Wave Height Distributions and Wave Grouping in Surf Zone

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Abstract

The main purpose of this paper is to propose a model for prediction of the spatial distributions of representative wave heights and the frequency distributions of wave heights of irregular waves in shallow-water including the surf zone. In order to examine the validity of the model, some experiments of irregular wave transformation have been made. In addition, an attempt has been made to clarify the spatial distribution of wave grouping experimentally. Especially the present paper focusses finding the effects of the bottom slope and the deep-water wave steepness on the wave height distribution and wave grouping.

1. Introduction

Battjes (1972) calculated the wave setup of irregular waves by a model of breaking waves that the Rayleigh distribution as the wave height distribution in deep-water is cut off partly by the breaker height determined by the local water depth. Afterwards Battjes (1978a) calculated the changes of the root-mean-square wave height and the wave setup of irregular waves on an arbitrary bottom slope by solving the energy balance equation, and it was reported that the calculated results agreed reasonably with the experimental ones. In that model of wave breaking the energy dissipation due to wave breaking was formulated by the bore model and the wave height distribution was cut off partly similarly to the previous model.

Since it is not realistic that the cumulative distribution function of the wave height is discontinuous at the breaking limit as Battjes's model, Goda (1975) proposed a model of irregular wave breaking that the portion of high waves in the Rayleigh distribution cut off partly is transfered to the portion of low waves with a certain probability due to breaking.

In the models of irregular wave breaking proposed by Battjes (1972) and Goda (1975), the Rayleigh distribution is modified with the breaker height determined by the local water depth, and the spatial propagation of waves with energy dissipation is not considered. On the other hand, it is considered in the model proposed later by Battjes (1978a), but still his model holds a problem that the cumulative distribution function of the wave height is discontinuous at the point of the wave height of breaking limit.

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In this paper, the representative wave heights such as the significant and mean wave heights and the wave height distributions are predicted under the assumption that the wave height change of an individual irregular wave is equal to that of monochromatic waves having the same wave height and wave period and it can be superimposed independently. This method of analysis will be called the individual wave analysis method hereafter.

In addition to the height and period of irregular waves, a phenomenon of wave grouping that large waves continue successively is important, because it seems to be one of main factors for collapses of coastal structures such as rubble mound breakwaters. It was recently reported by Johnson et al. (1978) that even if energy spectra of irregular waves are same, a degree of damage is not same but depends on wave grouping of irregular waves. Accordingly, characteristics of wave grouping in shallow-water must be investigated as an important factor of design waves. The characteristics of wave grouping in shallow-water have not been studied little so far.

2. Calculation method for wave height change of irregular waves in shallow-water

2.1 Formulation for wave height change of individual wave

The wave height change of an individual wave defined by the zero-up-cross method is formulated under the following assumptions: (1) an individual wave among irregular waves is transformed independently, (2) the small amplitude wave theory is applied to wave shoaling before wave breaking, (3) the breaker height is calculated by modifying the breaker index for monochromatic waves proposed by Goda (1970), (4) the bore model adopted by Battjes (1978b) is used in a modified form after breaking, and (5) the temporal change of the mean water level is considered by introducing the effect of surf beats in addition to the spatial change of the mean water level due to wave setup.

The representative wave heights and the wave height distributions are predicted by superimposing the calculated wave height of an individual wave.

Based on the assumption (2), the following equation of the shoaling coefficient K_s is used:

$$K_s = \left[\left\{1 + \frac{4\pi d/L}{\sinh 4\pi d/L}\right\} \tanh \frac{2\pi d}{L}\right]^{-1/2}, \tag{1}$$

and based on the assumption (3), Goda's breaker index

$$H_b/L_o = A\left[1 - \exp\left\{-1.5 \frac{\pi d}{L_o} (1 + 15 \tan^{4/3} \theta)\right\}\right], \tag{2}$$

is applied, in which A=0.17 in the case of monochromatic waves, $\tan\theta$ is the bottom slope, and L_o the deep-water wave length, L the wave length in shallow-water, d the mean water depth, and H_b the breaking wave height.

Concerning the change of the wave height after breaking, the bore model adopted by Battjes (1978b) is used. Both factors of the bottom slope and the deep-water wave steepness can be introduced to this model by using the surf similarity parameter. The non-dimensional wave height in the surf zone is expressed as follows:

$$\left.\begin{aligned}\tilde{H}^{-4} &= (1 - \frac{4}{9}K)\tilde{d} + \frac{4}{9}K\tilde{d}^{-7/2},\\ K &= (\frac{2}{\pi})^{1/2}B\gamma^{1/2}\xi_o^{-1},\\ \gamma &= 0.7 + 5\tan\theta \qquad (0.01 \le \tan\theta \le 0.1),\end{aligned}\right\} \tag{3}$$

in which $\tilde{H} = H/H_b$, $\tilde{d} = d/d_b$, d_b is the breaking water depth, and $\xi_o = \tan\theta/\sqrt{H_o/L_o}$, the surf similarity parameter in deep-water.

The water depth d in the above equations should be taken as the mean water depth. Battjes (1978b) assumed that the mean water level in the surf zone has a gradient equal to 1/5 of the bottom slope with zero setup at the breaking point; therefore, $\tan\theta$ in Eq. (3) becomes as gentle as 0.8 times of the actual bottom slope.

In this paper, the wave height change is firstly calculated by Eqs. (1), (2) and (3) for the still water depth h, and secondly the wave setup $\bar{\eta}$ is calculated numerically from deep-water to the shoreline by using the obtained wave hiehgts and the following equation:

$$\frac{d\bar{\eta}}{dx} = -\frac{1}{(h+\bar{\eta})}\frac{d}{dx}\left\{\frac{1}{8}H^2\left(\frac{1}{2} + \frac{2k(h+\bar{\eta})}{\sinh 2k(h+\bar{\eta})}\right)\right\} \tag{4}$$

The calculated value $\bar{\eta}$ is added to the still water depth h, and the calculation for the wave height change is repeated for the mean water depth $d = h + \bar{\eta}$. The repeated calculation is closed when the difference of successive wave setup becomes as small as 1% after repeats of several times.

Since the calculated value of $\bar{\eta}$ obtained from Eq. (4) using the measured wave height seem to give an overestimate compared with the experimental results, Eq. (4) is modified with the multiplication factor of 0.6.

Energy dissipation due to wave breaking is composed of various factors such as the horizontal roller, the bottom friction and the tubulence with air entrainment (Sawaragi and Iwata (1974)). Eq. (3) is derived from the energy balance equation in which the energy dissipation due to wave breaking is simplified by simulating that due to a bore. Since Eq. (3) can not always fit the experimental results well in all cases, the value of B in Eq. (3) is determined so that the spatial distributions of the wave height agree with the experimental results by Nakamura et al. (1966), Saeki et al. (1974) and Singamsetti et al. (1978) as follows:

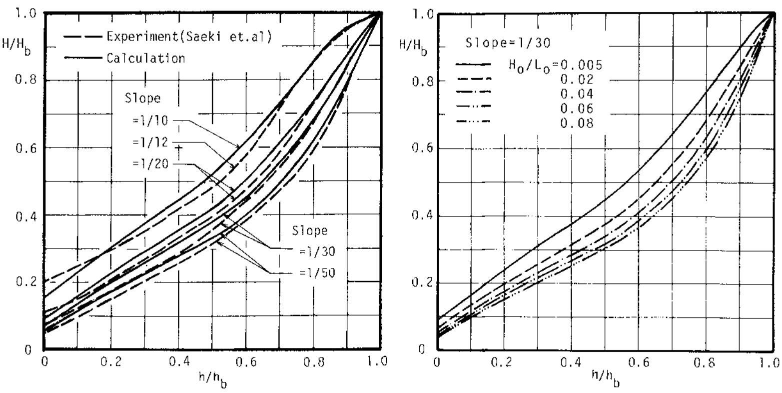
(i) In cases that $\tan\theta > 1/20$,

$$\left.\begin{array}{lll} B = 1 & \text{for} & 0.9 \leq \tilde{d} \leq 1.0, \\ B = 13-40\tilde{d}/3 & \text{for} & 0.6 \leq \tilde{d} \leq 0.9, \text{ and} \\ B = 5 & \text{for} & \tilde{d} \leq 0.6. \end{array}\right\} \quad (5)$$

(ii) In cases that $\tan\theta < 1/20$,

$$\left.\begin{array}{lll} B = 11-10\tilde{d} & \text{for} & 0.6 \leq \tilde{d} \leq 1.0, \text{ and} \\ B = 5 & \text{for} & \tilde{d} \leq 0.6. \end{array}\right\} \quad (6)$$

Fig. 1(a) shows the wave height decay of monochromatic waves with decrease in the still water depth after wave breaking in dimensionless form for the deep-water wave steepness of 0.02 with a parameter of the bottom slope from 1/10 to 1/50, in which the experimental results by Saeki et al. (1974) are plotted herewith. Fig. 1(b) also shows the wave height decay after breaking for the bottom slope of 1/30 with a parameter of the deep-water wave steepness from 0.005 to 0.08. It is seen from these figures that the wave height decay becomes remarkable with decrease in the bottom slope and increase in the deep-water wave steepness.



(a) (b)

Fig. 1 Wave height decay after wave breaking

2.2 Different points between transformations of breaking irregular waves and monochromatic waves

One of different points between irregular and monochromatic wave transformations in the surf zone is the breaking limit of an individual wave; that is, the breaking wave height of an individual wave of irregular waves is smaller than that of monochromatic waves having the same wave height and period (Kimura and Iwagaki (1978), Sawaragi and Iwata (1981)). The other point is the generation of temporal variations of the water surface having some long-periods, so called surf beats; therefore, they may affect transformations of breaking irregular waves.

With respect to the first point, the reasonable value of A in Eq. (2) is investigated by experiments, and for the second point the following empirical formula of the root-mean-square value of surf beats ζ_{rms} based on the field observation by Goda (1975) is used:

$$\frac{\zeta_{rms}}{H_o} = \frac{0.01\,H_o}{\sqrt{\frac{H_o}{L_o}\left(1+\frac{d}{H_o}\right)}} , \qquad (7)$$

in which H_o denotes the significant wave height in deep-water and L_o the wave length in deep-water corresponding to the significant wave period.

In this paper, the surf beat elevation ζ is added to the mean water depth by generating random numbers of the normal distribution with the mean value of 0 and the root-mean-square value of ζ_{rms} by Eq. (7).

2.3 Calculation method for wave height change of irregular waves

As input data, wave heights and periods obtained from the wave record at a constant water depth by the zero-up-cross-method, the bottom slope and the still water depths at all points to predict wave heights are given in this paper.

After calculation of the wave height change of an individual wave for the still water depth and surf beats, the root-mean-square wave heights $\overline{H^2}$ are calculated to obtain the wave setup $\overline{\eta}$ from Eq. (4) with $\overline{H^2}$ instead of H and the wave length corresponding to the significant wave period. Then, the wave height of an individual wave is calculated for the still water depth h, the obtained wave setup $\overline{\eta}$ and the surf beat ζ. Repeating this procedure, the calculation is closed when the difference of successive wave setup becomes less than 1% of the wave setup, and the representative wave heights and the frequency distributions of wave height are obtained. Fig. 2 shows the flow chart for this calculation.

In this calculation program, several supplemental water depths in addition to the water depths of input data are adopted in order to solve Eq. (4) numerically, and many additional water depths are used for determination of the breaking point.

3. Experimental aparatus and procedures

A wave tank, 27m long, 50cm wide and 70cm high, at the Department of Civil Engineering, Kyoto University was used in the present experiments. A wave generator

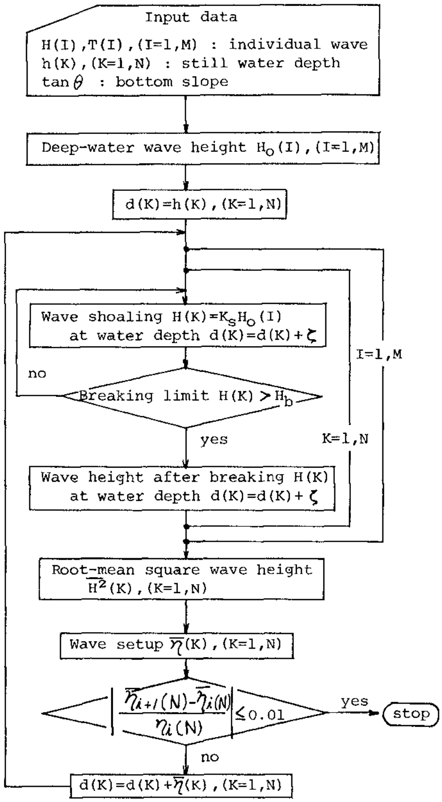


Fig. 2 Flow chart for calculation of wave height change of irregular waves

of irregular waves is installed at an end of the tank.

In order to examine at first the breaker height of an individual irregular wave, some experiments were carried out on the uniform slopes of 1/20 and 1/30 by taking photographs with a 16mm-cinecamera. An experiment for the 1/10 slope was not carried out because the data of breaker height of an individual wave tend to scatter (Kimura and Iwagaki (1978)). In these experiments, the elevation of the top of the slope was ajusted to be equal to the still water level in order to decrease the unsteady back wash which was considered as one of reasons for the difference from the breaking characteristics of regular waves. The range of the breaker depth to be measured by taking pictures was from 10 to 15cm. Irregular waves having the spectra of Pierson-Moskowitz type and two kinds of deep-water wave steepnesses were used. The water depth was kept to be 50cm at the uniform section. The films were analyzed by a film motion analyzer.

In order to examine furthermore the spatial distributions of wave heights and wave grouping of irregular waves with decrease in the water depth, experiments were carried out on the uniform solpes of 1/10 and 1/30 with the water depth of 45cm at the uniform section. Irregular waves having the spectra of Pierson-Moskowitz type and five kinds of deep-water wave steepnesses of 0.005 to 0.07 were used. Water surface elevations were measured by six wave gauges of capacitance type, which were installed at the water depths of 45,20,15,10,5 and 2cm in the case of 1/10 slope and 45,20,12,8,5 and 2cm in the case of 1/30 slope. Additional experiments for the slopes of 1/20 and 1/30 with the water depth of 50cm were carried out by using eleven wave gauges of capacitance type to investigate the change of wave grouping in detail.

Water surface elevations were recorded in a 14-channel analog data recorder. The records were digtized by an A-D converter with 0.04sec time interval and used in analysis.

4. Experimental results and discussions

4.1 Breaking characteristics of individual wave

Figs. 3(a) and (b) show the relationships between H_b/h_b and h_b/L_o on the uniform slopes of 1/20 and 1/30. An individual wave is defined by the spatial zero-up-cross method. The solid lines in the figures represent the results by Eq. (2), and the dashed lines show the results by putting the value of A in Eq. (2) as 0.13,0.14,0.15 and 0.16. The tendency that H_b/h_b decreases with increase in h_b/L_o is similar to that by Eq. (2). However, the experimental data are generally plotted somewhat smaller than the values by Eq. (2). Since the unsteady back wash seems to be weak in the present experiments, the data are plotted a little larger than those by Kimura et al. (1978) and Sawaragi et al. (1981). In determining the breaker height of an individual wave for numerical calculation, the value A in Eq. (2) is taken as 0.16.

4.2 Representative wave heights

Figs. 4(a) and (b) show the changes of the non-dimensional significant wave height $H_{1/3}/H_o$ and mean wave height $\overline{H}/H_o$ with decrease in the water depth on the uniform slopes of 1/10 and 1/30, respectively. An individual wave is defined by the temporal

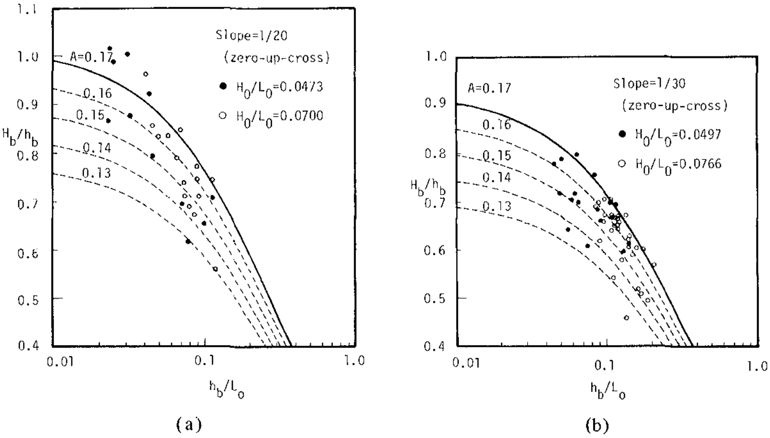


Fig. 3 Relationships between H_b/h_b and h_b/L_o for individual waves

zero-up-cross method, in which the mean water level is determined by simply averaging the water surface elevation. In very shallow water, however, there exist sometimes no intersecting waves with the mean water level. As a result, the significant wave height becomes large and the number of an individual wave decreases compared with the waves obtained from the water surface elevation after the effect of surf beats is eliminated. Therefore, the wave height distributions become different depending on whether the effect of surf beats is eliminated or not, as described later. The curves in Fig. 4(a) and (b) show the calculated results predicted by the individual wave analysis method.

It is found from these figures that the non-dimensional wave heights become large as the deep-water wave steepness, which is estimated from the significant waves at the constant depth by the small smplitude wave theory, becomes small and the bottom slope becomes steep. Concerning the significant wave height, the experimental results are a little larger than calculated ones when the relative water depth h/H_o is less than 1.0, and the calculated results in the region of wave shoaling using the shoaling coefficient by the small amplitude wave theory are a little smaller than the experimental ones. Concerning the mean wave height, the calculated results are somewhat larger than the experimental ones in the case of small deep-water wave steepness. Generally speaking, the predicted curves agree fairly well with the experimental results.

4.3 Wave height distributions

Figs. 5 and 6 show the frequency distributions of wave heights for the cases of

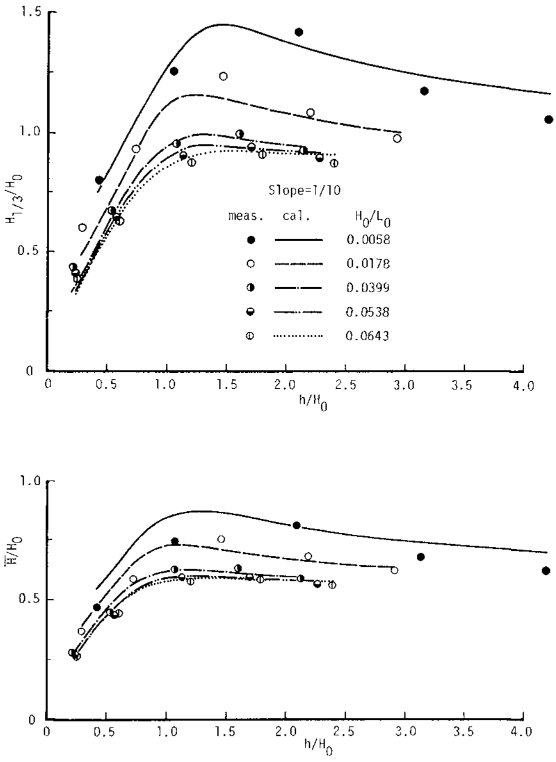


Fig. 4 (a) Change of significant and mean wave heights of irregular waves in shallow-water for 1/10 beach slope

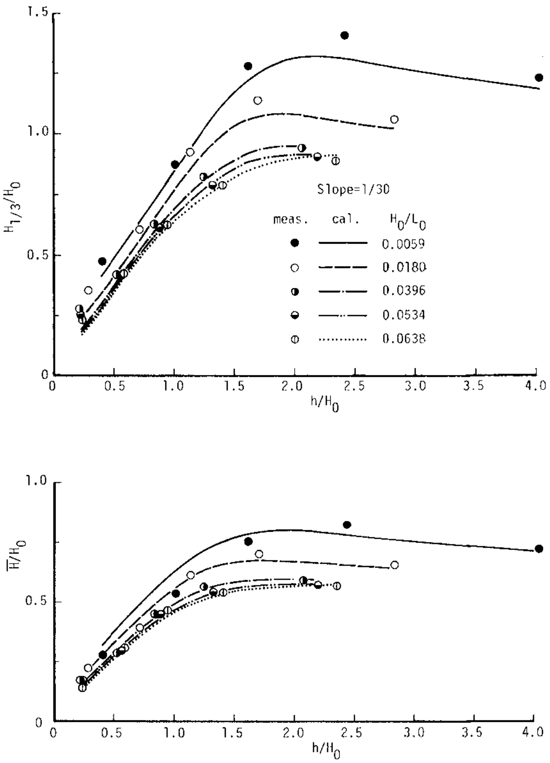


Fig. 4 (b) Change of significant and mean wave heights of irregular waves in shallow-water for 1/30 beach slope

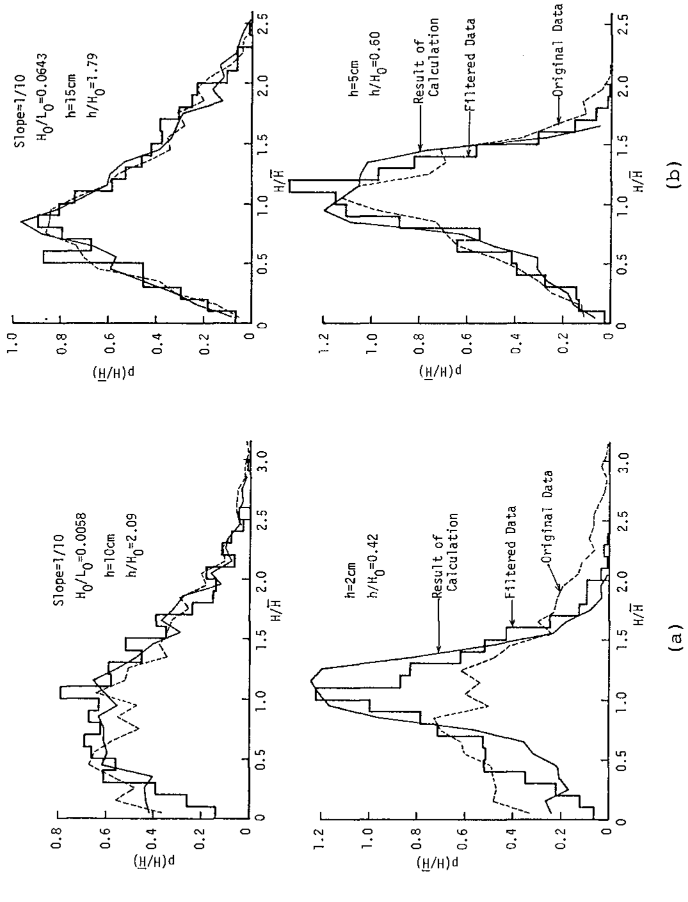


Fig. 5 Wave height distribution in shallow-water for 1/10 beach slope

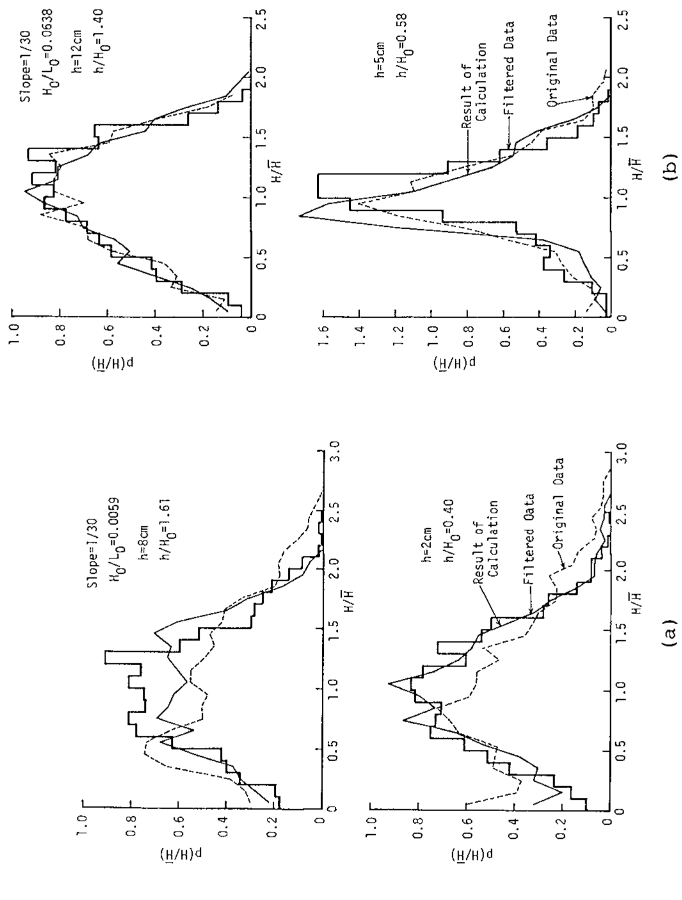


Fig. 6 Wave height distribution in shallow-water for 1/30 beach slope

the smallest and largest wave steepnesses in the experiments on the uniform slopes of 1/10 and 1/30, respectively, in which the wave height is normalized by the local mean wave height. In these figures the bent solid lines show the calculated results by the individual wave analysis method, the bent dashed lines show the experimental results obtained from the simply averaged mean water level denoted by "original data", and the histogram show the experimental results obtained from the "filtered data" of water surface elevations by cutting off the component waves of surf beats having the lower frequency than f_L Hz and the component waves having the higher frequency than f_U Hz. Cutting off of the high frequency components is for elimination of abrupt rising of the water surface due to splash with breaking and small disturbed waves after breaking. The frequency f_L was determined as $0.5f_p$ (f_p: the peak frequency) from the shapes of energy specta, and f_U was taken as $6f_p$ for convienience. An example of original and filtered water surface variations are shown in Fig. 7.

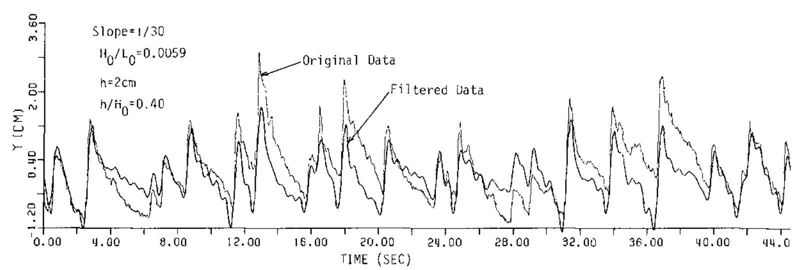


Fig. 7 Original and filtered water surface variations

In the case of large deep-water wave steepness as shown in Fig. 5(b) and Fig. 6(b), the shapes of the dashed line obtained from the original data and the histogram from the filtered date are almost same. It means that the effect of the filter is not so remarkable. However, in the case of small deep-water wave steepness as in Fig. 5(a) and Fig. 6(a), the shapes of the dashed line become flatter than those of the histogram, which means the effect of the filter is remarkable. Comparisons of the calculated wave height distributions with those of original data show fairly good agreement except for the case of small deep-water wave steepness in very shallow water. However, it is found that even in the case of small deep-water wave steepness, if the long-period variation due to surf beats and the high frequency components are removed from the original water surface variation, the calculated results agree well with the experimental ones. In other words, how to define an individual wave in very shallow water becomes important (Hotta (1980)).

4.4 Wave grouping

At present, several indicies are used to represent the degree of wave grouping, Goda (1970) used a run of wave heights $j(H_c)$ defined as the number of sequent waves, the heights of which exceed a definite value H_c and a total run $l(H_c)$ defined as the number of waves between the first excess over the definite level H_c and the next excess over the same level. Funk et al. (1979) used a Groupiness Factor defined as the coefficient of variation of the time history of wave energy.

In the following, the spatial changes of wave grouping with decrease in the water depth are examined by using the parameter of the mean length of run $\overline{j(H_c)}$ and the mean length of total run $\overline{l(H_c)}$. Figs. 8(a) and (b) show the changes of $\overline{j(\overline{H})}$ and $\overline{l(\overline{H})}$ on the uniform slopes of 1/10 and 1/30, respectively. Figs. 9(a) and (b) show the detailed experimental results of wave grouping for the bottom slopes of 1/20 and 1/30, respectively. The followings can be seen from Figs. 8 and 9: In the case of 1/10 slope the mean length of run $\overline{j(\overline{H})}$ and total run $\overline{l(\overline{H})}$ over the mean wave height are almost constant independently of the deep-water wave steepness when the relative water depth h/H_o is larger than 1.5 and decrease with decrease in the water depth. On the other hand, in the cases of gentle slopes as 1/20 and 1/30, the mean lengths of run $\overline{j(\overline{H})}$ and total run $\overline{l(\overline{H})}$ become large as the deep-water wave steepness becomes large. The mean lengths of total run $\overline{l(\overline{H})}$ in the case of 1/20 and 1/30 slopes decrease with decrease in the water depth in similar way to the case of 1/10 slope. However, the mean lengths of run $\overline{j(\overline{H})}$ take maximum values in the region $1.0 \leq h/H_o \leq 1.5$ and decrease rapidly with decrease in the water depth. This tendency is remarkable in the case of gentle slope and large deep-water wave steepness. The mean lengths of run $\overline{j(H_{1/3})}$ and total run $\overline{l(H_{1/3})}$ over the significant wave height also tend to take maximum slightly as well as $\overline{j(\overline{H})}$ and $\overline{l(\overline{H})}$.

According to the results of numerical simulation by Goda (1970), $\overline{j(H_c)}$ and $\overline{l(H_c)}$ become small with decrease in the spectral peakedness parameter Q_p. From the present experimental results, however, it is noted that $\overline{j(H_c)}$ and $\overline{l(H_c)}$ increase near the relative water depth of 1.5 in spite of decrease in the spectral peakedness parameter due to the rapid decay of a main peak and the grow of the low frequency components of the spectrum by wave breaking.

5. Conclusions

Among the transformation characteristics of irregular waves in shallow-water including the surf zone, the changes of the significant and mean wave heights, the wave height distributions and wave grouping have especially been investigated. In addition to the experiments of irregular waves, a model for prediction of wave heights named the individual wave analysis method has been proposed. The summaries of the results are described bellow.

(1) An individual wave of irregular waves tends to break compared with monochromatic waves having the same wave height and period in the experiments.

(2) Non-dimensional significant wave height $H_{1/3}/H_o$ and mean wave height $\overline{H}/H_o$ become large as the bottom slope becomes steep and the deep-water wave steepness becomes small.

(3) The predicted significant and mean wave heights are in fairly good agreement with the experimental ones.

(4) Comparisons of the predicted wave height distributions with those of the original experimental data show good agreement except for the case of small deep-water wave steepness in very shallow water. Even in such cases, if the long-period variation

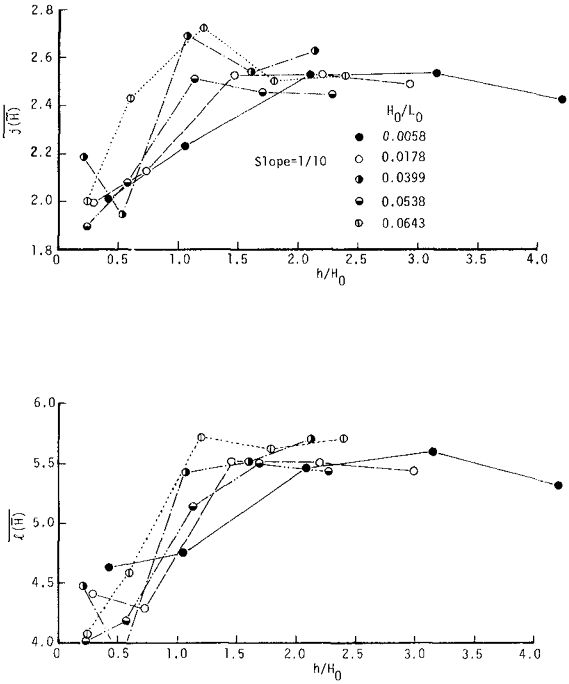


Fig. 8 (a) Change of wave grouping in shallow-water for 1/10 beach slope

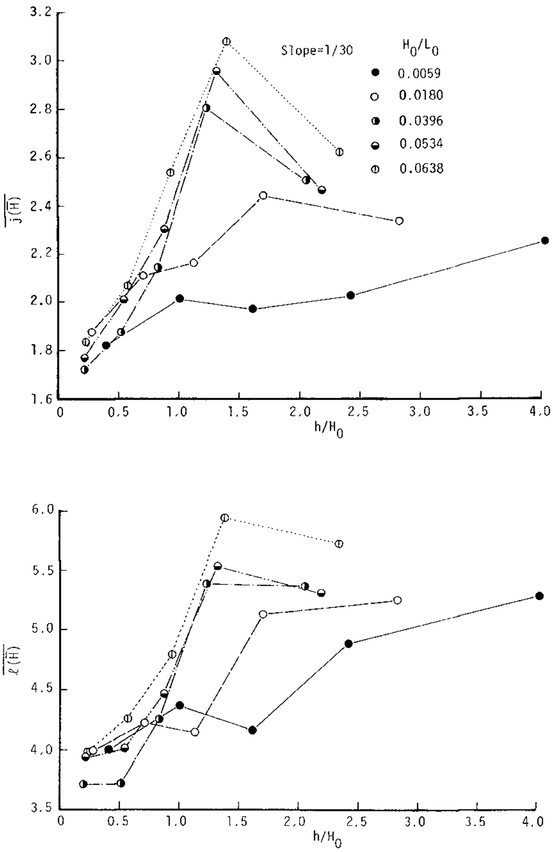


Fig. 8 (b) Change of wave grouping in shallow-water for 1/30 beach slope

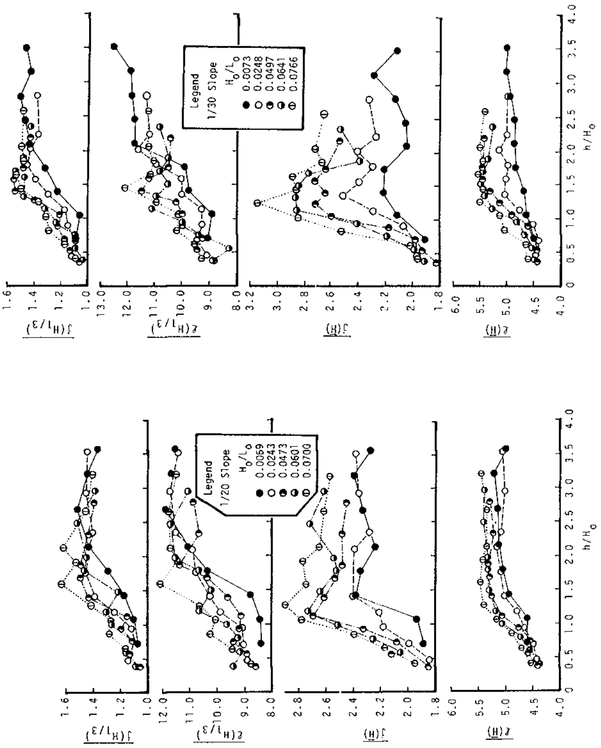


Fig. 9 Change of wave grouping in shallow-water for slopes of 1/10 and 1/30

due to surf beats and high frequency components of the water surface variation are removed, the calculated and experimental results agree well.

(5) Wave grouping becomes remarkable at the range of the relative water depth $1.0 \leq h/H_0 \leq 1.5$ with decrease in the bottom slope and increase in the deep-water wave steepness.

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