CHAPTER 2

A re-analysis of the spectra observed in JONSWAP

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ABSTRACT

The frequency spectra of wind-driven waves observed during JONSWAP are re-analyzed to establish whether the Toba formulation for the high-frequency tail ($\sim f^{-4}$) fits the data better than the Phillips formulation ($\sim f^{-5}$) used originally in the JONSWAP project. The results indicate that the f^{-4}-tail provides a better fit to the observed spectra. The proportionality factor in Toba's spectrum, which is theoretically expected to be a universal constant, is found to be uncorrelated with the growth stage of the waves.

Introduction

In so-called ideal conditions of wave generation (a steady, uniform windfield at right angles to a straight upwind coastline), the spectral distribution of wave energy over the frequencies appears to have a standard shape (see e.g. Hasselmann et al., 1973). In a pioneering paper, Phillips (1958) argued that there should be a range of frequencies where the spectral density $E(\omega)$ is saturated at a level determined exclusively by the local (radian) frequency (ω) and the gravitational acceleration (g). On dimensional grounds this saturation level should obey

$$E(\omega) = \alpha\, g^2\, \omega^{-5} \qquad (1)$$

with α a universal constant. However, Hasselmann et al. (1973) showed that α varied with the growth stage, instead of being a constant, as had been inferred by Phillips.

Another approach to the formulation of the saturation range is due to Toba (1972, 1973). It is based on his so-called three seconds power law for the significant waveheight (H_s) and the significant period (T_s). Stated in spectral terms, this law implies that the integral of the spectral densities, or the variance of the surface elevation (ε), and the peak frequency (f_m) are related according to

$$\varepsilon \sim g u_* f_m^{-3} \qquad (2)$$

in which u_* is the shear velocity of the wind over the water surface.

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On similarity grounds, Toba (1973) deduced from this that the saturation spectral density should be of the form

$$E(\omega) \cong \beta \; g \; u_* f^{-4} \qquad (3)$$

with β a universal constant.

Toba's formulation (3) and the JONSWAP-spectrum were published in the same year (1973), but the latter found initially much more recognition in the international literature than the former. This has begun to change only recently, with the presentation of further empirical evidence supporting (3) by Mitsuyasu et al. (1980), Kahma (1981), Forristall (1981), and Donelan et al. (1985), and with theoretical work of Kitaigorodskii (1983) and Phillips (1985).

In light of these developments, it was decided to re-analyse the original JONSWAP data set with the purpose of making a comparative investigation of the applicability of the formulations due to Phillips (1958) and Toba (1973). The JONSWAP data were chosen because they are supposed to represent virtually ideal generation conditions, and because the JONSWAP spectrum has widely been accepted in the oceanographic and engineering communities. Moreover, the data set is well documented.

The procedures and results of this study are only briefly described here. A full account will appear elsewhere (Battjes et al., 1986).

Data

The spectra which were analyzed in this study, obtained in the Joint North Sea Wave Project (JONSWAP, see Hasselmann et al., 1973), were kindly made available to the present authors for the purpose of a re-analysis. A total of 108 spectra from the 1969 campaign of JONSWAP was available. Nine double-peaked spectra were removed.

The spectral models

The observed spectra are studied in two frequency ranges: the high-frequency range for which the formulations due to Phillips (1958) and Toba (1973) were intended, and the full frequency range including the spectral peak. Only the results for the high-frequency range analysis are presented here.

For the high-frequency range (defined below), the following expressions are used:

Phillips (1958) : $E(f) = \alpha \; g^2 (2\pi)^{-4} f^{-5}$ (4)

Toba (1973) : $E(f) = \beta \; g u_* (2\pi)^{-3} f^{-4}$ (5)

Analysis procedure

Frequency range

The spectra observed in JONSWAP are given at frequencies between f_{min} = 1/128 Hz and f_{max} = 127/128 Hz, with a sample interval of 1/128 Hz. For the high-frequency range analysis, the lower and upper cut-off frequencies f_1 and f_2 were chosen as $f_1 = 1.5\, f_m$ and $f_2 = f_{max}$. For each observed spectrum, the value of the peak frequency f_m needed to determine f_1 was taken from Mueller (1976) (only for this purpose).

Parameter estimation and goodness-of-fit

The values of α and β were estimated simply as the average level of $(2\pi)^4\, g^{-2} f^5\, E(f)$ and $(2\pi)^3\, (gu_*)^{-1}\, f^4\, E(f)$ respectively, between the frequencies f_1 and f_2.

The best-fit values of α and β obtained for each observed spectrum are considered as a function of growth stage, for which the dimensionless peak frequency ν was chosen:

$$\nu = U_{10}\, f_m / g \tag{6}$$

A linear least-squares method for log scales was used to determine the coefficient r and exponent s of the assumed powerlaw relationship between α or β and ν:

$$\alpha = r\, \nu^s \tag{7}$$

and similarly for β.

To establish quantitatively which of the two models fits the JONSWAP observations best, the values of the integrated squared deviation V between observed spectral density (E') and model spectral density (E) were used:

$$V = \int_{f_1}^{f_2} [\, E'(f) - E(f)]^2\, df \tag{8}$$

However, this value is influenced by both the energy and frequency scales of the spectrum, which are irrelevant for a shape analysis. Therefore, a normalized squared deviation ($\hat{V}$) is considered:

$$\hat{V} = V\, f_m / \varepsilon_{1,2}^2 \tag{9}$$

where $\varepsilon_{1,2}$ is the total energy in the frequency range (f_1, f_2).

Results

Spectral parameters

The mean and standard deviation of the estimated values of α and β and the spectral shape parameters of the best-fit spectra are given in Table I, together with the coefficients and exponents of the power law variations with the dimensionless peak frequency (Eq. 7). A graphical impression of the variation of the scale parameters α and β with ν is given in the figs. 1 and 2.

spectral parameter		mean	standard deviation	r	s
Phillips	α	0.0204	0.0088	0.0563	1.06
Toba	β	0.129	0.0622	0.126	0.233

Table I - Mean and standard deviation of the spectral parameters α and β, and coefficients and exponents of the power law (7).

Goodness-of-fit

The joint variation of the normalized error measures $\hat{V}_P$ and $\hat{V}_T$ is given in fig. 3; the subscripts P and T refer to the Phillips and the Toba formulation, respectively. A histogram of the difference $Q = \hat{V}_P - \hat{V}_T$ is shown in fig. 4.

Discussion

Relative goodness-of-fit

It appears from an inspection of the results concerning $\hat{V}_P$ and $\hat{V}_T$ (fig. 3) and their difference $Q = \hat{V}_P - \hat{V}_T$ (fig. 4) that an f^{-4}-tail fits the observed spectra statistically significant better than an f^{-5}-tail. This is put in quantitative terms as follows. The mean and standard deviation of Q are 0.0948 and 0.0519 respectively, based on 82 observed spectra. With the Student-t test the hypothesis that the expected value of Q is larger than zero is accepted even at a confidence level as high as 99.9% (assuming the sample values of Q to be Gaussian-distributed).

The above quantitative assessment of the goodnes-of-fit of an f^{-4}- or an f^{-5}-tail cannot be compared with similar quantitative tests in the literature since such tests have not been carried out before, at least not to the knowledge of the present authors. Instead, several investigators have used a more graphic approach in which the observed spectra were multiplied with f^4 or with f^5. The functions thus obtained appeared to be more nearly a constant as a function of frequency with the f^4 multiplication than with the f^5 multiplication, e.g. Donelan et al., 1985; Forristall, 1981, Kahma, 1981; Mitsuyasu et al., 1980; Toba, 1973.

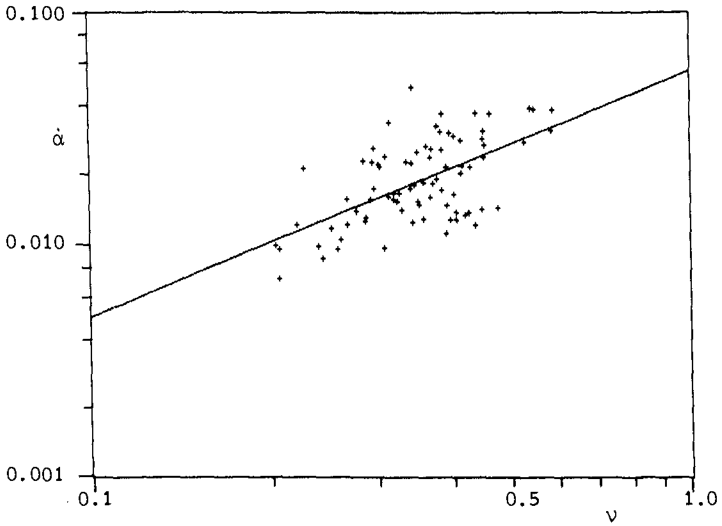

Fig. 1 - Data points: best-fit values of Phillips' α vs. dimensionless peak frequency ν. The straight line represents the best-fit power law relationship (7).

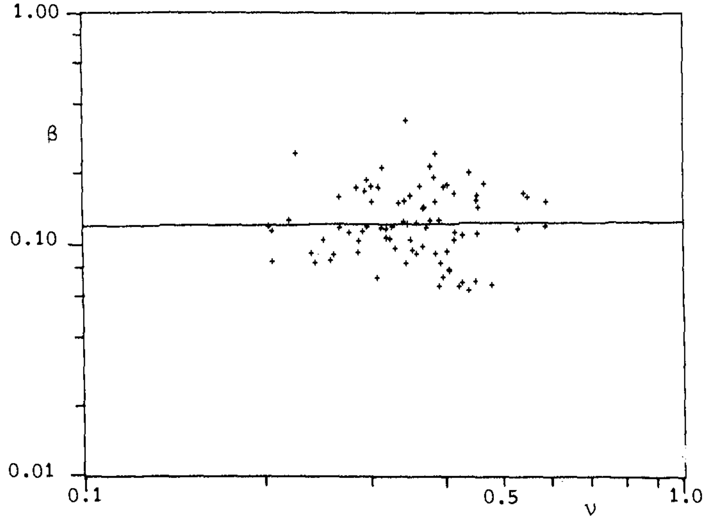

Fig. 2 - Data points: best-fit values of Toba's β vs. dimensionless peak frequency ν. The straight line represents the best-fit power law relationship (7).

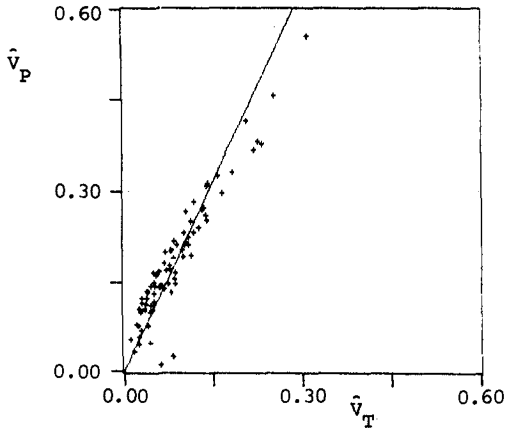

Fig. 3 - Data points: normalized error measures $\hat{V}_P$ (Phillips) and $\hat{V}_T$ (Toba) per observed spectrum. The straight line represents the best-fit proportional relationship.

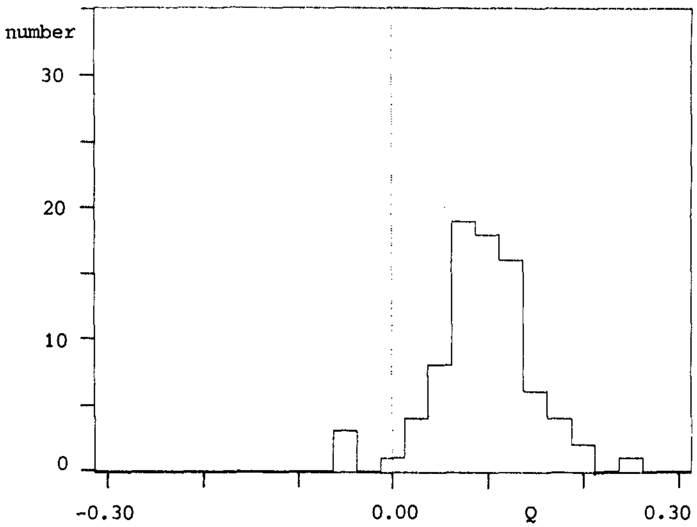

Fig. 4 - Histogram of difference in goodness-of-fit quantified with Q.

Spectral parameters

The discussion of the spectral parameters will focus on the energy scale parameter β and its possible variation with the dimensionless peak frequency ν, compared to the parameter α.

The best fit values of β have been plotted against ν in fig. 2, together with the calculated best-fitting power law relationship between the two. The data show a considerable scatter, but no trend of β with ν is visible. In fact, the hypothesis that the exponent (s) in the power-law relationship between β and ν is zero is accepted at a 99.9% confidence level, using a Student-t test (Draper and Smith, 1981). This observed lack of systematic variation of Toba's β with the growth stage parameter ν contrasts with the variation of Phillips' α (see fig. 1). It supports Toba's original model, insofar as this predicted that β should be a constant. However, in this context the unexplained relatively large scatter in the observed values is disturbing. (It is noted that the unexplained scatter did not decrease by correlating β with ν_*, the peak frequency scaled with g/u_*.)

Among the numerical results concerning Toba's β published previously, only those presented by Donelan et al. (1985) indicate a clear trend with ν. (Although Donelan et al. use a different formulation than the one used here, their results can be transformed into a relation between β and ν; the β-values so obtained have a mean value of about 0.11. Details of this transformation are given in Battjes et al., 1986.) A comparison of these results with those obtained in this paper is given in fig. 5. It is clear that the β-values found by Donelan et al. are correlated with ν, in contrast with those obtained in the present study from the JONSWAP data. The smaller scatter in Donelan et al.'s data suggests that the conditions during the observations of Donelan et al. were more homogeneous (with less unknown variability in geophysical factors) than those during JONSWAP.

The mean values of β from this study are compared in Table II with values reported in the literature. All entries in this table, except those of the present study and those referring to Donelan et al. (1985), have been obtained from Phillips (1985).

A striking outlier in Table II is the value of β from the laboratory data of Toba (1973). It is much lower than those obtained from the field data. Phillips (1985) ascribes this difference to strong wind-drift effects in the laboratory experiments. The values of β obtained from field data (except JONSWAP) vary from 0.06 to 0.11. As pointed out by Phillips (1985), there appears to be a remarkable geographically-based stratification in these results, the Pacific data being considerably lower than the non-Pacific data. The mean value of β from the present study, based on the JONSWAP observations (0.13, with a standard deviation of about 0.06) is somewhat higher than those from the other non-Pacific field data.

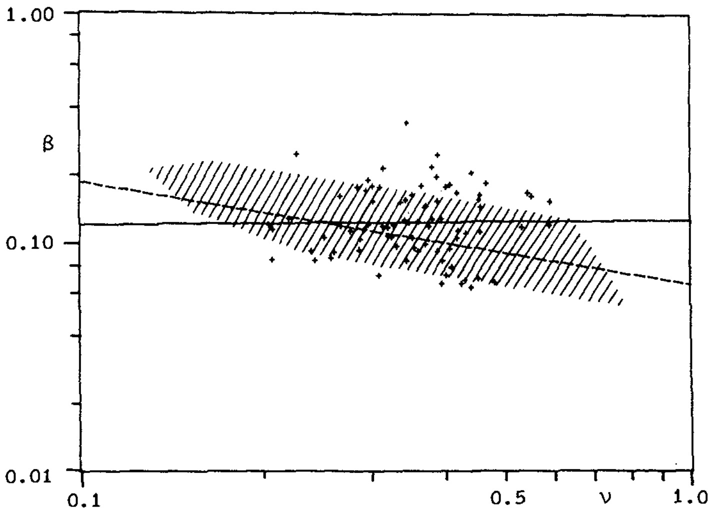

Fig. 5 - Data points and drawn line: see fig. 2. The hatched area and the dashed line represent the range of the transformed results of Donelan et al. (1985) and the corresponding best-fit power law relationship.

author	mean value of β
laboratory	
Toba (1973)	0.02
field	
Kondo et al. (1973)	0.06
Kawai et al. (1977)	0.062
Mitsuyasu et al. (1980)	0.087
Kahma (1981)	0.11
Forristall (1981)	0.11
Donelan (1985)	0.11
this study (JONSWAP)	0.13

Table II - Mean value of the energy scale parameter β from various sources.

CONCLUSIONS

A re-analysis of the original spectra measured in the JONSWAP campaign indicates that the high-frequency part of the wave spectrum can be better approximated with an f^{-4}-tail than with an f^{-5}-tail. The difference in goodness-of-fit is statistically significant at a level of confidence of 99.9%.

The estimated values of the energy scale parameter β in the Toba spectrum are found to be virtually independent of the stage of wave development represented by the dimensionless peak frequency ν. This finding is consistent with Toba's hypothesis that β should be a universal constant. However, it differs from the data of Donelan et al. (1985), whose results indicate a noticeable increase of β with a decrease in ν.

The β-values as determined from the JONSWAP data display a relatively large scatter. Their average value is slightly larger than the mean values reported in the literature.

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