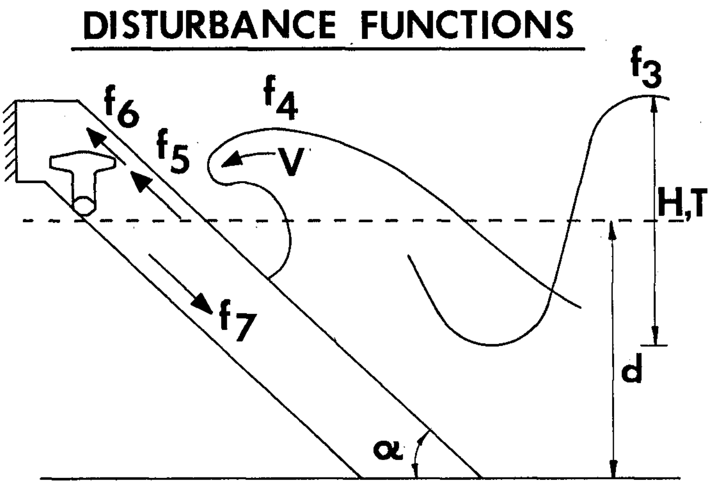

DISTURBANCE FUNCTIONS
f3
f4
f6
f5
V
H,T
f7
d
α

FIG. 4

TABLE 1 - Stability Functions

Function	Adjusting	Possible Factors
f_1	Effect of W as stabiliser (downrush)	1. Unit type 2. Interlock achieved 3. Surface friction
f'_1	Same for uprush	Similar
f_2	Effect of interlock (downrush)	1. Unit type 2. Method of placing 3. Surface friction 4. Extent to which disturbing force acts on adjacent units
f'_2	Ditto (uprush)	Similar but 4 may be very different
S	Factor of safety	Possibly made up from S_1. related to block type/size S_2. related to quality of wave data

TABLE 2 - Disturbance Functions

Function	Adjusting	Possible Factors
f_3	H approaching	H/d Period T (or steepness) Bed slope Reflections, α
f_4	V Velocity at face of armour	Type of breakwater, bed slope, wave steepness, resonance effects, α , porosity, crest detail
f_5	Drag coefficient	Shape of arms of unit Shape of voids
f_6	Uprush velocity in armour	Slope type of unit, porosity, roughness, armour underlayers
f_7	Downrush velocity	As for f_6, plus wave period or resonance effects crest detail
f_8	Degree of damage	Storm duration Wave spectrum Wave grouping

If we write the Iribarren formula in a form which keeps the stabilising and disturbing forces separate and insert the various functions f_1 .... f_8 and a factor of safety S these become clearer.

$$\frac{Wg}{\gamma_c}(\gamma_c - \gamma_w)(\mu\cos\alpha - \sin\alpha) \propto \gamma_w g H \left(\frac{W}{\gamma_c}\right)^{2/3} \qquad (7)$$

$$f_1 W(\gamma_c - \gamma_w)\, f_2(\mu\cos\alpha - \sin\alpha) = S\ f_3 f_4\ H\ f_5 f_6 f_7 f_8 \frac{W^{2/3}}{\gamma_c^{2/3}} \qquad (9)$$

We may also consider the up-rush case as below:-

$$f_1' W(\gamma_c - \gamma_w)\, f_2'(\mu\cos\alpha + \sin\alpha) = S f_3 f_4\ H f_5 f_6 f_8\ \frac{W^{2/3}}{\gamma_c^{2/3}} \qquad (10)$$

In equations 7, 9 and 10 the functions ($\mu\cos\alpha \pm \sin\alpha$) are included to emphasise the point that up-rush and down-rush are different effects. It must, however, be admitted that these functions of μ and α may not be correct and that it may be better to consider f_2 and f_2' separately.

Looking at equations 9 and 10 together with Tables 1 and 2, there will be many whose immediate reaction will be to despair of ever finding a solution. Indeed, in the sense of finding a single formula for all conditions, we may expect this to prove impossible.

In the same way that a structural engineer does not expect to use a single formula to analyse bending, shear, bond and torsion, we cannot expect to develop a single formula to represent all the phenomena involved in wave action on armoured slopes. We must therefore expect the development of a range of interrelated design formulae. In the meantime we must recognise the uses and the limitations of Hudson's and Iribarren's formulae.

Further Research

It is advocated that research into this subject should be carried out in a coordinated manner so that:-

(i) The new formulae can be developed as rapidly as possible.

(ii) Costs of the necessary model testing can be minimized and the greatest benefit obtained.

(iii) Testing methods and procedures may be standardized.

In addition to development of the formulae, it will be necessary to carry out statistical studies. Even if we identify and correct the empirical coefficients for all the phenomena and influences which are identified, we shall not be able to correct for all the random elements involved. We may therefore expect a certain scatter in our test results and in the performance of prototypes. It will be necessary to have some sort of quantification of the risk element involved in using the formulae.

To commence development of new formulae it is advocated that research should be directed at determining the effect of each of the many parameters (such as slope, water depth and wave steepness) on the stability of the armour layer - not by just adding for each parameter one new function which attempts to represent all the effects of varying that parameter, but rather by considering separately the many different effects. As we use hydraulic models for design of breakwaters it is natural to use then to test the effect of varying the different factors which should be taken into account in any stability formula. This method has serious weaknesses, in particular:-

(1) It can be extremely difficult to vary one factor while keeping all others constant.

(2) The results of flume tests, in the form of numbers of units moving or displaced, necessarily show an enormous scatter. This must to a large extent be because of the human element involved in their measurement; the counting of units relies very largely on the judgement of those conducting the tests. The results are not as accurate as the numbers might imply.

(3) The inherent soundness and stability of the model is likely to vary each time the model is rebuilt for new tests. Results are likely to vary randomly from test to test.

As indicated in Tables 1 and 2, changing one factor such as face slope can have a number of quite unrelated effects on different aspects of the disturbing and the stabilising forces.

This paper has indicated that the path ahead will not be easy but that it is important that we should at least attempt it. The means of progress will be an integrated programme of both theoretical research and model testing.

In planning such research it is suggested that effort should be directed chiefly to the problems of armouring rubble mound breakwaters in deep water. The work may well cast light also on the behaviour of breakwaters in the depths where breaking of waves limits the severity of wave attack but this is an area where experience gives engineers a better basis for design (even though not as yet a rational analysis).

Having posed the general problem in broad terms, it may be helpful to discuss in more detail three aspects which have not always received sufficient attention. These are the importance of contact friction, the wave shape characteristics and structural strength.

Importance of Contact Friction

When natural rock armouring is represented in scale model testing, natural stone of suitable shape and size is used, preferably from the quarry from which the armour will be obtained. Its contact friction is similar in value to the rock it models. Similarly, early tests of concrete armour units were carried out using model units of cement mortar which again gave reasonable representation of contact friction. New techniques have recently been developed for moulding model armour units from other materials (ref: 10 and 11). These techniques are quicker and cheaper and are more reliable in terms of dimensional accuracy and specific gravity of the units. However, these model units often have very different contact friction values to the concrete armour which they represent and laboratory tests (ref: 5 and 6) show that this difference has a major influence on slope stability. This is hardly surprising as a change in contact friction alters the direction of the forces acting between units at the limiting condition when movement occurs; it must therefore alter the natural angle of repose of the units and this has been shown to be so. Methods to measure the natural angle of repose of an armour slope are described by Iribarren (ref: 8), Hedar (ref: 12) and Gravesen (ref: 6).

Fig. 5 shows the relationship between angle of repose and angle of contact friction for dolos units with extrapolation carried out for maximum and minimum repose angles. The curve is based on the values obtained for concrete mortar and DHI*plastic units (ref. 6). Tests on model dolosse from two other laboratories (Types A and B) have given lower values of contact friction although measurement of their natural angles of repose was not possible owing to the few samples available. (The values of contact friction were similar to those reported by Gravesen for porcelain and glazed porcelain). Angles of repose for types "A" and "B" have been estimated by reference to possible extrapolations of values on the curve of Fig. 5.

Taking Iribarren's Modified Formula (3) and rearranging it we get:-

$$\frac{(\mu \cos\alpha - \sin\alpha)}{\mu} = H \sqrt[3]{\frac{K'\gamma_c}{W_r(S_r-1)^3}} \qquad (11.)$$

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RELATIONSHIP BETWEEN ANGLE OF REPOSE AND ANGLE OF CONTACT FRICTION FOR DOLOS UNITS

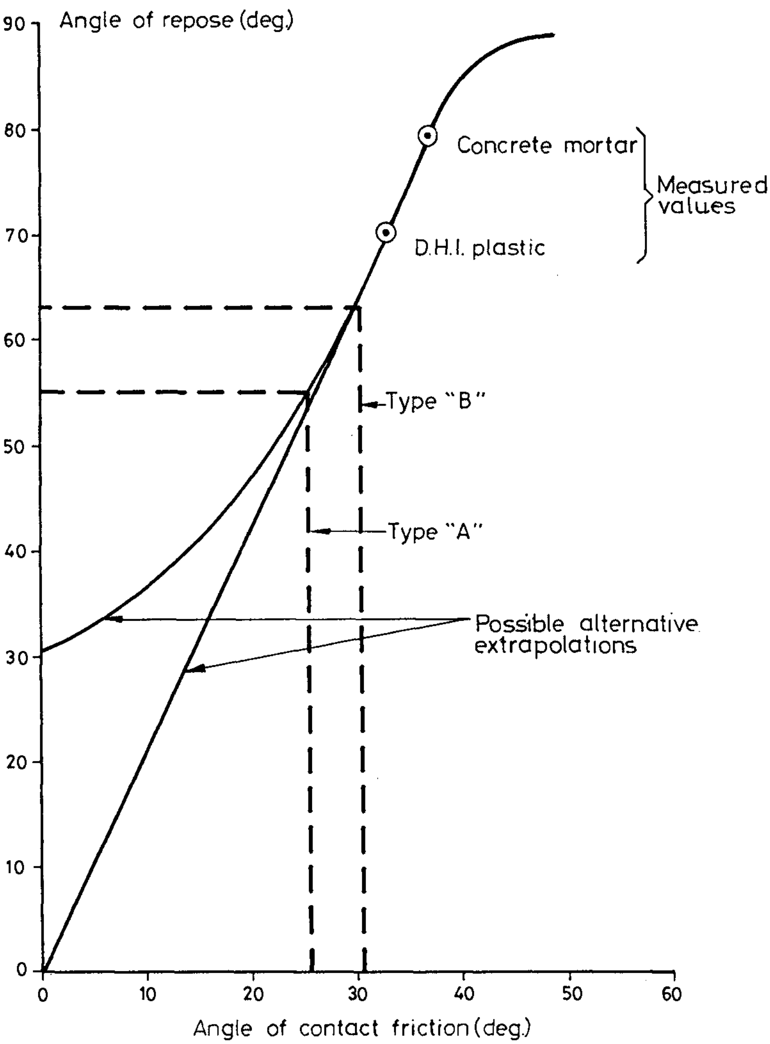

FIG. 5

If, for a given type and weight of armour, we consider the effect of contact friction and face slope angle we may consider C to be a constant where:-

$$C = \sqrt[3]{\frac{K' \gamma_c}{Wr}} \qquad (12)$$

and plot curves of H. C against ϕ (where $\phi = \tan^{-1}\mu$) for different slope angles; this is done in Fig: 6. The angles of repose of the various types of dolos mentioned have been plotted onto Fig: 6 and it can be seen that the influence of the natural angle of repose of armour units on their stability under wave attack is significant. It is not claimed that the results shown are precise, further tests are required, but it is fair to conclude that this effect is too important to be ignored as it has been by the majority of laboratories until now.

In view of the wide variety of materials being used for model armour units for test purposes, it is essential that contact friction should be represented correctly at least until we know how to correct for errors in modelling contact friction. It seems unlikely that mathematical analysis will assist us in this and it will be necessary to devise modelling techniques which enable contact friction to be varied without varying any other factors, such as specific gravity or hydraulic drag forces.

Wave Characteristics

Applying a significant wave height parameter to the Hudson equation does not seem suitable to a structure which has to withstand a spectrum including some waves which are considerably greater than this height. Care in selection of wave height was suggested by Hudson (ref: 1) with limits $H_{\frac{1}{3}} < H < H_{1/100}$ recommended by Morais (ref: 13).

Wave steepness is an important factor in terms of the disturbing force acting on the armour units. The steeper waves plunge whereas the flatter ones surge onto the breakwater.

Losada et al (ref: 14) carrying out tests on height-period interactions defined joint values of (H min, T) and (H, T min) which caused initiation of damage on breakwaters. (H min, T) therefore gives an optimum value of T for initiation of damage pertinent to each set of breakwater characteristics and also an optimum value of T for any wave height greater than H min, which causes the greatest amount of damage for that particular wave height.

Brunn (ref: 15) explained these optimum values of T as the occurrence of resonance between the wave period and the down-rush period. Peak forces perpendicular to the slope are set up by the collapsing-plunging wave repeatedly breaking, with the previous wave down-rush being in the same low position.

RELATIONSHIP BETWEEN ANGLE OF REPOSE AND SIGNIFICANT WAVE HEIGHT

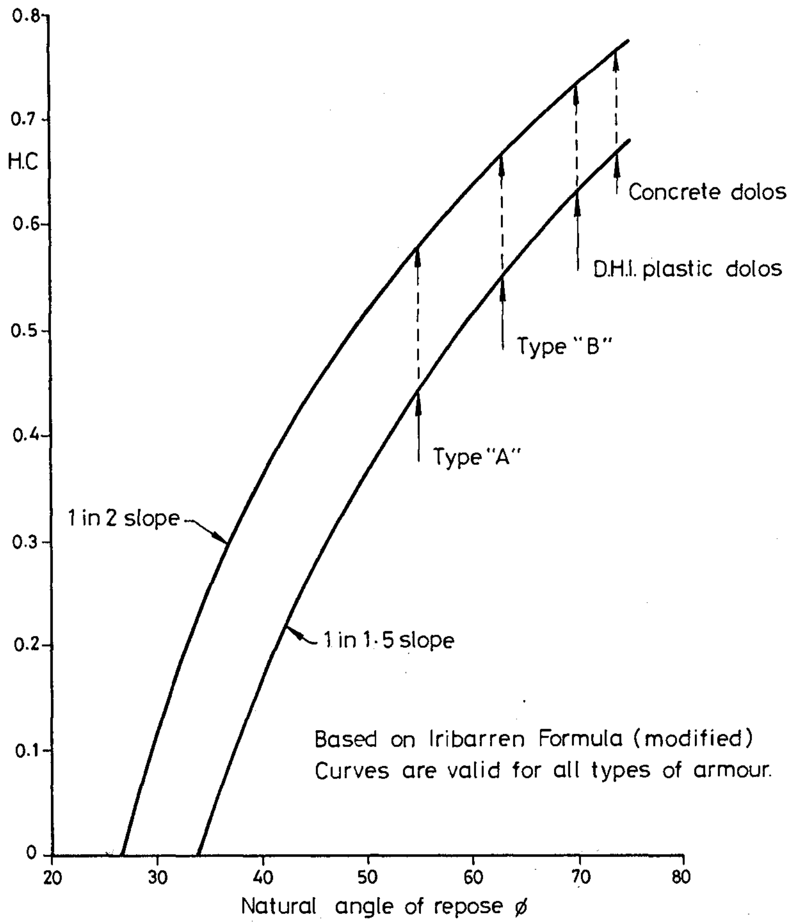

FIG. 6

Considering the rate at which energy is transmitted (wave power) towards a breakwater; it is directly proportional to wave period.

$$\text{Wave power} \qquad P = \frac{g^2 H^2 T}{32\pi} \ /\text{Sec.} \qquad (13)$$

However, it has been found that as the wave period increases there is an increase in wave reflection (ref: 2) and a corresponding decrease in near surface orbital velocities. An optimum wave period for maximum damage is therefore anticipated.

The wave form itself is influenced by depth limiting factors such as refraction, shoaling, bed friction, bed permeability and wave breaking. On interception with the breakwater the wave characteristics are modified by both the armour slope and the porosity of the breakwater cover layers. Consideration of these influences is therefore necessary prior to determination of H in a stability equation and for determination of the wave field in model tests.

It is envisaged that mathematical analysis could now contribute significantly to our understanding of the nature of many of these effects.

Structural Strength

With the recently developed interlocking units, which rely on linkage with adjacent units rather than on their own weight, there are greater possibilities of movement and oscillation of units under wave action. These oscillations result in impacts, and it is necessary for the units to be sufficiently sturdy to resist these impacts. This has been recognized since the initial development of such units as has the possibility that structural strength problems might increase with size of unit (assuming a consequent increase in wave height). Considerably more full scale testing of armour units is required to increase our understanding of the strength of large concrete units. It is not possible yet to measure the amplitudes, velocities and acceleration of the oscillations of the armour units or to predict the amount of movement which will cause breakage. However, model tests using breakable armour units with inserts of material with reduced strength have been used in hydraulic flume tests (ref: 11). The results were interesting and indicate that this could prove to be a valuable method for determining the strength requirements of interlocking armour units. It will however be necessary to find a modelling material (not just an insert) with correct strength characteristics as well as correct specific gravity and surface friction.

Until such techniques have been developed it would be advisable to include in our formula a specific factor of safety against the conditions under which model testing shows the degree of movement likely to result in breakage.

Conclusions

We may conclude in summary that:-

(i) Research into breakwater design and behaviour is now urgently needed to reduce reliance upon existing formulae which are no longer suitable for modern armour units.

(ii) An integrated programme of theoretical analysis and model testing will be required.

(iii) Future design methods will include more carefully controlled hydraulic model testing and the use of a range of new formulae.

(iv) Until the new techniques have been developed, tested and proved, we must recognize the limitations of the present formulae and be very careful indeed not to use them out of their intended context.

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LEGEND

Wr	=	armour block weight
K_D	=	Hudson's dimensionless stability coefficient
γc	=	concrete density
γw	=	water density
α	=	inclination of armour face from horizontal
H	=	design wave height
N	=	Iribarren's stability coefficient
A	=	incident wave height
P	=	armour block weight
$\mu = f$	=	friction coefficient of the armour blocks $f = \tan \phi$
ϕ	=	natural angle of repose
Sr = d	=	relative density of armour block $= \frac{\gamma c}{\gamma w}$
Ko, K'	=	dimensionless stability coefficients
Lp	=	wave length
T	=	wave period