AUTOMATIC CALIBRATION OF NUMERICAL TIDAL MODELS

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1. Introduction

Numerical models for the simulation of tidal waves in estuaries have become a standard tool of coastal engineers. Before they can be applied to practical problems, they have to be calibrated against nature. The basis for calibration is normally a representative set of tidal curves (natural field data), which has to be reproduced by the numerical model. Generally, the calibration is performed by empirically tuning certain parameters, until a fairly good agreement between measured and calculated quantities is obtained. In most cases, this is a "trial-and-error" process which may become very time-consuming, and which strongly depends on the intuition and experience of the user.

In order to make the process of calibration more effective, and to ensure that the best possible agreement between nature and numerical model is achieved, a calibration method has been developed to determine the parameters of the numerical model automatically by means of a mathematical method of optimization. The method is applied to a one-dimensional numerical model of the Elbe River.

2. Numerical Model

For the numerical river model, the common assumptions of vertically and horizontally averaged velocities, and of hydrostatic pressure distribution are made. The basic equations are derived from the conservative principles for momentum

$$\int \left(\frac{1}{gA}\frac{\partial Q}{\partial t} + \frac{1}{g}\frac{Q}{A^2}\left(\alpha + \frac{b_s}{b}\right) + \left(1 - \frac{\alpha}{g}\frac{Q^2}{A^3}b_s\right)\frac{\partial h}{\partial s} - I_s + I_v\right) ds = 0 \qquad (1)$$

and for mass

$$\int \left(b\frac{\partial h}{\partial t} + \frac{\partial Q}{\partial s}\right) ds = 0 \qquad (2)$$

where A stands for the stream area, b_s for the stream width, and b for the top width. s is the space coordinate along the river axis, Q the flux in this direction, and h gives the variation of the free surface.

The quantity I_S stands for the bottom slope, I_V for the friction losses, and α is a velocity coefficient.

These integral formulations are integrated directly. Adopting a finite element concept, and using linear trial functions in space and time for the description of the water level h and the flux Q, an implicit formulation of second order accuracy is obtained. The scheme is identical with that proposed by Preissmann (1960), and can alternatively be derived from a finite difference formulation for a finite control scheme. An analysis of the numerical properties of this scheme was presented by Evans (1977).

The numerical model was applied to the Elbe river, covering its tide-influenced length of 139.1 km from the North Sea (Cuxhaven) to Hamburg (St. Pauli) and further upstream to Geesthacht (Fig. 1). The model had to take into account islands and banks. For the discretization an average element length of about 1 km was chosen. A rather high resolution of the topography was thus obtained.

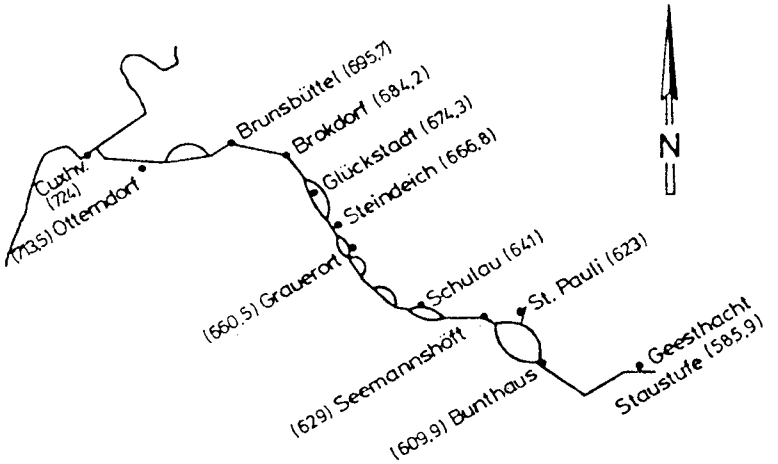


Fig. 1: One-dimensional Elbe model

For the description of bottom friction, the Taylor formulation

$$I_V = \frac{r|v|v}{g\ h} \tag{3}$$

was used, r is the friction parameter.

3. "Trial-and-Error" Calibration

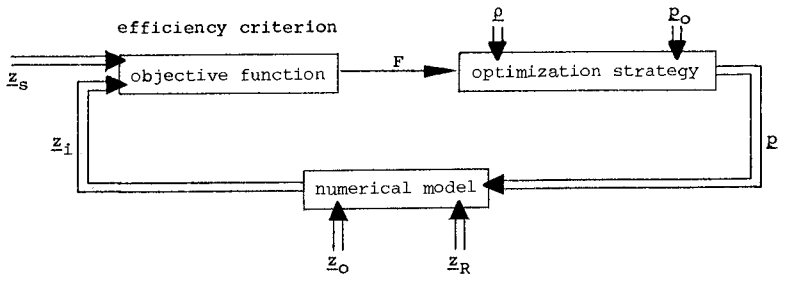
The numerical model calculates the time-dependent variation of the free surface, and the water flux along the river. The calculated values depend

on a chosen set of friction parameters. They have to be determined from a calibration of the model, which aims at hindcasting some typical measured states in nature. Normally the calibration is performed with respect to a set of tidal curves only, since data on velocities are more difficult to get. During the process of calibration, the friction parameters are varied empirically until a fairly good agreement between calculated and measured data is obtained. So the calibration is actually performed in a "trial-and-error" manner. In each interative step, the "degree of agreement" between computed and measured data should be determined according to some analytic criteria. This can be done efficiently on the computer only, unless the criteria for the calibration are strongly simplified.

The really crucial point of any "trial-and-error" calibration, however, is the estimate of a new parameter set for the next iteration step. It is difficult to make a good guess, due to the non-linearity of the numerical model. The model user needs a good deal of experience and intuition. In order to overcome this drawback, an automatic calibration method (Januszewski, 1980) was developed, based on the theory of mathematical optimization. This method is independent of any user's "feeling" and determines an optimal parameter set in accordance with any user-defined criteria and error bounds.

4. Optimized Calibration

The strategy for an automatic and optimal determination of friction parameters leads to a closed optimization system which is shown in Fig. 2. The numericl model is the basic element of this system. In the later application on the Elbe river, a numerical model will be used , for which the initial and boundary conditions $\underline{z}_o$ and $\underline{z}_R$ must be prescribed.



$\underline{z}_s$	field data	$\underline{p}$	varied parameters
$\underline{z}_i$	calculated state variables	F	performance index
$\underline{z}_o$	initial conditions	$\underline{\varrho}$	accuracy bounds
$\underline{z}_R$	boundary conditions	$\underline{p}_o$	starting parameters

Fig. 2: Strategy for Optimized Calibration

$\underline{z}$ is the vector of state variables which are the free surface and the fluxes or velocities at any cross-section. These quantities are either computed by the numerical model, giving the vector $\underline{z}_i$, or measured in the field, and then giving the vector $\underline{z}_s$. The computed vector $\underline{z}_i$ depends on the parameter vector $\underline{p}$ which contains the friction parameters.

In the closed calibration loop an initial parameter set p_0 has to be chosen as input for the numerical model, which calculates the vector $\underline{z}_i$. This vector has to be compared with the corresponding field data set in $\underline{z}_s$. For the Elbe model, this comparison was made for the water levels, as they were the only available field data. The analytic criterion, in accordance to which the comparison is being made, and the definition of which is open to the model user, is called the objective function. The comparison between calculated and measured state variables results in a scalar quantity, the performance index F. This is a measure for the agreement achieved between measured and calculated values.

The performance index is now the input for the optimization strategy, which generates a new set of parameters. This new parameter vector $\underline{p}$ is the output of the optimization strategy and in turn the input for a second cycle of the closed optimization system.

The iteration stops following a convergency test which is made during each cycle just before a new parameter set is calculated. For this test, the model user has to define the wanted accuracy bound ρ, according to which three tests are made:

a comparison of the actual values and those of the preceding iteration step, for all elements of the parameter vector $\underline{p}$

$$\underline{p}_\nu - \underline{p}_{\nu-1} \leq \varepsilon \tag{4}$$

a test on the achieved improvement of the performance index F

$$|F_\nu - F_{\nu-1}| \leq \eta_1 \tag{5}$$

and a test on the performance index itself

$$F \leq \eta_2 \tag{6}$$

Calibration is completely automized in the outlined closed loop. The model user influences the calibration run only by specifying the error bounds, and by defining the objective function. So he is still controlling the physics of the calibration, but has got rid of a lot of manual work.

4.1 Objective Function

The objective function describes the error between calculated and measured data. It should define a scalar non-negative quantity, the performance index F, which depends on the parameter set $\underline{p}$. The optimal parameter set $\underline{p}$ is obtained when the error between computed and measured data is minimized. The obtained minimum should be a global one.

For the application to the Elbe model, the objective function was defined as the sum of the squares of the differences between measured and calculated state values taken at all time steps for the tidal period and at

all locations for which field data were given.

$$F(\underline{p}) = \int_{t_o}^{t_1} (\underline{z}_s(t) - \underline{z}_i(t,\underline{p}))^T (\underline{z}_s(t) - \underline{z}_i(t,\underline{p}))\, dt \qquad (7)$$

The advantage of this formulation lies in the fact that the differences over the total time of the tidal period are equally weighted, i.e. the differences are minimized with respect to phase and amplitude.

In order to find the optimal parameter vector $\underline{p}$, the objective function has to be differentiated with respect to all components of the vector $\underline{p}$.

$$\left.\frac{\partial F(\underline{p})}{\partial p_j}\right|_{\underline{p}=\underline{p}^*} = 2\int_{t_o}^{t_1} (\underline{z}_s - \underline{z}_i)^T \left.\frac{\partial \underline{z}_i}{\partial p_j}\right|_{\underline{p}=\underline{p}^*} dt = 0,\ 1 \le j \le n \qquad (8)$$

A minimum for $F(\underline{p})$, at least in a local sense is obtained, when the derivative is equal to zero. The parameter vector $\underline{p}^*$ corresponds to min $F(p)$. This condition is fulfilled in three cases.

1) The derivative of computed state values

$$\frac{\partial \underline{z}_i}{\partial p_j}$$

is independent of the friction parameters, which means that the problem is not correctly posed.

2) The product

$$(\underline{z}_s - \underline{z}_i)^T \frac{\partial \underline{z}_i}{\partial p_j}$$

is equal to zero, which means that both functions $(\underline{z}_s - \underline{z}_i)$ and $\frac{\partial \underline{z}_i}{\partial p_j}$ are orthogonal, and thus independent of each other.

This result is meaningless.

3) The deviation $(\underline{z}_s - \underline{z}_i)$ is equal to zero. This is exactly the desired result for a perfect calibration.

The first derivative, however, does only give a necessary condition for a local minimum. A global minimum is obtained, if the second derivative is positive - semi-definite. This can be shown for each iteration step, as long as the objective function is based on an error-squared formulation.

The numerical model is integrated for discrete steps in the time domain, and thus the objective function too has to be formulated in a discrete manner.

$$F(\underline{p}) = \sum_{k=1}^{T_{max}} (\underline{z}_s - \underline{z}_{i,k}(\underline{p}))^t \underline{A} (\underline{z}_s - \underline{z}_{i,k}(\underline{p})) \qquad (9)$$

To this formulation, a symmetric matrix $\underline{A}$ is added, which allows for an arbitrary weighting of different elements of the error vector $(\underline{z}_s - \underline{z}_i)$.

Depending on the model user's experience and on the application he has in mind, it is also possible to use other objective functions. For the Elbe model

computations, the correlation coefficient $r_{j,si}$ was alternatively introduced:

$$F(\underline{p}) = \sum_{j=1}^{N} (1 - r_{j,si}) \qquad (1o)$$

which is based on an error-squared formulation, too. Other statistical measures can also be used.

The error computation must refer to all the tidal curves, as in the case of the Elbe model. It can be extended to include the velocities, too, or it can be restricted to tidal high and low water alone. The choice of the objective function is open to the user, and this means that automatic calibration does not become a black-box system.

4.2 Optimization Strategy

Optimization strategy is a mathematical procedure for finding the minimum value of the performance index. Due to the non-linearity of the St. Venant equations (1,2), it is impossible to find the minimum $\underline{p}^*$ of the objective function, i.e. $\underline{p}^* = \min F(p)$, in an explicit way. So the second method of Powell (1964), which is a method taken from the scope of non-linear programming, is applied for finding the minimum by a sequential search strategy. It is based on a direct searching technique which avoids the computation of derivatives for the objective function $F(p)$, which in turn depends on the non-linear St. Venant equations, and is rather complicated.

The principle of the searching strategy is explained by figure 3.

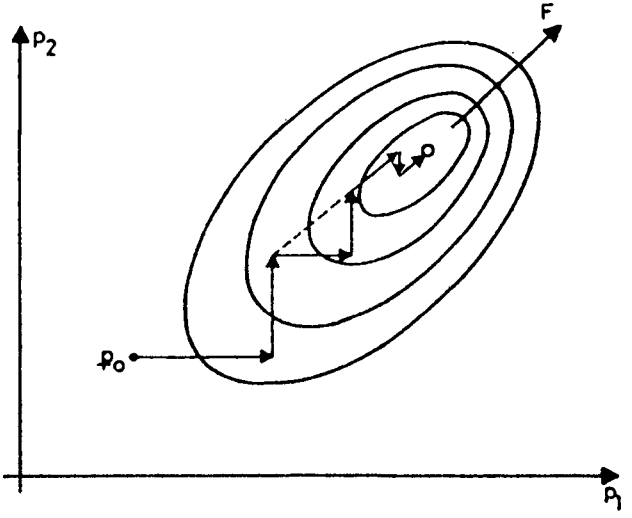


Fig. 3: Second Method of Powell

It is assumed that the objective function $F(\underline{p})$ is a convex function. In a three-dimensional representation which is chosen for reasons of simplicity of the graphical sketch, the objective function $F(\underline{p})$ = const. defines isolines in the p_1 - p_2 domain. Starting from an initial estimate $\underline{p}_0$, two linear independent searching directions are defined. The minimum of $F(\underline{p})$ is determined subsequently and independently in each direction. This leads to a new parameter vector $\underline{p}_1$. This procedure is repeated iteratively. To make the searching process efficient, a test is performed after each iteration step to find out whether the search directions should be the same for the next step, or whether they should be varied. A criterion on this is given by the determinant of the search directions. It will attain its maximum for orthogonal, and thus linear independent searching directions. So for each iteration step a new direction, which is defined from the starting value $F(\underline{p})$ to the computed value $F(\underline{p})$ at the end of the step, is introduced and tested. If the direction determinant, computed by omitting the first search direction and including the new one, is bigger than it was in the step before, the new direction is accepted.

The search for the minimum for $F(\underline{p})$ in each independent direction is effected by means of the method of quadratic interpolation, which avoids the computation of derivatives. An example for this strategy is given by figure 4.

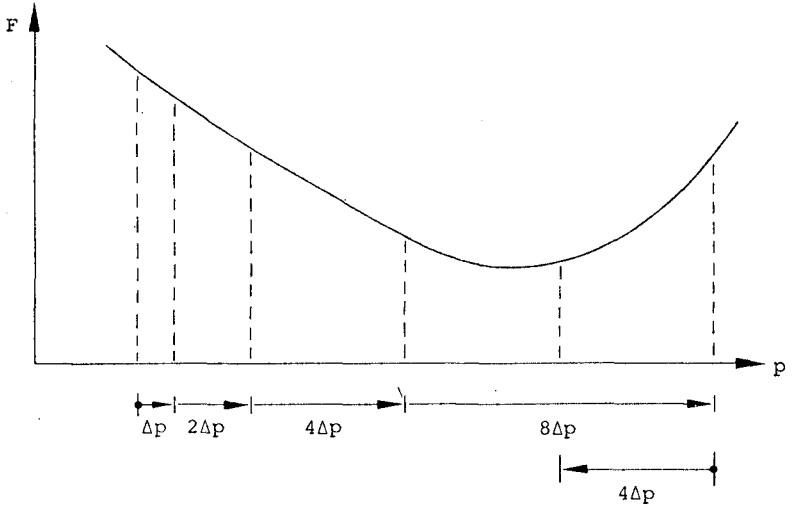


Fig. 4: Quadratic Interpolation

Starting with a value $F(\underline{p})$, neighbouring values

$F(\underline{p} + \Delta\underline{p})$, $F(\underline{p} + 3\Delta\underline{p})$, $F(\underline{p} + 7\Delta\underline{p})$, $F(\underline{p} + \ldots)$

are computed, until the last value is bigger than the preceding one. An

additional value is computed in the center of the last interval, and then the minimum is determined by quadratic interpolation. This procedure is rather efficient, as it basically corresponds to a binary searching strategy.

5. Results

The suggested method for optimized automatic calibration was applied to the section from Grauerort Geesthacht of the described Elbe river model (Fig. 1). This section, which is also given in figure 6, has a length of 74.6 km. The one-dimensional model for this part of the river was divided into 17 branches to include the islands and the highly branched Hamburg harbour (St. Pauli). Measured data were available at 5 locations along the river, which are marked by dots in figures 1 and 6. The model was run with constant inflow prescribed at Geesthacht, and a tidal curve given at Grauerort.

It was assumed that for this river section the friction parameters may be independently varied within 4 areas as shown in figure 6. At least one measured data set has to be available within each area, as otherwise no performance index can be determined for this area, and consequently the problem would not have a unique solution.

Figure 5 shows a comparison between "trial-and-error", or manual, and the optimized automatic calibration for the location Seemannshöft.

The calibration was performed for the first 12.5 hours of the tidal curves shown in figure 5, which correspond to the first tide on August 22, 1975.

The objective function for the manual calibration was the difference between measured and calculated values at tidal high and low water only. In the manually-performed calibration, friction parameters which depend on the flow direction were used. The obtained calibration is nearly perfect at high and low water, but not so good for the gradient at falling water.

For the automatic calibration, the friction parameters were assumed to be constant over the tidal period. Moreover, as a time-step of 10 minutes was used in the numerical model, the error was also minimized at about 75 points per tidal period, and per location, for which measured data were given. So amplitude and phase error equally weighted and in more detail were taken into account by this strategy. The phase error reduction leads to a much better agreement of the tidal curves. This will probably result in a much better representation of the velocities, the wave height, however, is too small.

The automatic calibration was performed to an accuracy of less than 0.005 m per location and time-step. The obtained friction parameters are given in the table of figure 6.

The variation of the parameters is explained from the typical cross section shapes of the river, which are approximately trapezoidal in area I, and much more complicated in areas III and IV. Moreover, the system is highly branched in the areas II to IV, and especially in the harbour area II simplifications with respect to the discretization have been made.

The given results were obtained by using the sum of the squares of the differences between measured and computed values as objective function.

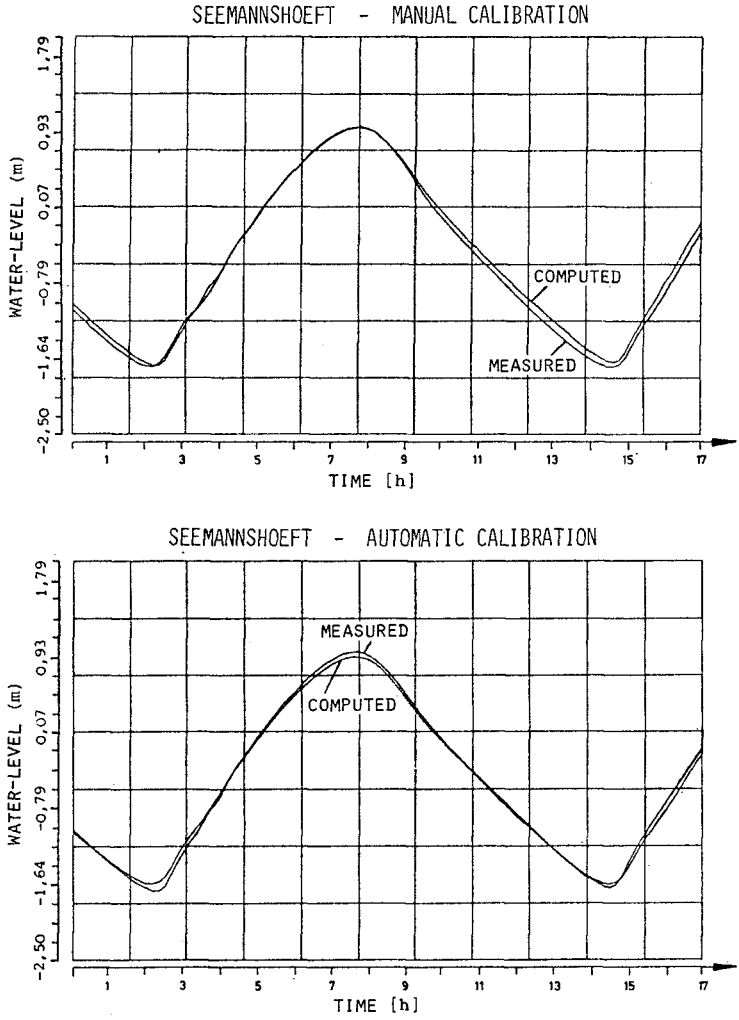
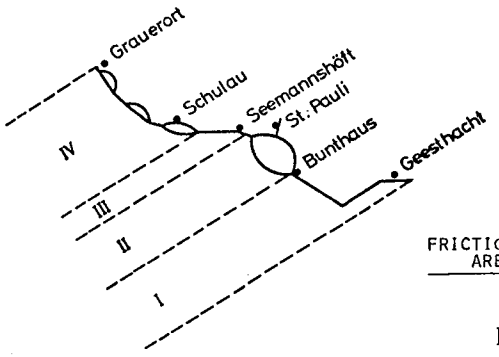


Fig. 5: Comparison Manual to Automatic Calibration



FRICTION TERM $I_R = \frac{r|v|v}{gh}$

FRICTION AREA	CALIBRATION MANUAL flood	MANUAL ebb	AUTOMATIC
I	0.007	0.012	0.00164
II	0.005	0.03	0.01829
III	0.004	0.004	0.00478
IV	0.005	0.004	0.00432

Fig. 6: Friction Parameters after Calibration

Alternatively, a calibration was performed, using the correlation coefficient (1o) as objective function. For the same error bounds, a correlation coefficient of o.99965 was obtained. The correlation as well as the relation of computed versus measured data is given in figure 7. The computation time for the calibration was reduced by about 2o % in comparison with the error-squared function.

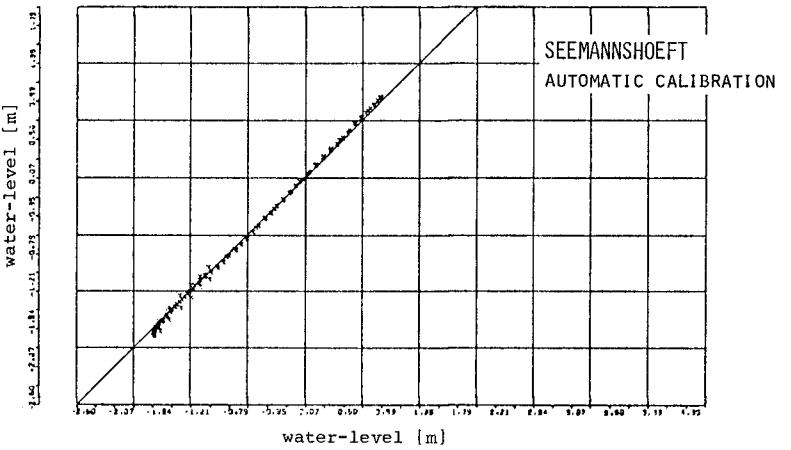


Fig. 7: Calibration using Correlation Coefficient

During calibration it was found that the numerical model was very sensitive to all parameter changes. Small increments on the optimal parameter set increased the value of the objective function considerably. This might easily lead to many poor estimates on new parameter sets in the "trial-and-error" calibration. It is confirmed by a comparison of the internal statistics, which was made over the number of computer runs for manual and automatic calibration. The optimization strategy required 33 runs for the given bounds, whereas about 60 variations of the parameter were necessary for the manual calibration. The manual calibration demanded for a careful and time-consuming data analysis, and repeated plotting. A trained engineer was kept busy by this job for weeks. The automatic calibration was performed within one computer run in one night. The obtained results cannot be improved, unless the error bounds are reduced. Moreover, the actual bound up to which the calibration was performed, is a useful information for the model user 's prognostic applications of the model.

6. Conclusion

A method from the field of mathematical optimization was applied for the calibration of a one-dimensional river model. The model starts from the St. Venant equations which are solved numerically. The optimization leads to a closed system, so that the calibration can be performed automatically within one computer run. This leads to considerable savings in manpower and computer time. The physical transparency for the hydraulic engineer is maintained. The engineer still defines the criteria according to which the calibration is performed. The parameter set finally obtained is the best and only combination of parameters which is obtainable within a given error bound.

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