Engineering Approach to Nonlinear Wave Shoaling

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ABSTRACT

Determination of a design wave height at a coastal structure requires calculation of a shoaling coefficient or determination of the maximum probable breaking wave height at the point of interest. In shallow water over a sloping bottom, low steepness waves are not accurately predicted by linear shoaling coefficients. Empirical breaking indices are inconsistent with both linear and nonlinear wave theories. Nevertheless, the coastal engineer must select a design wave in order to responsibly design the structure. A graphical procedure is presented herein to relate the equivalent deepwater wave to a breaking wave as it transitions into shoaling water. The procedure provides the coastal engineer with a more consistent understanding of the shoaling process. The results furthermore identify regions of relative depth and steepness where discrepancies arise when using linear shoaling coefficients that may significantly alter engineering design and laboratory studies.

INTRODUCTION

The purpose of this paper is to present a shoaling coefficient that can be used by coastal engineers to better describe the shoaling of finite height waves over sloping bottoms. The linear shoaling coefficient developed from Airy theory in conjunction with a $H_b = 0.78d$ breaking criterion predicts a wave height at the breaking point that is considerably lower than the breaker height predicted by empirical breaking indices commonly used in coastal engineering practice. Several investigators, cited later, have presented nonlinear shoaling curves to describe the phenomena, but they are not consistent with empirical breaker indices over sloping bottoms. These theories tend to predict greater wave heights at a given relative depth than suggested by empirical breaking coefficients. The coastal engineer requires a shoaling curve that is consistent with these commonly accepted breaking indices. This paper utilizes breaker indices as upper limits and the characteristics of theoretical and experimental nonlinear shoaling curves are used to develop a transition of wave height from deep to shallow water.

The problem is illustrated by the following example. Figure 1 shows a typical beach profile over which a long jetty is to be constructed. The design wave at various stations along the jetty is required to determine the armor unit size. For illustrative

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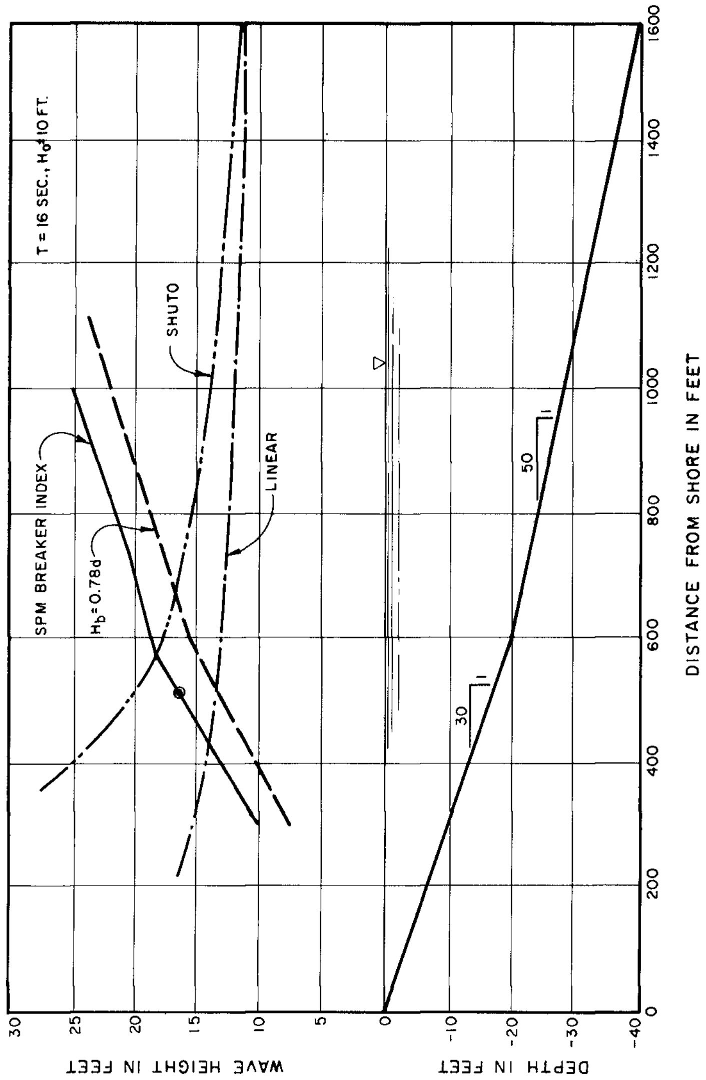

Figure 1. Example Problem for Nonlinear Shoaling.

purposes, a 16-second period is used with a 10-foot deepwater wave height. The dot-dash line represents the linear shoaling curve as determined by Airy theory. The circle represents the breaking height and depth as determined by empirical curves presented in Shore Protection Manual (1977). These data were developed by Goda (1970) and Weggel (1972).

The linear shoaling curve falls significantly below the empirically determined breaker point. Design wave heights shoreward of the breaker point can be determined by the limiting height criteria, but no consistent method exists to determine wave heights seaward of the breaker point. The linear shoaling curve is inconsistant with the breaker data. The theoretical nonlinear shoaling curve of Shuto (1974) is also plotted in figure 1. This curve predicts shoaling at a rate that exceeds the breaker point. While such a curve may be conservative, a shoaling curve more consistent with the breaker index is required for design applications.

EMPIRICAL BREAKER CURVES

Methods to determine breaker height and breaker depth over a given bottom slope for a given deepwater wave steepness are presented in Shore Protection Manual (1977). These procedures are widely used by coastal engineers. The work in this paper assumes these procedures to be representative of the best available data for use in engineering design.

Figure 2 is a graph for predicting breaking wave indices H_b/H'_o based on the work of Goda (1970). Goda reworked data of Iverson (1953) by correcting for side-wall friction and normalized this data using the nonlinear shoaling calculation of LeMehaute and Webb (1964) in deeper water and Iwagaki's (1968) procedure for more shallow regions. Figure 3 presents a graph which gives the ratio of breaker depth to height, d_b/H_b, based on the work of Weggel (1972). In his study Weggel consolidated breaking wave characteristics reported by a relatively large number of investigators.

The above two procedures account for the effects of bottom slope and implicitly include nonlinear shoaling effects. The procedures are widely accepted for use with monochromatic waves and are believed to provide reliable, conservative, estimates of breaking wave characteristics.

The empirical breaker data of figures 2 and 3 can be plotted as limit points in the form of a shoaling graph. The shoaling coefficient is defined as;

$$K_s = H/H'_o \qquad (1)$$

where H is the local wave height, and
H'_o is the equivalent deep water wave height.

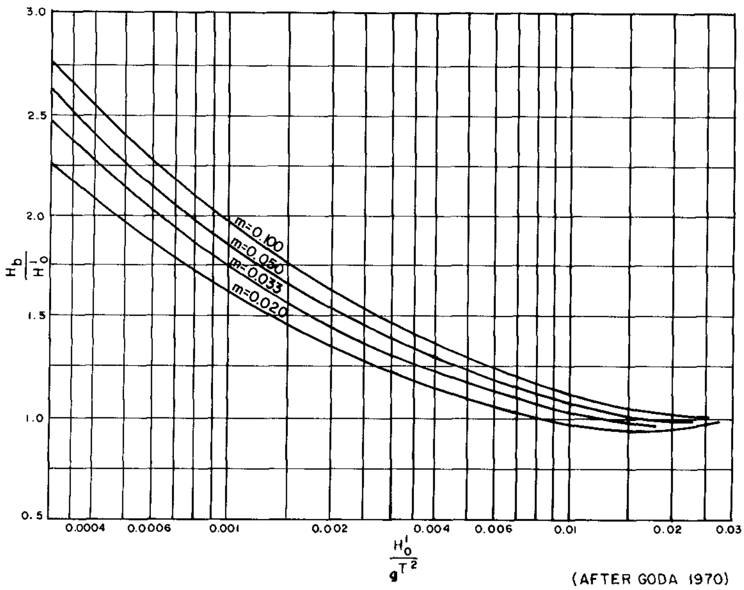

Figure 2. Breaker Height Index.

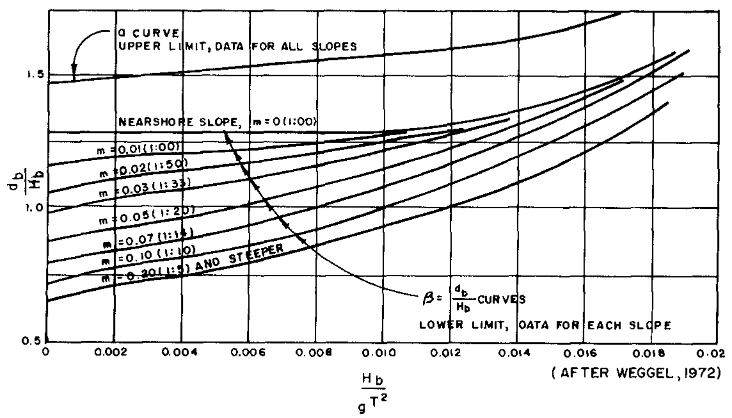

Figure 3. Relative Breaker Depth.

For linear theory, K_s, plotted in figure 4, is a function of relative depth, d/L_o, where d is the water depth, and L_o is the deep water wave length. For non-linear waves, K_s is also a function of bottom slope, m, and H_o/L_o. The limiting breaker height, H_b/H_o', determined from figure 2, can be used to find H_b/L_o for a given m and H_o'/L_o

$$H_b/L_o = (H_b/H_o')(H_o'/L_o) \qquad (2)$$

where H_b is the breaker height.

Finally, the relative depth can be determined using H_b/L_o found by equation (1) and d_b/H_b from figure 3.

$$d_b/L_o = (d_b/H_b)(H_b/L_o) \qquad (3)$$

Results of these calculations are presented in Table 1.

H_b/H_o' is plotted in figure 4 as a function of d_b/L_o for $.001 \leq H_o'/L_o \leq .14$ and $.02 \leq m \leq .1$. Isolines of equal H_o'/L_o connect through the limit waves for the various slopes, m. Isolines of m were drawn through the data and these are shown in figure 5. It is noted that for the lowest value of wave steepness the m = .1 and m = .05 slopes tend to converge. The reason for this is not clear, but could be attributed to wave reflection, data reduction methods, or laboratory scale effects. The data points plotted in figure 4 represent the maximum value of a shoaled wave height over a sloping bottom, m, for a given H_o'/L_o.

THEORETICAL AND EMPIRICAL SHOALING CURVES

The purpose of this paper is to develop a shoaling curve that is consistent with the limit breaking points shown in figure 4. Several investigators have used various wave theories to develop nonlinear shoaling curves. LeMehaute and Wang (1980) present a discussion of nonlinear shoaling and determined that no single theory can accurately account for wave shoaling from deep to shallow water. Sakai and Battjes (1980) present an excellent comparative review of several theoretical studies and present a shoaling curve based on the work of of Cokelet (1977). The Cokelet shoaling curve is compared with the empirical breaking data in figure 6. The third order Stokes curve of LeMehaute and Webb (1964); the hyperbolic curve of Iwagaki (1968); cnoidal curve of Svendsen and Brink-Kjaer (1972), Shuto (1974), and Yamaguchi and Tsuchiya (1976) all have a slightly lower shoaling rate compared with Cokelet theory, but from a practical standpoint are in very close agreement. Figures 7 and 8 compare the nonlinear shoaling curves of Svendsen and Brink-Kjaer, and Shuto, respectively, to the empirical breaking data. The cnoidal shoaling theory of Svendsen and Brink-Kjaer (1972) is based on

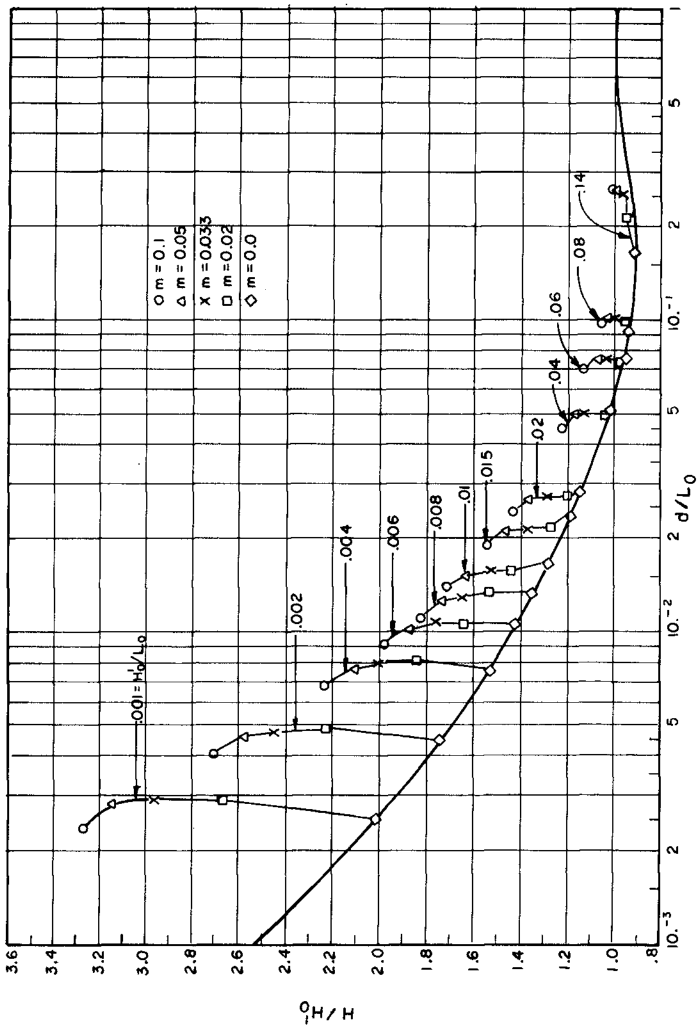

Figure 4. Breaker Points for Given Values of H'_o/L_o and m

Table 1. Breaker Points for Values of Wave Steepness and Beach Slope

m = .10				m = .033			
Ho/Lo	Hb/Ho	db/Hb	db/Lo	Ho/Lo	Hb/Ho	db/Hb	db/Lo
.001	3.275	0.74	.00242	.001	2.96	0.987	.00292
.002	2.70	0.75	.00405	.002	2.43	0.990	.00481
.004	2.24	0.76	.00683	.004	1.99	1.000	.00797
.006	1.98	0.77	.00914	.006	1.77	1.010	.01079
.008	1.83	0.78	.01137	.008	1.63	1.020	.01327
.010	1.72	0.79	.01360	.010	1.52	1.027	.01562
.015	1.54	0.81	.01870	.015	1.37	1.046	.02152
.020	1.44	0.84	.02410	.020	1.28	1.064	.02720
.040	1.23	0.93	.04580	.040	1.12	1.140	.05107
.060	1.13	1.04	.07040	.060	1.04	1.220	.07612
.080	1.06	1.16	.09823	.080	0.98	1.300	.10170
.140	1.00	1.87	.26200	.140	0.97	1.720	.23300
m = .05				m = .02			
Ho/Lo	Hb/Ho	db/Hb	db/Lo	Ho/Lo	Hb/Ho	db/Hb	db/Lo
.001	3.15	0.89	.00280	.001	2.66	1.082	.00288
.002	2.58	0.90	.00464	.002	2.22	1.086	.00482
.004	2.10	0.91	.00765	.004	1.84	1.094	.00806
.006	1.87	0.92	.01035	.006	1.64	1.100	.01085
.008	1.73	0.93	.01290	.008	1.52	1.106	.01340
.010	1.63	0.94	.01530	.010	1.42	1.112	.01580
.015	1.47	0.96	.02120	.015	1.27	1.126	.02147
.020	1.36	0.98	.02660	.020	1.20	1.140	.02740
.040	1.16	1.07	.04968	.040	1.04	1.190	.04950
.060	1.07	1.16	.07460	.060	0.97	1.247	.07260
.080	1.02	1.28	.10460	.080	0.95	1.310	.09960
.140	0.98	1.83	.25120	.14	0.96	1.577	.21200

Note: Values of H_o/L_o = .001 were extrapolated

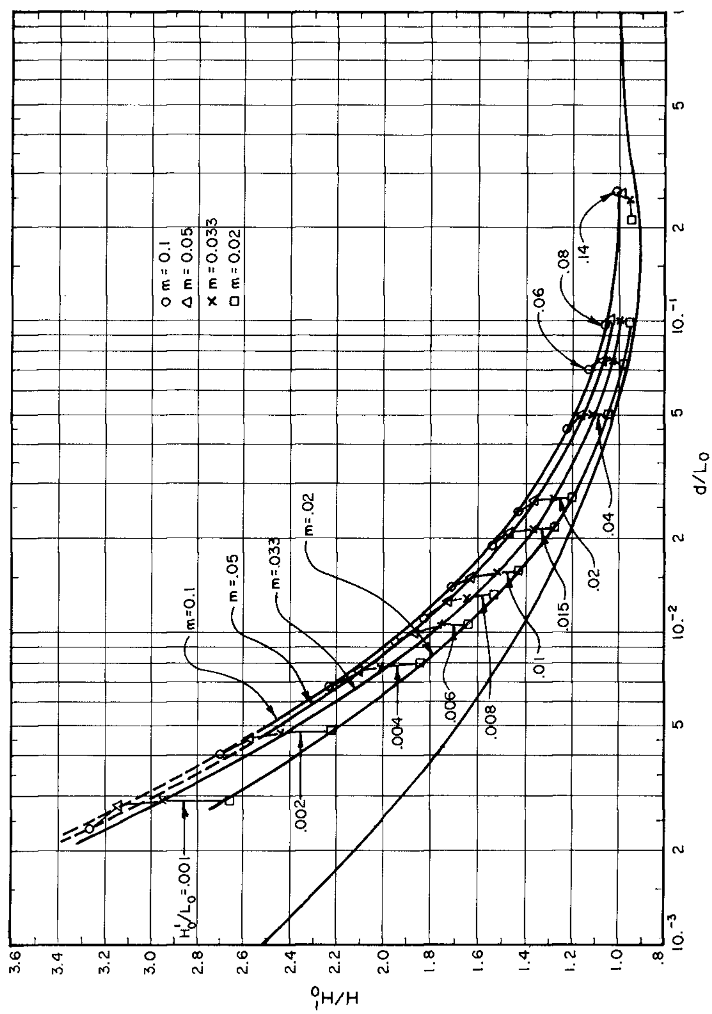

Figure 5. Isolines of Bottom Slope, m.

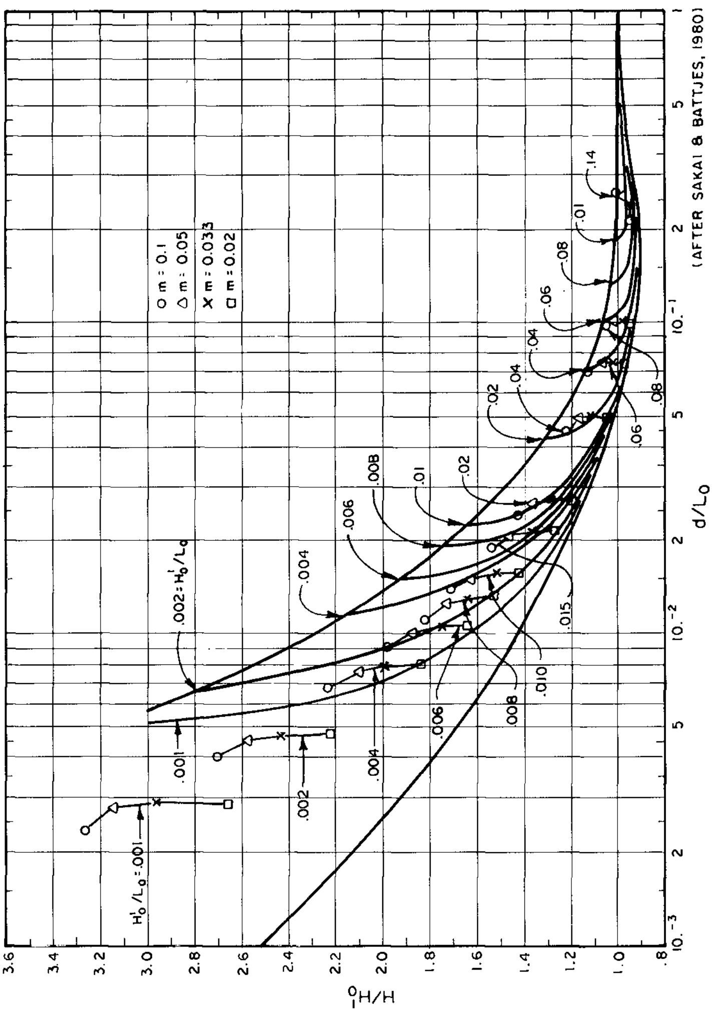

Figure 6. Nonlinear Wave Shoaling According to Cokelet's Theory.

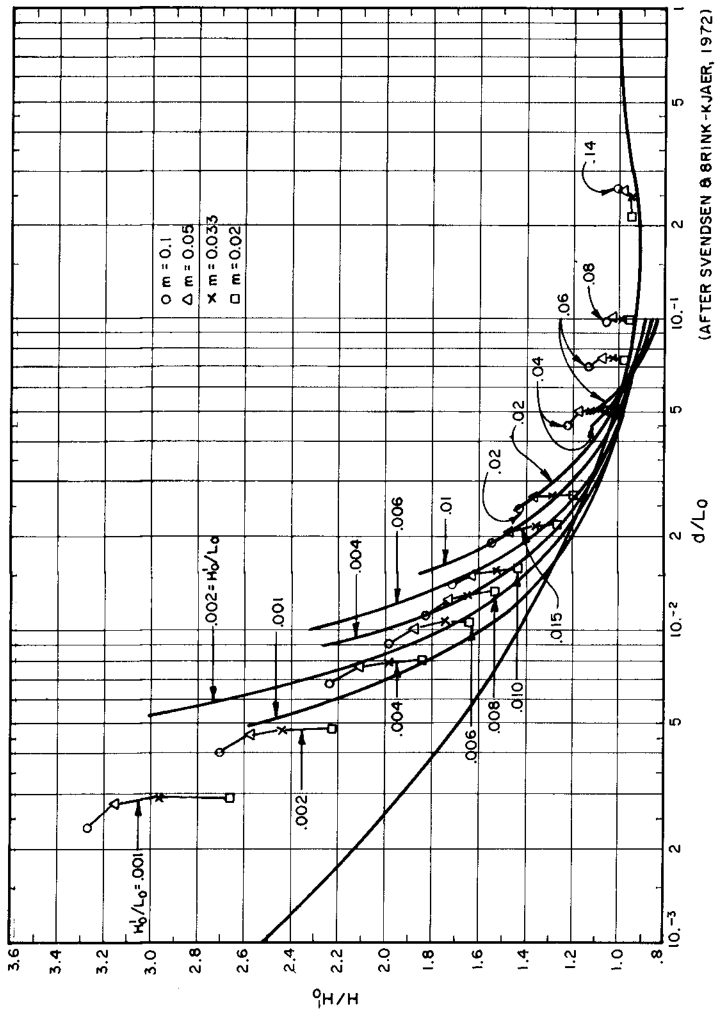

Figure 7. Nonlinear Wave Shoaling According to Cnoidal Theory.

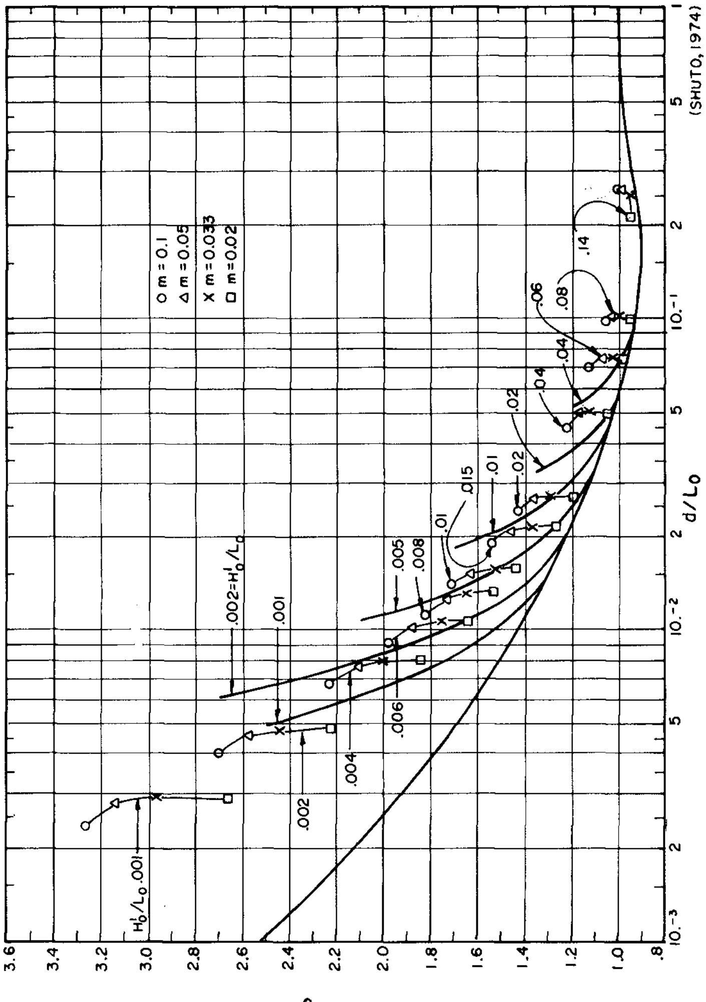

Figure 8. Nonlinear Wave Shoaling According to Shuto.

matching the wave energy flux according to cnoidal and sinusoidal wave theories at a relative depth $d/L_o = .1$. Experimental work done by Svendsen and Buhr-Hansen (1977) indicated that cnoidal wave theory more closely predicted their experimental results if wave height instead of wave energy is matched at $d/L_o = .1$. The effect of this change is to shift the cnoidal steepness curves, shown in figure 7, slightly to the right and upwards. Shifting the isolines of steepness towards the right moves them farther from the empirical breaker indices. Cnoidal wave theory predicts the data of Svendsen and Buhr-Hansen (1977) well but does not predict the breaker index data used to develop figures 2 and 3. This may be attributed to the free second harmonic waves generated by the sinusoidal motion of a piston-wave-generator, which were eliminated in Svendsen and Buhr-Hansen experiments. Sakai and Battjes found that the effect of finite height wave theory on deep water wave length has a relatively minor effect and that all of the above theories present shoaling curves that are in reasonably good agreement except near the breaking point.

Figures 6 through 8 clearly indicate that the above nonlinear approaches predict a significantly higher rate of shoaling than indicated by the experimental breaking limits. This indicate's that either a lower shoaling rate may exist or the emperical breaker indices are in error.
A theoretical nonlinear shoaling theory which accounts for beach slope was developed by Iwagaki and Sakai (1972). Iwagaki and Sakai presented theoretical nonlinear shoaling curves for several values of bottom slope, but limited these curves to values of $.006 \leq d/Lo \leq 0.0157$ and values of $H_o/L_o \leq 0.004$. Figures 9 and 10 compare the theoretical curves of Iwagaki and Sakai to the empirical breaker limits for values of $H_o/L_o = 0.001$ and 0.002 respectively. The qualitative agreement of these curves with the breaker limits is encouraging, however the limited range of conditions preclude their universal use.

Hydraulic model studies of wave shoaling have been conducted by Wiegel (1950), Iverson (1951), Ippen and Eagleson (1950), Eagleson (1956), Iwagaki (1968), Iwagaki and Sakai (1972), Walker (1974), Svendsen and Buhr-Hansen (1977), and Flick (1978). The more recent studies have been used primarily to verify various nonlinear shoaling curves cited in the previous section. The general result has been that the theories are comparable to the experimental data, except they tend to over predict shoaling near the breaker zone. The shoaling curve of Walker (1974) are compared with the limit breaking data in figure 11 for low values of wave steepness. This investigation shows a closer fit to the breaker limits than theory for low d/L_o.

SHOALING DIAGRAM

The form of the H_o/L_o curves suggests that a unique nonlinear shoaling curve exists for each beach slope. This is supported by the theoretical work of Iwagaki and Sakai (1972),

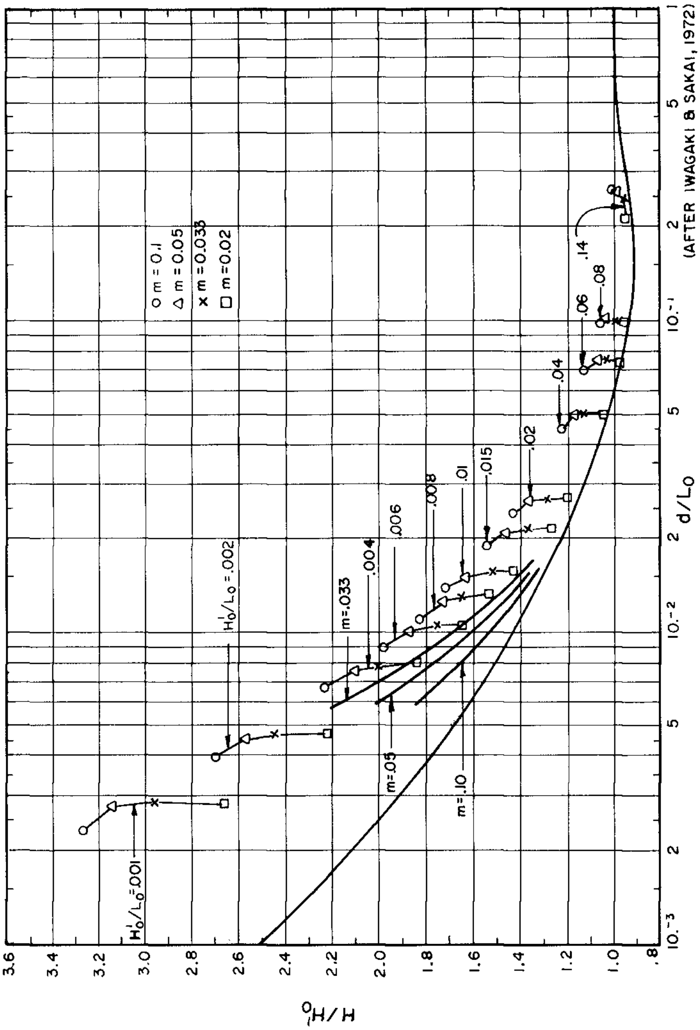

Figure 9. Nonlinear Shoaling According to Iwagaki and Sakai for $H'_o/L_o = .001$ and Several Values of Bottom Slope.

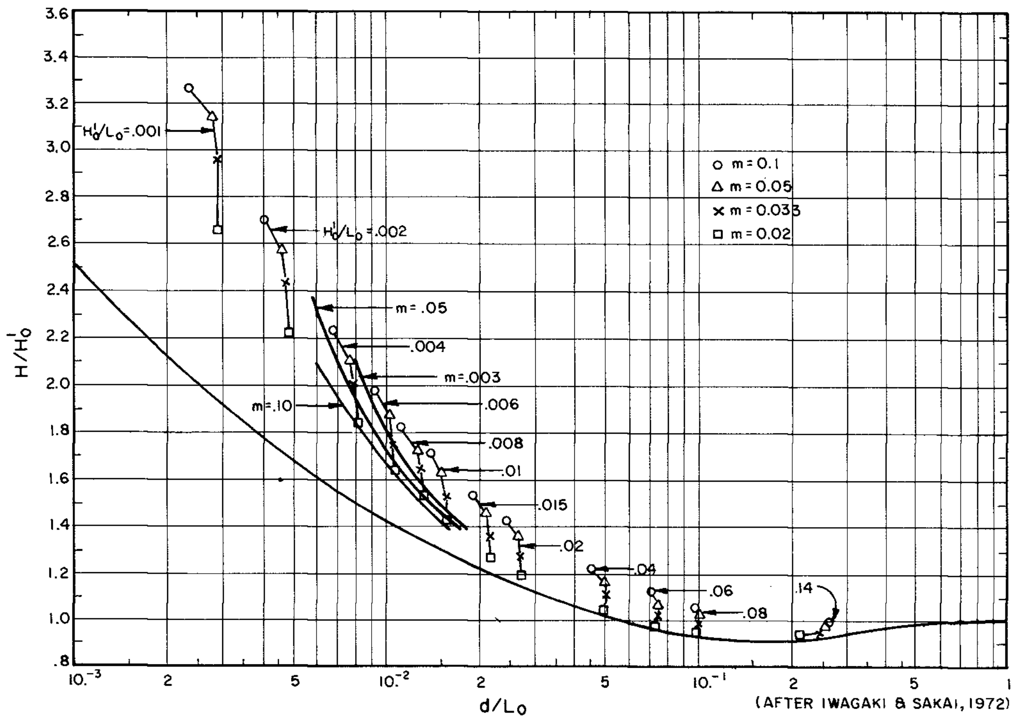

Figure 10. Nonlinear Wave Shoaling According to Iwagaki and Sakai for H_o'/L_o = .002 and Several Values of Bottom Slope.

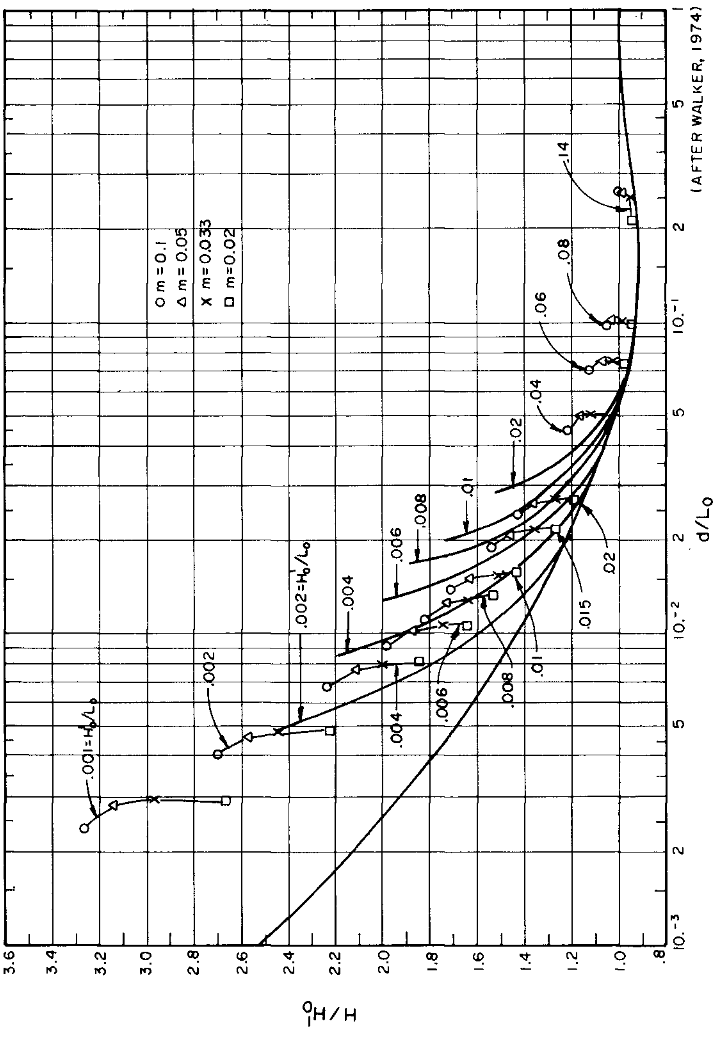

Figure 11. Nonlinear Wave Shoaling from Experimental Data.

but they only give curves for a limited range of values. The approach taken in this report was to draw isolines of steepness weighted towards m = .033 slope. This was done for several reasons. First m = .033 is found often in nature. Secondly, while the steepness curves were drawn to conform to the m = .033 slope, in most cases the curves also intersected, or nearly intersected, the m =.05 and m = .1 endpoints.

The isolines of wave steepness were drawn weighted towards the m = .033 endpoints starting from the linear curve at the point defined by Shuto (1974) were he found that linear shoaling no longer applies;

$$\frac{L_o H}{d^2} \sim \frac{30}{2\pi} \tag{4}$$

The resulting curves are shown in figure 12. While constructing these isolines of wave steepness, each line was compared to the theoretical curves of Cokelet, Shuto and Svendsen and BrinkKjaer (cnoidal) and the experimental curve of Walker. The theoretical curves predict a faster rate of shoaling than the empirical curve. Furthermore, the H_o/L_o curves are shifted towards the right hand side of the diagram (higher d/L_o values). In general the nonlinear shoaling curve based on Cokelet's theory predicts the highest rate of shoaling and is farthest from the empirical data. The experimental isolines of H_o/L_o = .002, and .004 given by Walker are closest to the breaker data. The H_o/L_o = .006 line given by cnoidal theory is closest to the empirical curve although the cnoidal curve still lies to the right of the empirical curve. The empirical curve for H_o/L_o = .001 is shifted considerably towards the left from any of the curves. The empirical H_o/L_o = .02 curve is very well predicted by the cnoidal theory except in the region where they intersect the linear shoaling curve. The empirical H_o/L_o = .04 appears to be an average of the curves given by cnoidal theory and by Shuto. For higher values of H_o/L_o the only curves for comparison to the empirical curves are those given by Cokelet. All of these curves lie considerably to the right of the empirical curves and exhibit a different type of assymptotic behavior to the linear curve.

Theoretically, Iwagaki and Sakai (1972) found that the effect of beach slope on shoaling is that the rate of shoaling is lower on steeper slopes than flatter slopes. However the experimental data indicate that waves reach higher breaking values for steeper slopes. Therefore the effect of weighting the steepness lines towards the m = .033 endpoint is to overpredict wave shoaling for the m = .05, and m = .1 slopes and underpredict the wave shoaling for the m = .02 slope. There are insufficient theoretical and experimental results to estimate the error involved in quantitative terms. However, the error appears to decrease for higher wave steepness values. In terms of predicting peak shoaling values (i.e. shoaling coefficients at breaking)

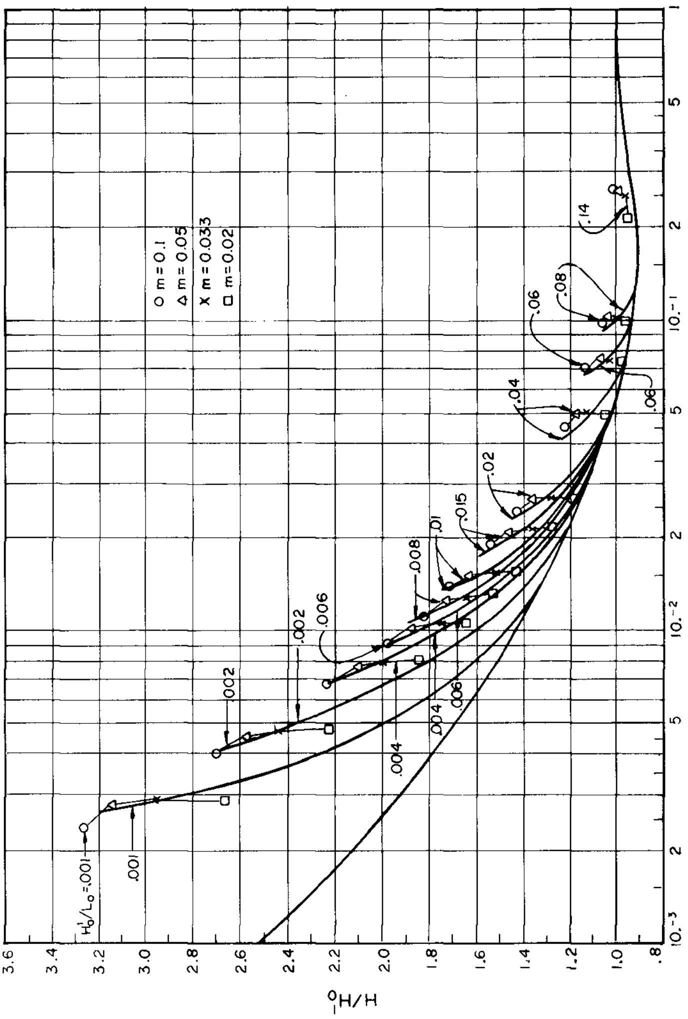

Figure 12. Isolines of H'_o/L_o.

weighting the curve towards m = .033 has the effect of underpredicting the breaker heights for the m =.02 slope, but predicts the breaker heights well for the other slopes. Again the largest error occurs for the lowest steepness values. Despite the fact that there is some error involved for the m = .02 it represents a better fit than linear theory. For more accurate estimates of breaker height and depth, one should use figures 2 and 3.

Superposing figures 5 and 12 results in a diagram for estimating nonlinear wave shoaling over a sloped beach. This curve is shown in figure 13. The linear curve is used for a flat beach (m = 0). This diagram is reasonably consistent with empirical breaking data now widely used for design purposes.

CONCLUSIONS

A nonlinear shoaling curve which provides the coastal engineer a reasonable means of determining wave heights over a sloped bottom seaward of breaking has been developed. The curve indicates that deviations from linear theory are relatively small for values of relative depth, $d/L_o > .05$. Deviations from linear theory reach a factor of 2 or more for $d/L_o >.003$. The curve indicates that wave shoaling is mildly dependent on beach slopes but insufficient data are available to quantify this effect.

The paper outlines the inconsistencies of various nonlinear wave theories compared with commonly used breaker indices. It is recommended that further experimental studies be conducted carefully to avoid adverse tank effects and that these studies be carried out over a range of beach slopes and wave steepnesses to verify nonlinear wave shoaling.

It is further recommended that design curves for wave runup, overtopping and similar phenonenom which use a linear shoaling coefficient to normalize wave height data be critically reviewed. As indicated in this paper use of the linear shoaling coefficient may lead to serious inconsistencies and unconservative designs.

Finally it is noted that the shoaling procedures in this paper apply to monochromatic waves.

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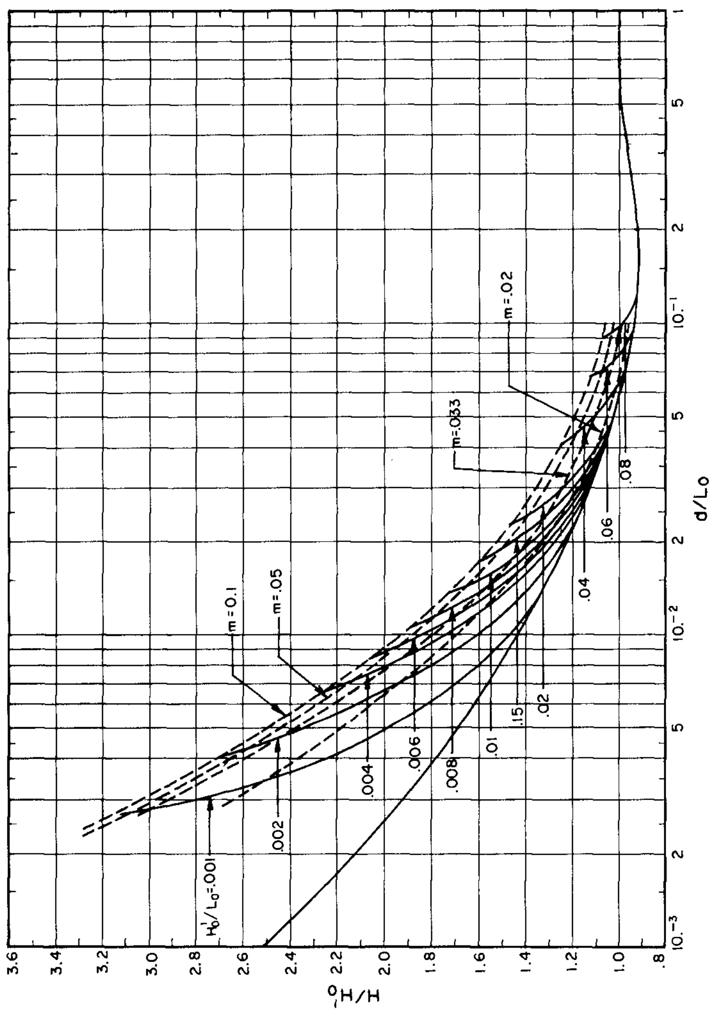

Figure 13. Nonlinear Shoaling Curve.

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