CHAPTER SIXTY SEVEN

THE INTERACTION OF SMALL AND FINITE AMPLITUDE LONG WAVES AND CURRENTS

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INTRODUCTION

The interaction of waves and currents is important for many engineering problems. For example, when considering forces on marine structures, the velocity and acceleration field must be defined, and thus the manner in which a current interacts with small and finite amplitude waves must be understood. When the current is large and oblique to the waves, the direction of the force on an offshore structure may change significantly with depth introducing a torsional moment. Wave refraction and the concomitant attenuation or amplification of waves are also affected by offshore currents. An example is the effect on incident waves of offshore currents induced by the discharge of cooling water from coastal-sited power plants. This current can modify the direction and magnitude of approaching waves, and by these changes the breaking waves at the shore and the nearshore sediment transport associated with these waves may be changed.

A number of theoretical studies have been conducted on various aspects of wave-current interactions; see Peregrine (1976). One theoretical study, Thomas (1981), will be used in this investigation. Careful experiments in this area are limited; several are: Iwagaki and Asano (1980), Sarpkaya (1957), and Thomas (1981). Each of these has given attention to certain aspects of small amplitude wave-current interactions. The experiments are difficult to conduct because of the problems inherent in introducing waves into a flume with a steady-uniform current or conversely a current into a wave tank with permanent waves. Certain features of these experimental problems can be seen through the following two examples. If a plunger-wave machine were used and located at one end of a flume in which a steady current is flowing, although the waves would be developing as they interact with the current, the previously steady current would be changed to an unsteady one by the periodic blockage of the flow by the plunger. If the waves are generated at one end of the tank and allowed to develop, and a current is introduced from the bottom of the tank, this current must expand to the full depth of the flow; hence, the waves propagate on a developing current. Therefore, comparisons to theory are, to some extent, difficult to realize, because the theory generally assumes wave-current interactions when each is fully developed.

This study basically had two objectives. The major objective was

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whether simple linear superposition could be used to describe wave-current interactions with weakly nonlinear finite amplitude waves. In essence, the question raised is: could the water particle velocities measured under a wave without a current be added to those measured with the current alone to yield the total velocity similar to that which was determined experimentally for the combined wave and current. The second was to investigate, using a simple means of introducing a current into the wave tank and withdrawing it, the effect of the configuration of the current inlet/outlet arrangement on the water particle velocities associated with the wave during the interval of wave development. For the possible currents generated in this study, conclusions could be drawn relative to these two questions.

This study is primarily experimental. The numerical method proposed by Thomas (1981) has been applied to the periodic wave measurements; as mentioned, this analysis will not be described herein, and the interested reader is referred to that publication for a discussion of the method. In this paper attention will be devoted to the experimental results and the question of superposition; results obtained using the numerical method proposed by Thomas (1981) will be included only as an adjunct to these.

EXPERIMENTAL EQUIPMENT AND PROCEDURES

Wave generation and measurement

Experiments have been conducted in a 40 m tilting tank which is 0.60 m deep and 1.10 m wide. The tank has glass sidewalls throughout with a stainless steel bottom which is flat to within approximately ± 0.1 mm. Circular rails attached to the top of the walls of the tank form precision tracks for an instrument carriage to which a wave gage can be attached. The wave machine used in this study is a vertical bulkhead generator located at one end of the tank and is driven by an electro-hydraulic system. The servo system which controls its motion consists of a servo controller, a function generator, and a feedback device. The function generator is a nonlinear function synthesizer with a microprocessor which stores a list of binary numbers corresponding to an arbitrary signal. The maximum voltage amplitude and the time are divided into 4096 parts; using these data a smooth signal can be obtained.

The motion of the wave machine was programmed for these experiments using the method described by Goring and Raichlen (1980) for the generation of long-nonlinear waves with a bulkhead wave generator. The boundary condition on the face of the plate for this technique is that the wave propagates away from the plate as it moves. Thus, the usual assumption of a negligible plate motion relative to its mean position is not necessary.

The variation of the water surface profiles with time were obtained using a parallel wire, resistance wave gage composed of 0.25 cm diameter stainless steel wire spaced 0.4 cm apart.

Inlet and outlet structures

The inlet and outlet structures were each constructed of lucite and were essentially boxes resting on the bottom of the tank extending across the width of the tank (110 cm), 61 cm in the direction of wave propagation, and 13.75 cm high. Inflow (or outflow) was brought into (or taken from) the flume by means of a 10.2 cm dia. pipe connected to one end of the inlet/outlet box. Straight vanes were used in the box to divide the front of the box and the pipe into six equal areas. Even with this attempt to distribute the flow uniformly across the width of the flume, flow separation occurred within the box leading to non-uniform flow conditions at the box exit. Nevertheless, as it will be shown, the velocity became well distributed through the depth of the tank at the measuring location.

Two pump-piping arrangements were used for these experiments. In the case of periodic waves the discharge was 0.02 cubic meters per second using a pump-piping system that permitted the flow to be reversed. In the experiment with solitary waves, where a larger velocity was desired, a discharge of 0.028 cubic meters per second was realized, but only adverse flows were possible, i.e., the box nearest the wave machine always collected the flow.

In the case of periodic waves the water depth was kept at 30.2 cm for all experiments and for the solitary waves the depth was maintained at 17.42 cm. For the latter, the smaller depth was necessary so that the mean current velocity would be a significant percentage of the water particle velocities in the wave. Considering the physical arrangement the water depth over the boxes was small for the solitary wave case and wave breaking occurred over the box; this will be discussed.

Measurement of water particle velocities

A two-dimensional laser-Doppler velocimeter (LDV) employing the reference beam technique was used to measure the water particle velocities at a location 21.6 m from the wave generator for experiments with periodic waves and 23.6 m from the wave generator for the solitary wave investigation. (These two locations are near the middle of the wave tank.) Two reference beams and a scattering beam were generated using a 5 mW helium-neon laser, and these were optically focused near the center of the wave tank. To provide a means for defining the direction of the velocity components the LDV was equipped with a frequency shifter consisting of two Bragg cells (which operate at a nominal frequency of about 40 MHz) and a frequency synthesizer with phase-locked loops. The frequency shift between the reference beams and the scattering beam created by the Bragg cells was 86.92 KHz. The laser and its associated optics were mounted to a carriage which was isolated from the wave tank. This permitted the laser to be moved vertically through the depth and along the wave tank in the direction of wave propagation.

PRESENTATION AND DISCUSSION OF RESULTS

In this section the results obtained using the LDV will be presented for both periodic and solitary waves. (The periodic waves generated were weakly nonlinear cnoidal waves.) For periodic waves the experimental results are compared to the velocities obtained from the linear superposition of independently measured velocities and to the numerical theory of Thomas (1981). The comparison of the results of experiments with solitary waves will be made only to the results obtained by the linear superposition of measurements of the wave alone and the current alone.

Cnoidal waves

Velocity profile for steady current alone

Measurements of the current velocity were made at the various depths at the same location where the wave measurements would be taken; 21.6 m (about 71.5 depths) from the wave machine. The velocity distribution is shown in Figure 1 where the abscissa is the velocity and the ordinate is the relative distance from the bottom. A universal velocity distribution is fitted to the data, for both favorable and adverse currents, with the assumption that the von Karman constant was 0.4. Since the flume was horizontal, the flow must be nonuniform and an independent estimate of boundary shear stress is not possible. Therefore, the coefficient B and the shear stress are obtained from the fitted line in the semi-logarithmic plot, the mean velocity U is obtained from the spacial integration of the velocity distribution. It is recalled that the coefficients in the velocity distribution expression normally used are A = 5.75 and B = 2.5; hence, the velocity distributions measured are in fair agreement with the usual logarithmic profile. (However, the boundary shear stress obtained in this manner gives small values of the Darcy-Weisbach friction factor so that full interpretation of the profile is difficult.)

SYMBOL	CURRENT	U (cm/sec)	$\sqrt{\tau_0/\rho}$ cm/sec	A	B
—○	FAVOR.	6.16	0.201	5.75	3.02
---●	ADVER.	6.26	0.146	5.75	4.34

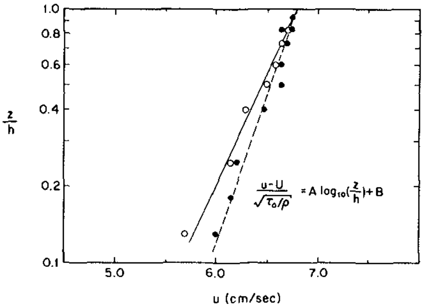

Figure 1 Velocity Distribution of Current Alone for Experiments with Periodic Waves.

Velocities for the wave without a current

Two sets of experiments were conducted to investigate the kinematic properties of the cnoidal waves, which were to be used in the current-wave experiments, propagating in the tank without the current. It was realized early in the experimental program that the inflow box near the wave machine could create an effect on the wave even though the velocity and wave measurements were made nearly 72 depths away.

Time histories of the water surface variation, the horizontal velocity, and the vertical velocity at mid-depth for the case without the inflow box are presented in Figure 2. (The cnoidal waves generated were weakly nonlinear with a wave period of 3.015 seconds and the ratio of wave height to depth of about 0.04.) It is noted that although the wave is not highly nonlinear, even with the large wave length to depth ratio (about 17), high frequency components are not apparent in either the water surface-time history (η vs t) or the time history of the horizontal velocity (u vs t). This primarily is due to the careful wave generation procedure used. For both the water surface and the horizontal velocity, the second, third, and fourth waves are similar and well formed. The theoretical cnoidal wave profile is shown in the upper

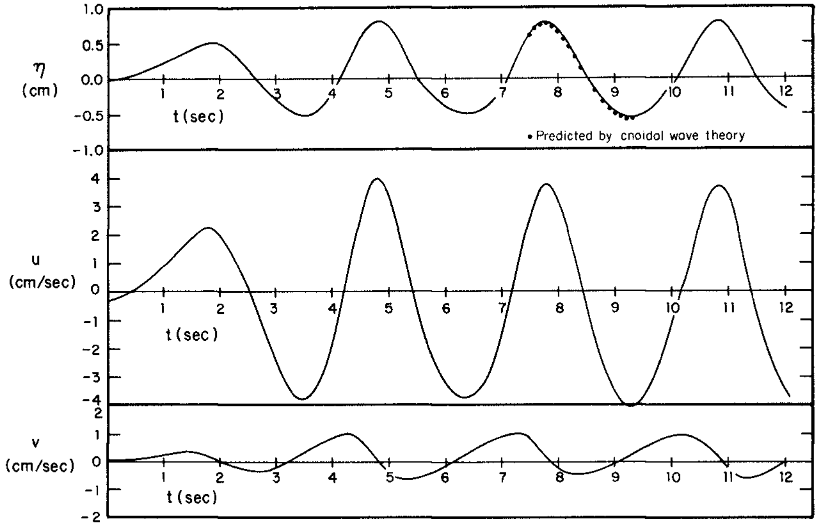

Figure 2 Water Surface, Horizontal, and Vertical Velocity-Time Histories at z/h = 0.5 for No Current (Without inflow box in place.

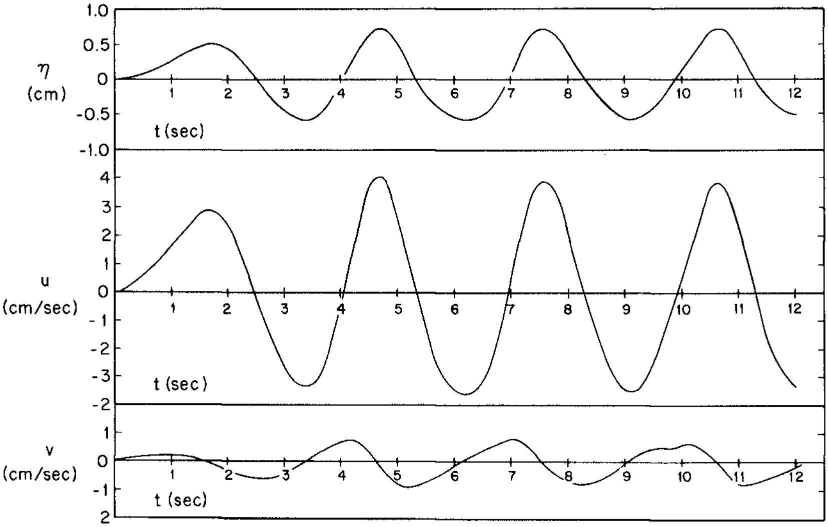

Figure 3 Water Surface, Horizontal, and Vertical Velocity-Time Histories at z/h = 0.5 for No Current (With inflow box in place).

portion of Figure 2, and it agrees reasonably well with the experiments. The magnitude of the vertical velocity, v, is significantly less than the horizontal velocity, i.e., of the order of about 20%, and its distribution appears skewed. The reason for this is not fully understood.

Similar time histories are presented in Figure 3 at mid-depth for the wave propagating over the inflow box but without a current. The primary effect is the vertical and the horizontal velocities are reduced compared to the corresponding conditions without the box. This may be an effect which is associated with the wave propagating over the box with only about five wave lengths to the measuring station.

In Figure 4 the variation of the maximum water particle velocities with relative depth measured under the crest of the wave is presented for the wave propagating in the bare tank and for the wave propagating in the tank with the inflow box; in the latter no current is imposed. The most obvious effect of the box occurs at elevations which are between the bottom and mid-depth. In the case with the box the velocities tend to increase as the bottom is approached whereas without the box the velocities remain relatively constant through the depth. Thus, the box seems to have an effect on the wave with regard to the depthwise distribution of its kinematic properties, perhaps more so than its effect on the wave profile. The data are compared to predictions from cnoidal theory and from small amplitude wave theory, and it appears that within the limits of experimental accuracy, the experiments without the box agree with the results of the small amplitude wave theory somewhat better than with those from the cnoidal theory.

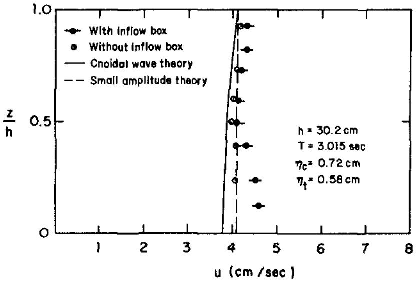

Figure 4 Depthwise Distribution of Maximum Horizontal Velocity Under the Second Crest of a Cnoidal Wave Train.

Waves with a favorable current

In this section experimental results obtained with the waves and the current traveling in the same direction (a favorable current) are presented. Similar to previous figures, the time histories of the water surface and the horizontal and vertical velocities at mid-depth are presented first in Figure 5. A comparison of Figure 5 and Figure 3 show that there is, at most, a difference of about 3% to 4% in the wave amplitude and the wave length between the second and third wave crest for the cases without and with the current. The crest height is reduced and the wave lengths are somewhat longer, as expected. Taken in totality, however, the effect of the current on the wave profile indeed is quite small and within the range of experimental error; results such as those obtained by Jonsson et al. (1970) indicate this also.

The depthwise distributions of the velocities are shown in Figure

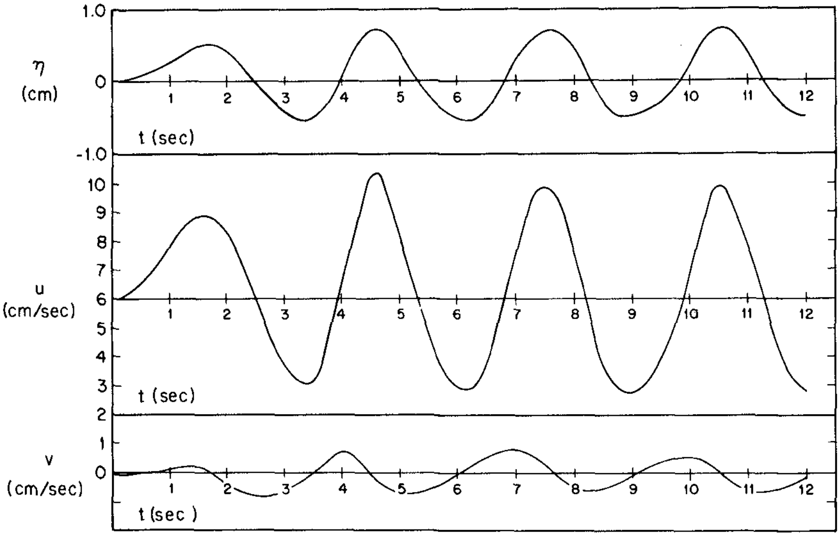

Figure 5 Water Surface, Horizontal, and Vertical Velocity at z/h = 0.5 for Wave and Current in Same Direction (Favorable Current).

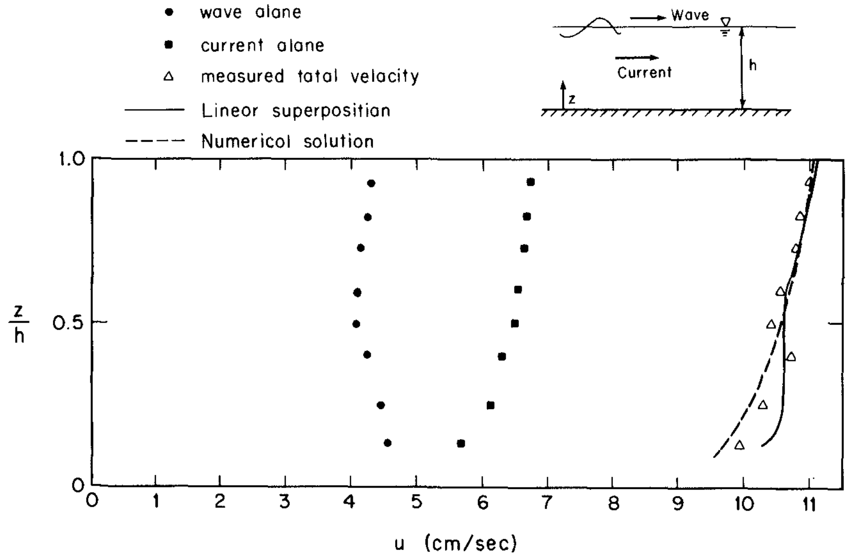

Figure 6 Distribution of the Horizontal Velocity for Wave and Current in Same Direction (Favorable Current).

6; the data are presented for the wave alone, the current alone, and the measured total velocity. The solid curve is obtained from superposing the measured data corresponding to the wave alone and that from the current alone. The dashed curve is the total water particle velocity predicted by the numerical solution proposed by Thomas (1981) which is based on the water particle velocity under the wave obtained from small amplitude wave theory. Above mid-depth the data agree well with each approach, and below it is difficult to establish the better agreement.

Waves with adverse current

For these experiments the direction of the current was reversed and is denoted as adverse. Data similar to those obtained for favorable currents are presented in Figure 7 showing the time histories of the water surface and the horizontal and vertical velocities at mid-depth. A close examination of the wave record indicates that the waves are somewhat steeper than they were for waves without the current or for waves traveling on a favorable current (see Figures 3 and 5, respectively). Perhaps more apparent are changes in both the horizontal velocity and the vertical velocity in terms of the steepness of the time history. The greatest differences between the kinematics of the waves propagating on a favorable and adverse current appear to be associated with the variation with time of the vertical velocities. This might be expected due to the small magnitude of the vertical velocity compared to the horizontal velocity for long waves, and hence, its sensitivity to small changes in the wave.

The measured velocity distributions are presented in Figure 8 for the wave alone, the current alone, and the total velocity. As for the case of the favorable current, linear superposition has been used based on the measured values along with the numerical solution proposed by Thomas (1981). In this case, the numerical solution appears to agree better with the data than the results of simple linear superposition. However, the differences are not large enough so that general conclusions can be drawn.

Solitary waves

In this section results obtained from experiments with solitary waves under the influence of adverse currents will be presented and discussed. The wave height was determined by the conditions that on the one hand a limited pump discharge was available for the solitary wave-current interaction investigation while on the other wave particle velocities were desired which would be of the same order of magnitude as the available current. The maximum average current velocity possible was 14.9 cm/sec and, to satisfy these conditions, a wave with a relative height (height/depth) of about 0.3 was used. Due to pump and piping restrictions only an adverse current could be generated. The velocity distribution for this current is presented in Figure 9 where the ordinate is the relative distance from the bottom and the abscissa is the velocity. Each data point shown is the result of averaging 10 different velocity samples at that elevation. Since each sample has a duration of one minute, in essence the data point is a temporal average of about 10 minutes of record. The averaging was necessary because of low frequency velocity fluctuations which apparently were caused by the outlet/inlet configuration. The inferred shear stress yields a friction

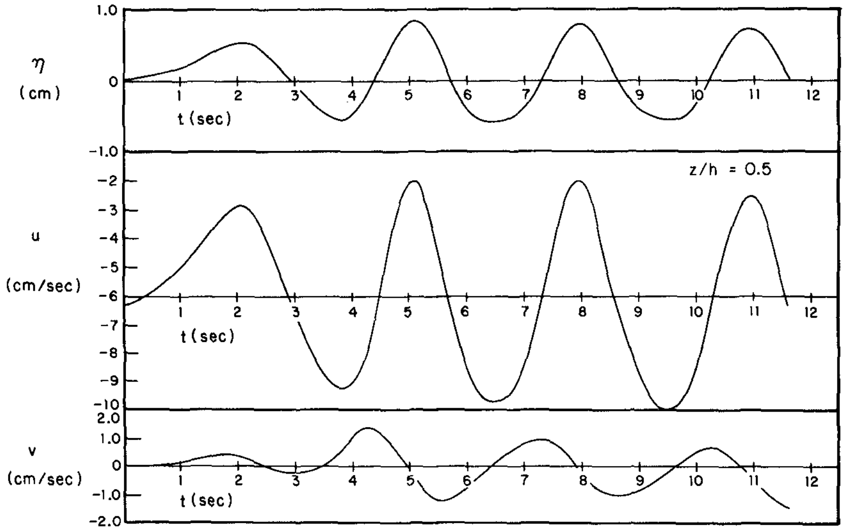

Figure 7 Water Surface, Horizontal, and Vertical Velocity at z/h = 0.5 for Wave and Current in Opposite Directions (Adverse Current).

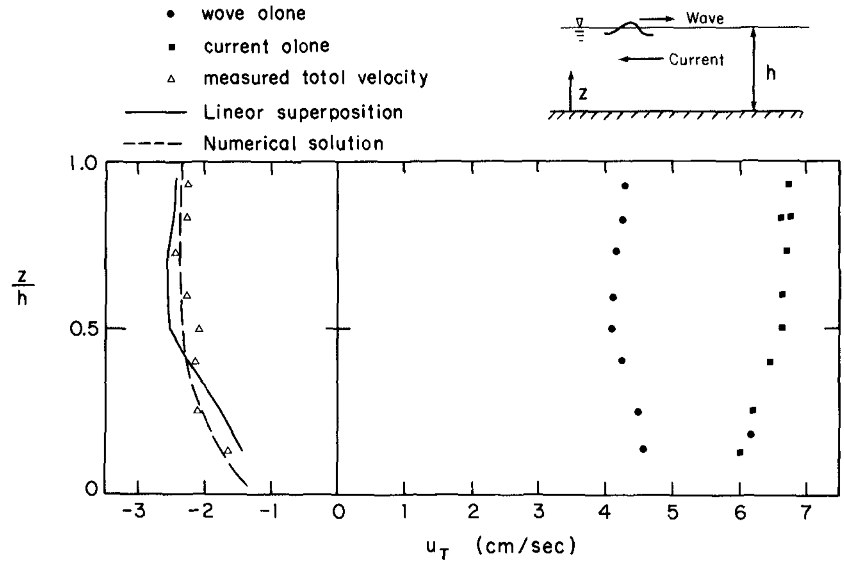

Figure 8 Distribution of the Horizontal Velocity for Wave and Current in Opposite Direction (Adverse Current).

factor smaller than predicted by usual means, but this is mitigated by the fact that the flume is horizontal and the current takes some distance to develop.

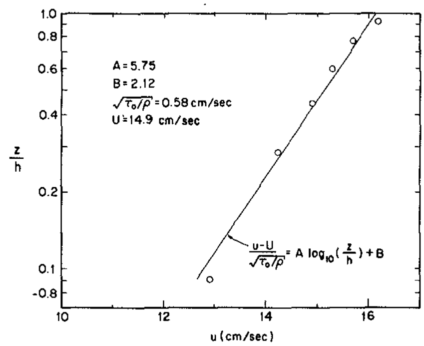

Figure 9 Velocity Distribution of Current Alone for Experiments with Solitary Waves.

The solitary wave is generated using techniques developed in other experimental studies where excellently formed waves with a negligible oscillatory tail were realized, e.g., see Lee, Skjelbreia, and Raichlen (1982). Water surface time histories of waves propagating without and with the adverse current have been obtained at relative distances from the wave generation of: x/h = 15.5, 75.8, and 135.5. In each case the wave broke as it propagated over the outlet box and then reformed into a lead wave which appeared to be solitary in shape followed by a group of oscillatory waves.

It is important to investigate the reproducibility of the wave generation arrangement used in these experiments, since the LDV is an instrument which can measure velocities only at one point at a given time. Therefore, to obtain the depthwise velocity distribution, the experiments must be repeated relocating the LDV for each measurement. In Figures 10 and 11 the wave profile is shown 135.5 depths from the wave generator (at the velocity measuring station) for the cases without and with an adverse current, respectively. For both current conditions the profile consists initially of a wave similar to a solitary wave followed by an oscillatory tail. Each of these records is for six different experiments, and the reproducibility is evident; even small oscillations in the record generally are reproduced well. It is interesting that in comparing the amplitude normalized with respect to the depth, for the wave with the adverse current the leading wave is about 2% greater in height than for the wave without the current.

Wave profiles were measured at several locations along the tank and the travel times between x/h = 15.5 and x/h = 135.5 were determined to compare wave celerities for conditions without and with the adverse current; these are shown in Table 1. (The experiments with the same last digit in the experiment number should be compared, e.g., WPA1 to WPC1, and etc.) If the mean current (14.9 cm/sec) is subtracted from the measured wave speed without a current the resultant celerity is within about 2% of that measured.

In Figures 12 through 15 time histories of the horizontal velocities are presented as measured at four different elevations and 135.5 depths from the wave generator. In each figure the measured horizontal water particle velocity for the waves propagating on a current is presented along with the water particle velocity time history which has been obtained by subtracting the measured velocity of the mean current from the measured horizontal velocity of the wave alone. Hence, the

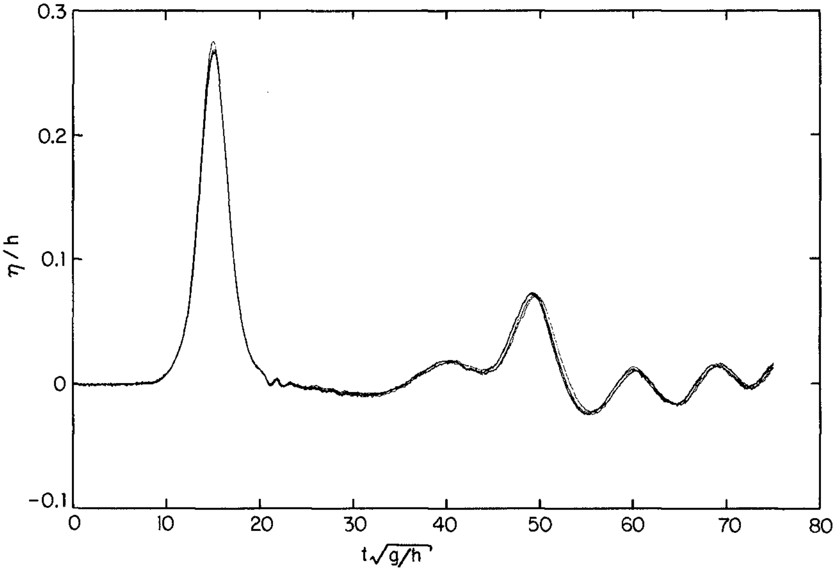

Figure 10 Water Surface-Time History at x/h = 135 with Outlet Box in Place, Without Adverse Current.

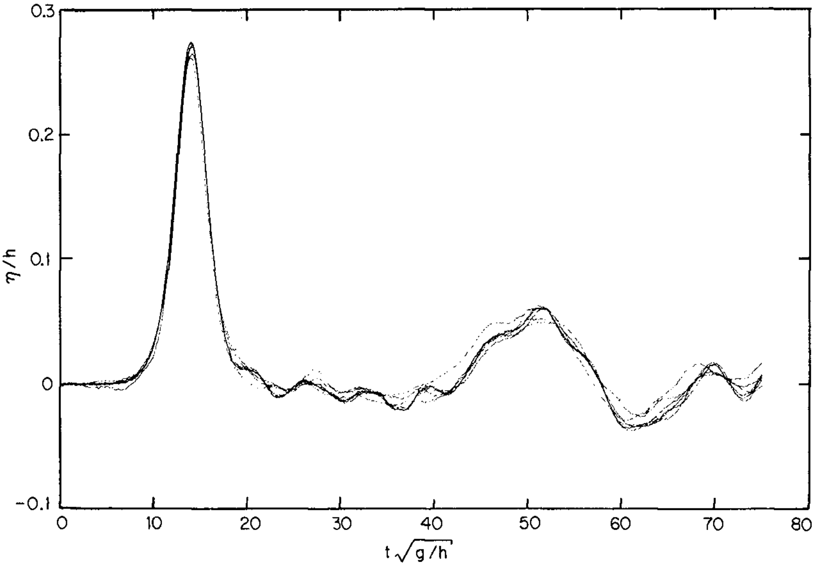

Figure 11 Water Surface-Time History at x/h = 135 with Outlet Box in Place, With Adverse Current.

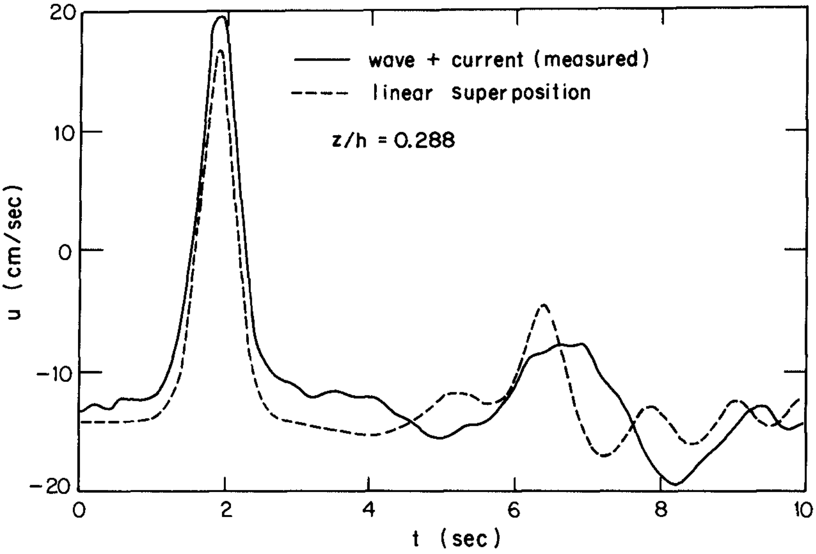

Figure 12 Horizontal Velocity Time History at z/h = 0.288.

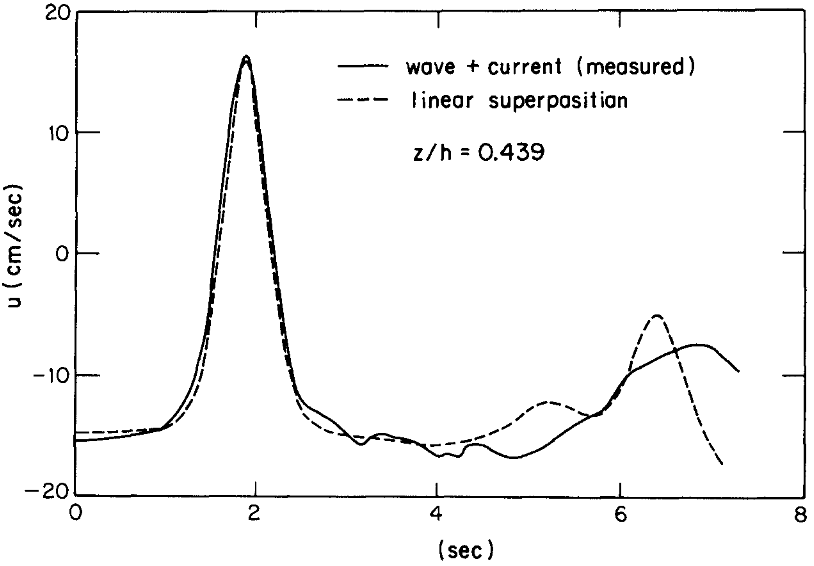

Figure 13 Horizontal Velocity Time History at z/h = 0.439.

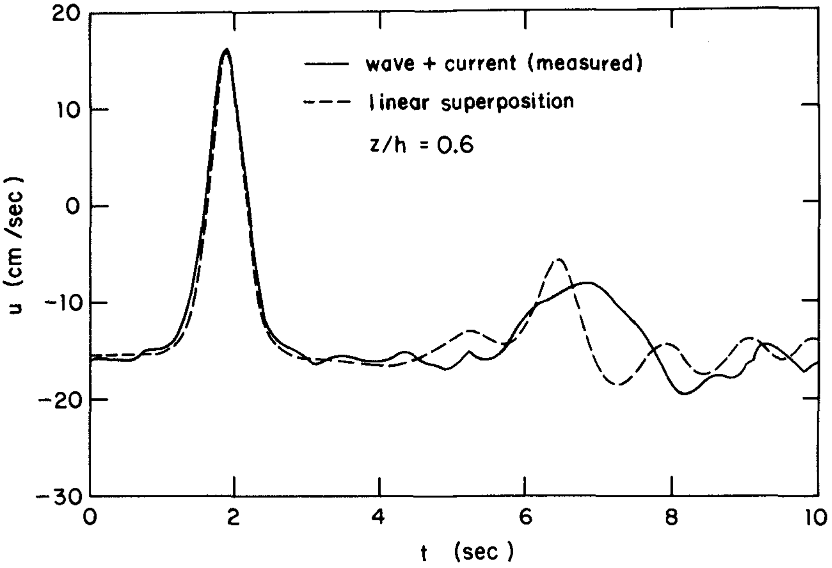

Figure 14 Horizontal Velocity Time History at z/h = 0.6.

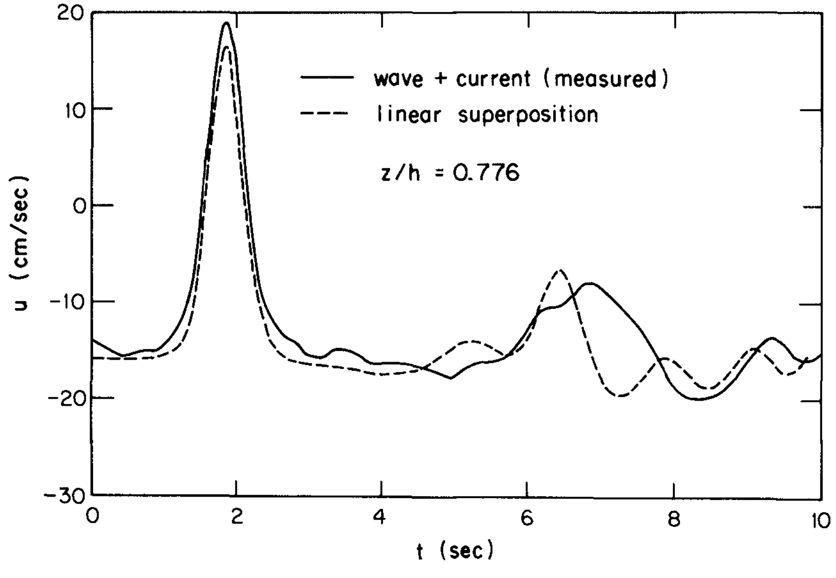

Figure 15 Horizontal Velocity Time History at z/h = 0.776.

Table 1 Celerities of the lead wave with and without a current.

Experiment	Current	C cm/sec
WPA1	no	154.96
WPA2	no	151.59
WPA3	no	149.42
WPC1	yes	140.48
WPC2	yes	138.16
WPC3	yes	134.26

dashed curve corresponds simply to linear superposition. There are differences which are apparent in these comparisons; however, the differences between the results of linear superposition and the measured total velocity for the lead wave are not large. In nearly all cases, the maximum velocity at the wave crest is underestimated by superposition by less than 10% to 15%. However, the trailing waves are significantly affected by the current so that superposition does not define the velocity field well in that region. (It should be noted that results from the method of Thomas (1981) were not compared to the experiments, since inherent to that method is the assumption that the water particle velocities under the wave for the condition without the current can be defined in a linear manner from harmonic components.)

In Figures 10 and 11, in which the variation of the water surface elevation with time was presented, similar differences between the wave profiles without the current and with the current were evident. Since this effect was significant in the trailing region of the wave, in this region differences between the velocities obtained by linear superposition and those measured with the current would be expected. Therefore, it appears the current affects the oscillatory waves which trail the main wave more than it affects the lead wave, and, thus, the velocity for the oscillatory tail cannot be constructed by simple linear superposition.

Examples of the time history of the vertical velocity components are presented in Figures 16 and 17 for relative depths of $z/h = 0.6$ and 0.78 at $x/h = 135.5$ and in each figure for conditions without and with the current. (Note the vertical scales in Figures 16 and 17 are different.) Several features are apparent. The ratio of the maximum vertical velocities at these two elevations is close to the ratio of the elevations themselves demonstrating the variation with depth of the maximum vertical velocities would be reasonably linear as predicted by linear long wave theory. The vertical velocity time histories at each of these two depths for conditions without and with the current are similar especially with respect to the velocity associated with the leading wave. As with the horizontal velocities most of the effect appears to be related to the velocities corresponding to the oscillatory tail.

CONCLUSIONS

The following major conclusions may be drawn from this investigation:

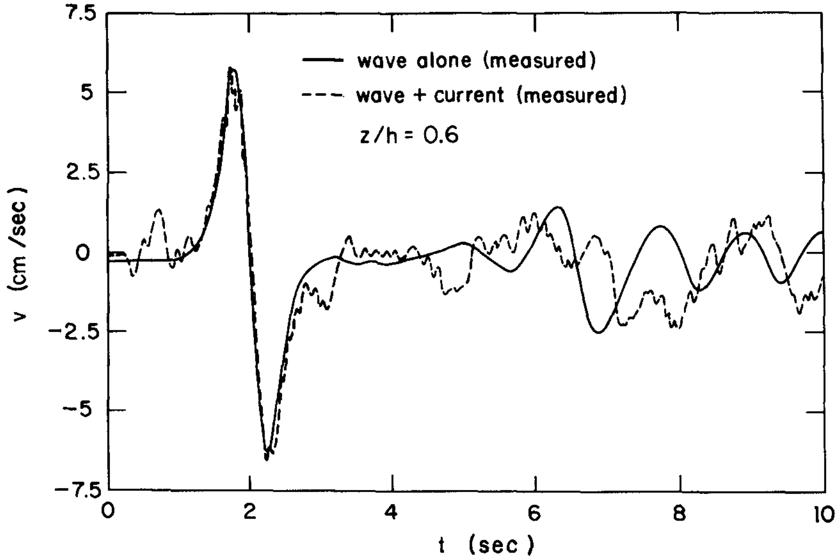

Figure 16 Vertical Velocity Time History at z/h = 0.6.

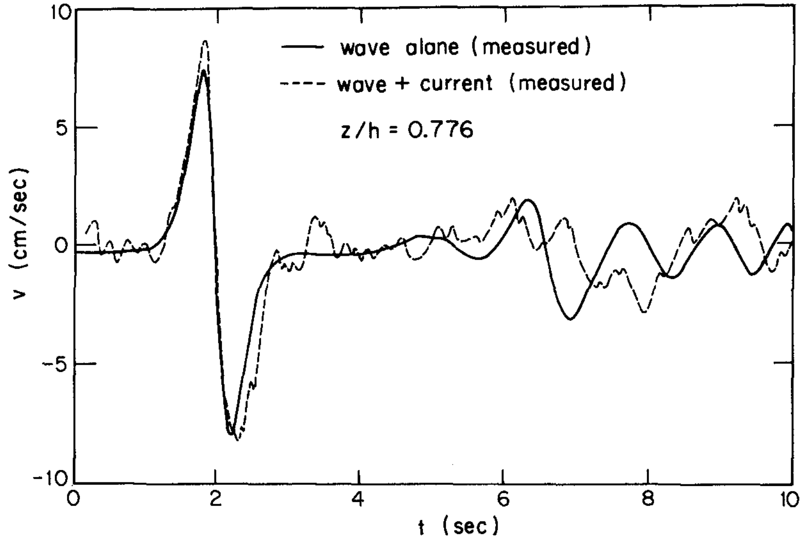

Figure 17 Vertical Velocity Time History at z/h = 0.776.

1. Even a very simple means of introducing a current into a wave tank for wave-current interaction studies can yield useful results. Indeed it may not be possible to investigate exactly wave-current interactions in the laboratory as the problem is formulated theoretically, since either the wave is developing on a permanent current or the current is developing while a permanent wave is propagating through it.

2. For engineering purposes, for waves of the order of magnitude investigated, linear superposition appears to adequately describe the maximum horizontal water particle velocities.

3. For the case of solitary waves where the oscillatory tail of the wave caused by the wave propagating over the inflow box was considerably changed by the current, the horizontal and vertical velocities were affected accordingly.

4. This investigation further demonstrates the importance of the LDV for "in situ" measurements of velocities in water waves.

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