CHAPTER THIRTY ONE

PREDICTION METHOD FOR THE WAVE HEIGHT DISTRIBUTION OFF THE WESTERN COAST OF TAIWAN

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Introduction

Taiwan Strait Iocates on the continental shelf of the western Pacific Ocean. The water depth is Iess than 100 meters. Furthermore, the bathemetry of the eastern side namely the offing of western coast of Taiwan shoals gradually. In consequence, in case of the wind blows from the north to south, waves in the deeper part of the strait refract to be north west direction while they are approaching the shore and local waves directly generated by the wind still keep the same direction of the wind. The situation is shown in Figure 1.

From September to April of the next year, anticyclones come from Mongolia causes monsoon in this area. The wind velocity in the monsoon sometimes exceeds 20 meters per second, but it is arround 10 meters per second in general. Howerer, the duration of winds over 5 meters per second has been recorded more than 50 days. Engineering works such as towing caissons for building breakwater as well as dredging offshore have to be done in these days. Furthermore, navigation operations should not be stopped unless the wind is too strong. Of course, waves are forecast every day, however, more precise information about the probability of the occurrence of certain wave height is of great significance.

In last conference, the authors submitted a probability density function of wave heights in this area. This distribution model is to be remended by considering energy loss in this paper, and concrete forecasting procedure is submitted for engineering and navigation practice.

The Distribution Model

In last conference which was held in Capetown the authors (Tang et. al. 1982) submitted a probability density function of wave heights off the western coast of Taiwan namely the eastern side of Taiwan Strait. The bathemetry of this area is rather flat and the prevailing wind direction is almost in the longshore direction. Waves in this area are considered to be the combination of the refracted waves from the deeper sea in the central part of Taiwan Strait and the local waves generated directly by the wind. The wave height is proportional to the square root of the sum of energies from refracted and local wave.

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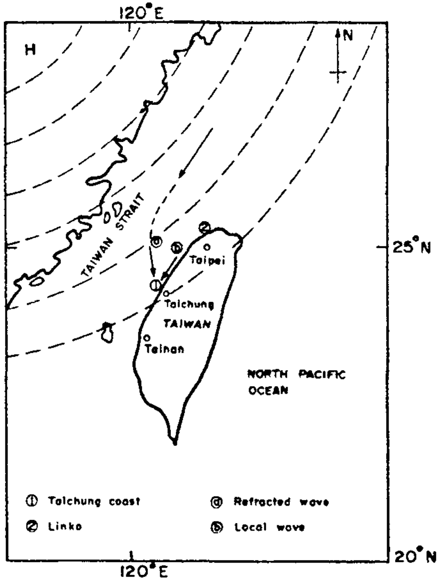

Fig.1 Wind and wave situation in the winter

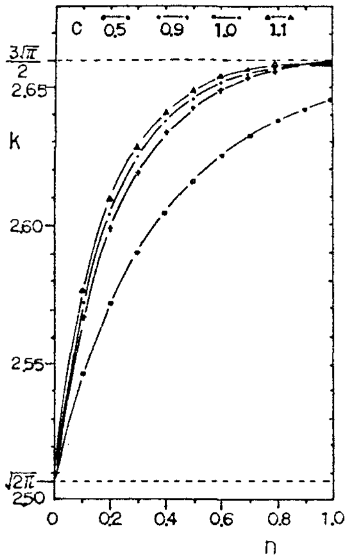

Fig.2 Relationship between k and n

Form such an assumption the wave height probability distribution was to be worked out as follows :

$$\psi(H) = \frac{H}{a-b}\left[\exp\left(-\frac{H^2}{2a}\right) - \exp\left(-\frac{H^2}{2b}\right) \right] \tag{1}$$

where
- a : energy of the refracted waves, $a = 4\,\sigma_1^2$
- b : energy of the local waves, $b = 4\,\sigma_2^2$
- $\psi(\cdot)$: probability density function
- H : wave height
- $\sigma_{1,2}^2$: area under the wave spectral density function curve, subscript 1, 2 denote the refracted and local wave respectively.

The ratio between the mean wave height $\overline{H}$ and the combined energy σ is

$$k = \frac{\overline{H}}{\sigma} = \sqrt{2\pi}\cdot\frac{1-n^{3/2}}{(1-n)\sqrt{1+n}} \tag{2}$$

where $\sigma = \sqrt{\sigma_1^2+\sigma_2^2}$, $n = \frac{b}{a} = \left(\frac{\sigma_2}{\sigma_1}\right)^2$

when $n \to 0$, it means that no local wave existing, $k = \sqrt{2\pi}$, as shown in Fig.2.

Above equation fits the reality better than Rayleigh's distribution in comparison with the measured data as shown in Fig.3.

Modification of The Model

In navigation and engineering practice, the wave height distribution must be predicted in advance of operation and being much more accurate is required.

In equation(1), the ratio of σ_1 and σ_2 is to alter the steepness and location of the peak of the curve sensitively. In addition, the wave energy dissipation should also be taken into consideration.

Denoting E_1 , E_2 to be the wave energy of refracted and local wave respectively, considering the energy dissipation, the total wave energy E at the location of interest is assumed to be the following linear combination.

$$E = r\,E_1 + c\,r\,E_2\ ;\ 0 < r \le 1\ ,\ c > 0\ ,\ c\,r \le 1 \tag{3}$$

From probability transform and convolution integration operations the revision of equation(1) is worked out as follows :

$$\psi(H) = \frac{H}{r(a-cb)}\left[\exp\left(-\frac{H^2}{2ra}\right) - \exp\left(-\frac{H^2}{2crb}\right) \right] \tag{4}$$

The expected value namely the average wave height and the variance, can be calculated respectively as

$$\overline{H} = \sqrt{\frac{\pi}{2}}\cdot\frac{r^{1/2}}{a-cb}\left[a^{3/2} - (cb)^{3/2} \right] \tag{5}$$

$$\sigma_H^2 = 2r(a+cb) - \frac{\pi r}{(a-cb)^2}\left[\frac{a^3+c^3b^3}{2} - (cab)^{3/2} \right] \tag{6}$$

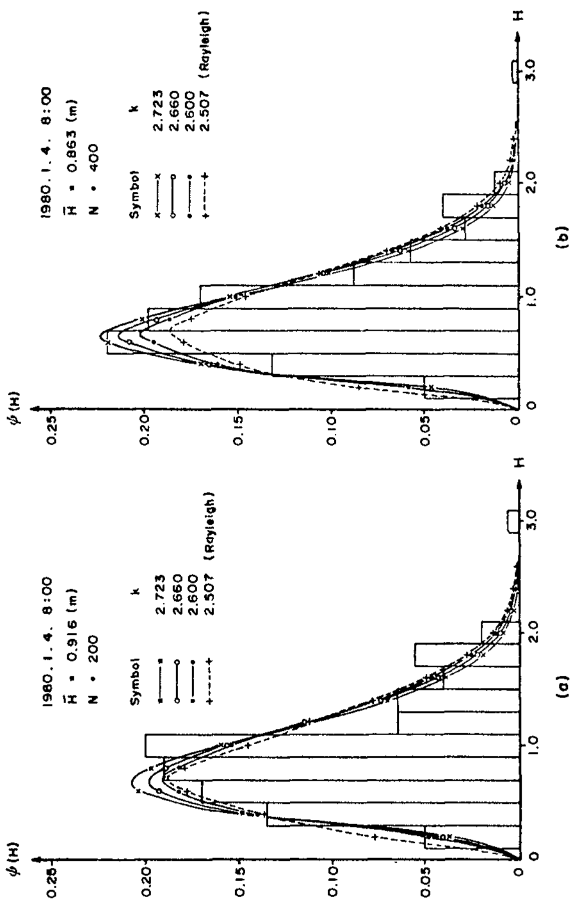

Fig.3 New probability density curve and measured data

The skewness and kurtosis of the curve are also to be evaluated as follows.
Skewness :

$$\sqrt{\beta_1} = \frac{1}{\{2(a+cb) - \frac{\pi}{(a-cb)^2}[\frac{a^3+c^3b^3}{2} - (c \cdot a \cdot b)^{3/2}]\}^{3/2}} \cdot \frac{\sqrt{\pi/2}}{a-cb} \cdot \{3[a^{5/2} - (cb)^{5/2}] - 6(a+cb)[a^{3/2} - (cb)^{3/2}] + \frac{\pi}{(a-cb)^2} \cdot [a^{3/2} - (cb)^{3/2}]^3\} \tag{7}$$

Kurtosis :

$$\beta_2 = \gamma - 3 = \frac{1}{\{2(a+cb) - \frac{\pi}{(a-cb)^2}[\frac{a^3+c^3b^3}{2} - (cab)^{3/2}]\}^2} \cdot \{8(a^2+cab + c^2b^2) - \frac{6\pi}{(a-cb)^2}[a^{3/2} - (cb)^{3/2}] \cdot [a^{5/2} - (cb)^{5/2}] + \frac{6\pi(a+cb)}{(a-cb)^2} \cdot [a^{3/2} - (cb)^{3/2}]^2 - \frac{3\pi^2}{4(a-cb)^4}[a^{3/2} - (cb)^{3/2}]^4\} - 3 \tag{8}$$

And the ratio k becomes

$$k = \frac{\sqrt{2\pi}[1-(cn)^{3/2}]}{(1-cn)\sqrt{1+cn}} \tag{9}$$

For evaluating the influence of coefficients k , r , c , on the shape of the curves, several calculations are made and illustrated in Figures 4 , 5 , 6.

These coefficients, however, should be determined by calculations of refraction effect, friction of sea bottom as well as wave — wave interaction. Such computations are too complicate to be done. Besides, the theories have not yet fully developed. Approximate evaluations are to be carried out by comparing calculated curve and measured data as shown in Fig.7.

Through these comparison, coefficient r, which represent the ratio of remainer of energy after dissipation, should be taked to be 0.95, and the coefficient c, which denotes the ratio of energy loss of refracted and local waves is of minor significant, so it can be consider as unity. The coefficient k, its definition is shown in equation (2) and (9), is merely the function of the ratio of local wind wave energy and the refracted wave, which will depend on the weather situation and can not be decided in advance and will not be a constant. The probability density function is as follows in consequence.

$$\psi(H) = \frac{H}{0.95(a-b)}[\exp(-\frac{H^2}{1.9a}) - \exp(-\frac{H^2}{1.9b})] \tag{10}$$

Prediction Procedure

If the weather forecasting data are available, the waves and wave height distribution off the western coast of Taiwan are to be forecasted as follows.

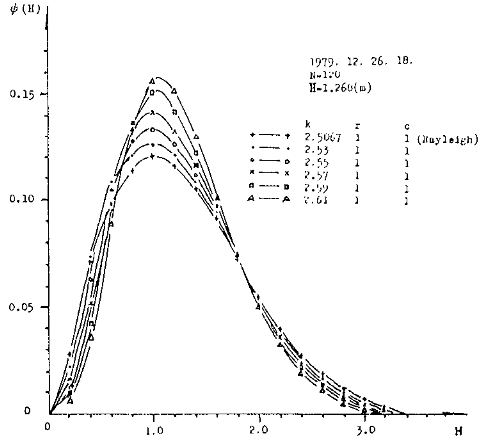

Fig.4 Coefficient k iufluences the shape of distribution(1)

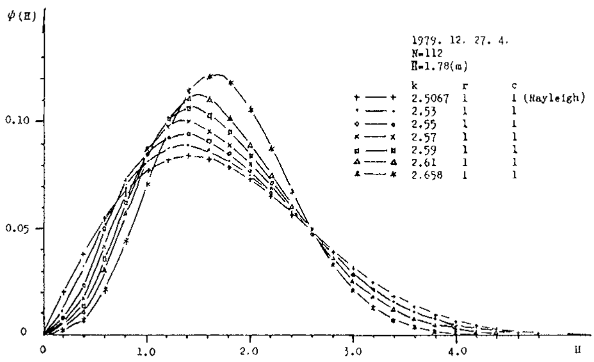

Fig.4 Coefficient k iufluences the shape of distribution(2)

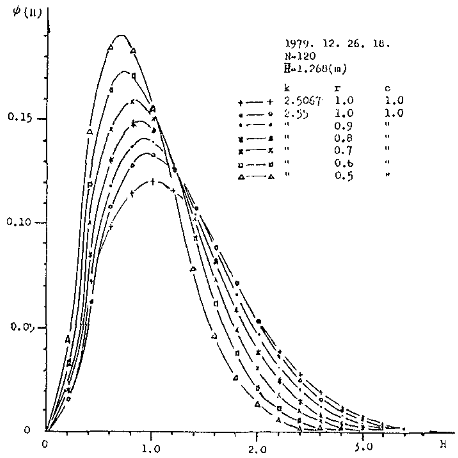

Fig.5 Coefficient r iufluences the shape of distribution(1)

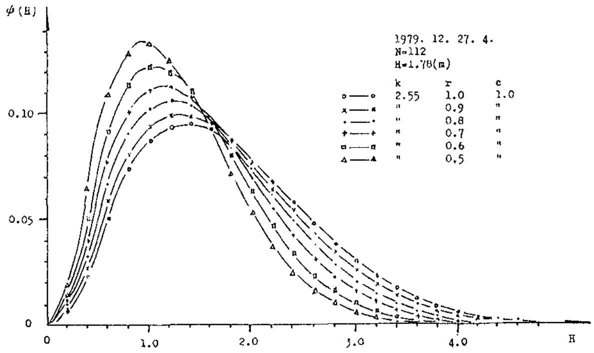

Fig.5 Coefficient r iufluences the shape of distribution(2)

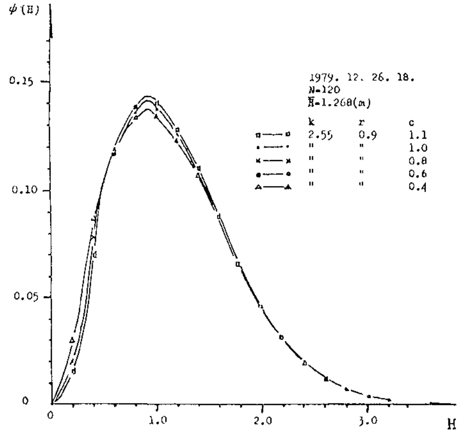

Fig. 6 Coefficient c iufluences the shape of distribution(1)

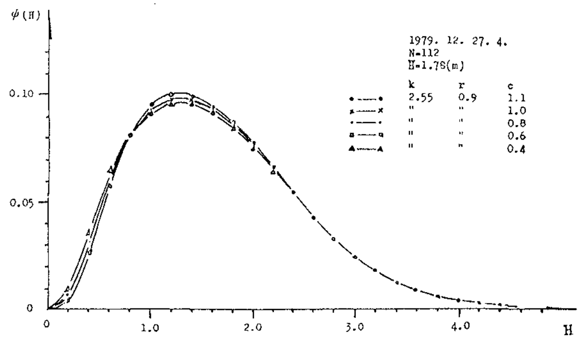

Fig. 6 Coefficient c iufluences the shape of distribution(2)

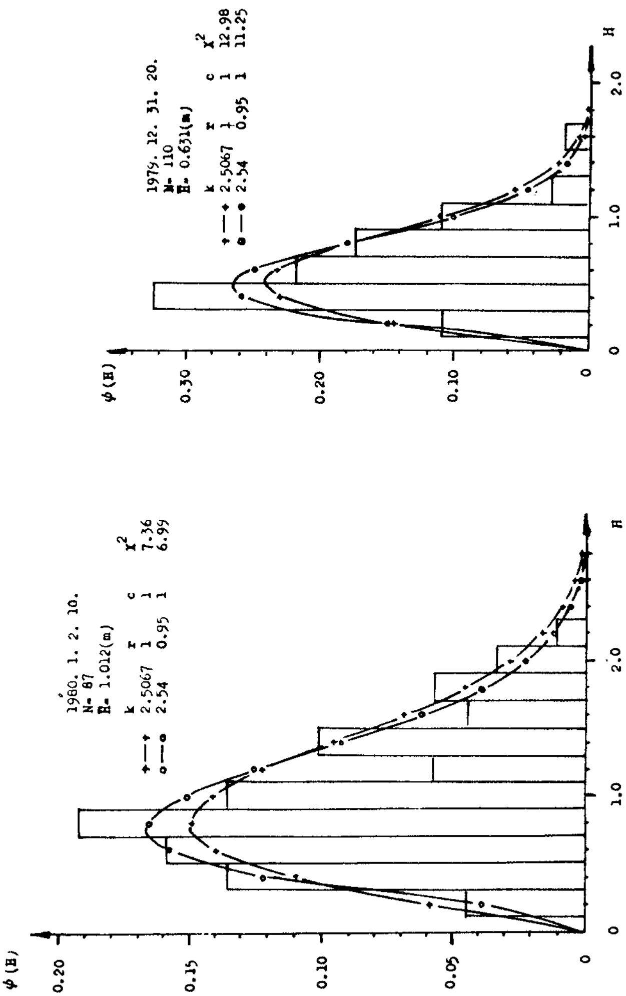

Fig.7 The measured data and the new distribution curve in comparison to Rayleigh's

Waves come from the East China Sea and Taiwan Strait are evaluated by Liang's (1976) element wave prediction model

$$E_1 = \frac{2}{\pi G}\int_r \int_\theta \overline{\xi}(r,\theta)\overline{U}^2(r,\theta)\cos^2[\beta(r,\theta)]\cdot \exp\left(-\frac{0.08\,r}{\overline{U}^2}\right) dr d\theta \tag{11}$$

E_1 : wave energy in the point of interest.
G : group velocity
$\overline{U}(r,\theta)$: wind velocity at 10 meters above sea level.
$\beta(r,\theta)$: the angle between the wind direction and the line.

Such definitions are illustrated in Fig.8.

In practical calculations, $\overline{\xi}$ can set to be constant. $H^2_{1/3,1}$ is to replace E_1 and $T_{1/3,1}$ can calculated from fetch and wind velocity.

$$H^2_{1/3,1} = \frac{2\overline{\xi}}{\pi G_{1/3}}\sum_r\sum_\theta \overline{U}^2(r,\theta)\cos^2[\beta(r,\theta)\exp\left(-\frac{0.08\,r}{\overline{U}^2}\right)\Delta r\Delta\theta \tag{12}$$

$$T_{1/3,1} = \frac{0.0552\cdot\pi\cdot\overline{U}}{g}\left(\frac{gF}{\overline{U}^2}\right)^{0.3269} \tag{13}$$

The local wave height $H_{1/3,2}$ and period $T_{1/3,2}$ are to be computed by following formula (Tang, 1970)

$$H_{1/3,2} = \frac{0.26\,U^2}{g}\tanh\left\{0.578\left(\frac{gD}{U^2}\right)^{3/4}\right\}\tanh\left\{\frac{0.01\left(\frac{gF}{U}\right)^{1/2}}{\tanh\left[0.578\left(\frac{gD}{U^2}\right)^{3/4}\right]}\right\} \tag{14}$$

$$T_{1/3,2} = \frac{2.8\,\pi U}{g}\tanh\left\{0.52\left(\frac{gD}{U^2}\right)^{3/8}\right\}\tanh\left\{\frac{0.0436\left(\frac{gF}{U^2}\right)^{1/3}}{\tanh\left[0.52\left(\frac{gD}{U^2}\right)^{3/8}\right]}\right\} \tag{15}$$

where U : local wind velocity
g : gravitational acceleration
D : water depth
F : fetch length

The wave height $H_{1/3}$ can be calculated by

$$H_{1/3} = \sqrt{r\,H^2_{1/3,1} + c\cdot r\cdot H^2_{1/3,2}} \tag{16}$$

According to Liang (1982) the period in this case can be calculated as follows

$$T_{1/3} = \frac{H^2_{1/3,1}\times T_{1/3,1} + H^2_{1/3,2}\times T_{1/3,2}}{H^2_{1/3,1} + H^2_{1/3,2}} \tag{17}$$

Examples of such prediction are shown in Fig.9

Finally, since

$$a = 4\,\sigma_1^2 = 4\left(\frac{H_{1/3,1}}{3.8}\right)^2 \tag{18}$$

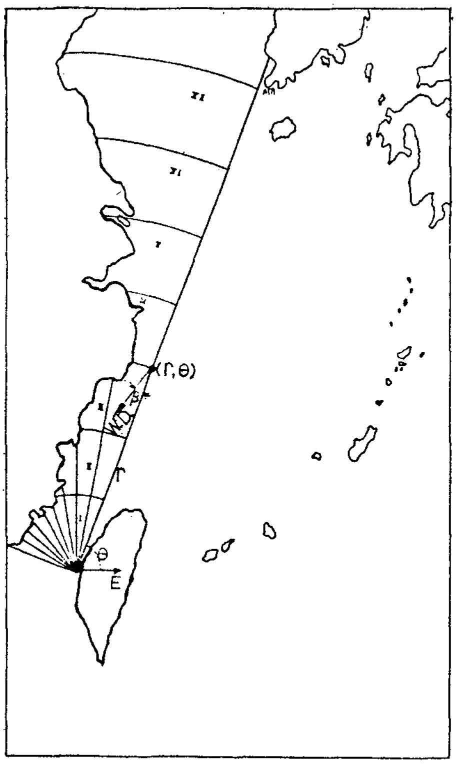

Fig.8 Illustration of element wave method

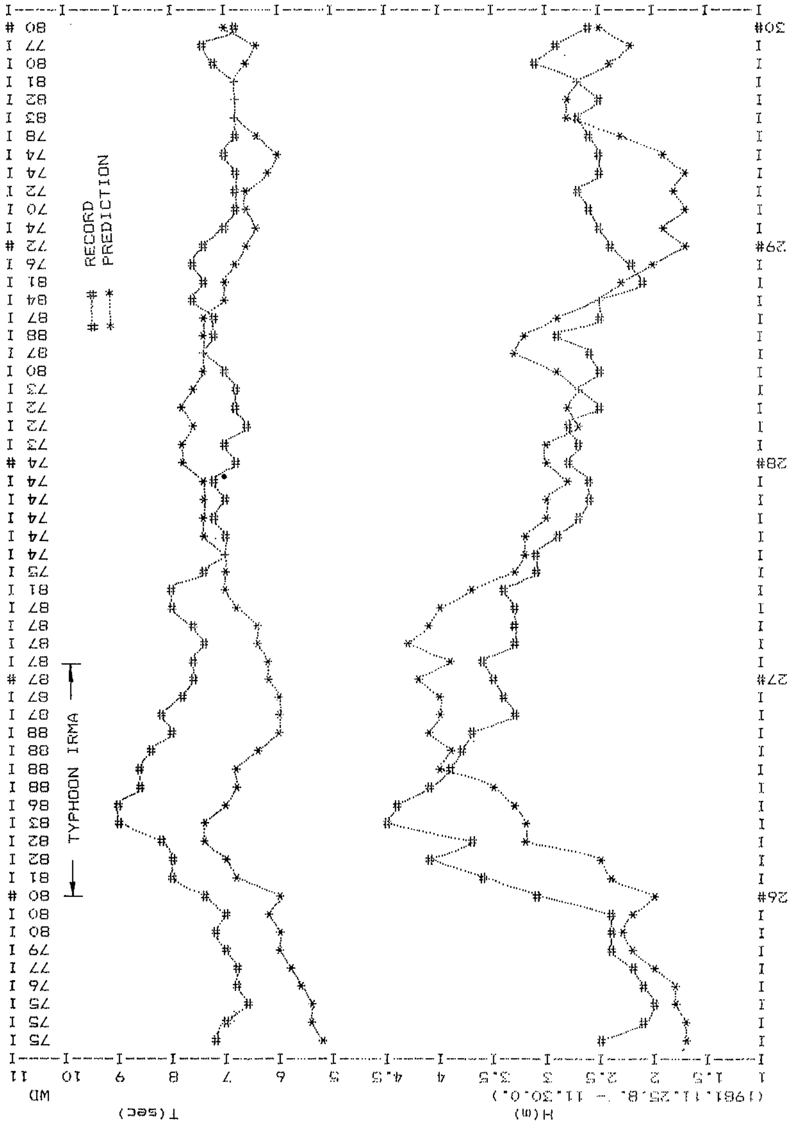

Fig.9 Results of wave prediction(1)

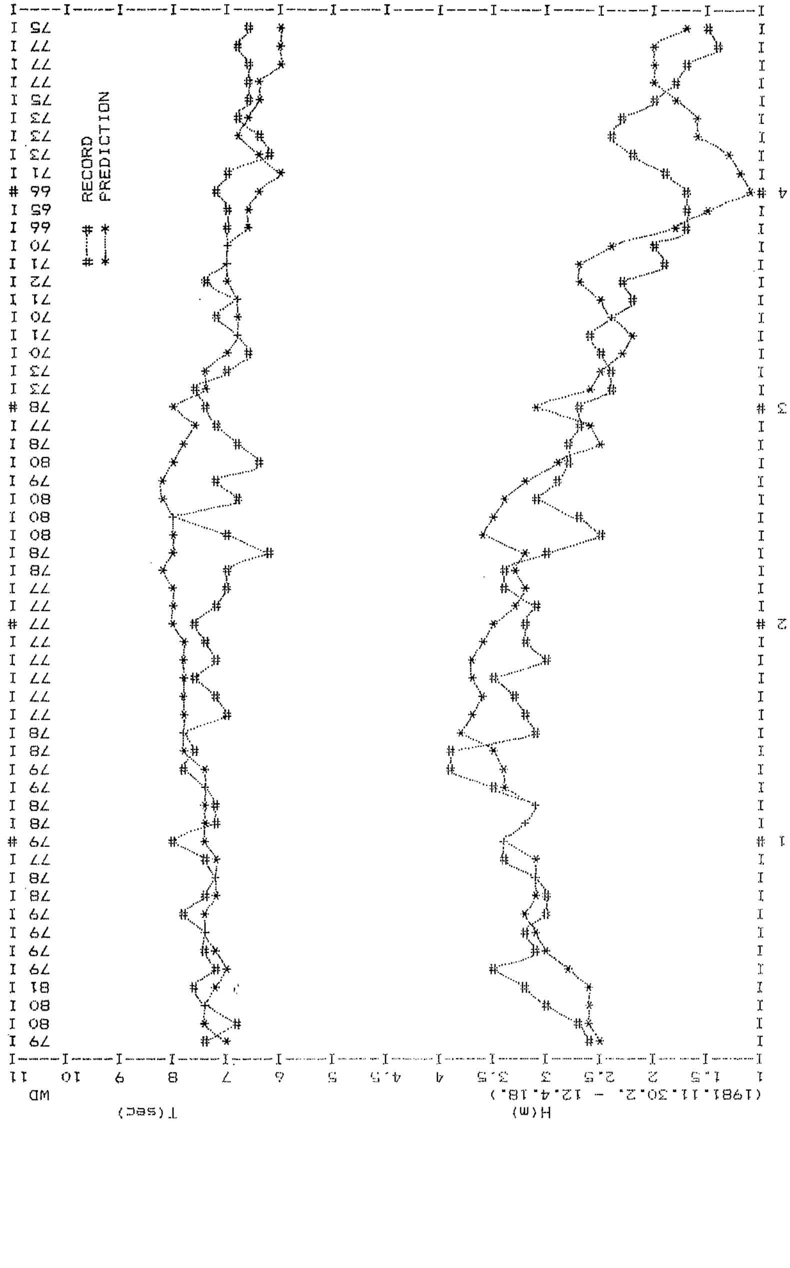

Fig.9 Results of wave prediction (2)

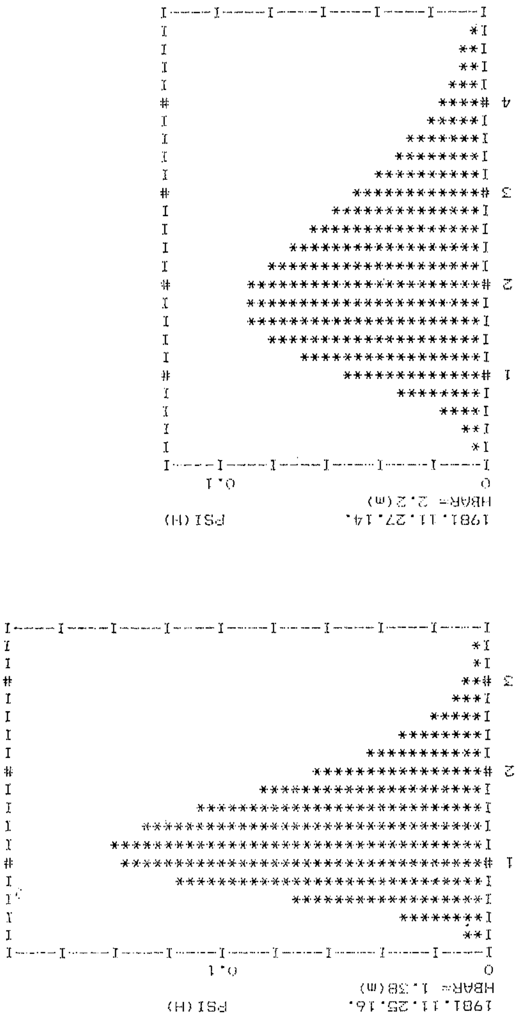

Fig. 10 Predicted wave height distribution

$$b = 4\ \sigma_2{}^2 = 4\ \left(\frac{H_{1/3,2}}{3.8}\right)^2 \tag{19}$$

after substitute into equation(10), the wave height distribution can be predicted as Fig.10.

Conclusion

The wave height distribution of the waves generated by monsoon is to be predicted by following procedure.

1. Calculate the waves by element wave prediction method from weather data.
2. Calculate local waves by empirical formulas.
3. Add wave energies linearly but the coefficient of energy remender is considered.
4. Calculate a and b by equations (18) and (19)
5. Substitute a , b to equation(10), the probability density equation is obtained.

These distribution model only can be applied in monsoon season, because the wind direction remains to be constant and the waves are moderate. Navigation and offshore engineering activities have to be done if the probability of waves exceeding certain height is negligible. However, during typhoon assailing, wave directions change frequently, navigation should be stop and no engineering work would take place. Neither the necessary to predict the probability of wave height nor the above model can be applied in such a case.

If the bathemetry and climate situation are similar to the offing of western coast of Taiwan, namely the sea bottom shoals gradually and the wind direction along the shore such as the southern part of German bight, this model might be adoptable.

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