

ANALOGOUS DISPERSION PROPERTIES OF SURF ZONE
AND ELECTROMAGNETIC WAVES

by
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ABSTRACT

Resonant interaction processes in the surf zone are marked by an anomalous dispersion property ($dc/df > 0$) as it is well known from resonance absorption processes of electromagnetic waves in dielectrics.

INTRODUCTION

At least since 1974 there has been an increasing number of publications dealing with so-called anomalous dispersion properties observed from deepwater wind waves (RAMAMONJIARISOA and COANTIC, 1976; LAKE and YUEN, 1978) as well as from waves in areas of transitional and shallow water depth (MASSEL and CHYBICKI, 1980; BÜSCHING, 1978). Most of the respective experiments are carried out by estimating the cross correlation or the cross spectral density functions from the signals of some wavegauges placed in the wave field appropriately.

From investigations in a large wind-water facility for deepwater wind waves for instance the most striking departure from theoretical values consists in the phenomenon of nondispersiveness $dc/df = 0$ especially at "phase-locked" higher frequency components with phase velocities close to the dominant wave celerity, see Fig. 1. There is, however, another deviation to be seen describing an anomalous dispersion $dc/df > 0$ in the low frequency range coincident with rather small energy

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densities. By contrast, the author's previous field investigations in the surf zone at heavy storm surge conditions have shown an anomalous dispersion property $dc/df > 0$ in the whole energy containing frequency range (BÜSCHING, 1978 a).

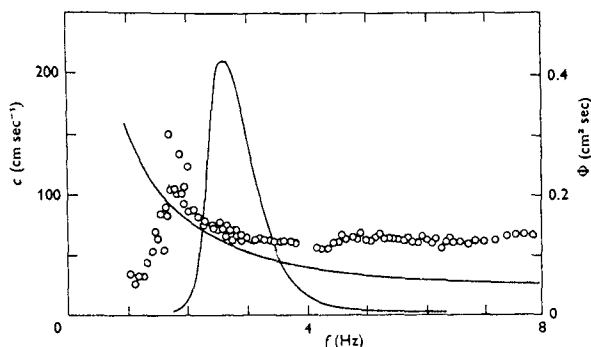


Fig. 1: The variation in phase speed of spectral components measured by RAMAMONJIARISOA (1974).

The theoretical phase speed $c = g/2\pi f$ and the spectral shape are also shown.

The anomalous dispersion of low frequency components in "deepwater" wind wave spectra is often attributed to the presence of wave groups travelling with the appropriate group velocity (PHILLIPS, 1978). It is, however, doubted by the author whether the longer period frequency components can really be regarded as deepwater components even in a "large" wind-water facility, and moreover it is questionable whether it can be distinguished between phase velocities on the one hand and group velocities on the other hand both in the same plot based on spectral analysis. In the following it will be demonstrated that resonant interaction effects are responsible for an anomalous dispersion property to occur in the very shallow portion of the surf zone. The present contribution refers to measurements previously evaluated in detail by the author, see BÜSCHING (1978 b).

ANOMALOUS DISPERSION OF SURF ZONE WAVES

Frequency Domain Data

In the upper parts of figures 2 and 3 two sets of synchronously measured energy spectra of water level deflexions are to be seen both characterizing different wind and water depth conditions at stations 100 m and 85 m distant from the shoreline respectively in a coast perpendicular measuring profile on the isle of SYLT/North Sea.

Although that measurements had been carried out at heavy storm surge conditions coherence $\overline{\gamma}^2$ is quite good at the energy containing frequency components (about $\overline{\gamma}^2 = 0.8$ and $\overline{\gamma}^2 = 0.7$ respectively). In this cases the phase velocity was calculated from the phase information of the transfer functions plotted below.

As to be seen from the graph at high coherence values the phase information can very well be approximated by a linear regression

$$\phi(f) = \phi_n + \alpha \cdot f \quad (1)$$

in which $\alpha \cdot f$ is due to the distance of the two measuring stations $\Delta x = 15$ m, and it can be shown from the conversion into phase velocity applying

$$c(f) = \frac{\Delta x \cdot 360^\circ}{\phi(f)} \cdot f \quad (2)$$

that the magnitude of the positive portion of ϕ_n decides on the degree of the anomalous dispersion property $dc/df > 0$.

As was found by the author in 1978 the anomalous dispersion gets more distinct with the water depth decreasing; hence, the magnitude of ϕ_n in Fig. 3 is greater than in Fig. 2:

$$\phi_2 > \phi_1 \quad (3)$$

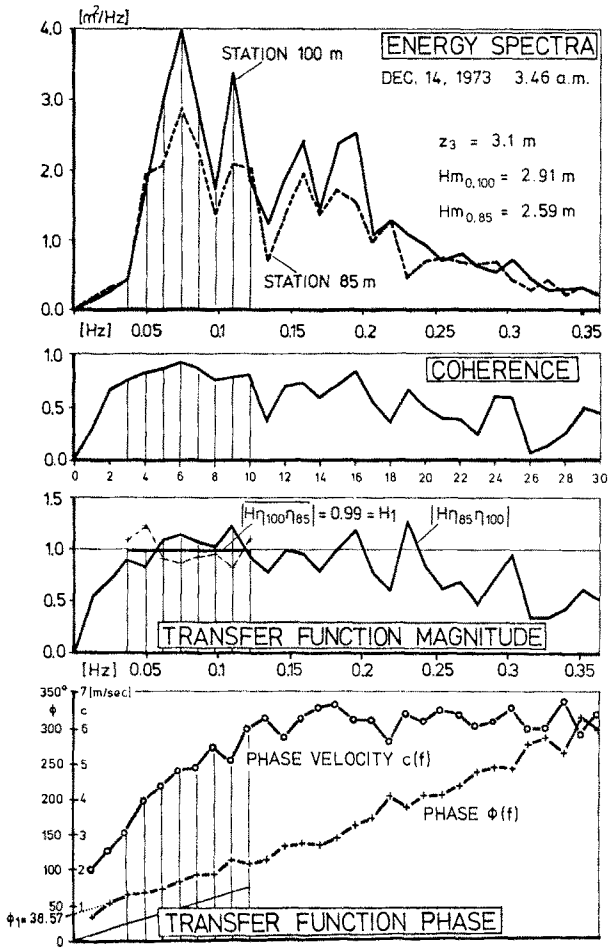


Fig. 2: Spectral functions of two wave gauge signals at a high tide water level. Gauge distances from shore 100 m and 85 m respectively.

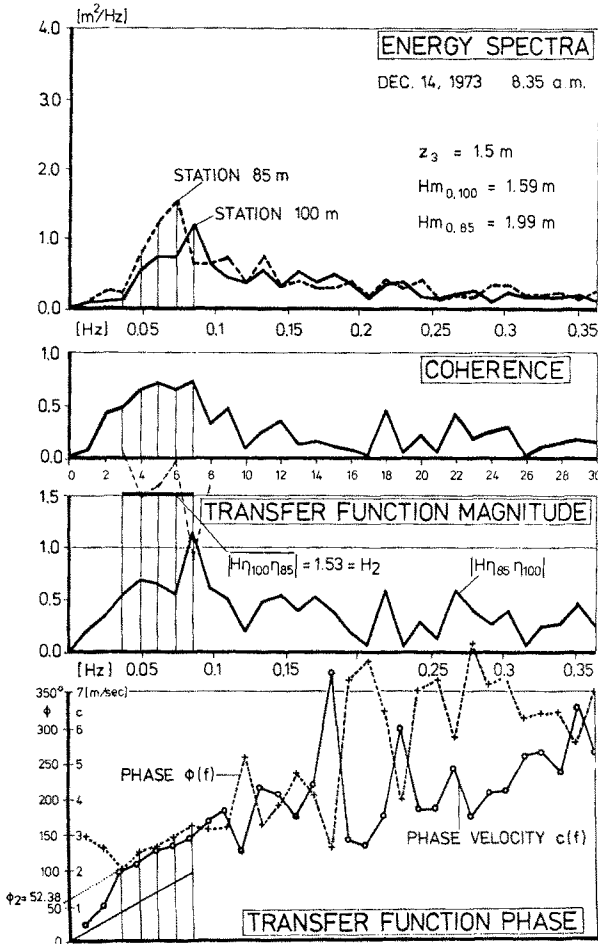


Fig. 3: Spectral functions of two wave gauge signals at a low tide water level.

Gauge distance from shore 100 m and 85 m respectively.

There is, however, another phenomenon to be seen from Fig. 3 representing a very shallow water depth condition. Obviously a peak shift to lower frequencies has happened on the rather short distance of $\Delta x = 15$ m in the upbeach direction, and additionally an increase in the maximum energy density can be watched. This fact also comes out clearly from the magnitude of the transfer function, if its inverse values are considered at high coherence only, resulting in an average value of

$$H_2 = |\overline{H\eta_{100}^{\eta_{85}}}| = 1.53 \quad (4)$$

Similar energy shifts to lower frequencies due to decreasing water depth are also to be seen from spectra published by SONU, PETTIGREW and FREDERICKS (1974) and GODA (1975). By contrast, at the condition of Fig. 2, marked by a greater water depth, energy density decreases from station 100 m to station 85 m accordingly resulting in an average value

$$H_1 = |\overline{H\eta_{100}^{\eta_{85}}}| = 0.99 < 1 \quad (5)$$

Time Domain Data

Increasing wave periods in the upbeach direction (in accordance with a peak shift to lower frequencies) could also be detected in the time domain from synchronously taken strip charts.

Figures 4 and 5 show the results of a zero crossing evaluation technique applied to the data of 6 and 5 measuring stations respectively still in operation in the same coast perpendicular measuring profile at normal weather conditions.

Besides the wave heights here the wave propagation velocity is plotted with reference to the water depth, the wave periods and the wave length respectively. Because of the lack of space here only two things shall be pointed out with respect to broken waves.

After breaking the wave deceleration is less than before breaking, and, as a consequence of an increasing "wave period", there is an anomalous

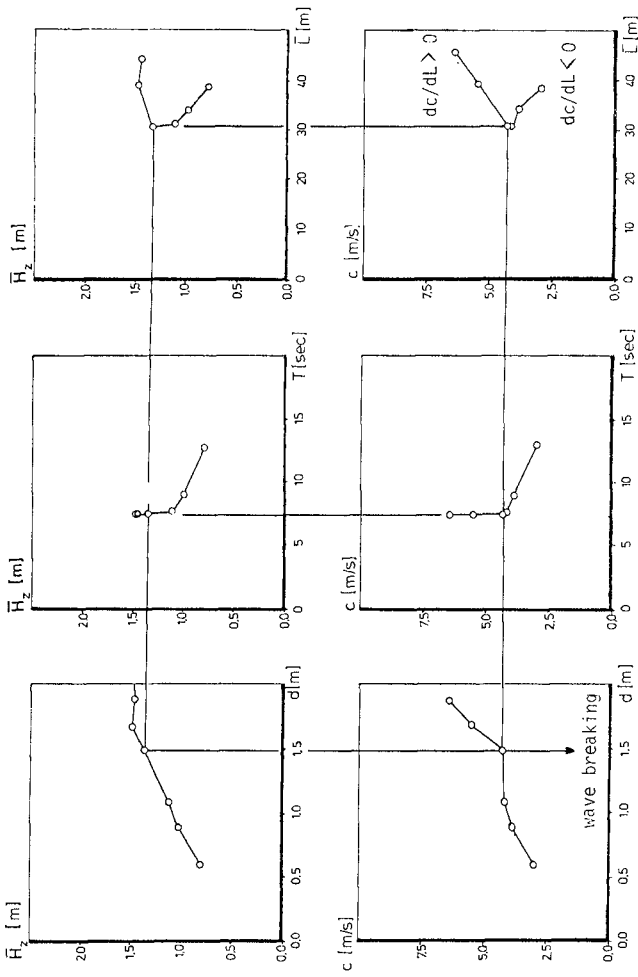


Fig. 4: Variation of mean wave heights \bar{H}_z and propagation velocity c with water depth, wave period and wave length respectively in a coast perpendicular measuring profile on April 3rd, 1973

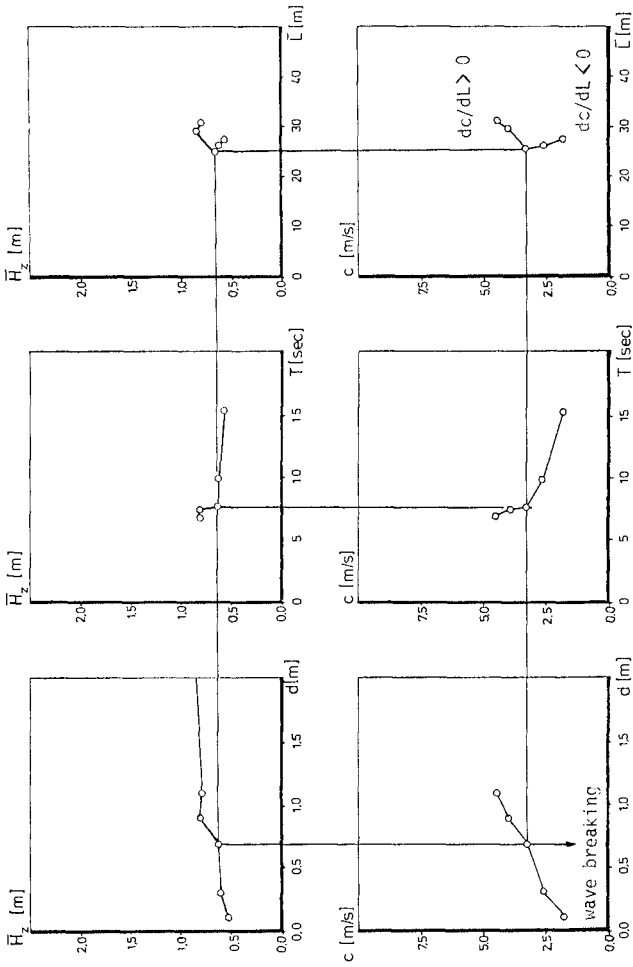


Fig. 5: Variation of mean wave heights \bar{H}_z and propagation velocities c with water depth, wave period and wave length respectively in a coast perpendicular measuring profile on March 18th, 1973

dispersion characterized by $dc/dL < 0$. The details of that measurements are also given in BOSCHING (1978 b).

ANOMALOUS DISPERSION OF ELECTROMAGNETIC WAVES

From electromagnetic waves it is well known that an anomalous dispersion of frequency components is due to a resonance phenomenon (resonance absorption in dielectrics). If one has an electromagnetic wave propagating through a medium made up of a number of oscillators (electrons or ions) per unit volume one gets a complex refractive index

$$n^* = n - ik \quad (6)$$

in which n is the real refractive index (also representing phase velocity) and $k = n \cdot \kappa$ is an absorption quantity (also representing the amplitude response near resonance).

As to be seen from Fig. 6 both components vary with wave length. By the equation (6) it is stated that resonance, absorption and anomalous dispersion are combined effects. If an incident component wave has nearly the same length or frequency as an oscillator of the medium resonance occurs. The oscillator absorbs the energy from the forcing incident wave. At resonance absorption represented here by k is a maximum, and in the vicinity of the spot of resonance always an anomalous dispersion $dn/dL > 0$ corresponding to $dc/df > 0$ (or $dc/dL < 0$ or $dn/df < 0$) appears. In a light spectrum this means that the sequence of spectral colours here is the inverse to the normal case: refraction of the red colour is a maximum instead of the violet colour. It is because of the coincident maximum of absorption that we are not as familiar with this kind of dispersion as we are with the normal dispersion property. In liquids and solids, due to molecular collisions and strong intermolecular forces, the absorption and dispersion curves become very much broadened, the broadening being very irregular owing to the varying molecular force field. Additionally damping is an essential quantity as it is true to all forced vibration processes.

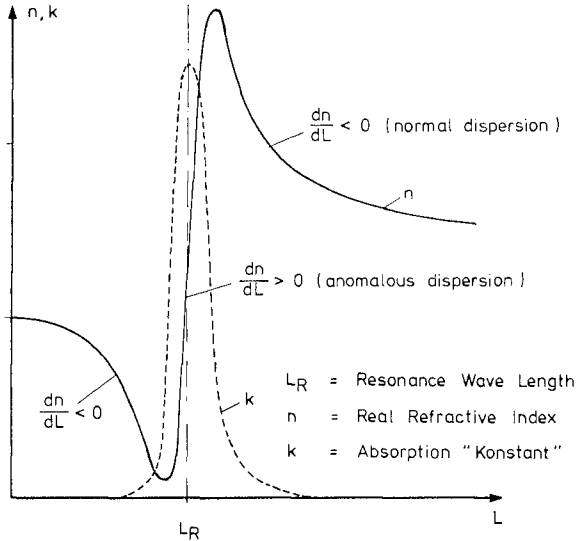


Fig. 6: Absorption above and below a region of anomalous dispersion centered about the resonance wave length

Moreover it is well known from thin film optics that dispersion properties depend on the actual film thickness, see for instance MAYER (1950).

With respect to the transmission of light through a gold film it can be seen from Fig. 7 that an anomalous dispersion property gets more distinct in the visible spectrum with the film thickness decreasing. At a rather thick film (solid line) the visible spectrum is dominated by a normal dispersion whereas at a very thin film there is an anomalous dispersion in the total visible spectrum.

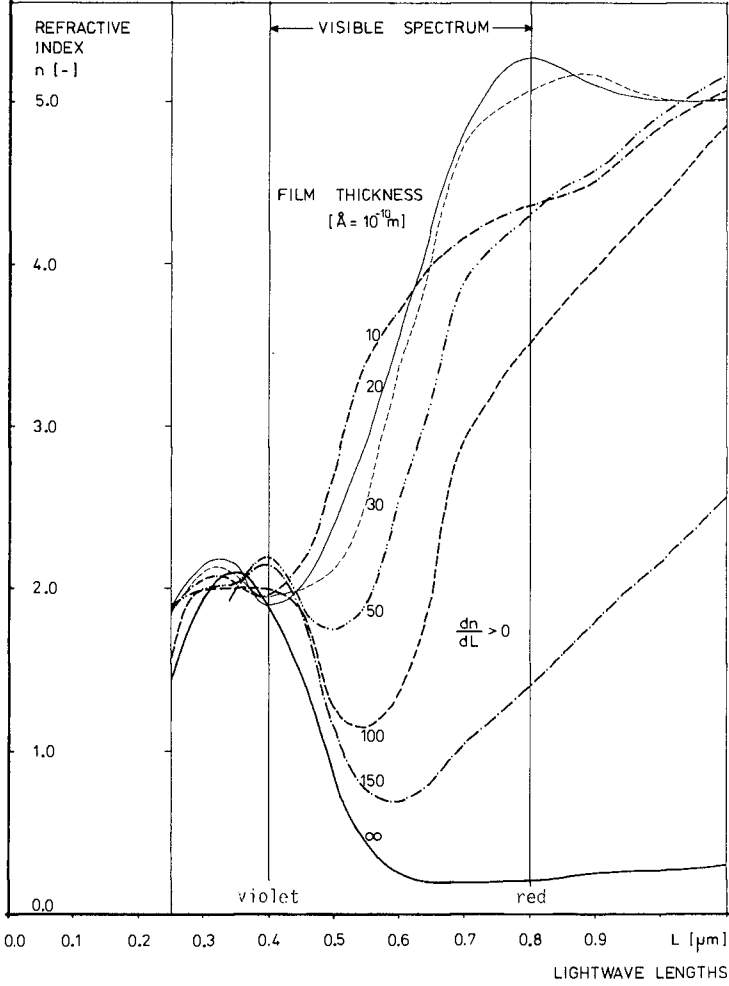


Fig. 7: Variation of dispersion properties at transmission of light through gold films with respect to decreasing film thicknesses

DISCUSSION AND CONCLUSIONS

Formerly the author was able to deduce an anomalous dispersion property combined with a peak shift Δf to lower frequencies from an equation

$$\Delta f = f_0 \cdot \frac{2 \cdot [c(x) - c_0]}{2 \cdot c_0 - c(x)} = f(x) - f_0 \quad (7)$$

see BÜSCHING (1980) in which f_0 and c_0 are the component frequency and celerity respectively at a known position in the surf zone, and $c(x) < c_0$ and $f(x) < f_0$ are caused by a superimposed (accelerated) current at a certain position on the same wave beam nearer to the shore. Vice versa a positive frequency shift can be produced in applying $c(x) > c_0$. The author is still convinced that superimposed currents do have an important effect on wave deformation processes particularly taking place in the post breaking zone. For example in the lower part of Fig. 8 there are shown some additional spectra measured synchronously at stations 100 m and 85 m from the shore. If the respective sets of spectra are connected with the coast normal component of the mean residual velocities measured at station 85 m (see middle part of Fig. 8) actually different frequency shifts can be seen. In the present case there is

- a) a negative frequency shift (red shift) combined with a seaward directed residual current (see first set of spectra),
- b) almost no shift in correspondance with minimal residual currents (see second set of spectra) and
- c) there is a positive shift (blue shift) coincident with a coastward directed residual current (see third set of spectra).

As, however, the additional phenomenon of an increasing energy density in the upbeach direction, especially at very low water depth conditions (first and third set of spectra), can not be explained in applying equation (7), in the following the question is considered whether the above demonstrated combined effects of resonance, absorption and anoma-

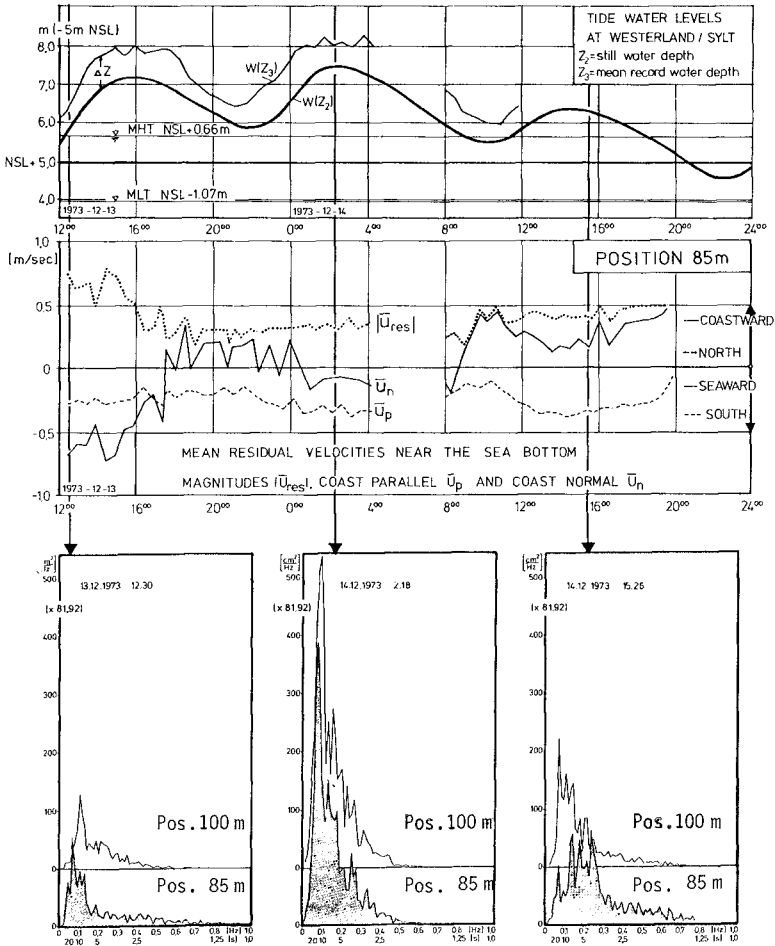


Fig. 8: Different frequency shifts by superimposed accelerated residual currents with respect to the onshore-offshore direction

lous dispersion, known from electromagnetic waves, also apply to surf zone characteristics.

In order to get a better understanding of that mechanism an inspection of the response characteristics of a single degree of freedom oscillator can be helpful, see Fig. 9.

As an analogue to the above shown transfer function this is also subdivided into magnitude and phase. The amplitude response characteristic as well as the phase characteristic both are plotted with reference to the frequency ratio

$$\eta = \frac{\text{forcing frequency}}{\text{natural frequency}} = \frac{f}{f_n} \quad (8)$$

instead of the frequency as it is used at the transfer functions.

Both components are significantly marked by the actual damping rate D which is well-defined by the respective families of curves.

Applying this model to the surf zone the forcing frequencies are those coming from offshore, and the volume of water present in the surf zone is characterized by some eigenfrequencies.

A similar concept is well known from edge wave investigations (GUZA and BOWEN, 1976; CHAPPELL and WRIGHT, 1978).

In order to check its validity in the following it is tried to compare the response characteristics of Fig. 9 with the transfer function data of Fig. 2 and 3 respectively, both representing different wave conditions and different anomalous dispersion properties.

If the values H_1 , Φ_1 and H_2 , Φ_2 are transferred to Fig. 9 conditions (1) and (2) can be differentiated by suitable damping rates D and corresponding frequency ratios η .

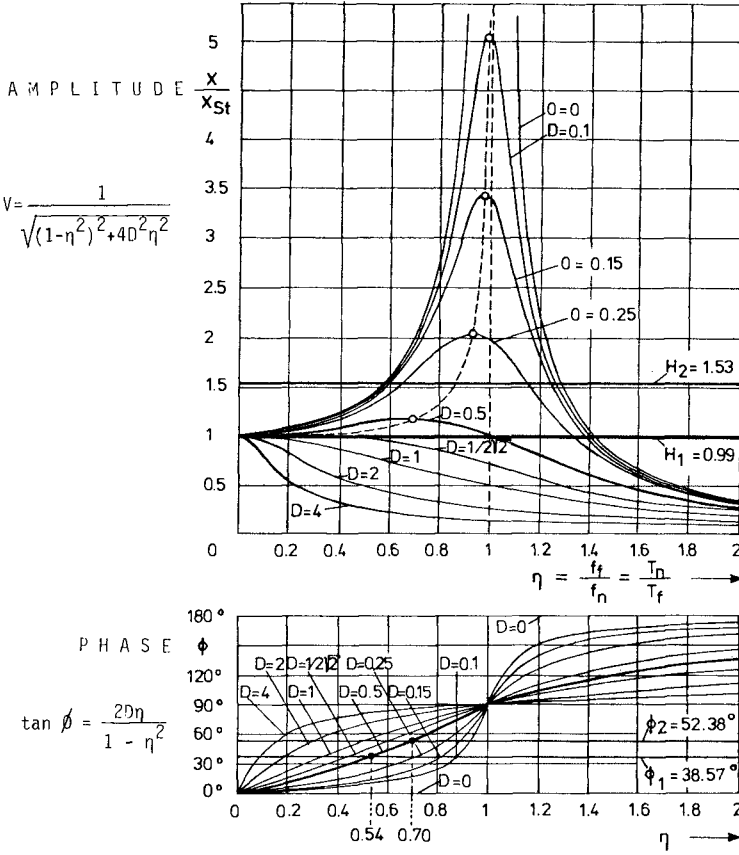


Fig. 9: Response characteristics of a single degree of freedom oscillator

At condition (1) a distinct amplitude and phase response can not be deduced because transfer function magnitude does not differ much from unity, and the phase angle is far from being $\phi = 90^\circ$. Amplitude and phase characteristics ($H_1 = 0.99$ and $\phi_1 = 38.5^\circ$) in this case get together at a frequency ratio

$$\eta_1 \approx 0.46 < 0.54 \quad (9)$$

corresponding to a damping rate

$$0.5 < D_1 < \sqrt{2}/2 \quad (10)$$

Herewith it can be concluded that damping dominates in this case.

By contrast, at condition (2) marked by a transfer function magnitude appreciably greater than unity ($H_2 = 1.53$) and a larger phase angle ($\phi_2 = 52.38$) it is found a frequency ratio

$$\eta_2 \approx 0.74 > 0.70 \quad (11)$$

and a corresponding damping rate

$$0.25 < D_2 < 0.5 \quad (12)$$

Hence, from this analysis there is an indication that resonant interactions including anomalous dispersion are more likely to occur in the surf zone at lower water depth.

This result, however, is in fact in accordance with the above shown example from thin film optics, see Fig. 7. Moreover the analogous behaviour of electromagnetic waves at resonance absorption and surf zone waves is underlined by the fact that the ratio film thickness divided by light wave length is similar to the ratio water depth divided by wave length in the post breaking zone.

REFERENCES

- RAMAMONJIARISOA, A., COANTIC, M., Loi expérimentale de dispersion des vagues produites sur une faible longueur d'action, C.R. Acad. Sci. 2B2 B, 111 - 114, 1976
- LAKE, B.M., YUEN, H. C., A new model for nonlinear wind waves. Part 1, Physical model and experimental evidence. J. Fluid Mech. 88, 33 - 62, 1978
- PHILLIPS, O. M., Strong interactions in wind-wave fields, Proc. NATO Conf. on Turbulent Fluxes through the Sea Surface, Wave Dynamics and Prediction, France, 1977
- MASSEL, S. R., CHYBICKI, W., On the dispersion relation for wind waves in shallow water, Abstracts EUROMECH 114 Gdansk, 23 - 25 Sept. 1980
- BOSCHING, F., Anomalous dispersion of Fourier components of surface gravity waves in the near shore area, Proc. 16th Int. Conf. Coastal Eng. Hamburg, 1978
- BOSCHING, F., Wave deformation due to decreasing water depth, Mitt. Leichtweiß-Institut, Techn. Univ. Braunschweig, H. 63, 1978
- BOSCHING, F., Doppler aspects of near shore wave transformation, Abstracts EUROMECH 114 Gdansk, 23 - 25 Sept. 1980
- SONU, C. J., PETTIGREW, N., FREDERICKS, R. G., Measurement of Swash Profile and Orbital Motion on the Beach, Proc. Int. Symp. Ocean Wave Measurement and Analysis, New Orleans, 1974
- GOOA, Y., Irregular Wave Deformation in the Surf Zone, Coastal Engineering in Japan, Vol. 18, 1975
- GUZA, R. G. and BOWEN, A. J., Resonant interactions for waves breaking on a beach. Proc. 15th Int. Conf. Coastal Eng., Hawaii, 1976

CHAPPELL, J. and WRIGHT, L. D., Surf zone resonance and coupled morphology. Proc. Int. Conf. Coastal Eng., Hamburg, 1978

MAYER, H., Physik dünner Schichten, Wissenschaftliche Verlagsgesellschaft M.B.H., Stuttgart, 1950