DESCRIPTION OF BEACH CHANCES USINC AN EMPIRICAL PREDICTIVE MODEL OF BEACH PROFILE CHANGES

Takaaki UDA* and Hiroshi HASHIMOTO**

ABSTRACT

In order to analyze beach profile changes due to longshore and onshore-offshore sand transport, here is proposed a new model named the "empirical predictive model of beach profile change", which is an application of the empirical eigenfunction method. The analysis of the profile data obtained at the Misawa fishery port in Ogawarako Coast over five years from 1973 to 1977 indicates that profile changes due to longshore transport and to onshore-offshore transport can be separated. The model is shown to be effective in the analysis of profile changes near coastal structures.

I. INTRODUCTION

Recently many investigations about predictive models of beach profile changes due to littoral drift have been conducted. Pelnard-Considère[1)] introduced an one-line theory and predicted the shoreline changes near a jetty, Bakker[2)] developed a two-line theory and analyzed beach profile changes due to longshore and onshore-offshore sand transport near a jetty.

The present authors[3),4)] applied the one-line model for coastline changes to the Fuji Coast and to the planning of artificial beach nourishment at Kanazawa Beach. As a consequence of these analyses, it was concluded that if coefficients of the model are suitably selected, the shoreline change due to longshore sand transport can be predicted fairly well. However, the one-line theory cannot predict the profile variation when both longshore and onshore-offshore sand transport are significant.

In this paper, in order to consider such problems more closely, we analyzed shoreline changes near the breakwater of the Misawa fishery port with the one-line theory, and also applied the empirical eigenfunction method to the analysis of beach changes.

II. BEACH PROFILE CHANCES DUE TO THE CONSTRUCTION OF MISAWA FISHERY PORT

The Misawa fishery port lies on the southern part of the 100 km long Ogawarako Coast, facing the Pacific Ocean in Aomori Prefecture, as shown in Fig. 1. The construction of the port was started in November, 1973

* Chief Research Engineer, Coastal Engineering Division, Public Works Research Institute, Ministry of Construction, Tsukuba, Japan

** Chief, Coastal Engineering Division, Public Works Research Institute Ministry of Construction, Tsukuba, Japan

Construction Bureau, Ministry of Transport, 1976. (in Japanese)
7) Winant, C.D.,D.L. Inman, and C. E. Nordstrom: Description of seasonal beach changes using empirical eigenfunctions, Jour. Geophys. Res., Vol. 80, No. 15, pp. 1979-1986, 1975.
8) Hashimoto, H., and T. Uda: Analysis of beach profile changes at Ajigaura by empirical eigenfunction, Coastal Eng. in Japan, Vol. 22, pp. 47-57, 1979.

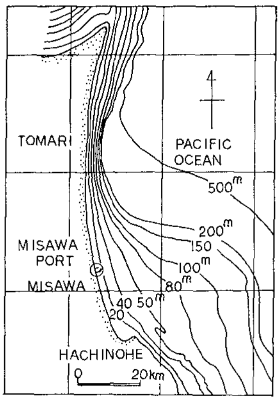

Fig. 1 Location of Misawa fishery port

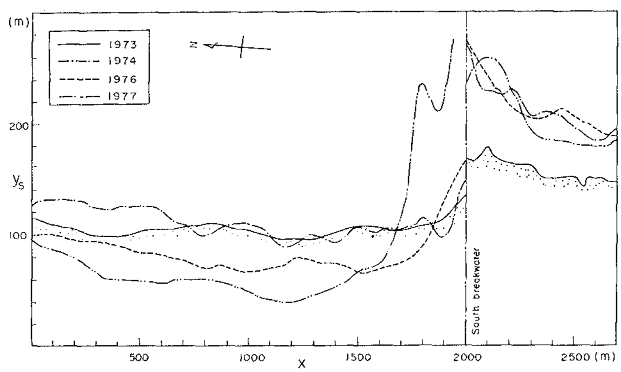

Fig. 2 Shoreline changes near Misawa port

and completed in December, 1977. Since the net sediment transport along this coast was toward the north,[5] accretion and erosion occurred, respectively, to the south and north of the breakwater of the port after its construction.

The beach profile near the port was measured once a year over five years from 1973 to 1977 at 50 meter intervals alongshore. Figure 2 shows the changes of the shoreline near the breakwater from 1973 to 1977. Over these five years, the shoreline advanced offshore by about 60 meters on the south coast of the breakwater, whereas it retreated landward by some 60 meters on the north coast. The shoreline near the north side of the breakwater advanced offshore more than 100m due to the diffraction of waves around the breakwater.

The detailed shoreline changes are shown in Fig. 3. The dates of construction stages are also summarized in Fig. 3. With the progress of the construction, the shoreline inside the port advanced offshore. Figure 4 shows typical examples of beach profiles along the P-P' section in Fig. 3. It is found that the accretion height amounted to approximately four meters in the port, and the profile changes extended from three meters above the mean sea level to eight meters below the sea level. The change of bottom elevation around the port was calculated from the measured profile data of 1973 and 1974, as shown in Fig. 5. Positive and negative values indicate accretion and erosion, respectively. According to Fig. 5, erosion occurred in the north and accretion in the south coast of the breakwater, respectively, implying sand movement from the south to the north at the head of the breakwater.

In addition, the relationship between the changes of the sectional area, A, of beach profile (such as the dotted area in Fig. 3) and the shoreline position, y_s, is shown in Fig. 6, and is approximated by

$$A = 11\, y_s \tag{1}$$

This relation indicates that the characteristic height of the beach change is about 11 meters.

III. WAVE CLIMATE OF OGAWARAKO COAST

The wave climate of Ogawarako Coast has been measured at Takahoko by the Ministry of Transport[6], which is located about 24 km north of the Misawa port. The wave height was measured at the depth of 17.5 meters using an ultrasonic wave gauge, and the wave direction was measured by the deformation of a ball caused by the water particle velocity. The directional probability density of the wave height calculated from the measured data over seven years from 1971 to 1977 is shown in Table 1. The prevailing directions of the incoming waves are ENE, E and ESE. We calculated an effective directional sum of the wave energy flux to define characteristic wave heights which would give the same wave energy flux as the directional sum over one year. It was found that characteristic wave heights were 1.45, 1.34 and 1.17 meters in the ENE, E and ESE directions, respectively, and the corresponding predominant periods were 9.0 seconds in the ENE and E, and 7.5 seconds in the ESE.

Since the shoreline near the breakwater is oriented about 5° counterclockwise from the north, the northward component of the wave energy flux has the magnitude of the same order as the southward component, although the northward component is slightly larger.

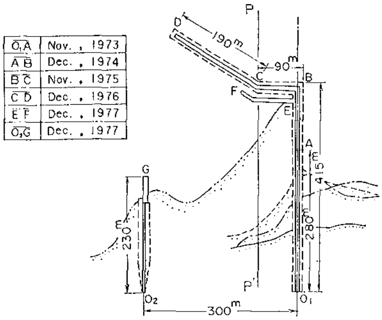

Fig. 3 Sketch of Misawa fishery port
(refer to the legend in Fig. 2)

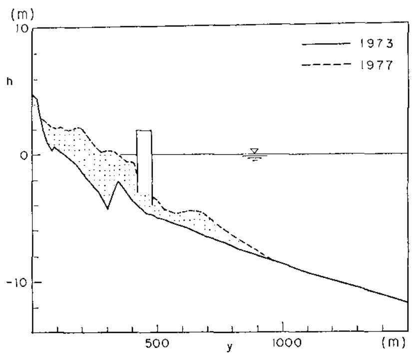

Fig. 4 Accretion near the port

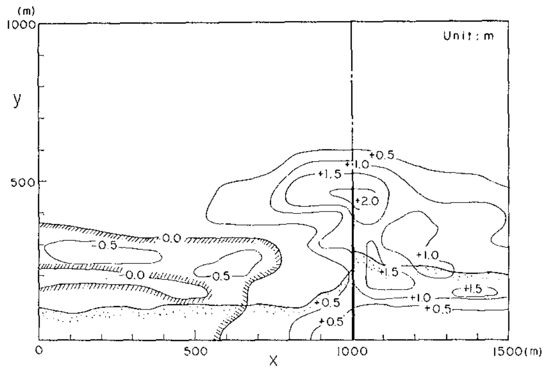

Fig. 5 Topographic changes around the port for one year from 1973 to 1974

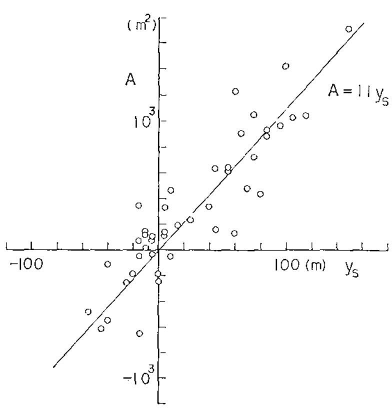

Fig. 6 Relationship between the changes of the beach section area, A, and the shoreline position, y_s

Table 1 Directional probability density of wave height calculated from the measured value from 1971 to 1977

wave height / direction	0.0 m ~ 05.	0.5 ~ 1.0	1.0 ~ 1.5	1.5 ~ 2.0	2.0 ~ 2.5	2.5 ~ 3.0	3.0 ~	Σ
NNE		0.001		0.001				0.002
NE	0.003	0.008	0.004	0.001		0.001		0.017
ENE	0.059	0.158	0.067	0.032	0.018	0.009	0.009	0.352
E	0.061	0.164	0.073	0.035	0.016	0.008	0.006	0.363
ESE	0.040	0.108	0.059	0.021	0.008	0.003	0.001	0.240
SE	0.005	0.012	0.006	0.002	0.001			0.026
Σ	0.168	0.451	0.209	0.092	0.043	0.021	0.016	1.000

IV. NUMERICAL SIMULATION OF SHORELINE CHANGES BY THE ONE-LINE THEORY

The one-line theory was applied to the shoreline changes near the Misawa port. The fundamental equations are,

$$\frac{\partial q}{\partial x}+h\frac{\partial y_s}{\partial t}=0 \tag{2}$$

$$q=F\left(-\frac{\partial y_s}{\partial x}-\tan\theta\right) \tag{3}$$

$$F=\frac{1}{8}\alpha w_0 K_r^2 H_i^2 C_{gi} \tag{4}$$

where q is the longshore sediment transport rate per unit length, h the characteristic height of beach change, y_S the position of shoreline, θ the wave direction, α the Savage coefficient, H_i the incoming wave height, and Cg_i the incoming wave group velocity.

First, the coefficient α was calculated from the volume changes and longshore component of wave energy flux at the site. The relation between volume changes per unit time, Q_x , and longshore component of wave energy flux, E_x , is expressed as follows:

$$Q_x=\alpha E_x \tag{5}$$

It was assumed to calculate the volume changes that northward sediment transport was fully blocked by the breakwater, and that shoreline changes outside of the survey area decreased exponentially with distance from the breakwater. The result is Q_x = 1.56 X 10^6 m^3 for four years. Since the longshore component of wave energy flux of this coast is northerly, and given by E_x = 1.93 X 10^6 ton.m/(m.year), we obtain α = 0.02 using Eq. (5). This value corresponds fairly well to the traditional value of Savage coefficient, α = 0.217.

The simulation of shoreline changes is carried out by solving implicit finite difference equations as follows,

$$\frac{1}{\Delta x}(q'_{i+1}-q'_i)+\frac{h}{\Delta t}(y'_{si}-y_{si})=0 \tag{6}$$

$$q'_i=F_i\left[-\frac{1}{\Delta x}(y'_{si}-y'_{si-1})-\tan\theta_i\right] \tag{7}$$

where the subscript i denotes a step of longshore distance, and the prime represents a time step. The boundary conditions, $q = 0$ at the breakwater and $y_s \to 0$ at $x = -2000$ meters were applied. The shoreline profile of the year 1973 was used as the initial condition. In this computation the time step was $\Delta t = 10$ days, the longshore width of the cell was $\Delta x = 50$ meters and the height of profile change was set $h = 11$ meters as discussed in the previous section.

For the seasonal changes of wave climate, the predominant wave height and wave direction for each month were given. The diffraction coefficient around the breakwater was once calculated for the initial topography to determine the longshore distribution of the coefficient, F, and this distribution was assumed unchanged.

Figure 7 shows the results of the shoreline simulation. Here, we set $\alpha = 0.18$ instead of 0.20, since $\alpha = 0.20$ resulted in overestimation of the shoreline changes. The calculated shoreline of 1976 agrees fairly well with the measured profile for $x < 1000$ meters although their difference increases near the breakwater. The measured and calculated shorelines of 1977 agree well for $x \leqq 1300$ meters, but the difference rapidly increases towards the breakwater.

Concerning the causes of the difference of measured and calculated shoreline changes, it should be mentioned that three-dimensional beach changes occurred in reality near the breakwater, since this south breakwater was extended in several stages and another breakwater was constructed 300 m north of it in 1977 as shown in Fig.3. The sand at the south of the breakwater moved along it and accumulated near its tip. Behind the breakwater, suspended sediment was also deposited in a relatively calm area. These phenomena are not considered in the calculation of coastline changes by the one-line theory, and it is necessary to use a three-dimensional model which takes into account the changes of wave, current and beach topography for the simulation in this region.

The shoreline change can be predicted to some extent, if suitable coefficient of the model are selected. However, the one-line theory does not have the capability for predicting changes of the onshore-offshore profile. Therefore, we apply the empirical eigenfunction method to the analysis of the beach profile changes in the next section.

V. EMPIRICAL PREDICTIVE MODEL OF BEACH PROFILE CHANGES

Winant et al.[7] studied beach changes at Torry Pine Beach by introducing an empirical eigenfunction method. They represented the bed elevation as a linear combination of products of functions of the offshore distance and of functions of time. The present authors[8] also applied this method to the analysis of data obtained at Ajigaura Beach. The results from the latter work indicates that the accretion caused by longshore movement of sand can be clearly separated from onshore-offshore movement.

The empirical eigenfunction method is essentially the same as the principal components analysis employed in the multivariate statistical

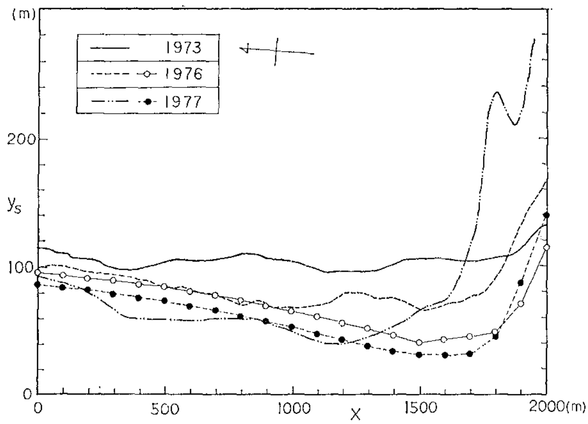

Fig. 7 Results of the shoreline simulation(The calculated shorelines of 1976 and 1977 are indicated by -o- and -●-,respectively. The measured profiles are shown by the solid, broken and chain lines.)

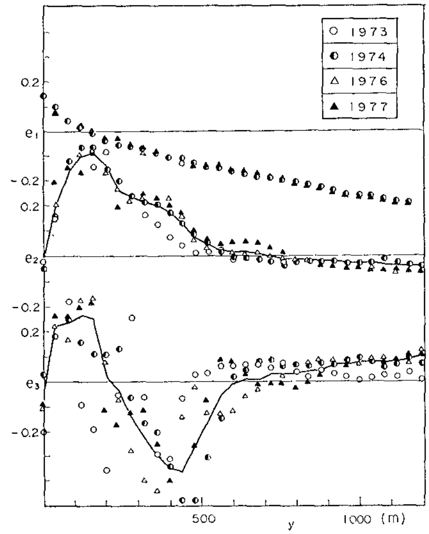

Fig. 8 Onshore-offshore profiles of the first, second and third eigenfunctions, e_1, e_2 and e_3

analysis. In the analysis of Winant et al., beach profile data at certain time intervals were used and the characteristic time changes were obtained. On the other hand, by analyzing beach profile data at certain longshore intervals, one can get the spatial characteristics of the profiles.

We now apply the empirical eigenfunction method to the bed elevation $h(x,y,t_0)$ which is a function of the offshore distance y, and longshore distance x at a certain time t_0. The expansion is expressed as follows,

$$h(x,y,t_0) = \sum_{k=1}^{n} e_k(y,t_0)\cdot C_k(x,t_0) \tag{8}$$

in which the empirical eigenfunctions e_k form an orthogonal set,

$$\sum_{y} e_m(y,t_0)\cdot e_n(y,t_0) = \begin{cases} 1, & m=n \\ 0, & m\neq n \end{cases} \tag{9}$$

In order to generate these functions, a symmetric matrix, A, is formed with the elements,

$$A_{ij} = \frac{1}{n_x n_y} \sum_{m=1}^{n_x} h(m\Delta x, i\Delta y, t_0)\cdot h(m\Delta x, j\Delta y, t_0) \tag{10}$$

where n_x and n_y are the total numbers of measuring points in the alongshore and offshore direction, respectively. The matrix A possesses a set of eigenvalues λ_k and a corresponding set of eigenfunctions $e_k(y,t_0)$, which are defined by the matrix equation,

$$Ae_k = \lambda_k e_k \tag{11}$$

The functions $C_k(x,t_0)$, which define the longshore changes of beach profile, are then evaluated as

$$C_k(x,t_0) = \sum_{y} h(x,y,t_0)\cdot e_k(y,t_0) \tag{12}$$

The method is useful to analyze spatial changes in beach profiles, such as the profiles around a breakwater or of a rhythmic topography. In some cases, it is desirable to use the data of changes of beach profile for the analysis of characteristic beach changes instead of the beach profile itself.

Although the functions e_k and C_k indicate beach characteristics at a certain time, they give the information of the characteristic beach changes. As far as accretion and errosion occur gradually, the functions e_k have approximately unchanged profiles and only the functions C_k change according to the time progress.

Figure 8 shows the onshore-offshore profiles of the first, second, and third eigenfunctions, e_1, e_2 and e_3. The solid line gives the average value over the four years indicated. The eigenfunction e_1 does not depend on the time. The eigenfunctions e_2 and e_3 show relatively larger time variations, but they can be reasonably approximated by the solid lines. Consequently, one can say that all the eigenfunctions e_k do not depend on the time for $k = 1, 2$ and 3.

The second eigenfunction e_2 has a positive value over a broad region of the shore and takes its positive maximum value near the shoreline as shown in Fig. 8.

The ratio of eigenvalues λ_k relative to the first eigenvalue λ_1 are $\lambda_2/\lambda_1 = 2.25 \times 10^{-2}$, $\lambda_3/\lambda_1 = 2.38 \times 10^{-3}$, and $\lambda_4/\lambda_1 = 7.38 \times 10^{-4}$. The contribution of each eigenfunction decreases by one order as compared to the preceeding eigenfunction.

Figure 9 shows the longshore changes of the function C_1 and C_2, which correspond to the first and second eigenfunctions, e_1 and e_2, respectively. The function C_1 shows almost no changes in 1973. However, the region having relatively small value expands gradually on the north coast of the breakwater year by year. This means that the beach slope of this region gradually became smaller. The function C_2, which indicates the longshore changes of the beach profile, is similar to the shoreline profile shown in Fig.2. The correlation between the function C_2 and the shoreline position y_s is drawn in Fig.10 and is given by

$$C_2 = \left\{ \begin{array}{ll} 0.065(y_s - 134), & y_s < 134m \\ 0.15(y_s - 134)^{\text{[illegible]}}, & y_s \geq 134m \end{array} \right\} \tag{13}$$

It is concluded from this that the second eigenfunction, e_2, corresponds to the profile changes due to the longshore sand transport.

We can calculate by the one-line theory the shoreline position, y_s, with a certain tolerance, which yields the value of the function C_2 through Eq. (13). Since the onshore-offshore profile is known at least approximately, we can then predict the three-dimensional beach changes due to the longshore sand transport from the products of the eigenfunction e_2 and the function C_2.

The function C_3 corresponding to the third eigenfunction e_3 is shown in Fig. 11. The absolute value of the function C_3 is smaller than that of the function C_2. As the third eigenfunction e_3 takes its positive value shoreward of the shoreline and a negative value in seaward, a positive value of the function C_3 implies accretion on the nearshore beach and erosion in the offshore zone as schematically shown in Fig. 12. On the south side of the breakwater, sand moved along the breakwater and accumulated at its offshore side. Accumulation also occurred near the shoreline but did not exceed that in the offshore. Therefore, the third eigenfunction is considered to correspond to the profile changes due to the sand movement along the breakwater.

The change of the function C_3 is supposed to be controlled by two factors which characterize respectively the profile changes near the shoreline and in the offshore zone. We selected the offshore distance y'' from the location of the four meter depth to the shoreline as shown in Fig. 12, and examined the relationship between y'' and the function C_3. The reason for the selection of this particular depth is that it approximately corresponds to the depth where the third eigenfunction e_3 takes its minimum value. As found in Fig. 13, we have the relation

$$C_3 = 0.0215(285 - y'') \tag{14}$$

This indicates the possibility that the onshore-offshore profile changes might be represented by change of the offshore distance y'' between two characteristic lines. Since the function C_3 is determined by Eq. (14) if the offshore distance y'' is given, and the profile of the third eigenfunction e_3 is known, the three-dimensional beach changes around the breakwater can be evaluated.

As a typical example, we calculated the profile of the beach that lies 200 meters north of the south breakwater, using the measured shore-

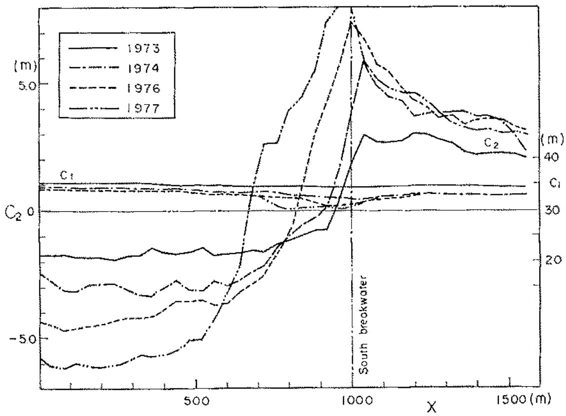

Fig. 9 Longshore changes of the functions C_1 and C_2 corresponding to the first and second empirical eigenfunctions

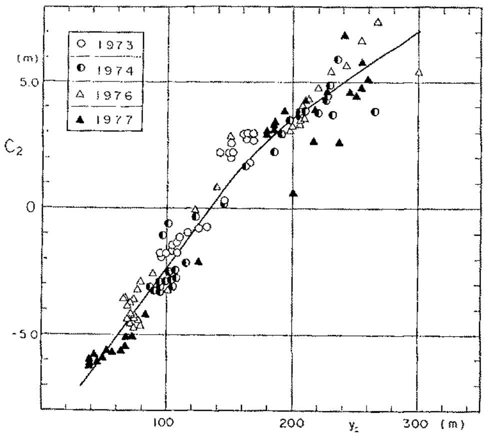

Fig. 10 Correlation between the function C_2 and shoreline position, y_s

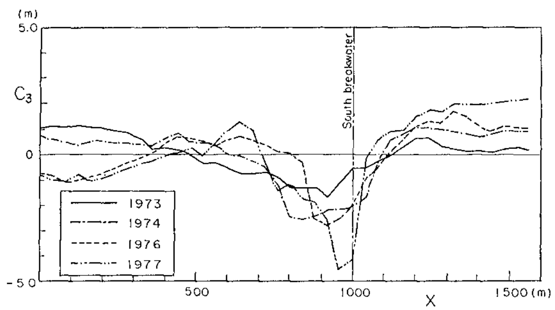

Fig. 11 Longshore changes of the function C_3 corresponding to the third eigenfunction

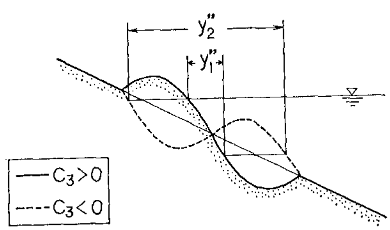

Fig. 12 A typical example of onshore-offshore profile changes

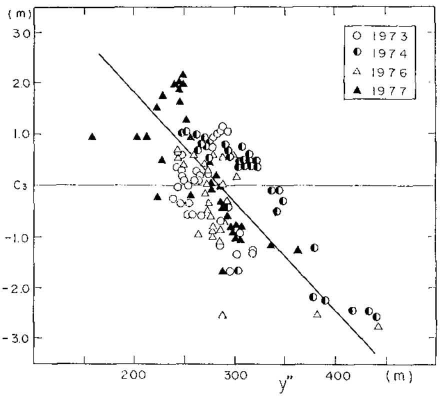

Fig. 13 Relationship between the distance y'' and the function C_3

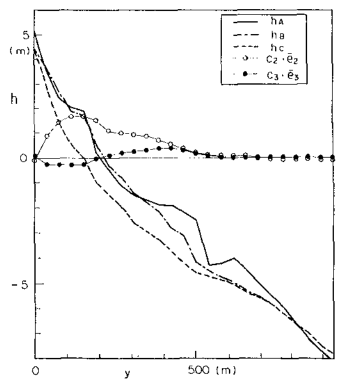

Fig. 14 Comparison of the measured beach profile and that calculated by the products of the eigenfunction e_k and the function C_k (h_A= measured beach profile, h_B= calculated beach profile, h_C= mean beach profile, C_2e_2, C_3e_3 = variation of beach profile corresponding to the second and third eigenfunctions)

line position. The shoreline position, y_s, and the offshore distance, y'', were given by 218 and 553 meters, respectively, at this location. By substituting these values into Eq. (13) and Eq. (14), C_2 and C_3 were found to be 4.16 and − 1.1 meters. In addition, C_1 is set to be 26.5 meters, which is the average taken over four years. Figure 14 shows the measured beach profile and the profile calculated by the products of the eigenfunctions e_k and C_k. The solid line, h_A, indicates the measured beach profile, the chain line, h_B, indicates the calculated beach profile, and the broken line, h_C, gives the mean beach profile. The calculated profile agrees fairly well with the measured one at the depth shallower than 2 meters, whereas thier disparity increases near the breakwater.

VI. CONCLUDING REMARKS

In order to discuss the beach profile change due to longshore and onshore-offshore sand transport, a new model was proposed, the empirical predictive model of beach profile change, by applying the empirical eigenfunction method. The model was applied to analyzing the data obtained at the Misawa fishery port.

As a consequence of this analysis, it was found that the first eigenfunction e_1 corresponded to the mean profile. The second eigenfunction e_2 turned out to correspond to the profile changes due to longshore sand transport, because its value was positive over a broad region of the shore, and the second time function C_2 corresponding to it, which gave longshore change of the beach profile, was correlated with the shoreline position, y_s. In addition, it was shown that the third eigenfunction e_3 corresponded to the profile changes due to the influence of the breakwater, because e_3 took positive values near the shoreline and negative values in the offshore zone, and the function C_3 was correlated with the offshore distance y'' of the four meter depth line from the shore.

Consequently, a basis of the two-line theory was substantiated, and the possibility of the development from a one-dimensional prediction model to a three-dimensional model of beach profile change was demonstrated.

REFERENCES

1) Pelnard-Considere, R.: Essai de theorie de lévolution des forms de rivage en plages de sable et de galets, Quatriemes Journées de l'Hydraulique, Paris, Les Energies de la Mer, Question III, rapport 1, Vol. 1, pp.289-298, 1956.
2) Bakker W. T.: The dynamics of a coast with a groyne system, Proc. 11th Conf. on Coastal Eng., pp.492-517, 1968.
3) Hashimoto, H.: Application of a prediction model for coastline changes to the Fuji Coast, Proc. 23th Conf. on Coastal Eng. in Japan, pp. 218-222, 1976. (in Japanese)
4) Tanaka, T., K. Noro, T. Suehiro, and T. Uda: An application of a prediction model for coastline changes to the planning of beach nourishment. Proc.25th Conf. on Coastal Eng. in Japan, pp. 213-217 1978. (in Japanese)
5) Sato, S., T. Kawanishi, N. Tanaka. and I. Irie: On the littoral drift of Ogawarako Coast, Proc. 19th Conf. on Coastal Eng. in Japan, pp. 7-11, 1972. (in Japanese)
6) Observation report of meteorology and wave climate in Ogawarako Province, Hachinohe Port Construction Office, Cecond District Port