CHAPTER FORTY FIVE

WAVE GROUP ANATOMY OF OCEAN WAVE SPECTRA

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ABSTRACT

This paper inquires into the questions of how wave groups are related to the wave spectrum, and how they differ in sea versus swell. Some results are presented in the form of a wave group model for sea spectra and for swell spectra. The models were developed from statistical analysis of a large number of wave records and apply to deep water only.

INTRODUCTION

A universal characteristic of sea (wind waves) and swell is the occurrence of sets of consecutive quasiperiodic larger waves called wave groups; these occur at intervals of approximately one to two minutes, vary in their energy and length, and are preceded and followed by generally nondescript low waves.

Wave groups have been recognized by coastal engineers in recent years as the cause of damage and destruction to vessels, offshore platforms, and shore structures due to high wave runs and to their periodicity. They constitute the principal components of the wave spectrum which the coastal engineer uses, yet information on their relationship to the spectrum has been quite incomplete. A further reason for interest in wave groups is their relationship to sea versus swell, or more specifically to initial wave steepness (H_o'/L_o in linear wave theory), particularly because the initial wave steepness controls many shallow-water wave variables, including breaker type, breaker height and depth, wave runup and overtopping, and beach profile response.

With these areas of practical concern in mind, we explore in this paper the following questions:

(a) How are wave groups related to the wave spectrum? (Or, given a spectrum what can be said about the

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characteristics of the wave groups occurring in the wave field?)

(b) How do wave groups differ in sea versus swell?

Some preliminary answers are given herein in the form of simple wave group models for sea and swell spectra. The models were constructed from statistical analyses of a large number of wave records for their wave group characteristics. The models apply to deep water only. Study of wave groups in shoal water is more complex due in part to differential shoaling of the various frequencies composing the spectrum and we have not probed this area in any depth. Such a study should logically follow and benefit from data developed for deep water.

In this paper a definition of wave group is given, wave group measures and associated wave record measures are specified, and some results from statistical analyses are presented leading to the models. The wave group definition was developed by Sedivy (1978), who also performed exploratory statistical analysis of wave records obtained from bottom pressure sensors in shallow water on the open California coast. Nelson (1980), applying this definition, probed wave group characteristics in deep water from a statistical analysis of records from a surface sensor on the California coast. It is Nelson's analyses that primarily provided the basis for constructing the deep water wave group models presented. The studies by Sedivy and Nelson were conducted at the Naval Postgraduate School in Monterey, California.

WAVE GROUP DEFINITION AND MEASURES

The analytical definition of the wave group presented by Sedivy (1978) is based on a comparison of the energy content in the group with that in the record, where energy is represented in terms of the statistical variance of the wave heights occurring in the group and in the record.

The procedure for identifying groups first involves computation of the variance of the whole record. The record, in digitized form, is then reanalyzed using a window of short duration over which a short-term variance is computed and plotted at the window midpoint. The window is moved along the wave record from beginning to end at one digital step at a time and produces a running short-term variance curve. The result of this procedure is illustrated in the upper diagram of Figure 1 for the record of mature swell shown. Those portions of the wave record where the running short-term variance values exceed the record variance indicate wave energy in excess of the record energy and identify possible wave groups.

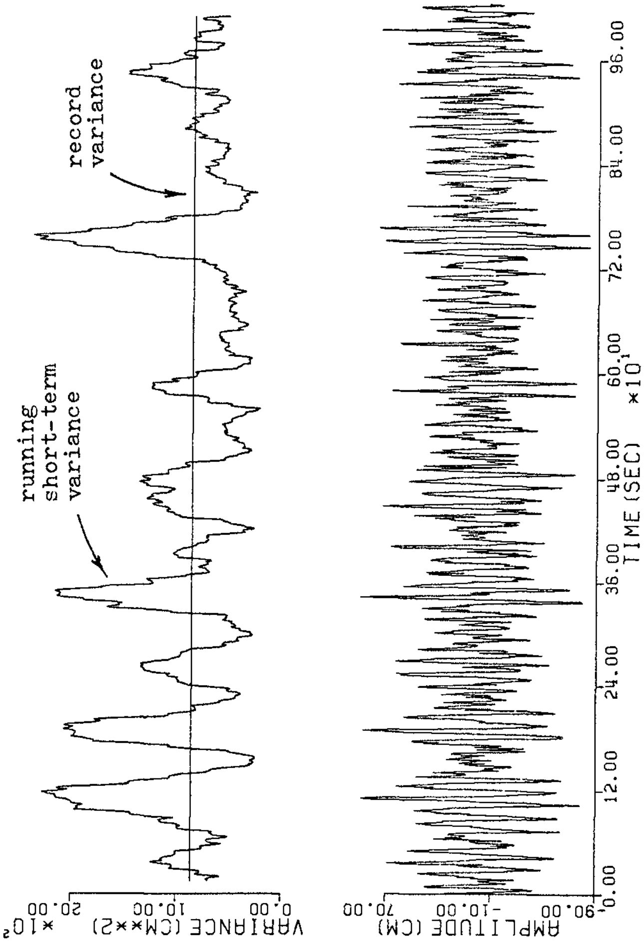


Figure 1: WAVE RECORD (lower) AND VARIANCE ANALYSIS (upper) (from Nelson, 1980)

In choosing a suitable window width W for computing the short-term variance, the finding by Thompson (1972) and by Smith (1974) that the average period of the waves in prominant wave groups approximates the spectral peak period T_R of the wave record indicates that W should be an integer multiple of T_R. Sedivy experimented with real and artificial wave records and settled on $W = 2T_R$ as optimum. Nelson (1980) conducted additional experiments by varying W over the range from 1/2 to 4 times T_R. He found that wave groups having high energy relative to the record energy were identified by all window widths and that the window width seldom affected the number of waves in the group. In low energy groups, however, as W was increased from 1/2 T_R to 4 T_R the number of groups identified decreased by approximately 50% and the number of waves per group increased somewhat. Nelson concurred with the choice of $W = 2\ T_R$ and concluded that it gives a short-term variance curve that is relatively smooth yet is reasonably sensitive to lower energy groups.

In addition to the requirement that the short-term variance must exceed the record variance, Sedivy specified three limitations on the definition of a wave group. First, the group must be composed of whole waves as defined by successive upcrossings of the mean water level by the wave-form. Since the short-term variance curve does not ordinarily cross the record variance "line" at an upcrossing, this specification was satisfied by placing the wave group boundaries at the first zero upcrossing met in moving away from the center of the wave group in either direction. Second, a wave group must contain a minimum of two waves; this limitation requires a minimum of order to the wave heights against a random wave field, and also rules out many occurrences where the short-term variance curve only just manages to rise above the record variance for a brief interval of time. Third, adjacent wave groups must be separated from one another by at least one-half the window width, otherwise they are treated as a single group; this condition was specified in order to prevent the possibility of including a given wave in two separate wave groups.

Following identification of the wave group using these procedures, various wave group characteristics may then be measured. Those wave group variables (designated with subscript G) that are dealt with in this paper, along with wave record measures (subscript R) and group-to-record parameters, are as follows:

<u>Wave group measures</u>

T_G Mean wave group period--average of the periods of the individual waves composing the group, computed from the group duration divided by the number of waves in the group; the waves composing a group tend to be periodic so that T_G approximates this periodicity.

V_G Mean wave group variance--computed by averaging the digital short-term variance values over the group duration.

N_G Number of waves per group--given by the number of intervals between upcrossings within the group.

Wave record measures

T_R Spectral peak period--reciprocal of the frequency of maximum energy density obtained from spectral analysis of the record (using the Fast Fourier Transform with four windows).

V_R Wave record variance--computed by digital time-series analysis (and checked by spectral analysis): the significant wave height H_R is related to the record variance by the relationship $H_R = 4\sqrt{V_R}$.

G_R Significant wave steepness--defined and described below; used in this study to categorize wave records by wave type, e.g., sea, young swell, mature swell, and old swell.

Group-to-record parameters--In order to compare wave group measures among records having different peak periods and energy levels, the group period and group variance were normalized by referencing them to the corresponding record measures as follows:

T_G/T_R Relative group period.

V_G/V_R Relative group variance.

The significant wave steepness of the record G_R was used as a measure of the wave type (Thompson and Reynolds, 1976). It is defined by analogy with the steepness H/L of monochromatic waves in deep water as given by the linear wave theory, where the linear theory wave height H and period T are replaced by the significant height of the record H_R and spectral peak period T_R, as follows:

$$\frac{H}{L} = \frac{H}{\frac{g}{2\pi}T^2} \equiv \frac{H_R}{\frac{g}{2\pi}T_R^2} = G_R$$

The wave type is then defined in terms of the significant wave steepness according to the table below:

Wave type	G_R	Significant height reduction (H_D/H_F)	Swell decay distance (naut. mi.)
Sea	1/12-1/40	1.00	0
Young Swell	1/40-1/100	1.00-0.50	0-250
Mature Swell	1/100-1/250	0.50-0.25	250-1600
Old Swell	< 1/250	< 0.25	> 1600

H_F = significant height in generating area
H_D = significant height at decay distance

The range of wave steepness for sea given in the table was evaluated from the Sverdrup-Munk-Bretschneider (SMB) wave generation graph (Bretschneider, 1958). The wave steepness boundaries between the types of swell were determined by specifying swell height reductions of 0.50 and 0.25 relative to the significant height of the waves in the fetch (column 3), and assuming wave generation in extratropical storms of average size. These height reduction factors, when entered into the SMB swell decay curves, yield both the swell steepness boundaries (column 2) and the approximate swell decay distances from the generating area (column 4). Since swell steepness diminishes with increasing travel time as well as travel distance from the generating area, the significant steepness may be considered a measure of the relative age of the swell (column 1), and provides the basis for designating swell as young, mature, or old.

WAVE RECORDS ANALYZED AND SOME RESULTS

The wave group models presented herein were constructed from the results obtained from statistical analysis of a large sample of ocean wave records. The wave records were recorded at an open ocean station off the central California coast that is exposed to an array of wave dimensions typical of the major oceans, ranging from locally generated wind waves to swell that has decayed over thousands of travel miles. The wave data were recorded by a Datawell Waverider accelerometer-type buoy and were digitized on magnetic tape at a sampling interval of one second. Each record analyzed was of 1,024 seconds duration, or approximately 17 minutes.

The sea surface sensor was positioned in a water depth of approximately 30 fathoms (55 meters). At that depth the linear theory shoaling coefficient K_S for 18-second waves, the longest spectral peak period dealt with, is 0.91. This value of the coefficient places the relative water depth d/L_o for waves of 18-second period toward the deep water boundary of the intermediate relative depth zone as conventionally defined. Accordingly, we consider the findings of this study to effectively apply to deep water for all wave periods dealt with.

A large number of wave records were screened from which 338 unimodal records were selected for analysis which met criteria requiring a single spectral peak and a relatively narrow bandwidth. These criteria were chosen to avoid complication in the selection of a short-term variance window used to identify wave groups and also to ensure as fully as possible that the waves in each record originated in a single generating area in order for the wave type to be determined.

The selected records are well distributed in their characteristics and cover spectral peak periods from 4 to 18 seconds, significant wave heights from 0.6 to 3.4 meters, and significant wave steepnesses from 1/15 (young sea) to 1/530 (very old swell). The 5,598 wave groups identified in these records were then statistically examined for their properties and relationships.

To convey an idea, in the limited space available, of the nature of the findings from these analyses, we now look at some of the results obtained by Nelson (1980). By way of example we will focus on the measure N_G, the number of waves composing a group, and examine its frequency of occurrence with respect to other factors as displayed in a series of graphs.

In Figure 2 the graph shows the cumulative frequency of occurrence, in percent, of N_G in different parts of the wave spectrum. The position of a wave group in the spectrum is determined by its relative group period T_G/T_R; values of the ratio near unity place the group close to the spectrum peak whereas groups having ratios <<1 and >>1 lie, respectively, well into the high frequency tail and the low frequency tail. The six curves in the figure are seen to form a tight-bundle, and we draw the conclusion from this and other data that the percentage distribution of N_G is fundamentally the same in the tails of the spectrum as at the peak, i.e., the spectrum can be sliced at any wave frequency and the percentage distribution of N_G among the wave groups occurring there can be expected, for a large group population, to be the same. The histograms for these six sets of data (not shown) are Rayleigh-like in form.

The distribution of N_G with respect to the relative energy content of wave groups V_G/V_R is shown in the cumulative distribution graph in Figure 3. As may be expected from the wave group definition, groups with the lowest energy content have an average group variance approximately equal to the record variance ($V_G \doteq V_R$). It is evident from this

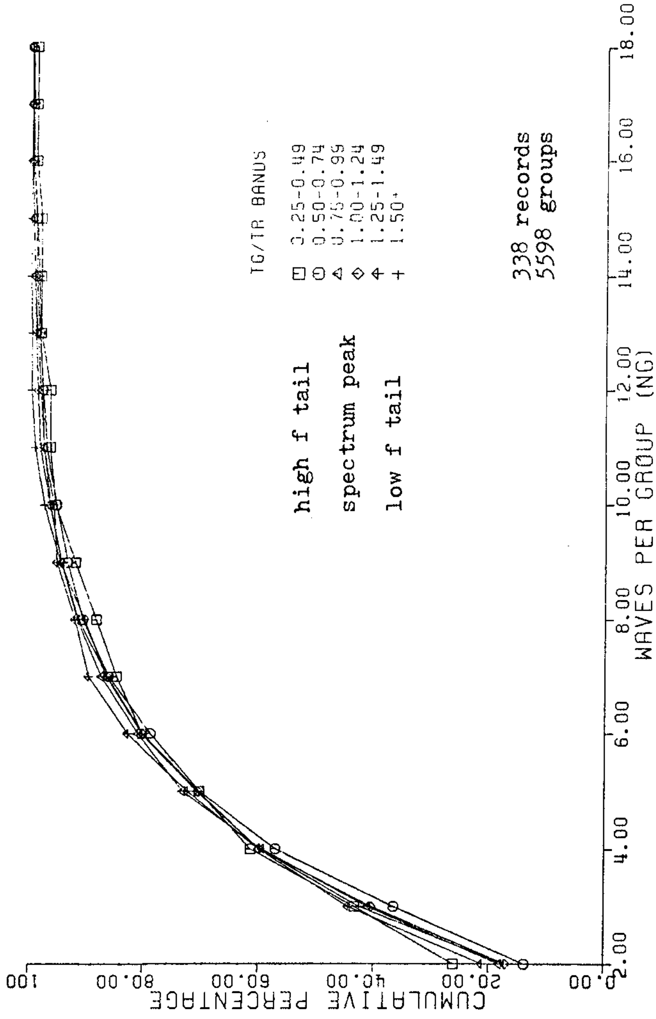


Figure 2: CUMULATIVE DISTRIBUTION OF N_G FOR T_G/T_R BANDS (from Nelson, 1980)

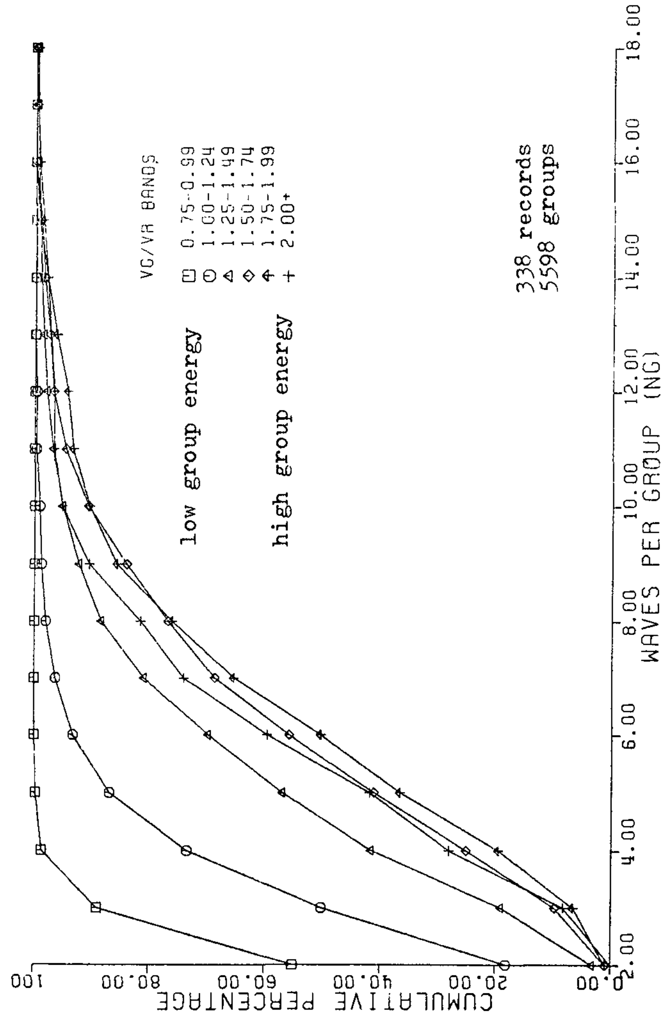


Figure 3: CUMULATIVE DISTRIBUTION OF N_G FOR V_G/V_R BANDS (from Nelson, 1980)

figure that N_G is sensitive to the relative energy content of wave groups. The lowest energy groups most frequently contain 2 waves, the minimum taken to constitute a group, and seldom contain more than 4 waves. The highest energy groups, on the other hand, rarely contain 2 waves, most frequently have 6 waves, and exceed 10 waves in 10% of occurrences. Histograms for the six energy levels (not shown) appear as nested curves having a Rayleigh-like form in which the distributions becomes less peaked and extend over a wider range of N_G as the group energy level increases.

The frequency of occurrence of N_G with respect to wave type, represented by wave steepness intervals, is illustrated in the cumulative distributions shown in Figure 4. It may be concluded that the older the swell the larger tends to be the number of waves in a group at any given probability level. The associated histograms (not shown) also have a Rayleigh-like form.

WAVE GROUP MODELS

Sea Model

Figure 5 presents a model of the relationship between wave groups and the energy density spectrum for the case of waves under generation by the wind in deep water.

The reader's attention is directed to the table in the lower part of the figure which describes the three areas of the spectrum. The table indicates that most wave groups in a large population are concentrated about the frequency of maximum energy density of the spectrum, i.e., their relative group periods T_G/T_R lie at or close to 1.0. In moving away from the peak and toward the tails of the spectrum, group periods deviate increasingly from the spectral peak period and the number of groups falls rapidly. Groups are rare at values of $T_G/T_R < 0.7$ and > 2.0 (Figure 6).

Wave groups falling in the tails of the spectrum not only occur infrequently but they also have a low relative energy level in which the average group variance is close to the variance of the record. As the spectral peak is approached, low energy groups increase in occurrence and are joined by groups containing increasing amounts of energy. At and very close to the peak of the spectrum are found those groups with the highest energy levels but also the greatest occurrence of groups at all energy levels.

With regard to the number of waves per group, the percentage distribution of N_G is the same in all parts of the spectrum (noted in Figure 2). Thus, the percentage frequency of occurrence of groups having, for example, ten or more waves is the same in the spectrum tails as at the peak, but groups actually occur much less frequently in the tails.

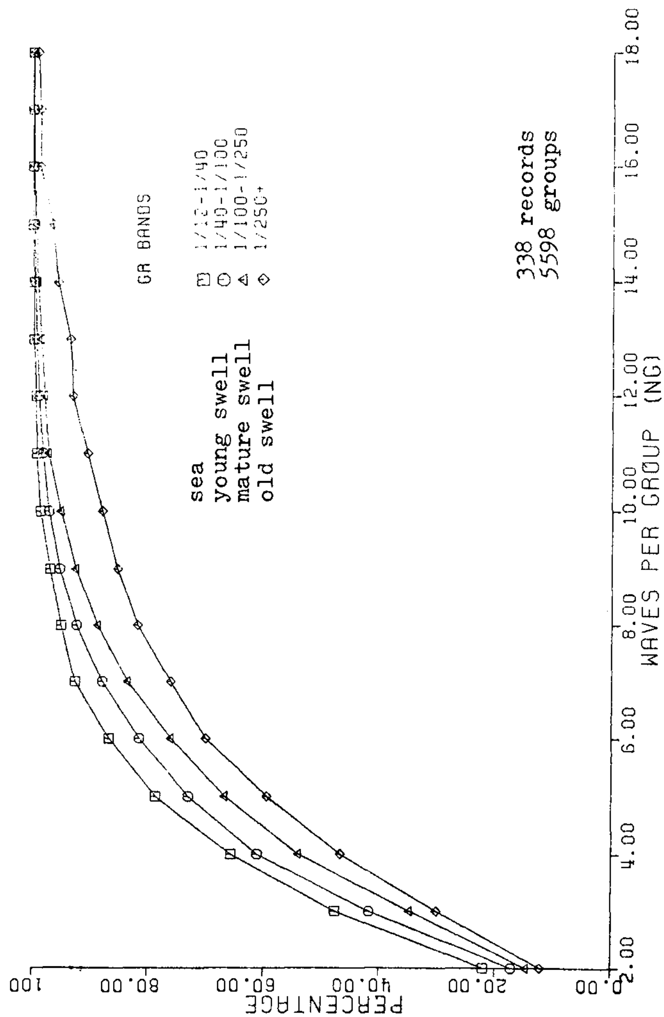
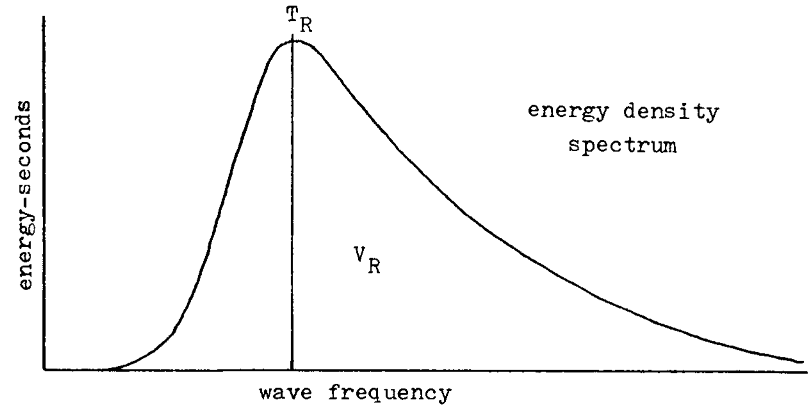


Figure 4: CUMULATIVE DISTRIBUTION OF N_G FOR G_R BANDS
(from Nelson, 1980)

<u>Sea and Swell</u>
As relative group energy (V_G/V_R) increases:

$T_G \longrightarrow T_R$
N_G increases



	Low f tail	Spectrum peak	High f tail	
Sea				
Distribution of groups:	few	max	few	
Group period, T_G:	$> T_R$	$\doteq T_R$	$< T_R$	
Group energy, V_G:	$\doteq V_R$	$\doteq V_R$ to $>> V_R$	$\doteq V_R$	
Waves per group, N_G:	Same percentage distribution at all frequencies (Fig. 2).			
Swell				
Distribution of groups:	few	many	max	few
Group period, T_G:	$> T_R$	$\doteq T_R$	$< T_R$	$<< T_R$
Group energy, V_G:	$\doteq V_R$	$\doteq V_R$ to $>> V_R$	$\doteq V_R$ to $> V_R$	$\doteq V_R$
Waves per group, N_G:	Same percentage distribution at all frequencies (Fig. 2).			

Figure 5: WAVE GROUP MODELS FOR SEA AND SWELL SPECTRA for unimodal spectra in deep water

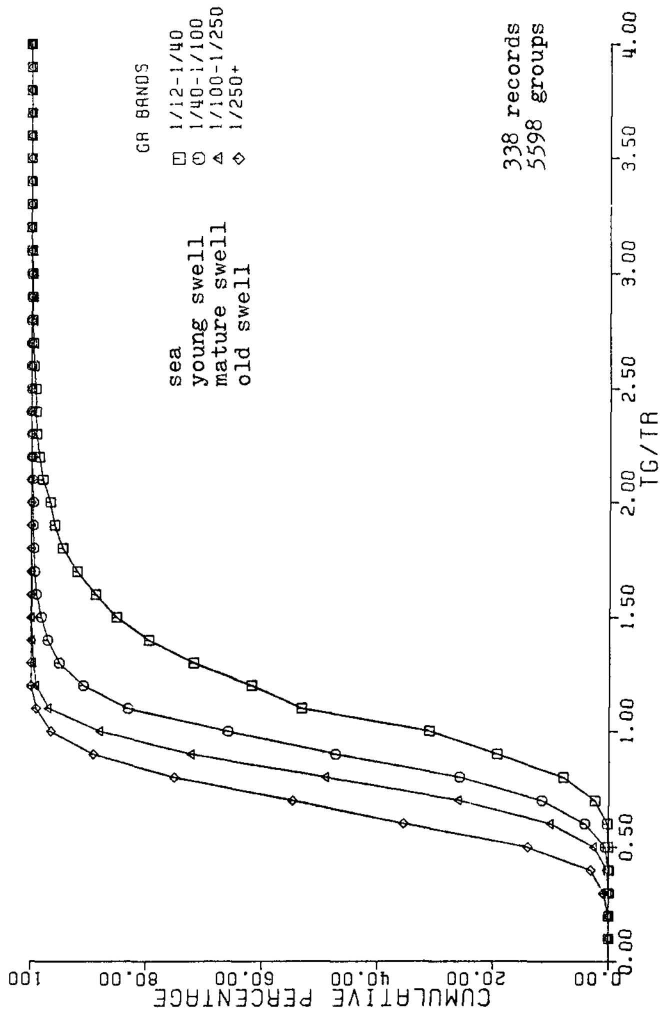


Figure 6: CUMULATIVE DISTRIBUTION OF T_G/T_R FOR G_R BANDS (from Nelson, 1980)

Directing attention to the statement to the left of the diagram in Figure 5, it may be noted further that in wave groups containing increasing amounts of energy relative to the wave record, the group period tends to approach the spectral peak period and the number of waves in the group tends to increase.

Swell Model

A general wave group model for swell in deep water is also presented in Figure 5. Swell is similar to sea in that the highest energy groups are most closely concentrated about the spectral peak and, indeed, control the position of the peak. However, swell contains many lower energy groups that differentially trail well into the high frequency tail of the spectrum. As a result, the most frequently occurring group period in a large population shifts away from the spectral peak toward shorter periods increasingly with increasing swell age.

The latter situation is illustrated in the cumulative distributions in Figure 6. The three swell curves represent normal distributions with similar standard deviations, but are displaced successively toward lower values of T_G/T_R with increasing swell age. For old swell, the median of the distribution is located at $T_G/T_R \doteq 0.7$. This means that in old swell having a spectral peak period of 14 seconds the most frequently expected group period should be about 10 seconds, although these are not the groups having the highest energy.

Swell groups are commonly observed to contain short waves due to apparent phase changes and other effects, and these bias the group period. Accordingly, when wave groups are filtered to remove anomalously short waves, most group periods increase substantially with the result that the median T_G/T_R value in any large population is shifted back toward unity for all swell types. A similar effect can be achieved by raising the record variance level that the running short-term variance must exceed in the definition of the wave group. This redefinition would have the additional effect of reducing the number of groups occurring in a given recording period and the number of waves per group.

Referring once again to Figure 5, the frequency of occurrence of the number of waves per group in swell is similar to that in sea in that the percentage distribution of N_G appears to be the same in all parts of the spectrum, and also in that N_G tends to increase as relative energy in groups increases. However, as the swell age increases the number of waves per group increases for any given percentage frequency of occurrence.

Further illustration, from a different perspective, of the relationship of swell groups to the wave spectrum is given in Figure 7 for a mature swell record. For each wave in the record the square of the height is plotted against the reciprocal of its wave period. Since wave energy is proportional to H^2, the graph is effectively a plot of energy versus frequency for every wave. Waves belonging to wave groups are coded according to the relative energy of the group; all waves occurring in the intervals between wave groups are designated "interval" waves and are separately identified. The curve shown in the figure is the wave spectrum (the energy scales of the H^2 values and the spectrum differ).

The manner in which the individual waves contribute to the wave spectrum is readily apparent in the figure. All waves having high and moderate energy lie close about the spectral peak frequency, and all are members of wave groups. There is also a number of low energy wave group members and these are clustered in the frequencies of highest energy density, although a few lie well into the high frequency tail. The interval waves, all of low energy, are most densely concentrated in frequencies above the spectral peak frequency, but extend well out along the high frequency tail in decreasing number and with diminishing range of energy. The occurrence of individual waves on the low frequency end is cut off sharply in this record.

The close association of individual waves with the wave spectrum shown can also be readily demonstrated by cumulating the H^2 values with frequency and constructing a histogram from this cumulative distribution having the same frequency interval as the wave spectrum. This H^2 histogram, if plotted in Figure 7, would resemble the wave spectrum closely (not shown due to clutter).

CONCLUSION

We hope that the wave group models for sea and swell presented herein will give practicing coastal engineers a better feel for wave spectra and the nature of the waves composing them, and that these models might also bring to the attention of theoreticians some areas that can benefit from application of their talents.

We recognize that the models are preliminary and believe that a great deal more is yet to be learned about wave groups from purely statistical analysis. Further work that profitably can be pursued includes: (1) Additional investigation of the relationships between swell groups and the wave spectrum in deep water, (2) examination of group-to-group relationships in deep water and how they are related to sea and swell, and (3) inquiry into all of the above relationships in shallow water.

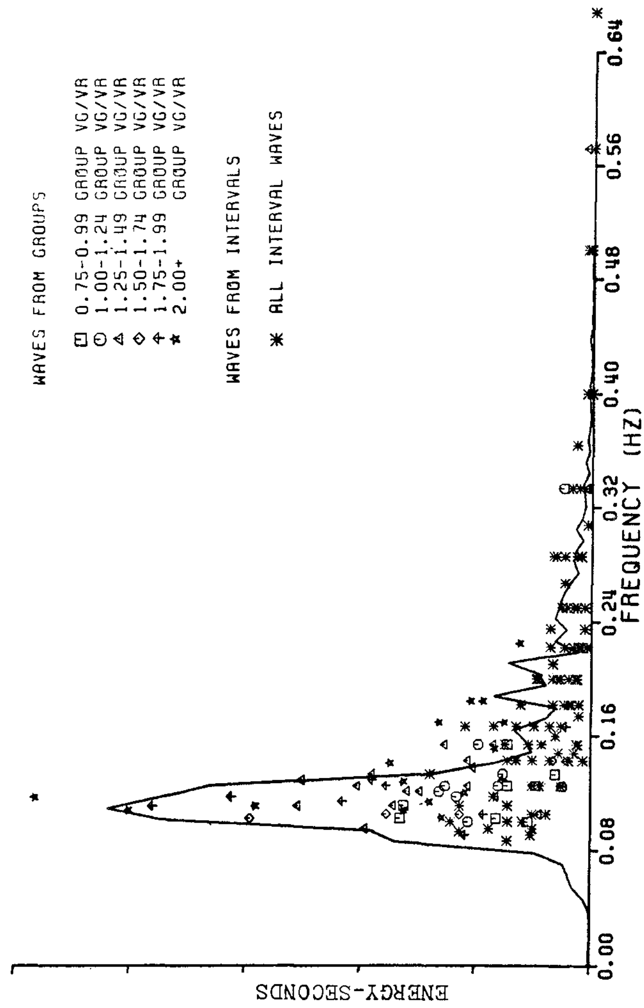


Figure 7: WAVE SPECTRUM WITH PLOT OF H^2 VS. f FOR INDIVIDUAL WAVES (modified from Nelson, 1980)

We gratefully acknowledge receipt from M.S. Longuet-Higgins, following the presentation of our paper at this conference, of a copy of his theoretical paper titled "Statistical properties of wave groups in a random sea state" scheduled for publication in the Philosophical Transactions of the Royal Society of London.

REFERENCES

Bretschneider, C.L., 1958. Revisions in wave forecasting: Deep and shallow water, Proc. 6th Int. Conf. on Coastal Eng., pp. 30-67.

Nelson, A.R., 1980. Statistical properties of ocean wave groups. Naval Postgraduate School, M.S. thesis, 133 pp.

Sedivy, D.G., 1978. Ocean wave group analysis. Naval Postgraduate School, M.S. thesis, 90 pp.

Smith, R.C., 1974. Ocean swell wave groups from wave record analysis. Naval Postgraduate School, 77 pp.

Thompson, W.C., 1972. Period by the wave-group method. Proc. 13th Int. Conf. on Coastal Eng., pp. 197-214.

Thompson, W.C., and F.M. Reynolds, 1976. Ocean wave statistics from FNWC spectral analysis. Proc. 15th Int. Conf. on Coastal Eng., pp. 238-257.