# STATISTICAL PROPERTIES OF RANDOM WAVE GROUPS

AKIRA KIMURA*

## ABSTRACT

This study deals with the statistical properties of the group formation of random waves determined by the zero-up-cross method. Probability distributions about
(1) the run of high waves
(2) the total run
(3) the run of resonant wave period
are derived theoretically providing that the time series of wave height and wave period form the Markov chain. Transition probabilities are given by the 2-dimensional Rayleigh distribution for the wave height train and the 2-dimensional Weibull distribution for the wave period train. And very good agreements between data and the theoretical distributions have been obtained. Then the paper discusses those parameters which affect the statistical properties of the runs and shows that the spectrum peakedness parameter for the run of wave height and the spectrum width parameter for the run of wave period are the most predominant.

## INTRODUCTION

It is often observed in the field observations that a large wave makes a group with another large waves. This characteristic of sea waves makes significant effects on several coastal engineering problems such as slow drift oscillation of vessel, stability of rubble mound, drainage of overtopping discharge and some other problems.

Arranging order of random waves is usually analized with a concept of the run. There have been two kinds of theoretical studies on the run of random sea waves. One is done by Goda(1) and the other is done by Ewing(2) and Nolte-Hsu(3). Goda derived the probability distribution providing the randomness of waves. On the other hand Ewing, Nolte-Hsu derived the probability distributions providing the narrow band spectrum. As the result of this assumption, succeeding several waves correlate each other. However several authors pointed that consecutive wave heights correlate each other, but the correlation of alternative wave heights diminishes nealy zero.(4),(5),(6) Then random wave height train seems to have a intermediate characteristics between these two theoretical assumptions.

Sawhny(7) examined the time series of crest-to-trough wave height by means of the Markov chain and found that consecutive three half waves

* Assistant Professor of Department of Civil Engineering, Faculty of Engineering, Tottori University, Tottori, Japan

correlate each other. Recently the author(8) showed that time series of zero-up-crossing wave height and wave period have properties very close to those of the Markov chain and that their transition probabilities may well be approximated with the 2-dimensional Rayleigh distribution and the 2-dimensional Weibull distribution respectively.

## RUN OF WAVE HEIGHT

Transition equation of the Markov chain is given as:

$$p_n = p_0 P^n \tag{1}$$

where $p_0$ is a initial distribution, $p_n$ is a distribution after n time transitions and P is a transition probability matrix. If wave height train $h_j$ (j=1,2,3, ...) are classified into the following states with reference to the standard wave height $h_*$:

$$\left.\begin{array}{ll} \text{State 1 :} & 0 < h_j \le h_* \\ \text{State 2 :} & h_* < h_j \le 2h_* \\ \cdots\cdots & \\ \text{State i :} & (i-1)h_* < h_j \le ih_* \\ \cdots\cdots & (j=1,2,3,\ \ldots\ ), \end{array}\right\} \tag{2}$$

then transition probability matrix is given by the following equations.

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13}\cdots \\ p_{21} & p_{22} & p_{23}\cdots \\ p_{31} & p_{32} & p_{33}\cdots \\ \cdots & \cdots & \cdots \end{pmatrix} \tag{3}$$

where

$$p_{ij} = \int_{(j-1)h_*}^{jh_*} \int_{(i-1)h_*}^{ih_*} p(h_1, h_2)\, dh_1 dh_2 \Big/ \int_{(i-1)h_*}^{ih_*} Q(h_1)\, dh_1 \tag{4}$$

$$(i,j=1,2,3,\ \ldots\ ),$$

in which

$$p(h_1, h_2) = \frac{4h_1h_2}{(1-4\rho^2)h_r^4} \exp\left[\frac{-1}{1-4\rho^2}\frac{(h_1^2+h_2^2)}{h_r^2}\right] I_0\left[\frac{4h_1h_2\rho}{(1-4\rho^2)h_r^2}\right], \tag{5}$$

$$Q(h_1) = \frac{2h_1}{h_r^2} \exp\left[-\frac{h_1^2}{h_r^2}\right] \tag{6}$$

$h_r$ is the rms wave height, $\rho$ the correlation parameter, $I_0$ the modified Bessel function of the 0-th order. Eq.(5) is the 2-dimensional Rayleigh distribution and eq.(6) the Rayleigh distribution. Correlation coefficient of consecutive wave height $h_1$ and $h_2$ is:

$$\gamma_h = \frac{E(2\rho) - 1/2(1-4\rho^2)K(2\rho) - \pi/4}{1 - \pi/4}, \tag{7}$$

in which K and E are the complete elliptic integrals of the first and second kind respectively. In fig.1 the curve(n=2) shows the relation between $\gamma_h$ and $\rho$. It follows that $p_{ij}$ in eq.(3) can be determined from the correlation coefficient of consecutive wave heights by using eqs.(4), (5),(6) and (7).

## ESTIMATION OF THE SUCCEEDING WAVE HEIGHT

The problem to estimate the succeeding wave height $h_2$ from the present wave height $h_1$ is treated as follows:

If the present wave height $h_1$ falls in the State i, $p_0$ is given as:

$$p_0 = (\underset{1}{0}, \underset{2}{0}, \underset{.....}{.....}, \underset{i}{1}, \underset{...}{...}) . \tag{8}$$

Substituting eq.(8) into eq.(1), distribution $p_1$ becomes

$$p_1 = (p_{i1}, p_{i2}, \cdots\cdots) . \tag{9}$$

Element $p_{ij}$ (j=1,2, ... ) is given by eq.(4). If $h_*$ in eq.(4) is sufficiently small, expectation of $h_2$ is:

$$\begin{aligned}\bar{h}_2 &= \int_0^\infty h_2\, p(h_2|h_1)\, dh_2 \\ &= \int_0^\infty \frac{2h_2^2}{(1-4\rho^2)h_r^2} \exp\left[\frac{-(h_1^2+h_2^2)}{(1-4\rho^2)h_r^2} + \frac{h_1^2}{h_r^2}\right] I_0\left[\frac{4h_1h_2\rho}{(1-4\rho^2)h_r^2}\right] dh_2\end{aligned} \tag{10}$$

where $p(h_2|h_1)$ is the conditional Rayleigh distribution. Fig.2 shows the relation of $\bar{h}_2/h_r$ and $h_1/h_r$. It shows that according to the increase of $\gamma_h$, $\bar{h}_2$ approaches $h_1$. But when $\gamma_h$ equals 0, $\bar{h}_2/h_r$ is always $\sqrt{2}/2$.

## RUN OF HIGH WAVES

The run of high waves which exceed the standard wave height $h_*$ is one of our greatest concern. In this problem, time series of wave height:

$$\cdots\cdots\, , h_{i-1}, h_i, h_{i+1}, h_{i+2}, \cdots\cdots \tag{a}$$

are classified into two states. One is $h < h_*$ and the other is $h \geq h_*$. If these states are distinguished with suffixes 1 and 2 respectively, eq.(3) is reduced to:

$$P = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}, \tag{11}$$

where

$$p_{11} = \int_0^{h_*}\int_0^{h_*} p(h_1, h_2)\, dh_1 dh_2 \Big/ \int_0^{h_*} Q(h_1)\, dh_1$$

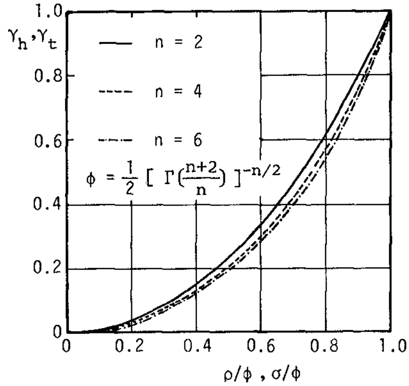



Fig.1 $\gamma_h \sim \rho/\phi$ , $\gamma_t \sim \sigma/\phi$

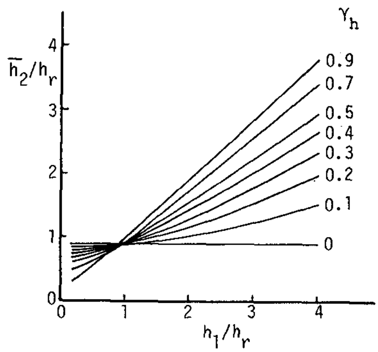



Fig.2 $\overline{h}_2/h_r \sim h_1/h_r$

$$\left.\begin{aligned} p_{12} &= \int_{h_*}^{\infty}\int_{0}^{h_*} p(h_1, h_2)\, dh_1 dh_2 \Big/ \int_{0}^{h_*} Q(h_1)\, dh_1 \\ p_{21} &= \int_{0}^{h_*}\int_{h_*}^{\infty} p(h_1, h_2)\, dh_1 dh_2 \Big/ \int_{h_*}^{\infty} Q(h_1)\, dh_1 \\ p_{22} &= \int_{h_*}^{\infty}\int_{h_*}^{\infty} p(h_1, h_2)\, dh_1 dh_2 \Big/ \int_{h_*}^{\infty} Q(h_1)\, dh_1 \quad , \end{aligned}\right\} \quad (12)$$

in which $p(h_1, h_2)$ and $Q(h_1)$ are given by eqs.(5) and (6).

The run of high waves starts when a wave height exceeds $h_*$ initially. Suppose $h_{i-1} < h_*$ and $h_i \geq h_*$, in the wave height train (a). The run starts from $h_i$. The initial distribution $p_0$ is (0,1). Substitution of $p_0$ and eq.(11) into eq.(1) gives:

$$p_1 = (p_{21}, p_{22}) \quad .. \ (n=1)$$
$$p_2 = (p_{21}p_{11} + p_{22}p_{21}, p_{21}p_{12} + p_{22}^2) \quad .. \ (n=2)$$
$$\cdots\cdots\cdots \quad .. \ \cdots\cdots .$$

$p_{21}$ is the probability that $h_i \geq h_*$ and $h_{i+1} < h_*$. Then the first element of $p_1$ gives the probability that the length of run is 1. But the first element of $p_2$ does not give the probability that the length of run is 2. That is, since $p_{21}p_{11}$ is the probability that $h_i \geq h_*$, $h_{i+1} < h_*$ and $h_{i+2} < h_*$, only $p_{22}p_{21}$ gives the probability that the length of run is 2. The elements which give the transition probability from State 1 should be precluded since they have no relation to the run of high waves. Finally transition probability matrix becomes:

$$P = \begin{pmatrix} 0 & 0 \\ p_{21} & p_{22} \end{pmatrix} \quad (13)$$

By substituting initial distribution and eq.(13) into eq.(1),

$$\left.\begin{aligned} p_1 &= (p_{21}, p_{22}) \\ p_2 &= (p_{22}p_{21}, p_{22}^2) \\ &\cdots\cdots\cdots \\ p_\ell &= (p_{22}^{(\ell-1)}p_{21}, p_{22}^{\ell}) \\ &\cdots\cdots\cdots \end{aligned}\right\} \quad (14)$$

By simple induction, probability distribution of the run of high waves is represented by:

$$P_1(\ell) = p_{22}^{(\ell-1)}p_{21} = p_{22}^{(\ell-1)}(1-p_{22}) \quad , \quad (15)$$

where $\ell$ is the length of the run. Mean length of the run is defined as:

$$\bar{\ell} = 1/(1-p_{22}) \tag{16}$$

Fig.3 (a),(b) and (c) show the probability distribution of the run of high waves for (a) $h_* = h_{mean}$, (b) $h_* = h_{1/3}$ and (c) $h_* = h_{1/10}$ respectively. When $\gamma_h$ equals 0 in each figure, the theoretical distribution corresponds to the Goda's theory.

By analogy, the probability distribution of the run of low waves which fall below $h_*$ consecutively can be given as:

$$P_2(\ell') = p_{11}^{(\ell'-1)} p_{12} = p_{11}^{(\ell'-1)}(1-p_{11}) , \tag{17}$$

where $\ell'$ is the length of the run of low waves. $p_{11}$ is given by eq.(12).

Fig.4 shows the mean length of the run of low waves for $h_* = h_{mean}$ (solid line), $h_* = h_{1/3}$ (dotted line) and $h_* = h_{1/10}$ (chain line). Mean duration in which high waves do not take place can be estimated from this figure.

TOTAL RUN [1]

From eqs.(15) and (17) the probability distribution of the total run can be introduced.

$$P_3(\ell_0) = \frac{(1-p_{11})(1-p_{22})}{p_{11} - p_{22}} (p_{11}^{\ell_0-1} - p_{22}^{\ell_0-1}) , \tag{18}$$

where $\ell_0$ is the length of the total run. Mean length of $\ell_0$ is:

$$\bar{\ell}_0 = 1/(1-p_{11}) + 1/(1-p_{22}) , \tag{19}$$

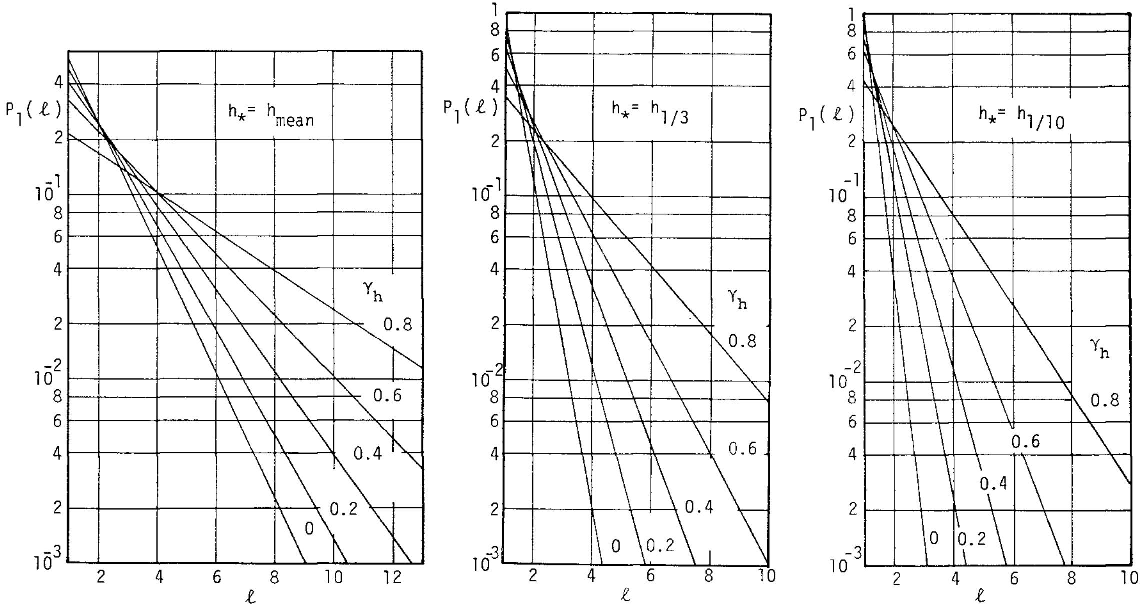
in which $p_{11}$ and $p_{22}$ are given by eq.(12). Fig.5(a) and (b) show the probability distribution of the total run for (a) $h_* = h_{mean}$ and (b) $h_* = h_{1/3}$.

Equations (15) to (19) are determined from $\gamma_h$ or $\rho$ with eqs(5),(6) and (12). Figs.6 and 7 show the theoretical relations of $p_{11} \sim \gamma_h$ and of $p_{22} \sim \gamma_h$ respectively, providing that $h_* = h_{mean}$(real line), $h_* = h_{1/3}$ (broken line) and $h_* = h_{1/10}$ (chain line).

VERIFICATION OF THE THEORETICAL PROBABILITY DISTRIBUTIONS

It is very difficult to obtain random sea waves which are long enough and statistically stationary to examine the properties of run. Therefore some numerical simulation techniques (9) have been used to generate random wave trains. Fig.8 shows the target spectra of five cases used in the numerical simulations. Slope of the spectrum changes from minus 4th to 8th power of frequency in the high frequency region. 5000 waves have been generated in each case.

Figs.9 (a) $\sim$ (e) show the comparison between data and the theoreti-




(a) $h_* = h_{mean}$ (b) $h_* = h_{1/3}$ (c) $h_* = h_{1/10}$

Fig.3 Probability distribution of the run of high waves

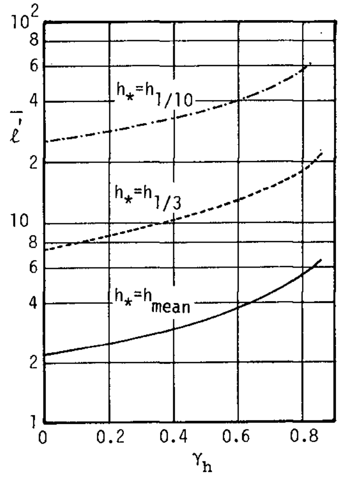
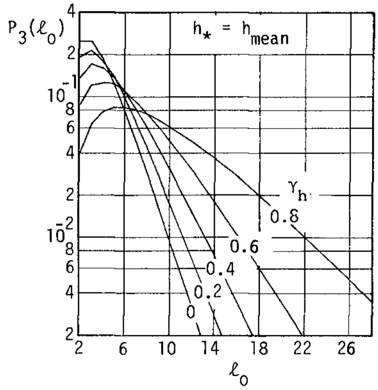
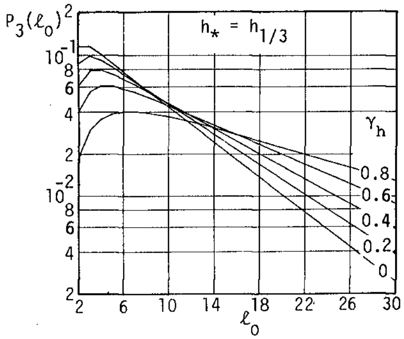



Fig.4 Mean length of the run of low waves




(a) $h_* = h_{mean}$




(b) $h_* = h_{1/3}$

Fig.5 Probability distribution of the total run

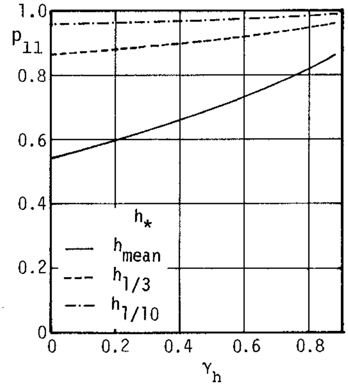



Fig.6 $p_{11} \sim \gamma_h$

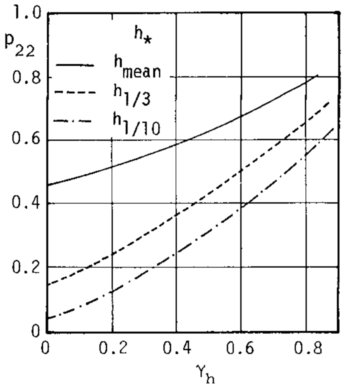



Fig.7 $p_{11} \sim \gamma_t$

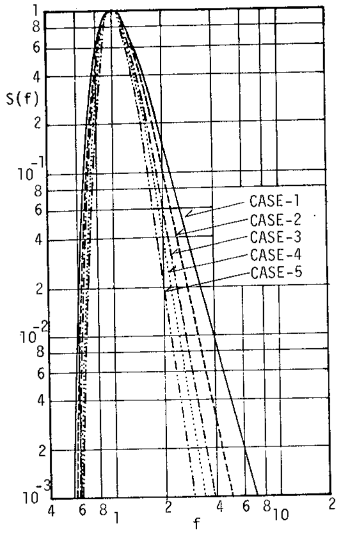
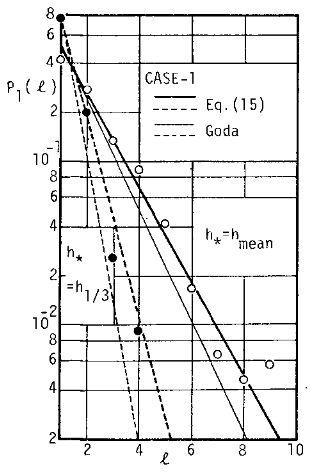
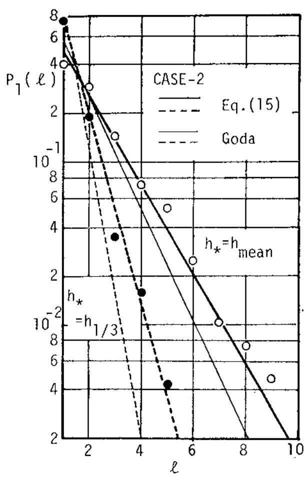


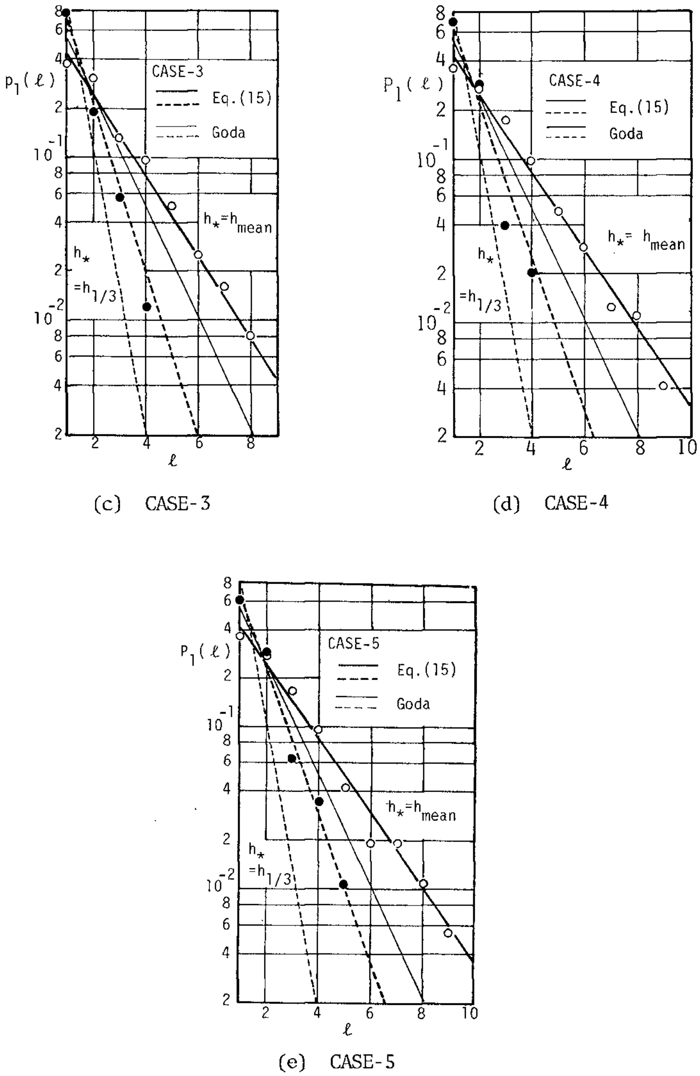

Fig.8 Target spectra




(a) CASE-1




(b) CASE-2




(c) CASE-3

(d) CASE-4

(e) CASE-5

Fig.9 Distribution of the run of high waves cal distribution of the run of high waves. The thick lines are the distributions from the theory developed here and the thin lines are those from the Goda's theory for $h_* = h_{mean}$ (solid line) and $h_* = h_{1/3}$ (dotted line). It can be seen that the plotted data agree very well with eq.(15). Figs.10 (a) ∿ (e) show the comparison of data and the theoretical distribution of the total run providing $h_* = h_{mean}$. The solid curve shows eq.(18) and dotted curve is the theoretical distribution from Goda's theory. Good agreements between data and eq.(18) have been obtained. Tables 1 and 2 show the comparisons of the mean length of the run of high waves and the total run between data and theories. With the increase of the correlation coefficient $\gamma_h$, mean length of data becomes longer while the Goda's theory gives the constant value which is considerably small. And the theory by Nolte-Hsu gives also small values compared with data. Estimations with the theory presented here give reasonable values in each case.

RUN OF WAVE PERIOD

Time series of random wave period determined by the zero-up-cross method forms the Markov chain approximately(8). Then almost same analysis is available for the run of wave period with that of wave height. However in such a problem as a resonant oscillation of structure, it is more useful to analize the run of wave periods which fall in the specified wave period band consecutively. For this purpose time series of wave period

$$\cdots\cdots\cdots\cdot,\ t_{i-1},\ t_i,\ t_{i+1},\ t_{i+2},\ \cdots\cdots \tag{b}$$

may well be classified into these three states:

$$\begin{aligned} &\text{State 1 :} && t_j < t_{*1} \\ &\text{State 2 :} && t_{*1} \le t_j \le t_{*2} \\ &\text{State 3 :} && t_j < t_{*2} \\ & && (j=1,2,3,\ \ldots\ ), \end{aligned} \tag{20}$$

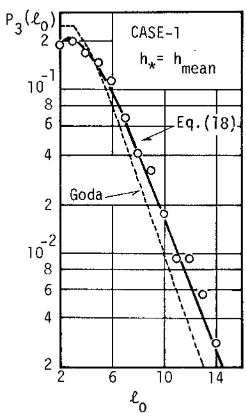
in which $t_{*1}$ and $t_{*2}$ are the low and high limit of the resonant period band of the oscillation system. The transition probability matrix reduces to the matrix of order 3: $(p_{ij})$ $(i,j=1,2,3)$. With the same discussion, the first and third row of the matrix, $p_{1j}$, $p_{3j}$ $(j=1,2,3)$ should be precluded to introduce the run of resonant wave period. Finally P becomes

$$P = \begin{pmatrix} 0 & 0 & 0 \\ p_{21} & p_{22} & p_{23} \\ 0 & 0 & 0 \end{pmatrix}. \tag{21}$$

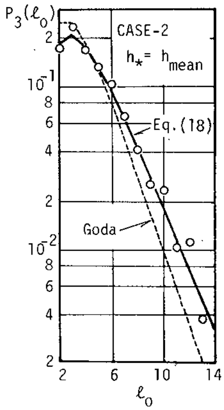
The run starts when a wave period falls in State 2 first; then initial distribution is (0,1,0) and the $\ell$th transition distribution is given as:

$$p_\ell = (p_{22}^{(\ell-1)}p_{21},\ p_{22}^{\ell},\ p_{22}^{(\ell-1)}p_{23}) . \tag{22}$$

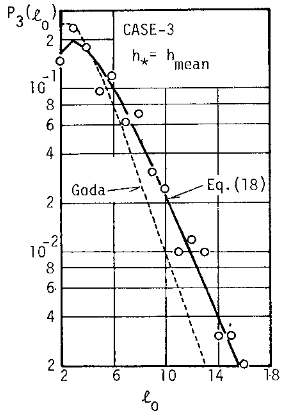
$p_{22}^{(\ell-1)}p_{21}$ and $p_{22}^{(\ell-1)}p_{23}$ are probability that consecutive $\ell$ waves fall in State 2, but $\ell$+1th wave falls in State 1 or 3 respectively. Then the



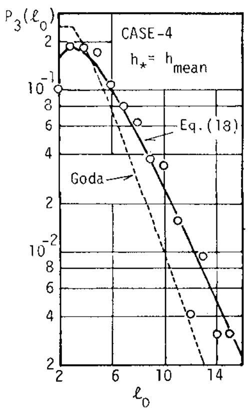

(a) CASE-1



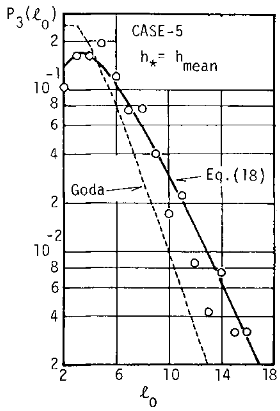

(b) CASE-2




(c) CASE-3




(d) CASE-4




(e) CASE-5

Fig.10 Distribution of the total run

Table-1 Mean length of the run of high waves

| | | $h_* = h_{mean}$ | | | | $h_* = h_{1/3}$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| Case | $\gamma_h$ | data | Eq.(16) | Goda | Nolte-Hsu | data | Eq.(16) | Goda | Nolte-Hsu |
| 1 | 0.19 | 2.20 | 2.08 | 1.84 | 1.33 | 1.28 | 1.33 | 1.15 | 1.12 |
| 2 | 0.23 | 2.29 | 2.15 | " | 1.47 | 1.29 | 1.37 | " | 1.19 |
| 3 | 0.29 | 2.34 | 2.28 | " | 1.64 | 1.29 | 1.44 | " | 1.28 |
| 4 | 0.33 | 2.42 | 2.37 | " | 1.78 | 1.37 | 1.50 | " | 1.36 |
| 5 | 0.38 | 2.45 | 2.46 | " | 1.92 | 1.53 | 1.57 | " | 1.44 |

Table-2 Mean length of the total run

| | | $h_* = h_{mean}$ | | | $h_* = h_{1/3}$ | | |
|---|---|---|---|---|---|---|---|
| Case | $\gamma_h$ | data | Eq.(19) | Goda | data | Eq.(19) | Goda |
| 1 | 0.19 | 4.66 | 4.55 | 4.03 | 9.33 | 9.87 | 8.61 |
| 2 | 0.23 | 4.67 | 4.67 | " | 9.47 | 10.12 | " |
| 3 | 0.29 | 4.94 | 4.90 | " | 10.00 | 10.63 | " |
| 4 | 0.33 | 5.17 | 5.10 | " | 9.95 | 11.07 | " |
| 5 | 0.38 | 5.36 | 5.32 | " | 10.71 | 11.57 | " |

probability that the length of the run becomes $\ell$ is given by the sum of these probabilities.

$$P_4(\ell) = p_{22}^{(\ell-1)} p_{21} + p_{22}^{(\ell-1)} p_{23} = p_{22}^{(\ell-1)} (1-p_{22}) \ , \qquad (23)$$

$$\overline{\ell} = 1/(1-p_{22}) \ . \qquad (24)$$

Transition probability of the time series of wave period may well be approximated with the 2-dimensional Weibull distribution (8),(10). Then $p_{22}$ is given by:

$$p_{22} = \int_{t_{*1}}^{t_{*2}} \int_{t_{*1}}^{t_{*2}} f(t_1,\ t_2)\ dt_1 dt_2 \Big/ \int_{t_{*1}}^{t_{*2}} R(t_1)\ dt_1 \qquad (25)$$

where

$$f(t_1,\ t_2) = \frac{n^2}{4A} \frac{(t_1 t_2)^{n-1}}{t_r^{2n}} \exp\left[-\frac{\phi}{2A} \frac{(t_1^n + t_2^n)}{t_r^n}\right] I_0\left[\frac{\sigma}{A} \frac{(t_1 t_2)^{n/2}}{t_r^n}\right] \ , \qquad (26)$$

$$R(t_1) = \frac{n}{2\phi} \frac{t_1^{n-1}}{t_r^n} \exp\left[-\frac{1}{2\phi} \frac{t_1^n}{t_r^n}\right] \ , \qquad (27)$$

$$A = \phi^2 - \sigma^2 \ , \qquad \phi = \frac{1}{2}\left[\ \Gamma\left(\frac{n+2}{n}\right)\ \right]^{-n/2},$$

in which $f(t_1,\ t_2)$ is the 2-dimensional Weibull distribution and $R(t_1)$ the Weibull distribution, $\sigma$ the correlation parameter, n the shape parameter, $t_r$ the rms wave period, $\Gamma$ the Gamma function. Correlation coefficient of consecutive wave period $t_1$ and $t_2$ is:

$$\gamma_t = \frac{\left[\Gamma\left(\frac{n+1}{n}\right)\right]^2 \left\{F\left[-\frac{1}{n},\ -\frac{1}{n};\ 1;\ \left(\frac{\sigma}{\phi}\right)^2\right] - 1\right\}}{\Gamma\left(\frac{n+2}{n}\right) - \left[\Gamma\left(\frac{n+1}{n}\right)\right]^2} \ , \qquad (28)$$

in which F is the hypergeometric function. It follows that $p_{22}$ can be determined from the correlation coefficient of consecutive wave period and the shape parameter with equations (25) to (28). But both parameters are closely correlated with each other as shown in Fig.11. Plotted data are obtained from the numerical simulations. Then $p_{22}$ can be determined by either of these parameters using the average relation between them. Fig.12 shows the relation between $p_{22}$ and $\gamma_t$ for $t_{*1} = 0.4t_{mean}$, $t_{*2} = 0.6t_{mean}$ (solid line), $t_{*1} = 0.9t_{mean}$, $t_{*2} = 1.1t_{mean}$ (dotted line) and $t_{*1} = 1.4t_{mean}$ , $t_{*2} = 1.6t_{mean}$ (chain line) for example.

Fig.13 shows the theoretical distribution of the resonant wave period providing $t_{*1} = 0.7t_{mean}$ and $t_{*2} = 1.2t_{mean}$. This is the period band in which dynamic response of a spring is greater than twice the static loading providing that the resonant period of the spring equals $t_{mean}$. Figs.14 (a) $\sim$ (e) show the comparison of data and the theoretical distribution. The agreement of data and the distribution is very well.

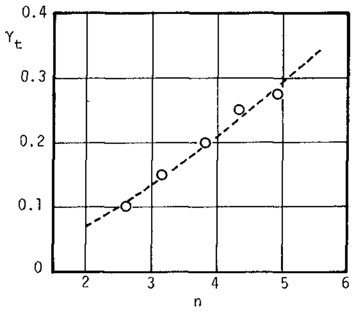



Fig.11 $\gamma_t \sim n$

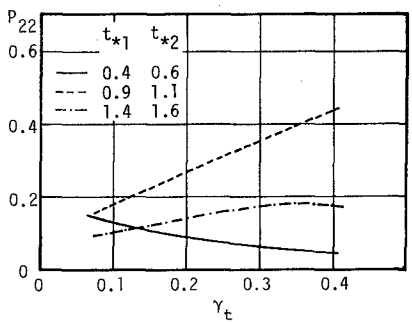



Fig.12 $p_{22} \sim \gamma_t$

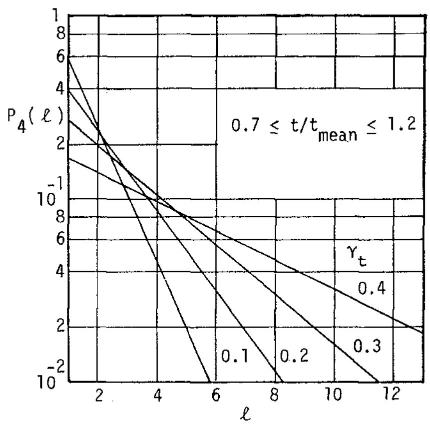
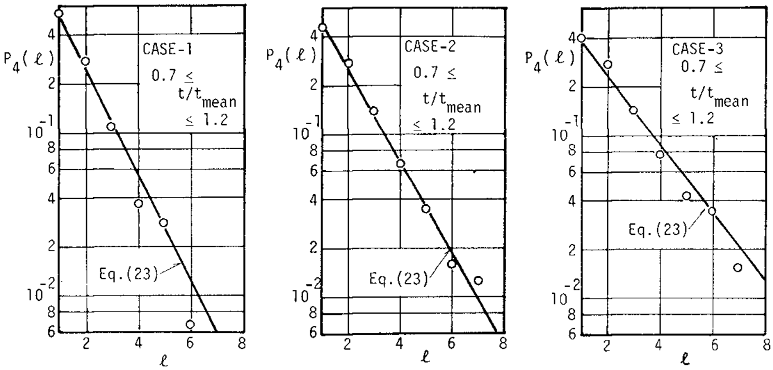
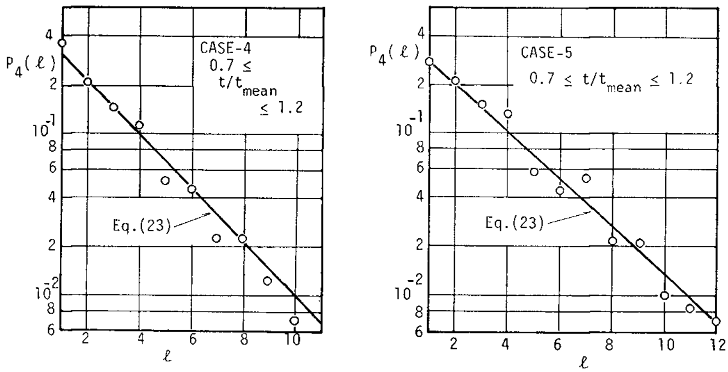



Fig.13 Probability distribution of the run of low waves




(a) CASE-1 (b) CASE-2 (c) CASE-3




(d) CASE-4 (e) CASE-5

Fig.14 Distribution of the resonant wave periods

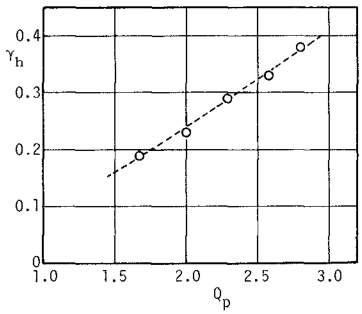



Fig.15 $\gamma_h \sim Q_p$

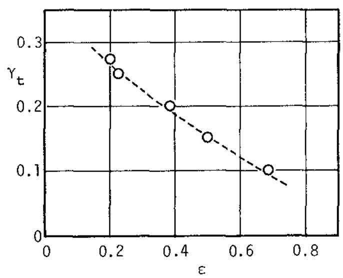



Fig.16 $\gamma_t \sim \varepsilon$

## ESTIMATION OF PARAMETERS

Statistical properties of the run of wave height can be determined from the correlation coefficient of consecutive wave height $\gamma_h$. But $\gamma_h$ changes with the width of the power spectrum. Fig.15 shows the relation between $\gamma_h$ and wave peakedness parameter $Q_p$ proposed by Goda(1). Plotted data are obtained from the numerical simulations. It is known from this figure that since $\gamma_h$ can be approximated from $Q_p$, statistical properties of the run of wave height can be estimated from $Q_p$.

On the other hand in determining the run of wave period, n and $\gamma_t$ are needed. But since both parameters are mutually correlated, statistical properties of the run can be determined by knowing either of them. Fig.16 shows the relation between $\gamma_t$ and the spectrum width parameter $\varepsilon$. $\gamma_t$ has close correlation with $\varepsilon$.

From Figs.15,16 it can be concluded that if the power spectrum of random waves is known, statistical properties of the run of wave height and wave period can be estimated.

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