CHAPTER ONE HUNDRED TEN

GRADING EFFECTS IN CONCENTRATION MEASUREMENTS
----

by:
J. van de Graaff [1] &
J.A. Roelvink [2]

ABSTRACT

In analysing measured sediment concentration distributions
the effect of the non-uniformity of the bottom material is
frequently not taken fully into account. Examples of
calculations are given to clarify the grading effect on
the parameters that determine the concentration
distribution (e.g. diffusion coefficient distribution).
Results of a preliminary test series indicate that the
particle size affects the diffusion coefficient involved.
$[\varepsilon(sediment) = \beta\ \varepsilon(fluid); \beta$ size dependent].

1 INTRODUCTION

The CERC formula is an example of an over-all description
of the longshore sediment transport rate. A limited range
of problems can be solved with the help of this formula
since it only describes sediment transports due to
wave-driven currents. In more general coastal engineering
problems not only wave driven currents but also currents
like tidal currents, ocean currents and wind driven
currents play an important role. To be able to tackle the
latter type of problems, one needs a transport formula to
calculate the transport as a function of boundary
conditions like water depth (h), wave height (H), wave
period (T), current velocity (v) and bottom particle size
(D50). The sediment transport rate for currents more or
less perpendicular to the orbital plane of the waves can
be simply described as:

----

[1] Senior Scientific Officer
    Delft University of Technology, The Netherlands
[2] Student Assistant
    Delft University of Technology, The Netherlands

$$S = \int_0^h v(z) * c(z) \, dz \qquad\qquad (1.1)$$

where: S   : sediment transport rate
       v(z)   : current velocity (function of z)
       z   : height above the bottom
       c(z)   : time averaged sediment concentration
       h   : water depth

As far as the authors know, only the so-called Bijker
formula (Bijker 1971; Van de Graaff and Van Overeem 1979)
describes the transport in a physical sense according
to Eq. (1.1) and can be used generally in coastal
engineering problems. However, although a ready made
formula is available, tests have been carried out all
over the world in measuring the time average sediment
concentration distribution over the water column under
wave and current conditions. (Among many others :
MacDonald 1977; Nielsen 1979; Bosman 1982.) Such
experiments are carried out since it is felt that the
concentration distribution as proposed by Bijker is not
valid under all conditions.

By analysing concentration measurements one tries to find
the underlying 'mechanism' which holds the particles in
suspension and causes the measured distribution. Since
the suspension mechanism is most likely diameter
dependent, it is necessary to take fully into account
the grading, the non-uniformity, of the bottom particles.

2 SUSPENSION MECHANISM AND 'THEORETICAL' DISTRIBUTIONS

Fig. 1 shows an example of a so-called time and bed
averaged sediment concentration distribution over the
waterdepth due to (random) waves running over a sloped
beach profile. The samples have been taken by suction
in a direction perpendicular to that of wave propagation.
The 'time and bed averaged' procedure has been developed
by Bosman 1982; it means that the suction time is long
with regard to the wave period ($\approx$ 100 T) and that the
suction pipe's intake nozzle is moved to and fro by a
carriage over a few ripple lengths ($\approx$ 5 λ). By this
procedure local effects (just above a ripple top quite
different concentrations do exist than do above a ripple
trough) are smoothed out.
In the example of Fig. 1, as well as in the other examples
of this paper, the several measuring points have been
measured simultaneously.

A distribution like in Fig. 1 is typical for random- as
well as for regular waves. The fact that close to the
(mostly) rippled bed high concentration rates are found,
is caused by the scraping effect of the orbital motion
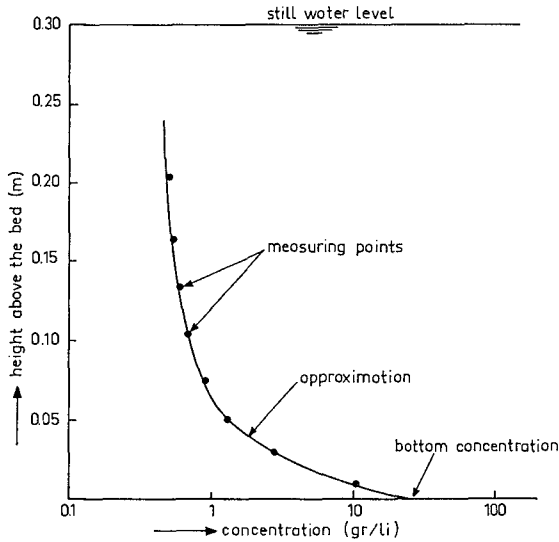of the waves over the bed.

Fig. 1   Concentration Distribution over
         Water Depth

Why and how the particles can reach the higher levels in
the vertical, is not clear yet. The vertical orbital
motion in the waves plays a convective role probably.

Whatever the mechanism may be, in this paper a simple
diffusion mechanism is assumed.
Assuming a steady state condition, in general the
concentration distribution over the depth can be described
by the expression:

$$w * c(z) + \varepsilon(z) * dc(z)/dz = 0 \qquad (2.1)$$

where: w   : fall velocity
        c(z)   : average concentration at level z
                   above the bed
        $\varepsilon(z)$: eddy diffusion coefficient
        z   : vertical upward directed ordinate; the
                   bottom being z=0

Assuming uniform material (w = constant) the distribution
of $\varepsilon(z)$ over the waterdepth defines the shape of the
concentration distribution line over the water depth. The
bottom concentration fixes the actual position of the
concentration profile shape.

As a final aim, the concentration distribution's characterizing parameters ($\varepsilon(z)$ distribution) and the bottom concentration should be related to the boundary conditions. In this paper, however, a more modest aim will be pursued: analysing the effects of non-uniform material on the apparent (z) distribution.

In the literature some basic $\varepsilon(z)$ distributions have been suggested all leading, in case of uniform bottom material, to typical concentration distributions. Three of those $\varepsilon(z)$ distributions will be mentioned, according to their names as found in literature.

a)   Coleman type
b)   Rouse type
c)   Bhattacharya type

In Table 1 the characteristics of the $\varepsilon(z)$ distributions are given together with the resulting concentration distributions.

	$\varepsilon(z)$ distr.	$\varepsilon(z)$	c(z)	
Coleman	constant	$\varepsilon(z)=const.=\varepsilon$	$c(z)=c_b \exp(\frac{-w}{\varepsilon} z)$	*
Rouse	parabolic	$c(z)=4\varepsilon_{max} \frac{z}{h}(\frac{h-z}{h})$	$c(z)=c_a (\frac{h-z}{z} \frac{a}{h-a})^{\frac{w\,h}{4\varepsilon_{max}}}$	**
Bhattacharya	triangular	$\varepsilon(z)=\frac{z}{h}\varepsilon_{max}$	$c(z)=c_a (\frac{h}{z}.\frac{a}{h})^{w\,h/\varepsilon_{max}}$	**

Table 1.

* c_b : bottom concentration (z=0)

** c_a : concentration at reference level z=a
             ($c(z) \to \infty$ for $z \to 0$)

The $\varepsilon(z)$ distributions as given in Table 1 hold for sediment. In river flow sediment studies, a distinction is made frequently between ε s(sediment) and εf(fluid). In general a simple direct relation between ε s and ε f is assumed:

$$\varepsilon s = \beta * \varepsilon f \qquad\qquad (2.2)$$

where: β : a proportionality factor.

A widespread discussion is going on concerning the actual β-value; values <1 as well as >1 have been mentioned. The case β>1 seems to get the greater support at the moment. (See for instance Van Rijn 1982). Furthermore it seems most likely that β is particle size dependent. For coarser material larger β values can be expected.
In a unidirectional flow the relation between ε f and ε s seems clear. It is felt that the turbulence in the masses of water is also the working agent in holding the particles in suspension (micro-scale). Under wave action a still less understood situation exists. Most likely the micro-scale turbulence is less important than the orbital motion induced processes (macro-scale). However, as

mentioned already, in the wave action case also a simple
diffusion process is assumed. With respect to ß values a
similar approach is chosen as for unidirectional flow,
whatever the real meaning of a ß value may be under wave
action.
In section 5 some results of preliminary tests are
reported concerning probable ß values.

3 RELEVANCY OF GRADING EFFECTS

Trying to fit the real concentration measurements of
figures like Fig. 1 with the distributions as predicted by
the standard cases of Table 1, leads to large differences.
In reality none of the given types holds for most of the
measured concentration distributions. A more appropriate
$\varepsilon(z)$ distribution should be found and is therefore one
of the ultimate aims of concentration analysis studies.
The 'theoretical' $\varepsilon(z)$ distributions of Table 1 can,
however, be used fruitfully to clarify the effect of the
grading of the bottom material.
Because of its simple behaviour a Coleman type $\varepsilon(z)$
distribution is chosen in the next example.

Starting from an arbitrary bottom concentration, the
resulting concentration distribution over the water depth
has been calculated for 5 different graded bottom
materials. Fig. 2 gives the particle size distributions
involved; standard log-normal distributions have been
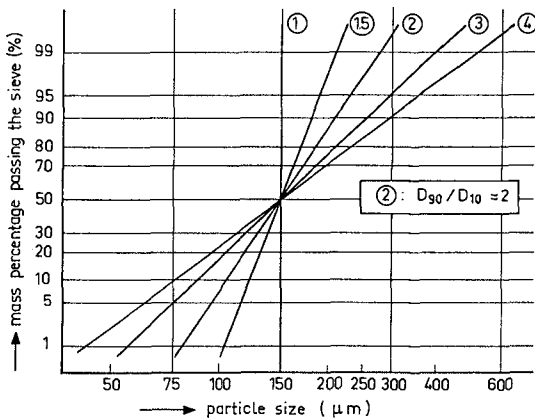chosen.

Fig. 2   Particle Size Distributions

In the calculation it is assumed that the particles in
suspension close to the bed (bottom concentration) have
the same particle size distribution as the original bottom
material. Furthermore that amount of sand is assumed to
exist of k fractions, all equal in weight. In each
fraction only particles between narrow limits are present
in conformity with the particle size distribution.
If k is large enough, the particle size distribution
within a certain fraction can be assumed to be constant.
For k=10 the characteristic particle size D(i) of the i-th
fraction (1 < i < 10) becomes:

$$D(i) = D(10*i-5) \qquad\qquad (3.1)$$

The fall velocity w(i) belonging to the particle size D(i)
can be computed with:

$$w(i) = 1/ \, 10^{(0.476 \log^2 D(i) + 2.180 \log D(i) + 3.226)} \qquad (3.2)$$

Formula (3.2) holds for salt water with a temperature of
5° C.
Assuming a constant $\varepsilon(z)$ value ε over the water depth,
the concentration distribution over the depth of each
fraction can be computed with:

$$c(z,i) = c(b,i) \, \exp\left(\frac{-w(i) \, z}{\varepsilon} \right) \qquad\qquad (3.3)$$

For each fraction the same ε value is assumed (thus in
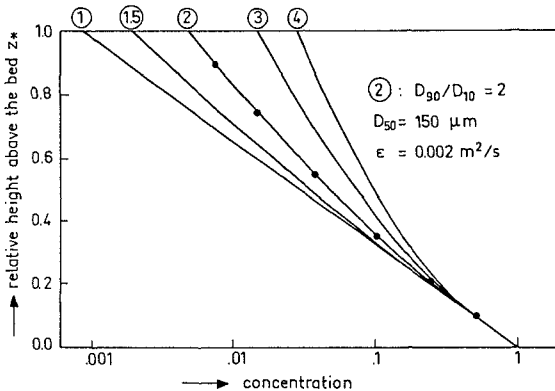this calculation ß=1 for each particle size).

Fig. 3   Concentration Distribution

The ultimate concentration c(z) at height z above the
bottom yields:

$$c(z) = \text{sum } c(z,i) \qquad\qquad (3.4)$$

Fig. 3 shows the result for $\varepsilon = 0.002$ m²/s, k = 10 and
the diameter distributions of Fig. 2, all with a D50 of
150 μm. Depending on the rate of grading, characterized by
the D90/D10 ratio, a variety of concentration curves can
be found. For normal beach material the D90/D10 ratio
ranges from 2 to 3.

From the calculated contributions of the fractions to the
ultimate concentration at level z above the bottom, the
apparent size distribution at that level can be computed.
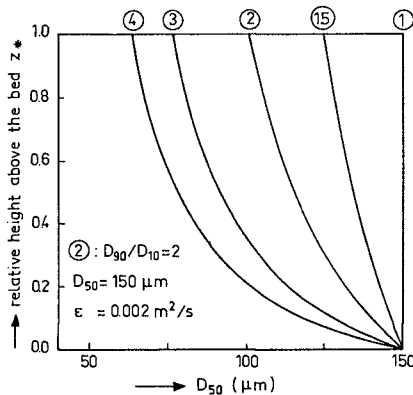Fig. 4 gives the D50 distribution for some rates of
grading.

Fig. 4   D50 Distribution over Water Depth

A quick, but rough, idea of the order of magnitude of the
mistake made when one does not take into account the
non-uniformity of the bottom material in analysing
measured concentration distributions, can be found as
follows.
Assume one has actual measuring points as indicated in
Fig. 3 (see on line (2)). Assume moreover that one does
not take into account the grading effect and one is
focussed on finding the constant ε value that explains
the position of the measured points as well as possible.
So one fits a straight line through the measuring points
in Fig. 3. From the slope of that line the apparent
(but wrong) ε value can be found. The ratio
ε(apparent)/ε(actual) is then a measure of the mistake
made. In Fig. 5 some results are given. To find
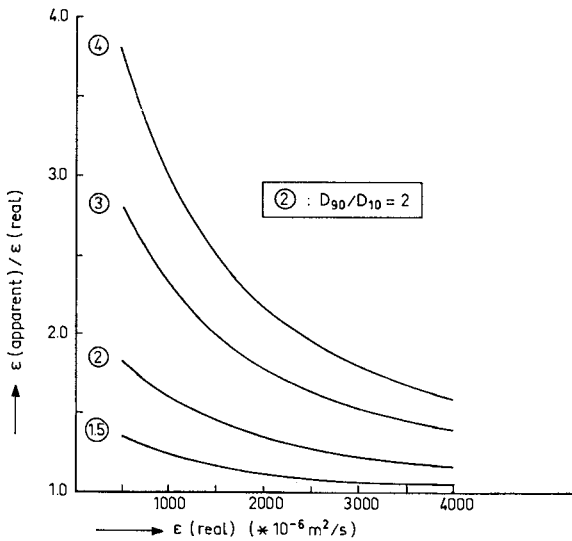ε(apparent) not a complete fit calculation has been

Fig. 5 Mistakes in ε Prediction

made, but quite simply, only the real concentrations
c(0.85) and c(0.15) have been taken into account
[c(0.85) means concentration at elevation z=0.85 h].
From Fig. 5 it can be seen that, depending on the rate
of grading and on the actual ε value a variety of
mistakes can be found. It seems clear that it is
absolutely necessary to take grading effects into full
account in analysing measured concentration distributions.

4 ACTUAL CONCENTRATION MEASUREMENTS

In Fig. 6 a measured concentration distribution is given
as found during a test in a wave flume of the Fluid
Mechanics Laboratory of the Delft University of
Technology. The boundary conditions are as follows:

    - horizontal bed with natural ripples
    - water depth h = 0.30 m
    - wave conditions: H = 0.10 m; T = 1.7 s
    - D50 = 97 μm ; D10 = 75 μm; D90 = 126 μm

The measured points have been approximated according to
the procedure as briefly described in Appendix I.
Starting with the (extrapolated) bottom concentration and
taking into account 20 fractions the ε(z) distribution
which describes the approximating curve precisely, can be
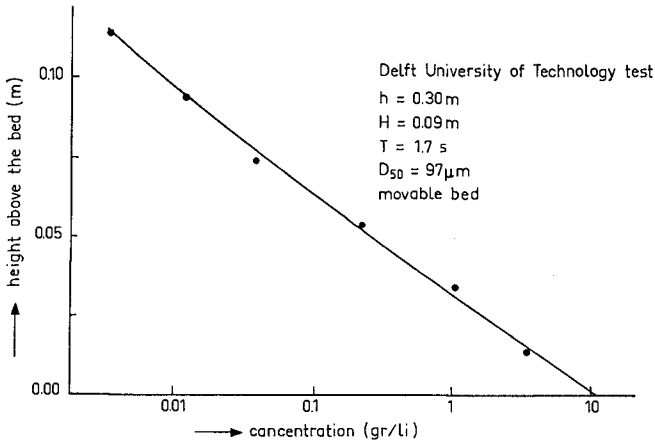computed. Fig. 7 presents the resulting ε(z) curve.

Delft University of Technology test

h = 0.30 m

H = 0.09 m

T = 1.7 s

$D_{50} = 97 \mu m$

movable bed

height above the bed (m)

concentration (gr/li)

Fig. 6   Concentration Distribution Movable Bed

graded material   uniform material

height above the bed (m)

Delft University of Technology test

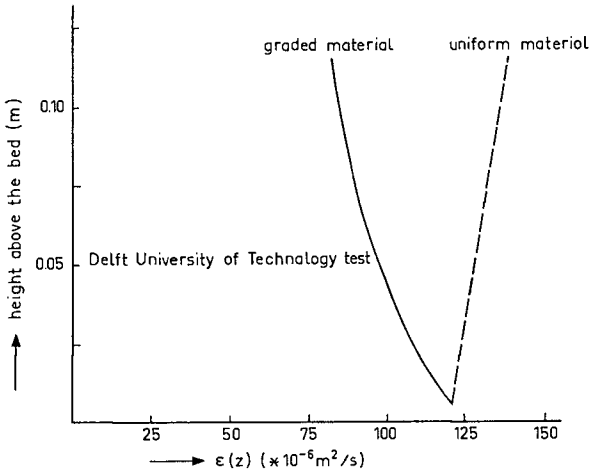$\varepsilon (z)$ $(\ast 10^{-6} m^2 / s)$

Fig. 7 $\varepsilon (z)$ Distribution

(In the same figure the result is given when one does not
take into account the non-uniformity of the bottom
material; again large differences can be found.)
The size distribution of the samples caught at the various
heights above the bottom has been analysed with the help
of the Delft University Settling Tube (see for details
Slot and Geldof 1984) of the Fluid Mechanics Laboratory.
Also the distribution according to calculations has been
determined. Fig. 8 gives the comparison between
measurements and calculations. Although some differences
are found, the conclusion is that the general trend is a
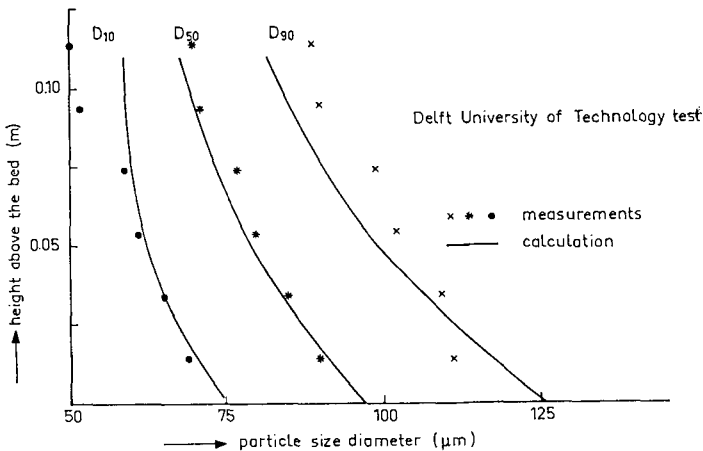rather good fit.

Fig. 8   D10 - D50 - D90 Distribution

In the calculations as described in this section, β=1 has
been assumed. The suggestion of size dependent β values
being greater than 1 as briefly discussed in section 2,
may be an explanation of the fact that, especially at
greater heights above the bed, the D90 values are
underestimated by the calculations and the D10 values
slightly overestimated.
In order to gain some insight into possible β values a
preliminary test has been carried out in one of the
flumes of the Delft Hydraulics Laboratory.

5 EFFECT OF GRAIN SIZE ON β-VALUES

The research on concentration distributions is one of the
main topics of the 'Sediment Transport' working group of
the Dutch Applied Research Program Rijkswaterstaat -
Coastal Research (TOW). This TOW program is carried out in

close cooperation between the Public Works Department, the
Delft Hydraulics Laboratory, the Delft University of
Technology and the Royal Dutch Meteorological Institute.
In the framework of the 'Sediment Transport' group a
single series of tests has been carried out by the Delft
Hydraulics Laboratory in order to get a first idea of the
effect of grain size on β-values. The results have been
analysed by the authors.
Normal movable bed tests are unsuitable to clarify the
mechanism involved. The rate of 'diffusion coefficient
production' in an actual case does not depend on the
boundary conditions (to be kept constant) like water
depth, wave height and wave period only, but also on the
form (height and length) of the bed ripples. These bed
ripples are, given the boundary conditions, highly
diameter dependent. A constant rate of wave and bed
induced diffusion activity, equal for several grain sizes,
can only be achieved with a fixed rippled bed, covered
with such a small amount of particles, that the original
bed form is not disturbed. In the tests with a horizontal
bottom, wooden fixed ripples have been applied; ripple
height: 0.017 m; ripple length: 0.07 m. Seven tests have
been carried out with nearly uniform sand; D50-values of
87, 112, 170, 192, 221, 280 and 346 μm respectively.
Although uniform material was desired, it turned out from
fall velocity tests in the settling tube, that some
variations existed. The D90/D10 ratio actually ranged from
1.3 to 1.5. A single test was carried out with well-graded
material: D50 = 217 μm; D90/D10 = 2.15.

In the tests one set of boundary conditions was
maintained. Water depth h = 0.30 m; (regular) wave height
H = 0.08 m; wave period T = 2 s. In each test a
restricted amount of sand was thrown into the flume. Two
to five series of simultaneously sucked samples in 4
points were gathered for each material tested. Fig. 9
shows an example of the vertical concentration
distribution. Mark the quite similar distribution shape
of the curves (1) and (2), despite the fact that for curve
(1) the concentrations are approximately a factor 15 less
than for curve (2). In case (1) less sand was thrown into
the flume.

In a first analysing round, real uniformity was assumed
for the 'uniform' material tests. Since in all cases
nearly straight lines were found for the concentration
distribution over the water depth (see Fig. 9),
pointing to a Coleman-type distribution, constant ε
values can be calculated with the help of the proper
formula of Table I. Fig. 10a represents the results,
showing a distinct dependency of the calculated ε values
on the settling velocity. A nearly linear relationship is
found. By extrapolation ($\rightarrow$ w=0) an (fluid)-value can
be calculated. For the tested materials β-values up to 1.8
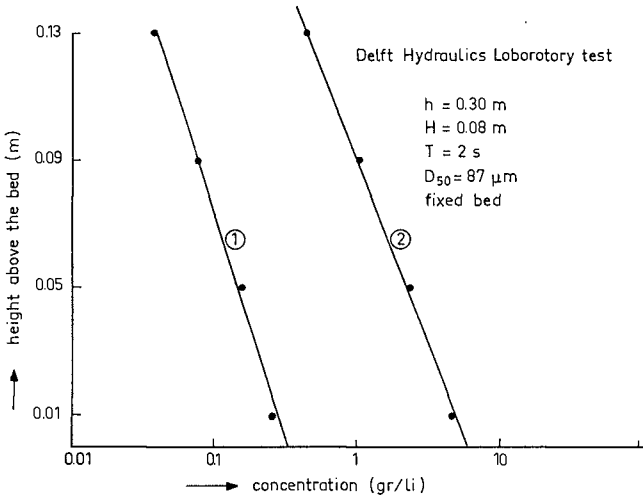can be found for w = 0.04 m/s (D50 $\simeq$ 300 μm).

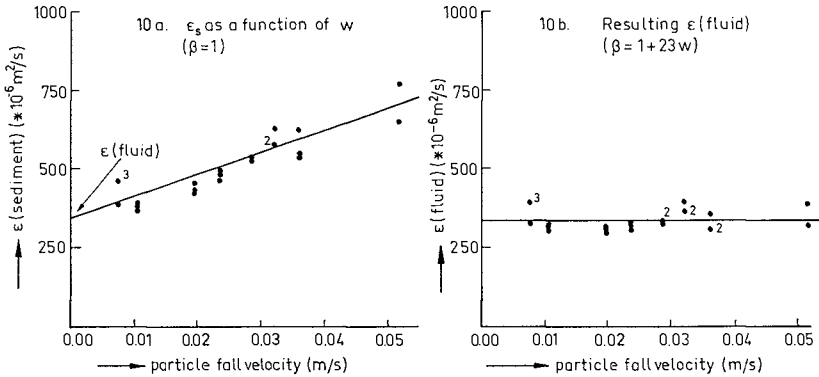**Fig. 9   Concentration Distribution**

Fig. 10   Diffusion Coefficient vs. Fall Velocity

In a second analysing round, the real grading of the
'uniform' materials was taken into account. As a
hypothesis a linear relationship (to be stated) between w
and ß was assumed. The fit procedure described in
Appendix I was applied. Due to the grading and the applied
fit procedure, a non-constant ε distribution over the
water depth is found this time. The variation over the
water depth, however, is not very large. An average value
over $0 < z < 0.09$ m has been calculated. By an iteration
process the most appropriate ε(fluid) value and the
ß-slope value m were calculated. Fig. 10b shows the
ultimate result.

$$\varepsilon \text{ (fluid)} = 0.000337 \text{ m}^2/\text{s}; \quad \text{ß} = 1 + 23.0 \ast w$$

$$(5.1)$$

The relationship that was found to exist between ß and w
holds for the set of boundary conditions during the test
series.
As long as a similar relationship has not been derived for
other sets of boundary conditions, the ß – w relationship
cannot be used generally. Tests with different boundary
conditions, to be carried out in the Fluid Mechanics
Laboratory of the Civil Engineering Department of the
Delft University of Technology, are being prepared.
Since a general ß – w relationship is not yet available,
the calculations as presented in section 4 cannot be
refined at this moment.

6 CONCLUSIONS

–   The non-uniformity of the bottom material strongly
        affects the concentration distribution over the
        water depth under wave action.

–   When the concentration distribution is described
        with the help of a diffusion equation, the
        diffusion coefficient seems to be particle-size
        dependent; ε (sediment) = ß ε (fluid), the ß
        coefficient being approximately a linear function
        of w.

–   However, it is not clear yet whether a similar
        ß – w relationship holds for quite different
        boundary conditions or not. Further research will
        have to clarify that topic.

7 REFERENCES

Bijker,E.W. (1971)
'Longshore transport computations'
Proc. ASCE, Journal of the Waterways, Harbors and Coastal
Engineering Division, WW 4, November 1971

Bosman,J.J. (1982)
'Concentration measurements under oscillatory water
 motion'
Delft Hydraulics Laboratory / TOW
Report on model investigation   M.1695 part II
Delft, February 1982

Graaff,J. van de and J. van Overeem (1979)
'Evaluation of sediment transport formulae in coastal
 engineering practice'
Coastal Engineering 3 pages 1-32
Elsevier Scientific Pub. Comp. Amsterdam 1979

MacDonald,T.C. (1977)
'Sediment suspension and turbulence in an oscillating
 flume'
TP 77-4 U.S. Army Corps of Engineers
Coastal Engineering Research Center, Fort Belvoir, Va.
April 1977

Nielsen,P. (1979)
'Some basic concepts of wave sediment transport'
Series paper 20, Inst. Hydrodynamics and Hydraulic
Engineering (ISVA) Technical University of Denmark

Rijn,L.C. van (1982)
'Computations of bed load and suspended load transport'
Delft Hydraulics Laboratory
Research Report S 487 part II
Delft, September 1982

Slot,R.E. and H.J.Geldof (1984)
'An improved settling tube system for sand'
Journal of Hydr. Res. (in preparation)

Appendix I

FIT PROCEDURE MEASURED CONCENTRATION DISTRIBUTIONS

In behalf of the calculation procedures of this paper
(e.g. ε - and D50- distributions over the water depth) one
needs a mathematical description of the concentration
distribution over the water depth. Mostly -compare Fig. 1-
a restricted amount of measuring points is available.
Besides that the measuring points show some scattering.
When applying fit procedures with many degrees of freedom,
unrealistically high bottom concentrations and strange
curves in the upper section of the vertical can be
found frequently.
This is unacceptable for the aims, stated in this paper,
and when the bottom concentrations have to be related to
the boundary conditions also.
The described fit procedure seems rather objective and
holds true for tests on model scale at least.

From Fig. 1 (z* vs. ln c(z)) it can be seen that:

        - highest concentrations occur near the bed
        - a gradual decrease in concentration occurs
          towards higher levels above the bottom.

Looking at the gradients as a function of z* for arbitrary
concentration distributions, one can schematize the
gradients in curves according to Fig. A1. (In Fig. A1 a
'standard' square is assumed. The vertical axis (z*)
ranges from 0 to 1 and the horizontal axis (d ln(c)/dz*)
also from 0 to 1. The actual gradients can be found by
multiplying with a negative factor). It seems to be better
-and easier- to impose some restrictions on the gradient
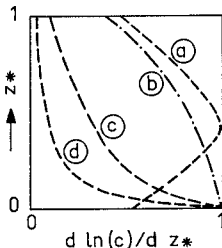curve than to do so on the original concentration curve.
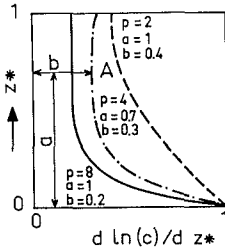

Fig. A1

Standard Square
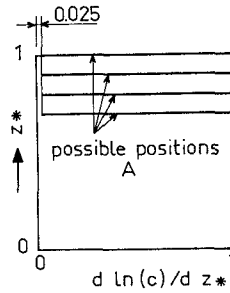
Fig. A2

Possible Fillings

Fig. A3

Extreme Positions
Point A

REQUIREMENTS OF GRADIENT CURVES IN STANDARD SQUARE

1) For $z*=0$, the gradient is 1 and at its maximum. At the bottom the largest gradients are assumed to occur. So curves like curve (a) in Fig. A1 are not acceptable.

2) No negative gradients. A negative gradient in the standard square at a certain level above the bottom means that the concentration increases towards higher distances from the bottom. That seems quite unlikely; experiments never showed any clear evidence to that. Also zero-gradients are unlikely. In the standard square minimal b values (see Fig. A2) of 0.025 are assumed. The value of 0.025 is subjective; however, slightly different values hardly affect the total result as it turned out after analysing a number of test cases.

3) No large gradients close to the bed in the standard square. In Fig. A1 curve (b) seems attractive. At the bed the largest gradient occurs; nevertheless close to the bed also rather large gradients occur. A distribution like curve (b) will never lead to unrealistically high bottom concentrations. However, from actual measurements it turned out that curves like (c) and (d) in Fig. A1 are more likely in some cases . However, with curve (d) extremely high bottom concentrations can be found. So it seems appropriate to make some restrictions. In Fig. A2 and A3 some possible filling-ups of the standard square are given. By assuming a maximum value of 8 for p (see next section), very high bed concentrations can be avoided.

RESULTS

The curves in Fig. A2 can be described by:

$$d(\ln(c))/dz* = q\ [\ b + (1-b)\ [(a-z*)/a]^p\ ] \qquad (A.1)$$

where: q   : (negative) scale factor
       b   : horizontal ordinate point A (0.025<b<1)
       a   : vertical ordinate point A
                  (in the procedure proposed a can be
                  1 or 0.9 or 0.8 or 0.7; these values
                  are rather subjective).
       p   : exponent of fit parabola for gradient
                  curve (p can be 2; 4; 6 or 8)

Integration of Eq. (A.1) yields:

$$\ln(c)=q[\ bz*-(1-b)(a-z*)/(p+1)[(a-z*)/a]^p +d\ ] \quad (A.2)$$

where: d is the integration constant

In a computer program the procedure starts with a=1 and
p=8. The 'best' values of the parameters q, b and d are
determined with a least squares method. If it turns out
that b < 0.025 or b >1.0, then b is fixed at 0.025 or 1.0
respectively and the procedure is restarted. The
parameters q and d are determined now. In the next step
it is checked whether a better result is found with a=1
and p=6, and so on. Next the procedure starts with a=0.9
and p=8 again. As stated the least a-value is taken to be
0.7.

From test series it turned out that the fit procedure
proposed seems to be rather objective. (Compare Fig. 1)