CHAPTER ELEVEN

Estimating Error of Coastal Stage Frequency Curves

Mark D. Prater[1], A.M. ASCE, Thomas A. Hardy[1],

H. Lee Butler[2] M. ASCE, and Leon E. Borgman[3] M. ASCE

Abstract

A computer intensive statistical procedure known as the bootstrap has been used to estimate the error in coastal stage-frequency relationships due to uncertainties in hurricane meteorological distributions. These stage-frequency relationships are developed through the use of a joint probability method, so that the probability of a storm event is the product of the probabilities of the individual independent components which comprise that storm event. The bootstrap technique provides an estimate of the error of the stage-frequency by determining the variation possible in each component's probability distribution. This variability is due to the construction of the distribution from a finite set of historical events. An example of the bootstrap is given and stage-frequency results and error estimates typical of a coastal region are shown.

Introduction

Efficiency in design of coastal protection is becoming more and more important. Development of coastal regions, costs of damages from storm induced water levels, and costs of protection from these waters are all increasing. Adequate protection for coastal regions is desired; however, due to monetary constraints what amount of water level protection that can be considered adequate becomes a question for which there is no easy answer. Therefore, inherent in any coastal protection project, there is a need to develop the best possible estimate of the stage-frequency relationship for the project area, as well as an estimate of the error in this relationship.

The tool in most widespread use for the development of coastal flood frequency, especially from hurricane induced water levels, is the joint probability method (4). However, a generally accepted method to judge the correctness of the produced flood frequency, or to estimate

[1] Research Hydraulic Engineer, Research Division, Coastal Engineering Research Center, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631

[2] Chief, Coastal Processes Branch, Research Division, Coastal Engineering Research Center, U. S. Army Engineer Waterways Experiment Station, P. O. Box 631, Vicksburg, MS 39180-0631

[3] Department of Statistics, University of Wyoming, Laramie, Wyoming 82070

the error of these frequencies, is not available. This paper introduces a method which can be used to develop error estimates from meteorological uncertainty for stage-frequency relationships derived through the joint probability method.

This paper will first outline the joint probability method as it is used to develop stage frequency relationships for regions whose extreme water levels are dominated by hurricane surges. Secondly, a computer intensive statistical tool known as the "bootstrap" (1,2) will be introduced and a procedure will be described which uses the bootstrap to obtain error estimates from meteorological uncertainties. Finally, an example will be given showing the use of the bootstrap technique for developing confidence intervals for a coastal stage-frequency curve.

Joint Probability Method

When developing a hurricane stage-frequency relationships, it is not necessary, or even possible, to model all hurricane windfield configurations that are possible in the region of interest. It is assumed that a storm's configuration can be completely and uniquely described by a set of parameters. Therefore, all possible hurricanes are represented by a large number of synthetic storms, each with its own specified configuration, defined by the values of its parameters.

Commonly, five parameters are used to describe the configuration of a hurricane windfield (Figure 1). The central pressure deficit is the difference between the ambient atmospheric pressure and the extreme low pressure in the center of the hurricane, and is directly related to the intensity and magnitude of the storm. The radius of maximum winds is the distance from the center of the storm to the region of the strongest winds, and is a measure of the size of the hurricane. The forward speed is the storm's rate of translation. The direction of storm motion and location of landfall determine the track of the storm.

General relationships can be expressed which relate the movement and distribution of wind magnitudes and directions with specific values of the above parameters (6). If these parameters can be shown to act independently of one another, the probability of a hypothetical storm occurring is the product of the probabilities of the component parameter values. If independence of parameters cannot be justified, the joint probability method and the bootstrap can still be used, but procedures become more complex. For simplicity, independence will be assumed in the following discussion.

Probability distributions for hurricane parameters are derived by ranking historical values observed for each parameter. A plotting position method such as the Weibull formula (3) is then used to relate rank of parameter value to a probability of exceedance for that value (Figure 2).

Specific values of each parameter are then chosen to represent the entire range of parameter values obtainable in the area of interest. These representative values are chosen through a series of sensitivity tests by simulating the storm surge at a specified location with the

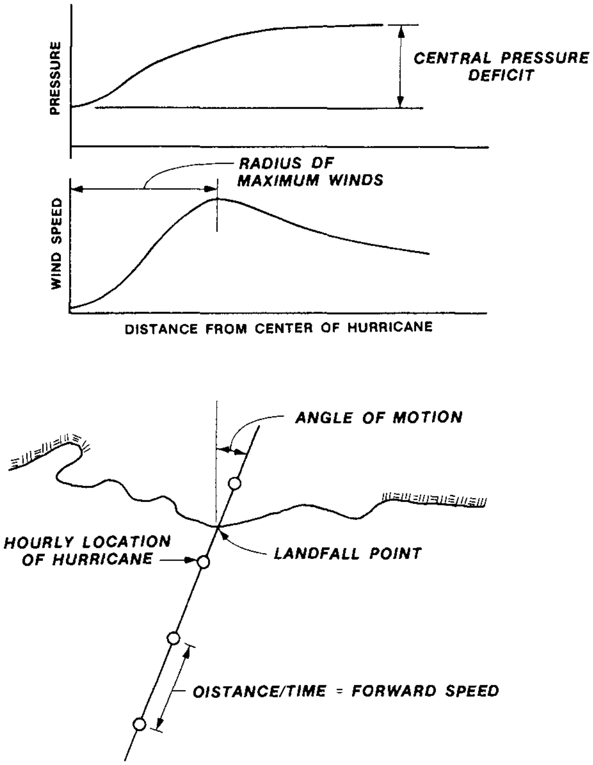
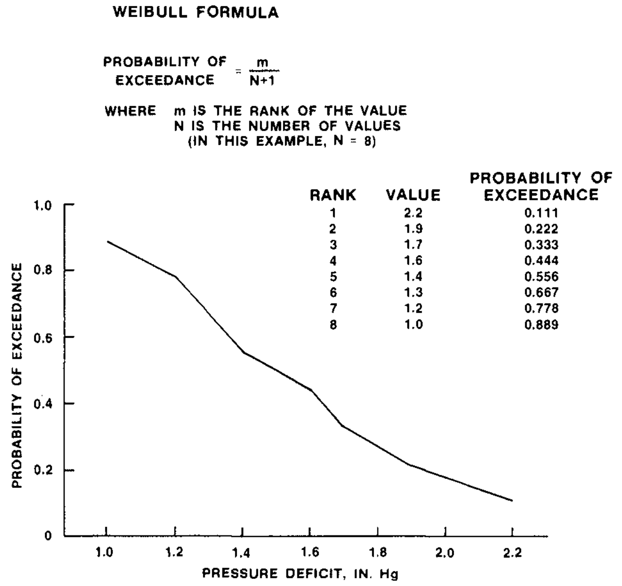


Figure 1. Schematic of Hurricane Parameters



WEIBULL FORMULA

$$\text{PROBABILITY OF EXCEEDANCE} = \frac{m}{N+1}$$

WHERE m IS THE RANK OF THE VALUE
N IS THE NUMBER OF VALUES
(IN THIS EXAMPLE, N = 8)

RANK	VALUE	PROBABILITY OF EXCEEDANCE
1	2.2	0.111
2	1.9	0.222
3	1.7	0.333
4	1.6	0.444
5	1.4	0.556
6	1.3	0.667
7	1.2	0.778
8	1.0	0.889

Figure 2. Exceedance Distribution From Plotting Positions

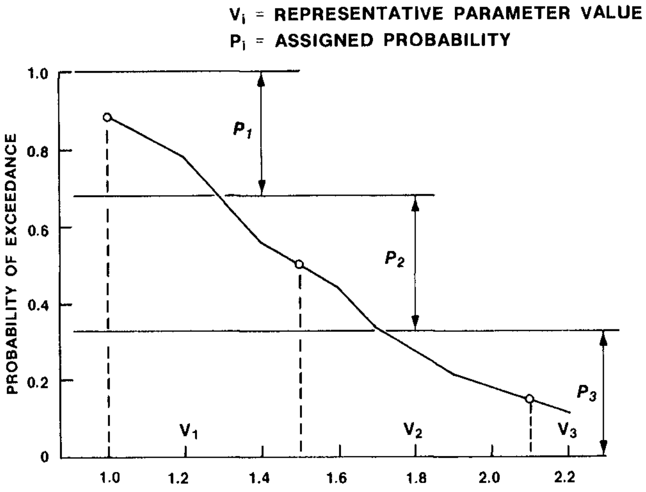


Figure 3. Parameter Value Probability Assignment

use of a parameterized model which produces a windfield from the hurricane parameter values, and a numerical hydrodynamic model. The purpose of these tests is to determine the variability of the surge level purely due to the variation in the value of a specific parameter. Often, all of the other parameters are assigned mean values while the remaining parameter is varied through the range of possible values. If the resulting surge levels vary greatly, more parameter values may be needed to represent the parameter probability distribution. Fewer values are needed for representation if little variation in surge is observed or if the variation in surge can be defined mathematically, i.e., by a linear function. The amount of probability which a parameter value receives is proportional to the amount of the probability distribution that the value is to represent (Figure 3).

The total number of storms simulated is the product of the number of representative values chosen for each parameter. If three values are chosen for each parameter, then (3)(3)(3)(3)(3)=243 different synthetic hurricanes will be be modeled, with each simulation resulting in a surge time history. The total number of storms will vary greatly with the number of values chosen to represent each parameter. Often, the outcome of a hurricane simulation is not reported as a surge time history, but as the maximum surge height obtained. The effect of the hurricane, in this case the maximum surge height, is assumed to have the same probability as the hurricane itself. However, this refers to the probability of the maximum surge height from an individual event occurring, not the probability of the same maximum surge height occurring from any other event. For example, a hurricane is given a probability P1; therefore, its maximum surge height of H is given probability P1. Another storm can occur with probability P2 which also produces a surge height of H . The probability of the H-level surge height, however, is neither P1 or P2, but the accumulative probability of all events causing a surge height of H .

To include the effect of tidal action on the total water level experienced at the coast, all of the computed surge records are convolved with a tidal record which is representative of the tides that the hurricane might encounter at the study site. This is possible if the water depth and other physical features allow the linear addition of surge and tide level to obtain a total water level. The hurricane surge and the tide both behave as shallow water waves, and thus their behavior depends on the water depth. In shallow areas the height of the surge or tide is a significant portion of the total water depth. In deep water, the surge and tide heights are a small fraction of the water depth, so that the behavior of the surge would be the same whether the tide is present or not. In most cases this linearity condition is met. The length of record used for the convolution process can vary from a minimum of a lunar month as long as 18.6 years, which is the cycle length for tidal activity. The length of tidal record is a decision which must be made on a case-by-case basis. For the following example, one lunar month was used.

The process of convolution involves creating all possible combinations of the surge and the tide records. This is achieved by adding the surge height time history produced through simulation to a continuous tidal record of equal length to form a total water level record. In most cases, convolution is done by sampling the tidal and surge records at small time intervals, in this case one hour. The tidal record is then shifted one hour and the process is repeated (Figure 4). A lunar month is roughly 672 hours, so a convolution of a single synthetic storm surge record produces 672 different surge-tide combinations. Since the occurrence of a storm and a tide in nature is a random event, a storm can occur at any time of the month. Thus the probability of any hourly tidal height occurring is uniform and equal to 1/672. When all of the storm surge records have been treated in the above manner, using the previous example, (243)(672)=163,296 different surge-tide combinations will have been produced. The maximum water level is then found for each combination, and the water level's probability is the product of the probabilities of the producing storm and tide.

The final stage of this process is the construction of a water level vs. return period relationship, usually expressed as a curve. An array of intervals is created, each labeled with a unique water level value and spaced by a specified increment, for example 0.1 ft (0.03 m). The large number of maximum water levels are sorted and their probabilities are added to the appropriate interval. This process generates a water level probability distribution (Figure 5). The cumulative summation of the probabilities at each height produces an exceedance distribution, thereby providing the probability of a specified water level being equaled or exceeded.

A major factor not yet mentioned is the average recurrence interval for hurricanes. This value is found by dividing the number of recorded hurricanes into the record length. If the occurrence of hurricanes can be taken to be a Poisson process, then the average return interval, or return period between occurrences of a specified water level or greater, is the average recurrence interval divided by the exceedance probability. This produces a water level vs. return period relationship which is often the end result of the stage-frequency study (Figure 6).

The magnitude of error needs to be addressed before the above relationships should be used. The stage-frequency curve would give an exact reproduction of the behavior of the water level if absolutely no errors or uncertainties were introduced. This unfortunately is not possible. The parameterization of the hurricanes by only five parameters may not completely describe the windfield. The windfield and storm surge model may be either biased or inaccurate. These are factors which should be considered but are not the focus of this paper. A remaining problem is in the selection and assignment of parameter value probabilities. In the northern latitudes of the Atlantic Ocean for example, adequately monitored hurricane occurrences are scarce, and developing accurate parameter distributions with limited data becomes difficult. It is through the uncertainty of these parameter distributions that the error estimates on the stage-frequency curve are derived.

Bootstrap Method

The bootstrap (1,2) is a recently developed statistical tool which utilizes the computational speed of modern computers. This technique

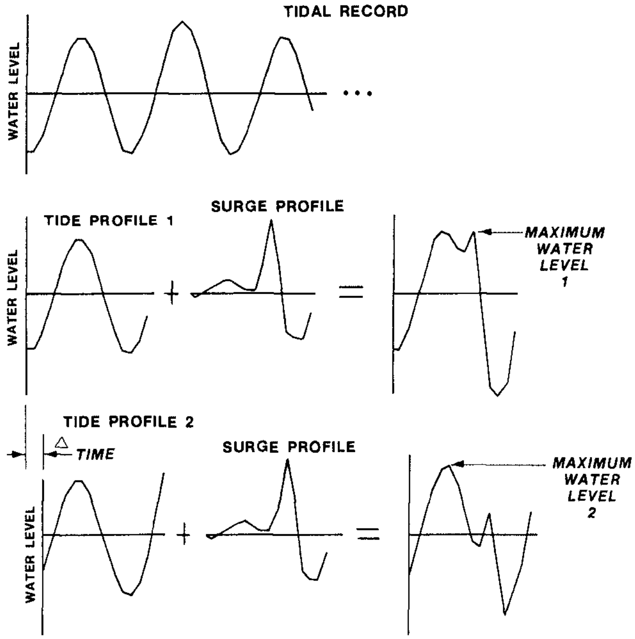


Figure 4. Tide Plus Surge Convolution Process

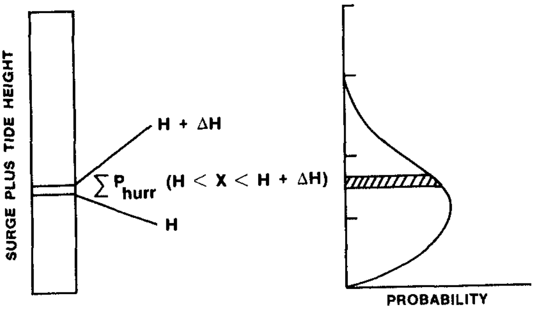


Figure 5. Probability Distribution for Total Water Level

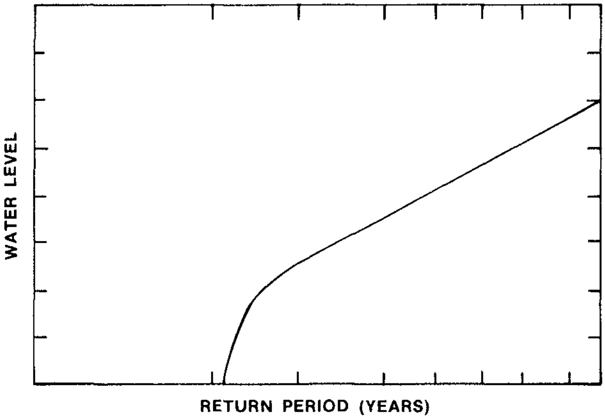
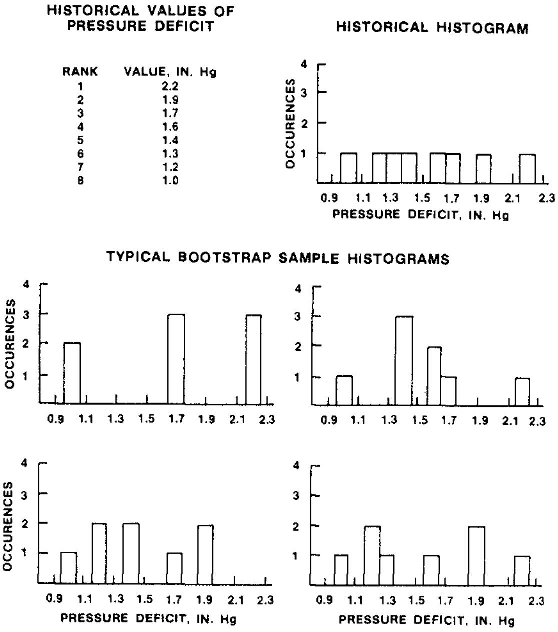


Figure 6. Stage-Frequency Curve From Joint Probability Method



RANK	VALUE, IN. Hg
1	2.2
2	1.9
3	1.7
4	1.6
5	1.4
6	1.3
7	1.2
8	1.0

Figure 7. Variability of Synthetic Samples

is used here to develop stage-frequency relationships while accounting for the uncertainties of the parameter distributions due to a finite record length of hurricane occurrences. This section of the paper will give some insight on how the method works.

In the northeast coast of the United States, for example, only eight values of hurricane central pressure deficit have been recorded. The central pressure deficit is the most influential of the five hurricane parameters, and is directly responsible for the intensity, and therefore the surge height produced by the hurricane simulation. A stage-frequency curve developed on only eight values may be unreliable; therefore, the uncertainty inherent in such a small sample must be accounted for. These eight central pressure values can be thought of as being a random sample from a population whose distribution is unknown. It is desirable to estimate the unknown distribution as best possible, as well as calculate the possible difference, or error, between the true and estimated distributions. It is also advantageous if this can be done without the constraint of using a theoretical distribution.

The bootstrap method approximates the unknown population distribution with the historical sample distribution. This is what is done in most joint probability studies. However, instead of having only one historical sample from which to develop distributions, the bootstrap technique proposes to sample from the assumed population many thousands of synthetic "historical" sets. Each synthetic set generated by the bootstrap mimics how the parameter values might have occurred historically, since the synthetic bootstrapped set and the historical set are assumed to have been sampled from the same population. In this manner, the variation of the synthetic distributions about the assumed population can be quantified. The underlying assumption of the bootstrap is that all relationships between the assumed population and the synthetic samples also apply to the actual unknown population and the historical sample.

The bootstrap is performed in the following manner. For our example, the eight historical values are, in effect, placed in a box. A value is then randomly selected from the box, recorded, and replaced. This process continues until eight values, the size of the original set, have been chosen (Figure 7). This set of values constitutes a synthesized sample, from which a probability distribution of the sample can be formed. This new distribution is as likely to exist as the original historical distribution, since they both were assumed to have been sampled from the same population. The above process is repeated over and over, on the order of several thousand times, thus creating a large number of equally likely distributions. The variation among the synthesized distributions is caused by creating distributions from small samples, even though the population distribution is assumed to be known. This variability is not observed with historical sets since only one set of each parameter exists. This variability, estimated through the bootstrap process, indicates that the single historical parameter set should not be taken as an exact representation of the population distribution, but does not show the variability of the sample distributions.

The bootstrap is then applied to the remaining four parameters, creating several thousand synthesized probability distributions for each parameter. The stage-frequency curve discussed earlier was developed from the distributions of the five historical parameter sets. Other stage-frequency curves could just as well be generated from the synthetic parameter distributions. No additional hydrodynamic simulations need to be made in order to create these new curves. The change of a probability distribution for a parameter only changes the probability assigned to the parameter values, not the values themselves.

A new bootstrapped stage-frequency curve is generated by choosing one synthesized probability distribution at random for each parameter. A new probability assignment is made for each parameter value, and a new probability is calculated for each hurricane previously simulated. The tidal convolution process can be by-passed since only the probabilities of the water levels change, not the heights. Finally, a stage-frequency curve is created which can be considered as likely a representation of the true, unknown stage-frequency curve as was the stage-frequency curve generated from historical sample sets. If this process is repeated 1000 times, a family of 1000 such stage-frequency curves will be generated.

It is common for water levels to be determined at return periods of 10, 25, 50, 100, 200, 500 and 1000 years. After the bootstrapping process, 1000 possible values of water level, one from each of the bootstrapped stage-frequency curves, is obtained at each return period. If the 1000 values found at each return period are ranked in decreasing order, the 500th value will be by definition the 50th percentile, or median, value. This value will be the best estimate of the water level for the given recurrence interval. Estimates of confidence intervals can be obtained from other percentile levels. The 90th percent confidence interval can be estimated as the interval between the 95th percentile value (rank 950) and the fifth percentile value (rank 50). When the above process is done at each return period, an estimate of the uncertainty of a stage-frequency curve is obtained (Figure 8). This uncertainty is purely due to the small sample size of the hurricane parameters.

The bootstrap was used to estimate the error in stage-frequency curves during a project performed at the U. S. Army Engineer Waterways Experiment Station by the Coastal Engineering Research Center (CERC) for the U. S. Army Engineer District, New York. The details of the study are found elsewhere (5) and will not be discussed here.

For the study performed at CERC, it was found that 4000 synthetic parameter distributions and 1000 synthetic stage-frequency curves gave very stable results, stability defined as little change in final result with increased number of bootstrapped samples. The stability also increased as the historical sample size increased. The bootstrap procedure is fairly simple to program and implement, and when performed on a CRAY-1 computer, cost less than a single hurricane simulation.

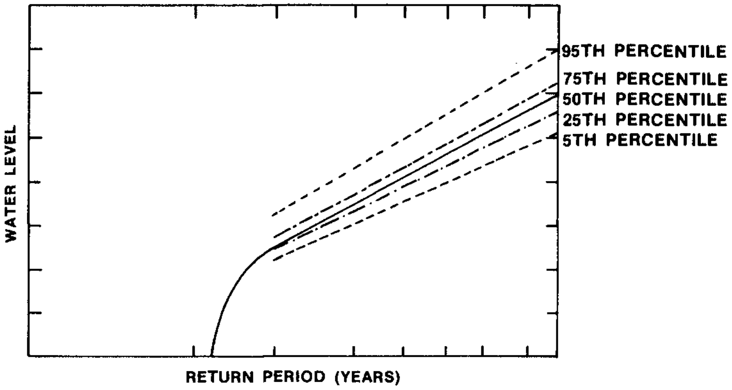


Figure 8. Bootstrapped Stage-Frenquency Confidence Intervals

Summary and Conclusion

A bootstrap procedure has been introduced which estimates the error due to meteorological uncertainty in coastal stage-frequency curves produced by the joint probability method. The procedure consists of assuming that the unknown population distribution for each hurricane parameter is the same as the parameter distribution obtained from historical data. Synthetic parameter distributions are created by repeated sampling from this assumed population. The variation of the synthetic parameter distributions is due to the small sample sizes. Synthetic stage-frequency curves are then generated by randomly selecting a synthetic distribution for each parameter. A large number of these stage-frequency curves are then used to estimate confidence intervals. The bootstrap method is flexible, simple to program, and inexpensive to implement.

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Appendix.-References

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