CHAPTER TWENTY THREE

RUN-UP OF PERIODIC WAVES ON BEACHES OF NON-UNIFORM SLOPE

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ABSTRACT

Run-up of periodic waves on gentle or non-uniform slopes is discussed. Breaking condition and run-up height of non-breaking waves are derived by the use of the linear long wave theory in the Lagrangian description. As to the breaking waves, the width of swash zone and the run-up height are obtained for relatively gentle slopes (less than 1/30), on dividing the transformation of waves into dissipation and swash processes. The formula obtained here agrees with experimental data better than Hunt's formula does. The same procedure is applied to non-uniform slopes and is found to give better results than Saville's composite slope method.

1. INTRODUCTION

Attempts are recently made to establish, in the prediction of beach process, a two-dimensional model which is more realistic than the conventional one-line or two-line models [Watanabe(1981)]. In the modelling to be developed, no sufficient knowledges have yet been accumulated on the swash zone which is the most remarkable output of the beach process. As the first step to fill this lack, run-up of periodic waves is discussed both on uniform gentle slopes and on beaches of non-uniform slopes which are often seen at natural sandy beaches.

Many studies have been made on run-up of periodic waves on uniform slopes, as are summerized below. Run-up of periodic waves is roughly divided into two types; run-up of surging (standing) waves and that of breaking (progressive) waves.

For non-breaking waves, the run-up height and the breaking condition as a standing wave can be estimated well by the modified Miche's(1944) solution [Le Méhauté et al.(1968), Van Dorn(1966), Takada(1970)] and Miche's condition(1951).

On the other hand, for breaking waves, the run-up height is empirically correlated directly to the offshore wave characteristics and the beach slope [Hunt(1959), Savage(1958)]. This is partly due to a lack of

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knowledge about breaking waves. In addition, most of them deal with the run-up height on dikes which have relatively steep slopes.

Recently, some studies are performed on the size of the swash zone. Roos & Battjes(1976) showed from run-up experiments on relatively steep slopes (greater than 1/7) that the run-up height R was estimated well by Hunt's formula(1959) and that the run-down height R_r was expressed as $R_r = R(1-0.4\xi)$, in which ξ was the surf similarity parameter. Guza & Bowen (1976) and Van Dorn(1978) showed that the swash parameter $\varepsilon = a_s\sigma^2/gs^2$ is almost constant for breaking waves. Although some people tried to discuss the run-up height in relation to the wave set-up [Van Dorn(1976), Michi & Watanabe(1982)], they did not formulate the results.

In the case of non-uniform slopes, one can use Saville's composite slope method (1958) to calculate the run-up height for breaking waves. Taylor(1980) applied this method to natural beaches by expressing beach profile as a parabolic curve. However, the physical meanings of this method is not always clear.

In the present study, the authors first obtain the run-up height of surging waves and the breaking condition as a standing wave on non-uniform slope by the use of the linear long wave theory in the Lagrangian description. The non-uniformity of profiles means steps, bars and foreshores with different inclination from those of inshore. Secondly, for breaking waves, we divide the wave deformation after breaking into two processes. One is the "Dissipation Process" and another is the "Swash Process". Combining these two processes in terms of wave height, we derive the run-up height on uniform gentle slopes. This technique is also applied to non-uniform slopes and is compared with Saville's method.

2. RUN-UP OF NON-BREAKING WAVES

Run-up height, R, of a linear standing wave on a uniform slope connected to the water of a constant depth, h, is given by the following equation, as was analysed by Shuto(1972) by using the Lagrangian description.

$$\frac{R}{H_i} = \left[J_0^2\left(4\pi\frac{\ell}{L}\right) + J_1^2\left(4\pi\frac{\ell}{L}\right)\right]^{-1/2} \qquad (1)$$

where H_i and L are the height and length of the incident wave in the channel of constant depth, ℓ the horizontal distance between the shoreline and the toe of slope, and J_0 and J_1 the Bessel functions of the zeroth and the first order. As ℓ tends to ∞, the relationship becomes

$$\frac{R}{H_i} = \sqrt{\frac{\pi\sigma}{s}}\left(\frac{h_1}{g}\right)^{1/4} \qquad (2)$$

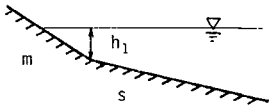
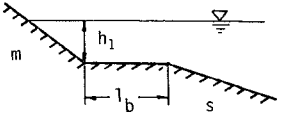
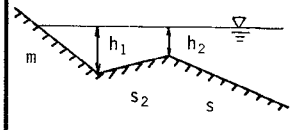
where σ is the angular frequency and s the angle of slope. In order to replace H_i by the height of incident wave in deep water H_0, the law of energy conservation is applied. Then Eq.(2) is reduced to Eq.(3) which is similar to Miche's solution(1944).

$$\frac{R}{H_0} = \sqrt{\frac{\pi}{2s}} \qquad (3)$$

Table 1. Solutions of run-up height and breaking condition of standing waves.

$$\frac{R}{H_0}=\frac{1}{Z\sqrt{\frac{\pi}{2s}}} \qquad \left(\frac{H_0}{L_0}\right)_{cr.}=z\sqrt{\frac{2s}{\pi}}\frac{m^2}{\pi}$$

J_0, J_1, N_0, N_1 : Bessel and Neumann function of the zeroth and the first order.

Profile	Value of coefficients
	$z=\sqrt{\left(\frac{J_0(d)N_1(e)-J_1(d)N_0(e)}{J_0(e)N_1(e)-J_1(e)N_0(e)}\right)^2+\left(\frac{J_0(d)J_1(e)-J_1(d)J_0(e)}{N_0(e)J_1(e)-N_1(e)J_0(e)}\right)^2}$ $d=\frac{2\sqrt{2\pi}}{m}\left(\frac{h_1}{L_0}\right)^{0.5}$ $e=\frac{2\sqrt{2\pi}}{s}\left(\frac{h_1}{L_0}\right)^{0.5}$ $L_0=\frac{g}{2\pi}T^2$
	$z=\sqrt{\left(\frac{u\,N_1(e)-v\,N_0(e)}{J_0(e)N_1(e)-J_1(e)N_0(e)}\right)^2+\left(\frac{u\,J_1(e)-v\,J_0(e)}{N_0(e)J_1(e)-N_1(e)J_0(e)}\right)^2}$ $u=J_0(d)\cos kl_b-J_1(d)\sin kl_b$ $v=J_1(d)\cos kl_b+J_0(d)\sin kl_b$ $d=\frac{2\sqrt{2\pi}}{m}\left(\frac{h_1}{L_0}\right)^{0.5}$ $e=\frac{2\sqrt{2\pi}}{s}\left(\frac{h_1}{L_0}\right)^{0.5}$
	$Z=\sqrt{\left(\frac{u\,N_1(q)-v\,N_0(q)}{J_0(q)N_1(q)-J_1(q)N_0(q)}\right)^2+\left(\frac{u\,J_1(q)-v\,J_0(q)}{N_0(q)J_1(q)-N_1(q)J_0(q)}\right)^2}$ $u=J_0(p)\frac{J_0(d)N_1(e)-J_1(d)N_0(e)}{J_0(e)N_1(e)-J_1(e)N_0(e)}+N_0(p)\frac{J_0(d)J_1(e)-J_1(d)J_0(e)}{N_0(e)J_1(e)-N_1(e)J_0(e)}$ $v=J_1(p)\frac{J_0(d)N_1(e)-J_1(d)N_0(e)}{J_0(e)N_1(e)-J_1(e)N_0(e)}+N_1(p)\frac{J_0(d)J_1(e)-J_1(d)J_0(e)}{N_0(e)J_1(e)-N_1(e)J_0(e)}$ $d=\frac{2\sqrt{2\pi}}{m}\left(\frac{h_1}{L_0}\right)^{0.5}$ $e=\frac{2\sqrt{2\pi}}{s_2}\left(\frac{h_1}{L_0}\right)^{0.5}$ $p=\frac{2\sqrt{2\pi}}{s_2}\left(\frac{h_2}{L_0}\right)^{0.5}$ $q=\frac{2\sqrt{2\pi}}{s}\left(\frac{h_2}{L_0}\right)^{0.5}$

A similar manipulation yields the following breaking condition of standing waves which is analogous to Miche's result(1951), from the breaking condition obtained by Shuto(1972).

$$\left(\frac{H_0}{L_0}\right)_{cr.} = \sqrt{\frac{2s}{\pi}}\,\frac{s^2}{\pi} \qquad (4)$$

Run-up on a slope of more complicated shape can be obtained as follows. Divide the whole beach into segments of uniform slope with the inflection points of bottom slope as their end points. For each segment, solutions of the linear standing long waves in the Lagrangian description are determined, corresponding to its own slope. These local solutions are connected to build the solution over the whole slope. The continuation is carried out for the horizontal and vertical displacements of the water particles which are on the boundary between the two different slopes at the initial instant. As for the procedure, one may read the reference (22). Once the solution is established, the run-up height is correlated with the height of incident waves in deep water, as is shown above.

Solutions are obtained for bi-linear sloped beach, step-type beach and bar-type beach. Equations are summerized in Table 1. Numerical examples for the offshore slope s = 0.03 and the foreshore slope m = 0.1 are shown by circles and triangles in Fig.1. The dotted lines in Fig.1 are the result given by Eqs.(3) & (4), by setting s = m = 0.1. Even though the equations are functions of not only m but also s, R/H_0 and $(H_0/L_0)_{cr}$ are mainly governed by m. One may consider that the solutions of R/H_0 (or $(H_0/L_0)_{cr}$) for m = 0.1 and s = 0.03 are between the values of Eq.(3) (or Eq.(4)) for s = 0.1 and for s = 0.03. However, this is not

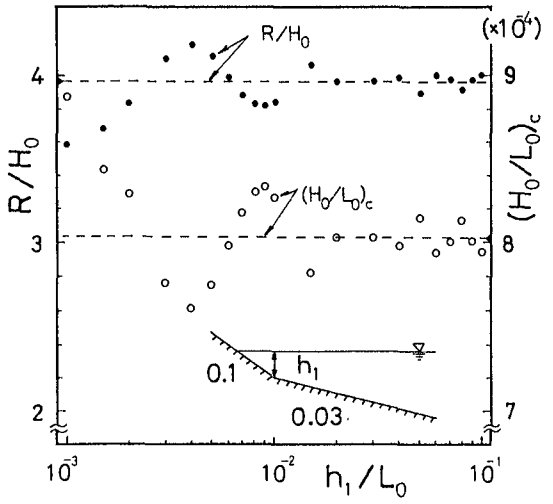


Fig.1(a) Numerical Examples of run-up height and breaking condition of standing waves (bi-linear sloped beach)

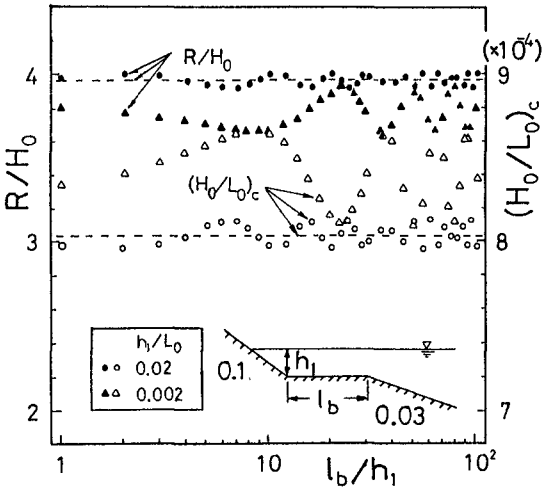


Fig.1(b) Numerical examples of run-up height and breaking condition of standing waves (step-type beach)

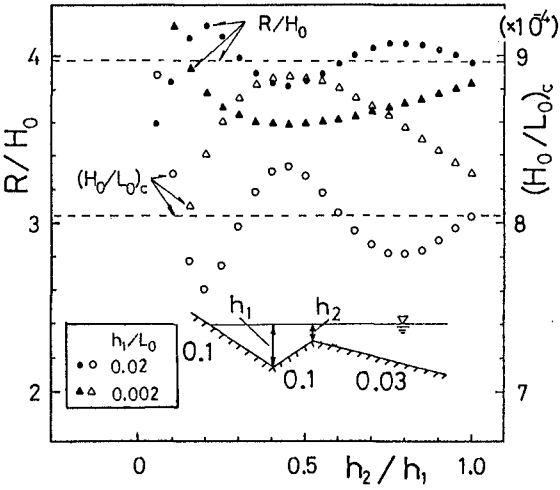


Fig.1(c) Numerical examples of run-up height and breaking condition of standing waves (bar-type beach)

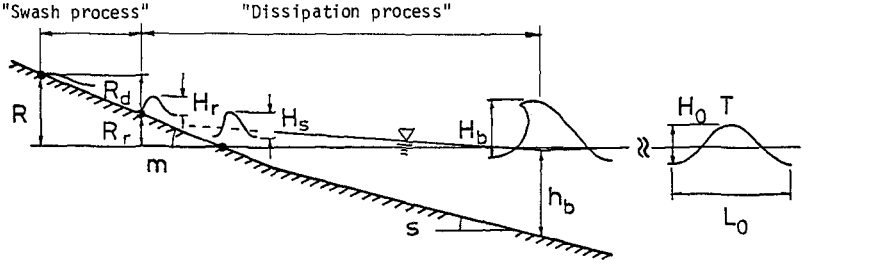
true in case of the standing waves in contrast to the case of the breaking waves. This result suggests that the foreshore slope is the most important in problems of the run-up, the breaking condition and the reflection of standing waves.

3. RUN-UP OF BREAKING WAVES

Deformation of progressive waves after breaking is divided into the following two processes; i) the dissipation process in which the kinematic and potential energies are consumed due to breaking, and ii) the swash process in which the kinematic and potential energies alternates to each other on a dry bed. The run-up height and the width of swash zone are calculated by continuing the wave height between the two processes. Figure 2 gives the definition sketch.

3.1 Dissipation process: decay of wave height due to breaking

According to the experimental curves of Sasaki & Saeki(1974) for waves after breaking on a uniform slope, the decay in wave height is almost independent of the wave steepness. It is also shown in their results that if h/h_b is smaller than 0.5, H/H_b is linearly proportional to h/h_b with the coefficient of proportionality $\beta = 0.6$, independent of the beach slope. Assume that their experimental curves can be extended to the points where the curves intersect the beach profile. The height, R_r, of the points of this intersection which are hereafter called as the beginning points of run-up are given by



H_0 : Offshore wave height
T : Wave period
L_0 : Offshore wave length
H_b : Breaking wave height
h_b : Breaking water depth
s : Tangent of slope in the surf zone
H_s : Wave height at the shoreline of still water level
H_r : Wave height at the beginning point of run-up
m : Tangent of foreshore slope
R : Total run-up height above still water level
R_d : The width of swash zone
R_r : The height of the beginning point of run-up

Fig.2 Definition sketch.

$$\frac{R_r}{h_b} = \frac{1}{\beta}\left(\frac{H_s}{H_b} - \frac{H_r}{H_b}\right) \tag{5}$$

The wave height, H_s, at the shoreline of still water level is given by black circles in Fig.3, from experimental results of Sasaki & Saeki(1974) and Roos & Battjes(1976). It is expressed by

$$\frac{H_s}{H_b} = \begin{cases} 3.04\ s^{1.07} & \text{for } 0.01 < s < 0.2 \\ 0.55 & \text{for } 0.2 < s \end{cases} \tag{6}$$

With these informations, R_r can be obtained as a function of H_r, the wave height at the beginning point of run-up if the breaking wave height H_b and the breaking water depth h_b are given.

For a non-uniform slope for which no somple relations similar to Eqs.(5) and (6) are available at present, numerical computation such as proposed by Mizuguchi(1980) may be used.

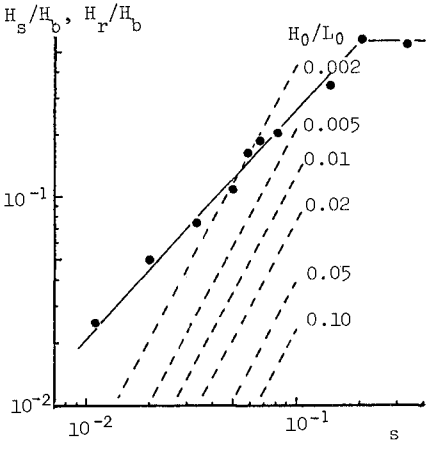


Fig.3 Relative wave heights H_s/H_b, H_r/H_b and beach slope.

3.2 Swash Process: run-up of waves

Trajectory of the wave front when waves are climbing a uniform slope is given by Freemann & Le Méhauté(1964) as follows.

$$x_f(t) = -\frac{1}{2} gAm\left(t - \frac{u_s}{gAm}\right)^2 + \frac{1}{2}\frac{u_s^2}{gAm} \tag{7}$$

$$A = \frac{1+f/(a^2m)}{(1+2a)(1+a)} \tag{8}$$

where $x_f(t)$ is the horizontal coordinate of the wave front measured from

the beginning point of run-up, t the elapsed time after the wave front passes the beginning point of run-up, u_s the water particle velocity of rarefaction waves at the beginning point of run-up, a the ratio of the wave propagation velocity $c = \sqrt{g(h+\eta)}$ to the water particle velocity, and f the friction coefficient defined by $\tau = \rho f u^2$. According to the experiments of Iwagaki et al.(1966), a = 0.26 and f = 0.005 - 0.01 for a smooth bottom in case of m > 0.1.

Trajectory of the wave front when waves are descending is given as follows, on continuing the solution of Brandtzaeg(1964) with Eq.(7).

$$x_f(t) = \frac{u_s^2}{2gAm} - \frac{2z}{f'} \ln\left[\cosh\sqrt{\frac{f'g(m-\tan\gamma)}{2z}}\,(t-\zeta T)\right] \qquad (f' \neq 0) \quad (9)$$

$$x_f(t) = \frac{u_s^2}{2gAm} - \frac{1}{2} g(m-\tan\gamma)(t-\zeta T)^2 \qquad (f' = 0) \quad (10)$$

where ζT is the time when the maximum run-up height appears, T the wave period, z the thickness of water of descending waves, f' the friction coefficient and γ the angle between water and beach surface. With the assumptions that f = f'=0 (or A = 1) and that the water surface displacement at the shoreline varies linearly with respect to time, we have $\zeta = 0.46$.

The value of u_s is assumed to be given as follows in terms of H_r.

$$u_s = F\sqrt{gH_r} \qquad (11)$$

Figure 4 shows empirical relationships between u_s and H_r. The experimental results are taken from one of the present authors(1982) for single bores on m = 1/20, Iwagaki et al.(1966) for solitary waves on m > 1/10 and Roos & Battjes(1976) for regular wave trains on m = 1/3 and 1/7.

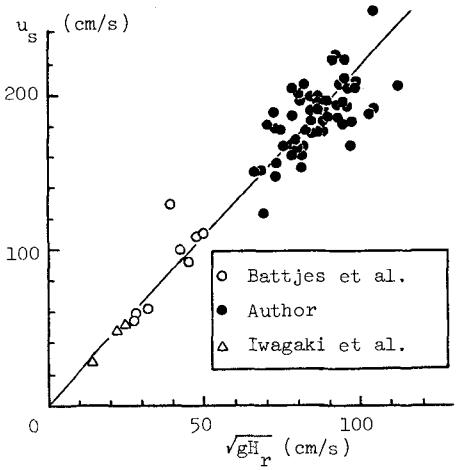


Fig.4 Relationship between u_s and $\sqrt{gH_r}$ (for smooth fixed bottom)

It was assumed $\zeta = 0.46$ in case of regular waves. From the figure, the coefficient of proportionality which is a kind of Froud number is 2.2 on an average for both single waves and regular wave trains. This is nearly equal to the wave front condition of Iwasaki & Togashi(1969).

From Eqs.(7) and (11), H_r and R_d are derived as follows.

$$H_r = \frac{gm^2T^2}{F^2} A^2\zeta^2 \tag{12}$$

$$R_d = \frac{gm^2T^2}{2} A\zeta^2 \tag{13}$$

Figures 5 and 6 compare the calculated results of H_r and R_d for $F = 2.2$, $f = 0.005$ and $\zeta = 0.46$ with experimental results of Roos & Battjes(1976), Van Dorn(1976) and Michi(1982). Since the value of $a = 0.26$ according to to Iwagaki et al.(1966) makes the calculated value too big in case of a gentle slope, the followings are used for convenience.

$$a = \begin{cases} 0.26 & \text{for} \quad 0.1 < m \\ 0.13\, m^{-0.3} & \text{for} \quad 0.01 < m < 0.1 \end{cases} \tag{14}$$

In the result of Van Dorn(1976) who discussed R_d, the calculated values were about twice the experimental results in case of a steep slope. From Eq.(13), swash parameter $\varepsilon = a_s\sigma^2/gm^2$ is calculated as $\pi^2\zeta^2 A$ (= 2.09 A) and is found to be a function of foreshore slope and friction factor.

Equation (12) (or Eq.(13)) shows that H_r (or R_d) is mainly governed by the foreshore slope m and the wave period T, whereas H_s is a function of the slope s alone. For a comparison, values of H_r/H_b for different wave periods are drawn by dotted lines in **Fig.3** which is originally given to show the relationship H_s and the slope angle.

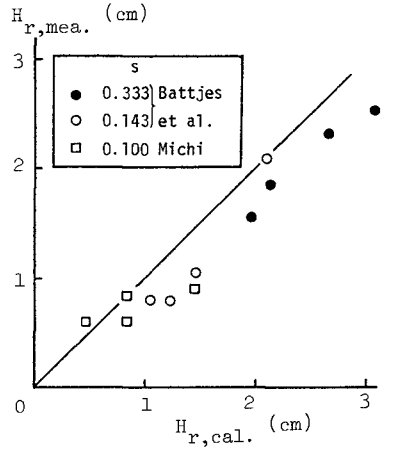


Fig.5 Comparison of H_r between measured and calculated. (for smooth fixed bottom)

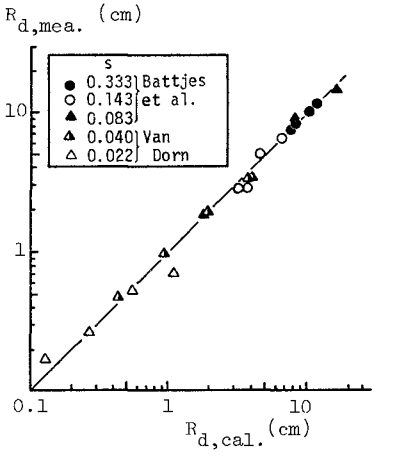


Fig.6 Comparison of R_d between measured and calculated (for smooth fixed bottom)

3.3 Run-up on a gentle, uniform slope

Run-up height R is given by a sum of R_r and R_d.

$$\frac{R}{H_0} = \frac{0.46}{\beta} s^{-0.12}\left(\frac{H_0}{L_0}\right)^{-0.2}\left[3.04\, s^{1.07}\right. \tag{15}$$

$$\left. - \frac{2.94\pi\zeta^2 A^2}{F^2} s^{-0.09} m^2 \left(\frac{H_0}{L_0}\right)^{-0.75}\right] + \pi A \zeta^2 m^2 \left(\frac{H_0}{L_0}\right)^{-1}$$

for $0.02 < m < 0.33$, $0.01 < s < 0.1$ and $0.003 < H_0/L_0 < 0.07$

The first term on the right-hand side is R_r/H_0 which is equal to the wave set-up and the second term is R_d/H_0, the width of swash zone. Breaking conditions are determined by the following formulas obtained from the same data as Goda(1970) used.

$$\frac{H_b}{H_0} = 0.68\, s^{0.09}\left(\frac{H_0}{L_0}\right)^{-0.25} \quad \text{for } 0.01 < s < 0.1,\ 0.003 < H_0/L_0 < 0.07 \tag{16}$$

$$\frac{h_b}{H_0} = 0.46\, s^{-0.12}\left(\frac{H_0}{L_0}\right)^{-0.20} \tag{17}$$

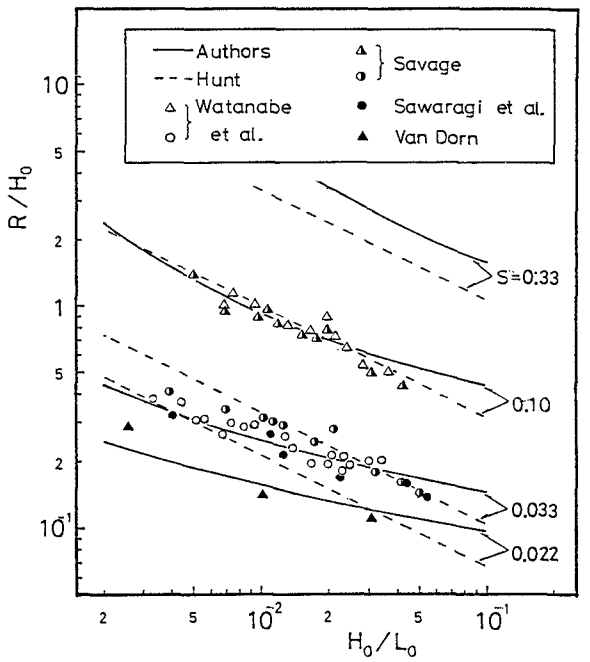


Fig.7 Comparison of calculated R/H_0 with experimental data.

Figure 7 compares experimental results of Michi & Watanabe(1982), Van Dorn(1976), Sawaragi et al.(1976) and Savage(1958) with the calculated ones for $\beta = 0.6$, $F = 2.2$, $f = 0.005$ and a obtained from Eq.(14). Calculated run-up heights agree better with experimental results than Hunt's formula(1959) does. Figure 8 shows the relative magnitude of R_r/H_0 and R_d/H_0 for s = 1/10, 1/30 and 1/90. The gentler the slope is and the greater the wave steepness is, the contribution of the wave set-up R_r is the greater, although the greater wave steepness does not always yield the greater R_r/H_0.

The discussion above is for a fixed bottom of smooth surface and is not directly applicable to a natural beach which is of movable and rough surface. According to Sawaragi's experiments(1962), effects of bottom roughness is not dominant on the breaking condition and is negligible for a natural, sandy beach.

Effects of bottom roughness in the swash process is shown in Fig.9. Equation (13) is used to calculate the run-up height for different values

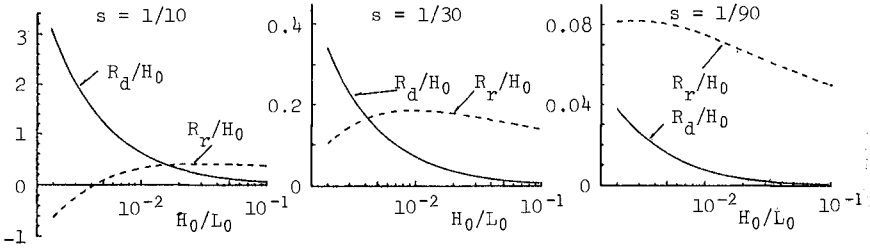


Fig.8 R_d/H_0 vs. R_r/H_0 for various slopes.

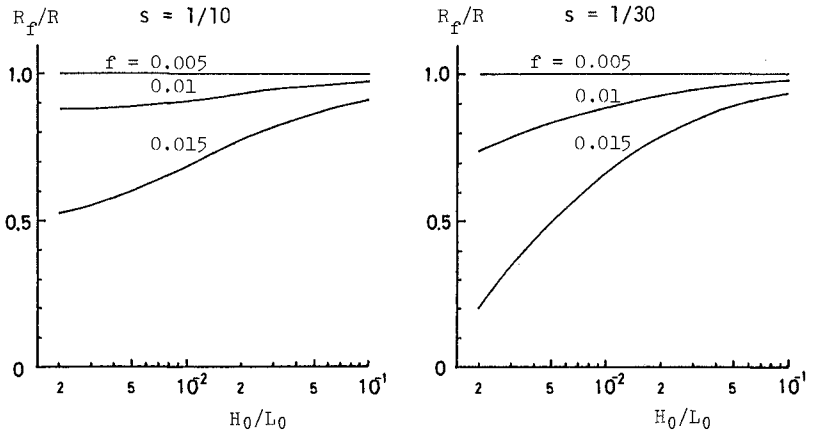


Fig.9 Effect of the value f on run-up height.

of f and the results are made dimensionless, divided by the run-up height on a smooth bottom for which f is 0.005. Tsuchiya et al.(1978) reexamined Savage's data(1958) and obtained the effect of roughness on run-up height, which is similar to Fig.9. However, we must take notice of the fact that f used here is somewhat different from the original definition of the bottom shear stress but is a virtual friction coefficient which includes several other effects such as turbulence effect caused by breaking in the name of the friction coefficient. No method is available to determine the value of f from hydraulic conditions near the bottom.

From prototype experiments by the Central Research Institute of Electric Power Industry, relationships similar to Fig.4, 5 and 6 are shown in Fig.10,11 and 12. As for the experimental channel, one may read the reference(8). All experiments were conducted with a constant still water depth of 4.5 m and on initial slope inclination of 1/20. The slope consists of movable sands with the median diameter of 0.27 mm. Foreshore slopes formed by waves ranged from 0.033 to 0.186. In these experiments, the value of F is also about 2 on an average. The friction coefficient is found to be 0.8 so as that calculated values of R_d and H_r give the best fit to the measured data. Values of f as well as a should be experimentally determined, after all.

3.4 Run-up on a non-uniform slope

The same procedure is applied to the run-up on a non-uniform slope, based on the following assumptions.

i) Dissipation process does not vary by changing foreshore profiles.

ii) Water particle velocity is continuous at the point where profile changes and the value of a depends on local slopes.

Examples are given in case of bi-linear sloped beaches, the inflection point of which coincides with the shoreline. If $R_r > 0$, Eq.(15) can be used. For $R_r < 0$, the following equation is used.

$$\frac{R}{H_0} = \frac{F^2}{2A_2}\left(\frac{H_r}{H_0}\right) - \frac{A_1}{A_2}s\left(\frac{x_1}{H_0}\right) \tag{18}$$

$$\frac{x_1}{H_0} = \frac{-0.676}{\beta}s^{-1.21}\left(\frac{H_0}{L_0}\right)^{0.05}\left[2.07s^{1.16}\left(\frac{H_0}{L_0}\right)^{-0.25} - \frac{H_r}{H_0}\right]$$

$$2\zeta\left(\frac{H_0}{L_0}\right)^{-1/2} = \frac{F}{A_1 s}\sqrt{\frac{2}{\pi}}\left(\frac{H_r}{H_0}\right)^{1/2} + \sqrt{\frac{2}{\pi}}\frac{A_1 s - A_2 m}{A_1 A_2 sm}\sqrt{F^2\left(\frac{H_r}{H_0}\right) - 2A_1 s\left(\frac{x_1}{H_0}\right)}$$

where A_1 and A_2 are the values of A obtained from Eqs.(8) and (14) for s and m, x_1 the horizontal distance between the beginning point of dry bed and the inflection point of the slope.

Figure 13 shows comparisons of the calculated results with experimental results of Toyoshima et al.(1964) and Saville(1958). Full lines in Fig.13 are Eqs.(15) & (18) and broken lines are calculated by Saville's composite slope method(1958). The composite slope method always gives smaller values. The present method agrees farely well with the experimental results, although the effect of the foreshore slope on the deformation of waves after breaking is not taken into consideration.

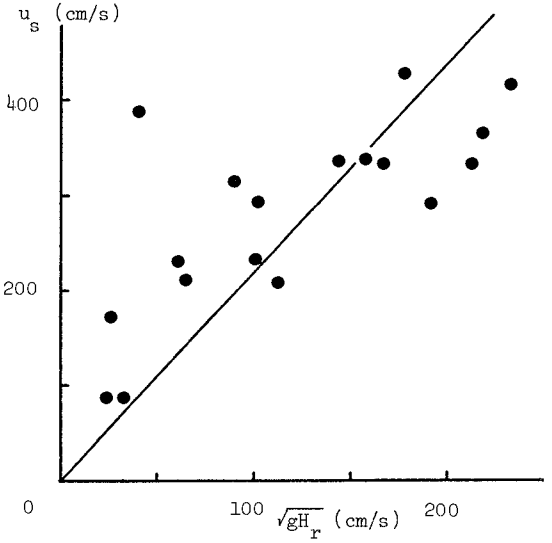


Fig.10 Relationship between u_s and $\sqrt{gH_r}$ (for movable sand bottom)

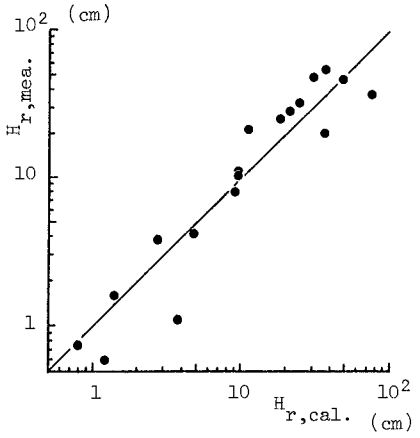


Fig.11 Comparison of H_r between measured and calculated. (for movable sand bottom)

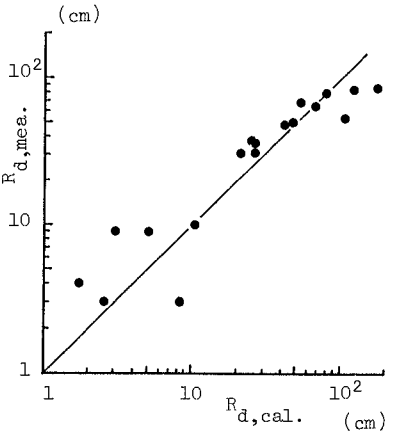


Fig.12 Comparison of R_d between measured and calculated. (for movable sand bottom)

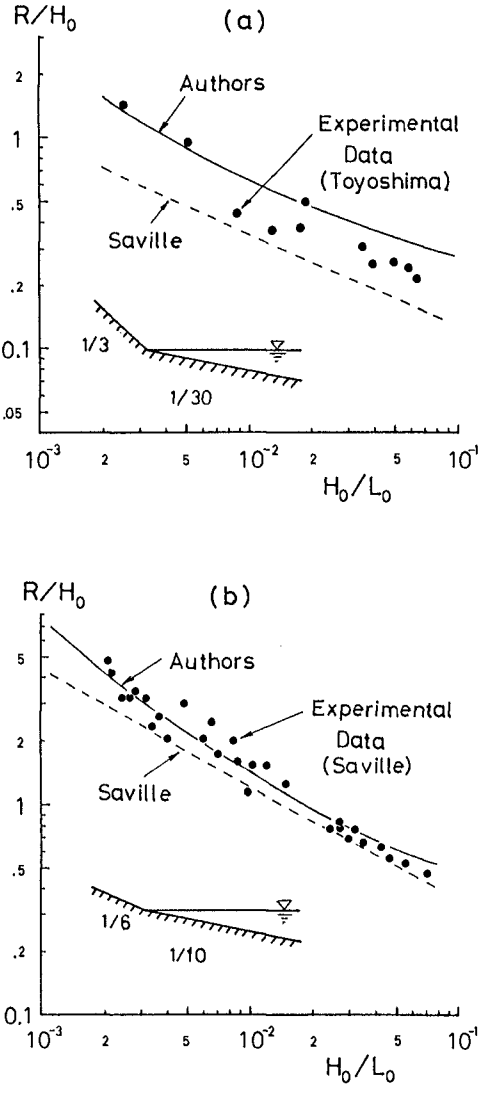


Fig.13 Comparison between Saville's method and the present method.

4. CONCLUSIONS

Run-up of periodic waves is discussed, *considering the wave deformation,* especially for *gentle and non-uniform slopes* (with steeper foreshore slopes, steps and bars) which are often seen at *natural beaches.* Results are summerized as follows.

(1) The run-up height and the breaking condition for non-breaking waves on non-uniform sloping beaches are derived as shown in Table 1. Foreshore slopes are found to be dominant in these values. The procedure to obtain the solution can be applied to understand the behavior of long period waves in the surf zone.

(2) For breaking waves, run-up phenomena can be explained well by connecting dissipation and swash processes.

i) The width of swash zone is mainly determined by the foreshore slope and wave period. It can be calculated by Eq.(13) if the foreshore slope is within the range of 1/3 - 1/50.

ii) For gentle uniform slopes (1/10 - 1/100), the run-up height formula , Eq.(15), obtained by setting m = s can be used instead of Hunt's formula(1959) which is originally applicable to steeper slopes than 1/10.

iii) The procedure to connect dissipation and swash processes is applied to calculate the run-up height in the case of steeper foreshore profiles which are generally found at natural beaches. The procedure gives not only better results than Saville's method(1958), but also a physically sound explanation of run-up. We can also obtain the size of the swash zone which is closely connected with the stability of dikes built near and above the shoreline and which is also important in estimating sand movement near swash zone.

iv) Further studies are necessary for wave run-up on more complicated slopes and for estimation of values of friction coefficient.

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