CHAPTER TWELVE

Numerical Simulation of Storm Surges by Multi-Level Models

by

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Abstract The numerical simulation of storm surges in shallow bays is investigated by multi-level models such as the 2-level and FE-FD models which are used to consider the velocity profiles in wind-induced currents. These models were applied to hindcast the storm surge caused by Typhoon 7916 in Osaka Bay with the result that the water surface elevations and currents induced by the storm surge were successfully simulated.

1. Introduction

In the recent years, rapid advancement in both the physical sciences for marine modeling and in the numerical calculation techniques for the governing differential equations have made new developments in fields such as storm surges, tides, mixing in the upper ocean and the air-sea interactions possible. We investigate here numerical simulation models of the storm surge which is one of the most complex, large scale and disastrous natural air-sea interaction phenomena in the ocean and coastal zones.

Such numerical calculation techniques have advanced greatly, particularly in the fields of computer-aided engineering and mathematics. Until now, the FEM(Finite Element Method), BEM (Boundary Element Method) as well as the classical FDM(Finite Difference Method) have prevailed as the techniques most used to solve differential equations in fluid dynamics. However, a number of types of numerical schemes have been proposed to carry out such calculations more accurately and stably, paticularly in the case of the application of the FDM to a time-evolutional equation.

The physical modeling for storm surges is generally constructed by combining the wind and pressure fields of an atmospheric low pressure area with its accompanying tides and progressive waves of long periods. Assuming incompressive, viscous fluid, the basic equations for storm surges can be described by the Navier-Stokes equations. The most typical modeling is the so-called 1-level model based on depth-integrated equations in which velocity profiles are uniform.

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Considering, however, the dynamics of storm surges in partially enclosed shallow water, the flow structure and patterns of induced currents should be three dimensional because of the co-existence of wind driven currents and their return flows. In such regions, not only the nonlinearity of governing equations and complex geometrical features of coastlines but the three dimensionality of flow structures must be taken into consideration, so many types of models have been proposed.

Previous studies of the numerical simulation of storm surges have been made in three directions. The first direction has been to search for optimal schemes for time discretization in the FDM, which has resulted in schemes such as the leap-frog scheme (explicit) and the ADI (Alternating Direction Implicit) scheme. These are usually adopted for their time-effeciency scheme. In the second direction, researchers have made efforts to reproduce coastlines in calculations by using triangular grid systems. Thacker(1979) proposed the irregular-grid, finite-difference techniques for curving coastlines by approximating partial derivatives using the slopes of planar surfaces associated with the triangular components of the grid. This offers the advantages of greater flexibility than the coordinate transformation method (Wanstrath et al(1976)) and greater economy than the finite-element method (Pinder and Gray(1977), Tanaka(1978)), which have been used to include curving coastlines in calculations. The third direction has been to study mathematical modeling in order to introduce the three-dimensionalities of flow featuers into calculations of storm surges in shallow bays and estuaries. Koutitas and O'Connor(1980), examined the computer modeling of three-dimensional coastal currents induced by winds by the application of a composite finite-element/finite-difference procedure (referred to here as it the FE-FD model). The authors(1982) have modified the FE-FD model into a 3-level model which is more economical and more applicable generally than the previous multi-level models.

In this paper, we discuss the applicabilities and present limitations of the multi-level models by comparison of the theoretical results generated and the actual currents and tides observed at the storm surge caused by Typhoon 7916 in Osaka Bay, Japan.

2. Multi-Level Models

2.1 The 2-level model

The vertical distribution of the horizontal velocity component of wind-induced current in partially enclosed shallow water have been investigated theoretically and experimentally. It has been shown that there is a conversion point of mean velocity in the range from 0.2 to 0.4 in nondimensional depth (Baines and Knapp(1965)). This suggests that bottom stresses evaluated in the 1-level model are questionable in their magnitude and direction. To overcome these inconsistencies, the authors (8) have proposed the 2-level model, in which the upper level represents the wind-driven current and the lower, the return flow.

Using the definition sketch and notations for the 2-level model shown in **Figure 1**, the model equations for both the upper and lower levels, and for the continuity equation are written as
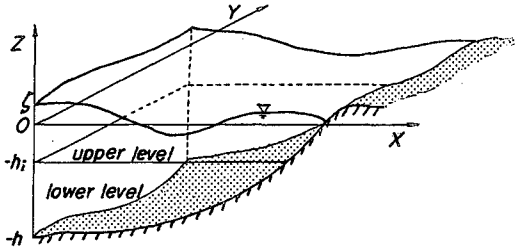
Figure 1. Definition sketch and notations for the 2-level model.

a) for the upper level

$$\frac{\partial M_u}{\partial t} + (NL)_{ux} + (uw)_i = fN_u - gH_u\frac{\partial \zeta}{\partial x} - \frac{1}{\rho_w}\frac{\partial p}{\partial x} + \frac{\tau_{sx}}{\rho_w} - \frac{\tau_{ix}}{\rho_w} + (HM)_u \quad (1)$$

$$\frac{\partial N_u}{\partial t} + (NL)_{uy} + (vw)_i = -fM_u - gH_u\frac{\partial \zeta}{\partial y} - \frac{1}{\rho_w}\frac{\partial p}{\partial y} + \frac{\tau_{sy}}{\rho_w} - \frac{\tau_{iy}}{\rho_w} + (HM)_u \quad (2)$$

b) for the lower level

$$\frac{\partial M_l}{\partial t} + (NL)_{lx} - (uw)_i = fN_l - gH_l\frac{\partial \zeta}{\partial x} - \frac{1}{\rho_w}\frac{\partial p}{\partial x} + \frac{\tau_{ix}}{\rho_w} - \frac{\tau_{bx}}{\rho_w} + (HM)_{lx} \quad (3)$$

$$\frac{\partial N_l}{\partial t} + (NL)_{ly} - (vw)_i = -fM_l - gH_l\frac{\partial \zeta}{\partial y} - \frac{1}{\rho_w}\frac{\partial p}{\partial y} + \frac{\tau_{iy}}{\rho_w} - \frac{\tau_{by}}{\rho_w} + (HM)_{ly} \quad (4)$$

where the discharges per unit width are defined by

$$M_u = \int_{-h_i}^{\zeta} u\,dz, \qquad N_u = \int_{-h_i}^{\zeta} v\,dz, \qquad M_l = \int_{h_i}^{h} u\,dz, \qquad N_l = \int_{h_i}^{h} v\,dz$$

and $H_u = \zeta + h_i$, $H_l = h - h_i$, (NL) and (HM) indicate the nonlinear and horizontal mixing terms respectively, $(uw)_i$ and $(vw)_i$ are the mass fluxes at the interface, and f is the Coriolis parameter. The shearing stresses at the sea surface and the sea bottom are expressed in quadratic form, using the drag coefficients C_D and C_f, as

$$\left.\begin{array}{ll} \tau_{sx} = \rho_a C_D W_x \sqrt{W_x^2 + W_y^2}, & \tau_{sy} = \rho_a C_D W_y \sqrt{W_x^2 + W_y^2}, \\[2mm] \tau_{bx} = \rho_w C_f U_l \sqrt{U_l^2 + V_l^2}, & \tau_{by} = \rho_w C_f V_l \sqrt{U_l^2 + V_l^2} \end{array}\right\} \quad (5)$$

The shearing stress at the interface is given in the linear form.

$$\tau_{ix} = -2C_i\rho_w\nu_i(U_u - U_l)/h, \qquad \tau_{iy} = -2C_i\rho_w\nu_i(V_u - V_l)/h \quad (6)$$

where, ν_i is the eddy viscosity. The coefficient C_i of Eq. (6) has been determined as $3/2$, assuming a steady, uniform current in the enclosed domain with uniform water depth.

The continuity equation is given by

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}(M_a + M_l) + \frac{\partial}{\partial y}(N_a + N_l) = 0 \tag{7}$$

The model equations mentioned can be integrated numerically by the finite-difference technique.

2.2 The FE–FD model

Using the Navier–Stokes equation for fluid elements and the continuity equations for a water column, neglecting the horizontal diffusion of momentum and using the notations shown in **Figure 2**, where H is the total water depth, h the water depth in the M.W.L. and ζ the water elevation induced by the storm surge, the fundamental equations can be described as
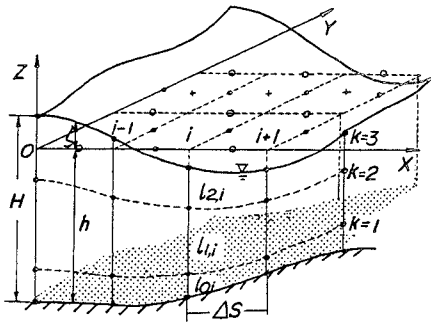
Figure 2. Definition sketch and notations for the FE–FD model.

$$L_x(x,y) = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv + g\frac{\partial \zeta}{\partial x} - \nu_i\frac{\partial^2 u}{\partial z^2} = 0 \tag{8}$$

$$L_y(x,y) = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu + g\frac{\partial \zeta}{\partial y} - \nu_i\frac{\partial^2 v}{\partial z^2} = 0 \tag{9}$$

The continuity equation is written as

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}(\int_{-h}^{\zeta} u dz) + \frac{\partial}{\partial y}(\int_{-h}^{\zeta} v dz) = 0 \tag{10}$$

As to the three-dimensional problem, the coastal area is first covered by a regular-grid network in the x-y plane, and the water depth is divided into the same number of elements in the vertical water column. Next, the finite-element method is applied to Eqs.(8) and (9) and the finite difference method of fractional step is applied to Eq.(10) to obtain the components of currents $(u_k(x,y), v_k(x,y))$ at the

nodal point k and water surface $\zeta(x,y)$. In the case of the three—level model, the equations of motion are, consequently, written in matrix form, by

$$[A_1] \cdot [u] = [B_tFINAL-BRACKET, \qquad\qquad [A_y] \cdot [v] = [B_y] \qquad\qquad (11)$$

in which, for example, the matrices and vectors of the x—components are respectively expressed as

$$[A_x] = \begin{bmatrix} l_{x,1ij}/3\Delta t & l_{x,1ij}/6\Delta t & 0 \\ l_{x,1ij}/6\Delta t & (l_{x,1ij}+l_{x,2ij})/3\Delta t & l_{x,2ij}/6\Delta t \\ 0 & l_{x,2ij}/6\Delta t & l_{x,2ij}/3\Delta t \end{bmatrix} \qquad (12)$$

$$[u] = \begin{bmatrix} u_{1ij}^{n+1} \\ u_{2ij}^{n+1} \\ u_{3ij}^{n+1} \end{bmatrix} \qquad [B_x] = \begin{bmatrix} B_{1x} \\ B_{2x} \\ B_{3x} \end{bmatrix} \qquad\qquad (13)$$

where B_x is the vector of known variables calculated at the time step of n-1/2 in the fractional method, whose components include the atmospheric pressure gradients, and the shearing stresses at the water surface, interfaces and bottom.

3. Hindcast of Storm Surge Caused by Typhoon

3.1 The calculation conditions

Using the 2—level and FE—FD models, the storm surges caused by Typhoon 7916 are hindcasted in the region shown in **Figure 3**. This typhoon which passed through Osaka Bay on Novemver 30, 1979 caused peculiar tidal changes in the inner part of the bay. The tidal records measured at 9 points along the bay are shown in **Figure 4**. Resurgences are remarkable immediately after the peak of storm surge.

The computation was made by the leap—frog staggered system, in which the horizontal grid spaces were chosen as Δs=1.5 km in order to approximately cover the two channels in computation, and time space Δt= 20sec. And the depths at two channels were adjusted to cover the same area of the cross section. Considering that the mean water depth of the bay is 28m, the upper level depth in the 2—level model is determined as h_i = 8m. The open boundary conditions are given by the surface elevations which were estimated by interpolation of the observed anomaly of meteorological tides at the points Nushima, Kainan, Akashi and Higashi—Futami, and the so—called fixed boundary conditions are used along the coastlines.

It is important in numerical modeling for storm surges to determine the unknown physical parameters such as the drag—cefficients at the water surface C_D, the at the bottom C_f, the friction coefficient C_f, and the eddy viscosity ν_i, which are expressed practically as,.
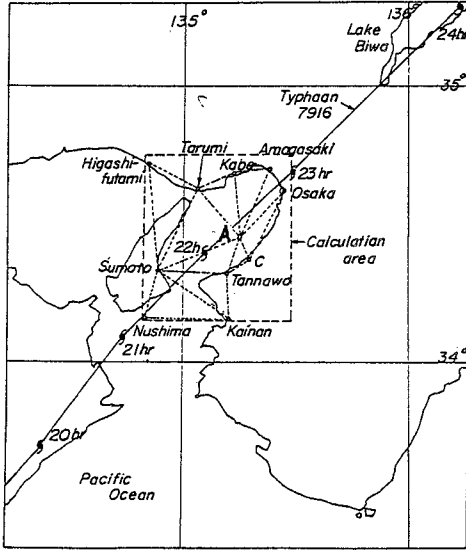
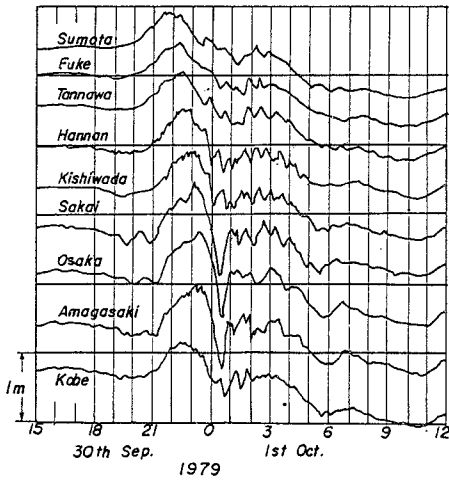Figure 3. The computation region.

Figure 4. Observed tidal records showing anomalies in meteorological tides caused by Typhoon 7916 in Osaka Bay.

$$(C_{xD}, C_{yD}) = 1.50 \times 10^{-3} (W_x^{-0.19}, W_y^{-0.19}) \quad \text{for } W_x, W_y \leqq 8 \, (\text{m/sec}) \tag{14}$$

$$(C_{xD}, C_{yD}) = 2.04 \times 10^{-4} (W_x^{0.74}, W_y^{0.74}) \quad \text{for } W_x, W_y > 8 \, (\text{m/sec}) \tag{15}$$

respectively and

$$C_f = 0.0026 \quad , \quad \nu_i = 0.01 \, (\text{m}^2/\text{sec}) \tag{16}$$

where Eqs. (14) and (15) for the drag coefficient at the surface are those proposed by Mitsuyasu (1981).

3.2 Computation of tides

The dominant tides in the Seto Inland Sea including Osaka Bay are the M_2 semi-diurnal lunar tide which is a semi-diurnal lunar tide coming in through the Kitan Channel in the south of the bay which flows out through the Akashi Channel in the west. The duration of the tide is about 2 hours and its amplitude reduces to 1/3 in the bay. The S_2 tide is also similar to the M_2 in these tendencies. We here consider only the semi-diurnal tides (M_2 , S_2) whose amplitudes and phases are listed in Table 1.

Table 1 Amplitudes and phases of tides

Points	Amplitudes (M_2 + S_2)	Phases (deg.)
Nushima	0.629 m	179.8°
Kainan	0.661 m	188.2°
Higashi-Futami	0.285 m	294.6°
Ei	0.334 m	327.7°

The computation of tides was carried out by the 1-level model for four periods to obtain the initial flow conditions prior to the computation for storm surges.

3.3 Typhoon model

The wind and pressure gradients of a typhoon are expressed practically by Fujita model as

$$\left. \begin{array}{ll} \dfrac{\partial P}{\partial x} = -\dfrac{P_o r_o x}{(r^2 + r_o^2)^{3/2}}, & \dfrac{\partial P}{\partial y} = -\dfrac{P_o r_o y}{(r^2 + r_o^2)^{3/2}} \\[4mm] W_x = C_1 F_x - C_2 G_x, & W_y = C_1 F_y + C_2 G_y \end{array} \right\} \tag{17}$$

In this case, the wind field associated with the moving speed of a typhoon (V_x, V_y) (m/sec) and the baloclinic wind field are expressed, respectively as

$$\left. \begin{array}{l} F_x = V_x \exp\left(\dfrac{-r\pi}{5 \times 10^5}\right), \qquad F_y = V_y \exp\left(\dfrac{-r\pi}{5 \times 10^5}\right) \\[4mm] G_x = (\sqrt{3}y/2 + x/2)\,\{\sqrt{f^2/4 + (\rho_w/\rho_a)gP_o r_o/(r_o^2 + r^2)^{3/2}} - f/2\,\}, \\[4mm] G_y = (\sqrt{3}x/2 - y/2)\,\{\sqrt{f^2/4 + (\rho_w/\rho_a)gP_o r_o/(r_o^2 + r^2)^{3/2}} - f/2\,\} \end{array} \right\} \tag{18}$$

where $r = \sqrt{x^2 + y^2}$ is the distance from the center of typhoon, P_o (m) the pressure head corresponding to the atmospheric pressure drop, and r_o, C_1 C_2 are the typhoon parameters for which values for Typhoon 7916 were 60 (km), 0.6 and 0.6, respectively.

3.4 Computations for the storm surge

a) The 2-level model

Using the finite-difference method, the computation of storm surge was performed by the 1- and 2-level models. The calculated anomalies in meteorological tides at points, Kobe, Osaka and Sumoto are shown in Figure 5 with the astronomical tides. Figure 6 shows the flow patterns calculated by the 2-level model. We conclude that the results obtained by the 2-level model agree well with the observed tides around the peak of storm surge, including its forerunners. Disagreement at the resurgences appearing immediately after the peak can be seen in the results of both models, which may be caused by the rapid changes in its pressure fields. This fact has already been suspected by the authors (1981).

The current measurements have been carried out at point A shown in Figure 3, using magnetic current meters at three points of which the depths are 3, 9 and 16m. Comparison between the calculated and observed west-east components of currents at the point 3m deep is shown in Figure 7. In contrast to the agreement for surface elevation, both currents calculated by the 1- and 2-level models do not agree well with the observations at point A. The effects of the 2-level modeling are, however, remarkable in this figure in that changes in current velocities with time calculated by the 2-level model approach those observed.
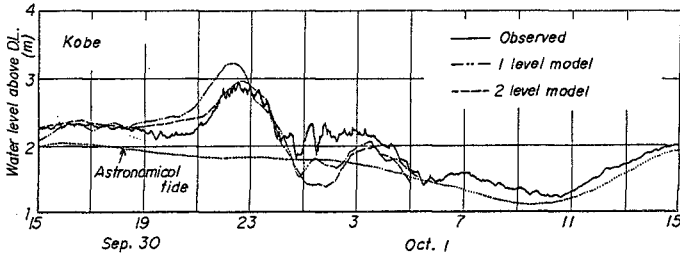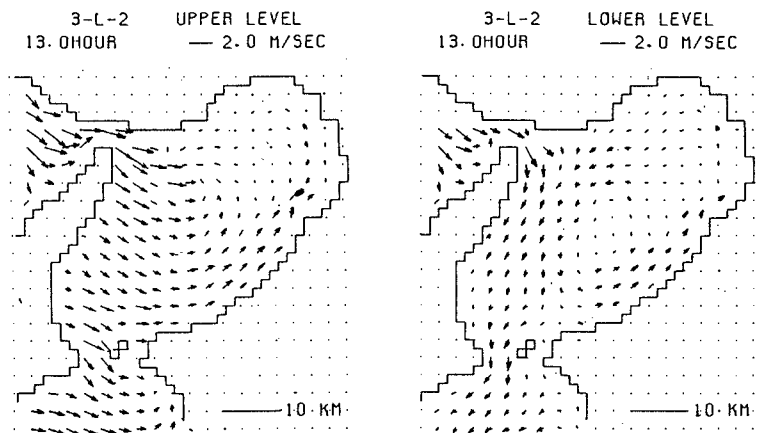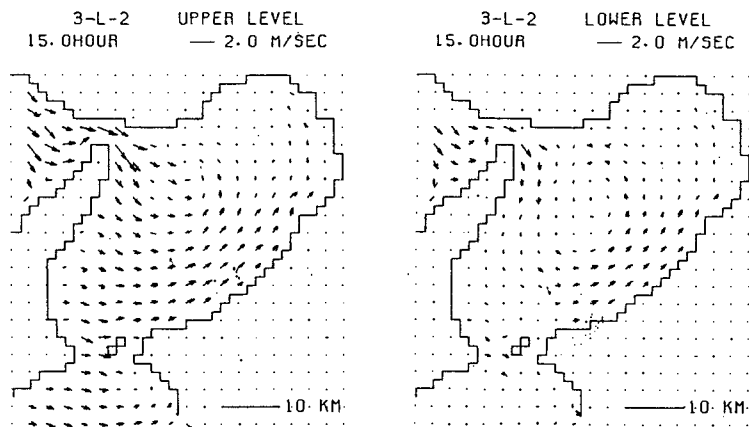
(a) At Kobe
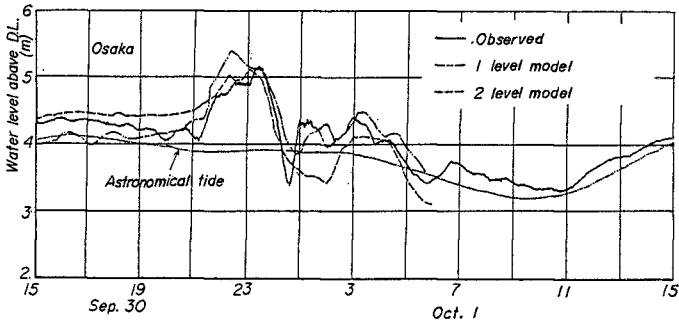
(b) At Osaka
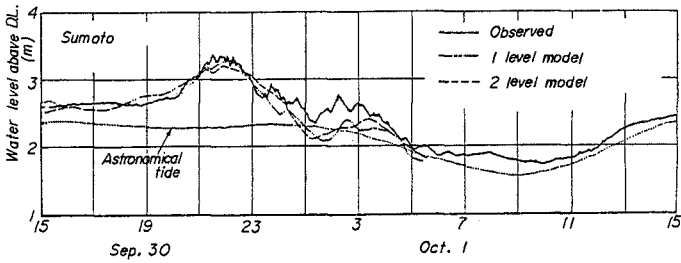
(c) At Sumoto

Figure 5. Comparisons between computations and observations
of storm surges.

(a) At 22:00 Sept. 30

(b) At 0:00 Oct. 1

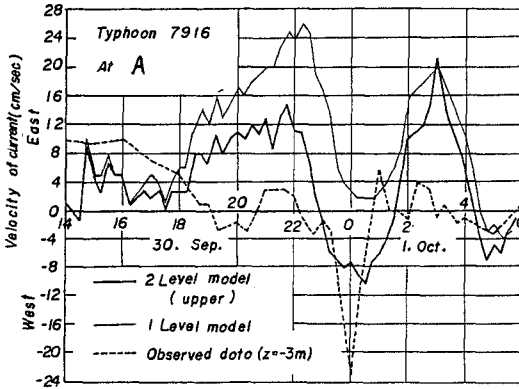Figure 6. Flow patterns computed by the 2-level model.

Figure 7. Comparison of calculated and observed east-west velocity components at point A.

b) The FE–FD model

Koutitas and O'Connor have made reference to the importance of an accurate estimate of the value for the vertical eddy viscosity ν_i when applying the FE–FD model to a wind-induced flow. It is important, of course, to make clear its value physically and empirically. In this paper, the FE–FD modeling procedure is employed as a tool of new approach to establish a 3-level model which has simplicity and generality, so that the same values for ν_i used in the 2-level model can be used, without further discussion. The ratios of thickness of the upper, middle and lower layers to the water depth at each level (element) are set respectively as 3:6:1, and the shearing stresses at the surface are determined as

$$S_x = \rho_a C_D W_x \sqrt{W_x^2 + W_y^2} = \rho_w \nu_i \frac{\partial u}{\partial z}\Big|_{\text{surface}}, \left.\right\}$$

$$S_y = \rho_a C_D W_y \sqrt{W_x^2 + W_y^2} = \rho_w \nu_i \frac{\partial v}{\partial z}\Big|_{\text{surface}} \left.\right\} \qquad (19)$$

In addition the non-slip condition can be used at the bottom because the bottom shearing stresses can be automatically determined from the velocity gradient. This simplicity is one of remarkable features of the model.

The computational results for water surface elevations and currents in the vertical direction (east-west and north-south components) at point A are shown in Figures 8 and 9, respectively. Comparison with the results of the 1- and 2-level models shows that the water surface elevations are over-estimated at the inner part of the bay. In Figure 9, the solid lines show the velocity profiles of the currents due to the anomalies in the meteorological tides and the dotted lines, those caused by the storm surge. The flow patterns in the upper and lower levels at 13hr (actually 22:00 Sept. 30) and 15hr (actually 0:00 Oct. 1) are shown
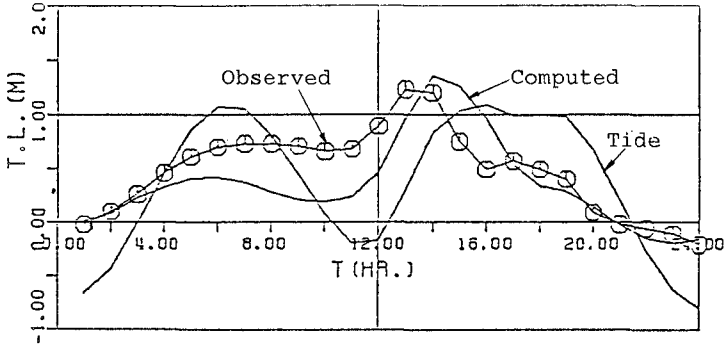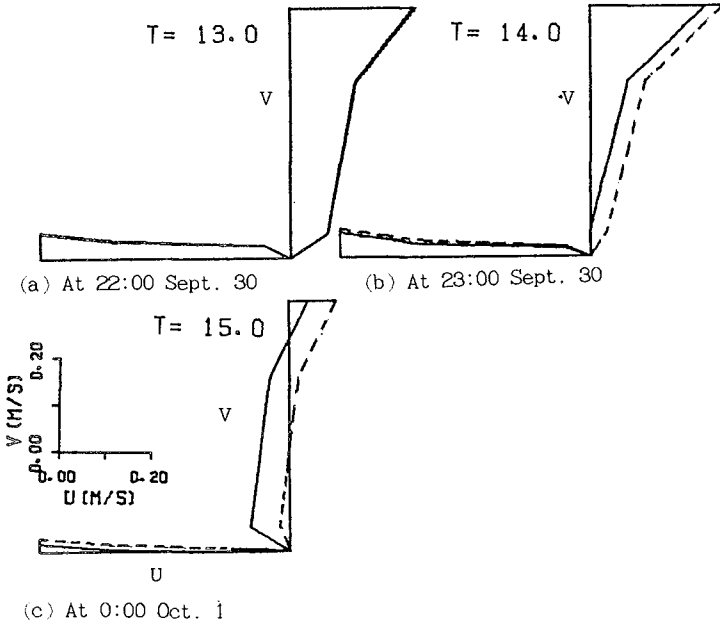
Figure 8. Surface disturbances computed by the FE–FD model
         at point A.

(a) At 22:00 Sept. 30   (b) At 23:00 Sept. 30

(c) At 0:00 Oct. 1
Figure 9. Velocity distributions calculated by the FE–FD
         model at point A.

in **Figure** 10, in which opposite currents in the upper and lower levels
are recognized clearly, but no opposite ones exist in the results from
the 2-level model as shown in **Figure 6**. Comparison of the calculated and
observed current vectors is shown in **Figure 11**, in which the solid lines
are the observed current vectors, the dotted lines calculated ones by
the FE-FD model. 1-L and 2-L express the vector obtained by the 1- and
2-level models, respectively. The current vectors calculated by the
FE-FD model are in better agreement with the observed ones than those
calculated by the 1- or 2-level models, however the strong current to
the southeast observed at 15hr (actually at 0:00 Oct. 1) is still not
completely simulated.
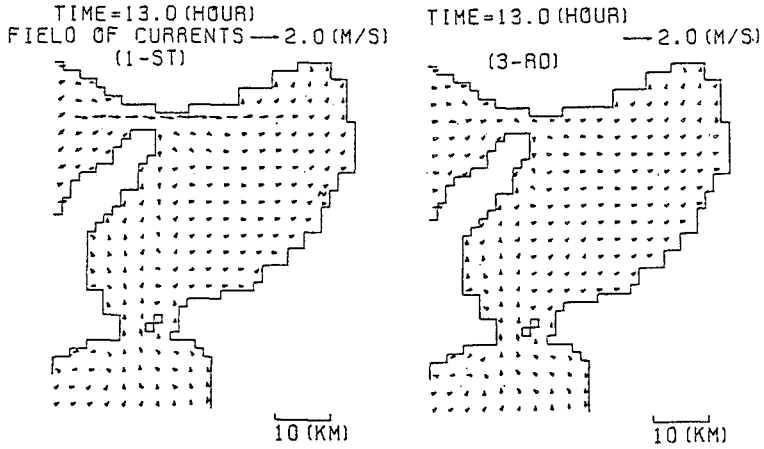
4. Discussions and Conclusions

Thus far, we have presented the multi-level model numerical
calculation methods and the results of storm surge hindcasting, with
considerations. The multi-level models used in this paper are
fundamentally based on the concept of introducing the effects of
three-dimensionality of currents into the calculations for the storm
surge.

Using the 2-level model, which is a modification of the 1-level
model, water surface elevations and currents caused by storm surges in
shallow bays can be simulated by estimating the shearing stresses at the
interface between the upper and lower levels more reasonably. As the
above formulation, of course, depends on the accuracy of the eddy
viscosity estimate, the optimal coefficients in the model should be
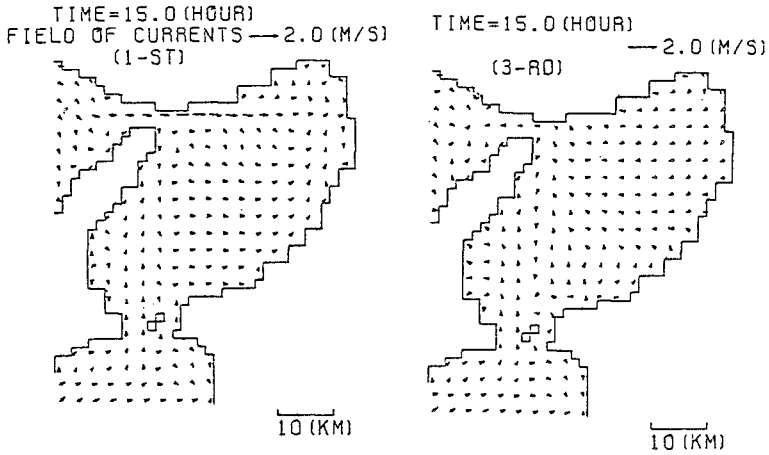found practically by applying the model proposed to many cases of storm
surges.

The FD-FE model proposed here deals with the modified 3-level model
which has more simplicity and flexibility than previous multi-level
models. This model is so economical in calculation time that the actual
prediction of storm surges including their induced-currents as well as
their associated tides even without using a so-called "super computer"
may be feasible in the near future. But this model as yet requires
practical improvement to determine reasonable formulas for shearing
stresses at the bottom and the water surface.

It is concluded that the multi-level models are applicable to the
numerical prediction of storm surges in shallow bays. As to further
developments in the multi-level models, we think that it is possible and
useful to introduce the BEM (Boundary Element Method) into this model
instead of the finite-element technique to calculate the equations of
motion and the irregular-grid finite-difference method for solving the
continuity equation.

TIME=13.0 (HOUR)
FIELD OF CURRENTS ——2.0 (M/S)
(1-ST)

TIME=13.0 (HOUR)
——2.0 (M/S)
(3-RO)

10 (KM)

10 (KM)

(a) At 22:00 Sept. 30

TIME=15.0 (HOUR)
FIELD OF CURRENTS ——2.0 (M/S)
(1-ST)

TIME=15.0 (HOUR)
——2.0 (M/S)
(3-RO)

10 (KM)

10 (KM)
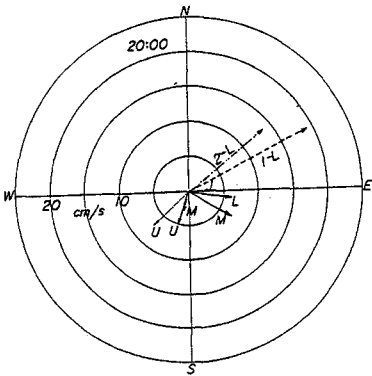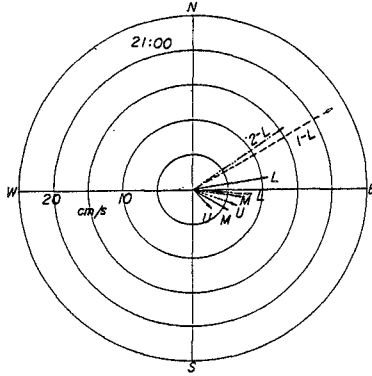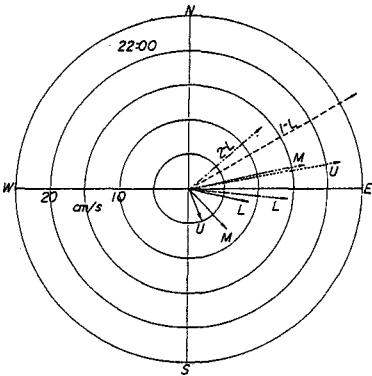
(b) At 0:00 Oct. 1

Figure 10. Flow patterns in the upper and lower levels.
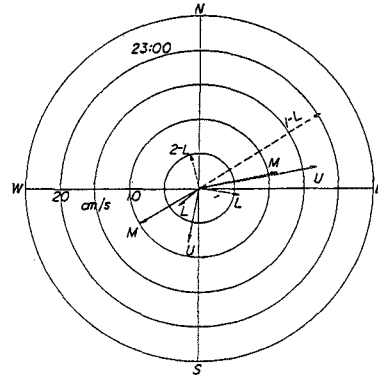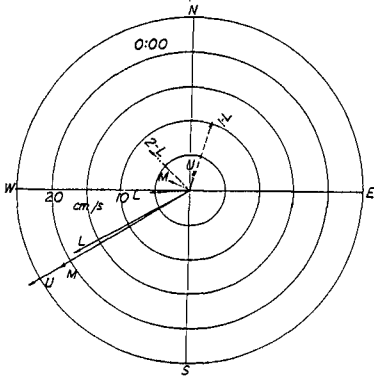
(a) At 20:00 Sept. 30

(b) At 21:00 Sept. 30

(c) At 22:00 Sept. 30

(d) At 23:00 Sept. 30

(e) At 0:00 Oct. 1

Figure 11. Comparisons between calculated and observed velocity vectors at point A.

Appendix-References

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