CHAPTER 3

MATHEMATICAL AND PHYSICAL WAVE DISTURBANCE MODELLING COMPLIMENTARY TOOLS

by

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ABSTRACT

This paper presents comparisons between physical and numerical model reproductions on the basis of comprehensive wave disturbance studies of a major spanish port.

Mathematical modelling has reached in many cases a degree of reliability comparable to that of a physical model, but it is essential that both types of modelling systems are validated against measurements.

1.- INTRODUCTION

During the past decade, the advancement of digital computers and numerical techniques have made possible a deterministic numerical modelling of wave penetration in coastal areas and ports, i.e. a mathematical reproduction of wave time series resulting from the combined effects of refraction, diffraction and (partial) reflection of irregular waves. Such tools have now reached a degree of reliability comparable to that of a physical model. Even though further developments of the numerical tools are still needed, the coastal and port engineer is already now faced with the question whether to consider physical and numerical models as alternative or as complimentary tools.

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On the background of comprehensive wave disturbance studies of a major Spanish port, this paper analyses strengths and weaknesses of the two tools applied (physical and numerical model), and outline the contribution that the individual tools would make to the decision basis required by the port authority, today and in the future. The analysis contains qualitative and quantitative comparisons between physical and numerical model reproductions of the same port layout and wave conditions.

2.- PHYSICAL MODELLING

2.1.- Introduction

Complete similitude between hydraulic scale models and their prototypes is not possible unless the scale is such that the model is as large as its prototype. Some priorities must be established in order to simulate correctly the most important forces of the system considered.

In Coastal Engineering projects the forces on system elements consist of the kinetic reaction due to the inertia of an element's mass, gravity, viscous shear, surface tension, elastic compression and the pressure forces.

For overall similarity, the ratio of inertia forces, model to prototype, must equal the ratio of the vector sum of the active forces.

If gravitational forces predominate the ratio (Froude Number) model/(Froude Number) prototype, must be equal to 1, where the

Froude Number = $\dfrac{V}{(gL)^{1/2}}$

If viscous fores predominate the ratio

(Reynolds Number) model /(Reynolds Number) prototype, must be equal to 1, where the Reynolds Number = $\dfrac{LV}{(\mu/\rho)}$

If Surface Tension effects predominate the ratio

(Weber Number) model /(Weber Number) prototype, must be equal to 1, where the Weber Number =

If Elastic Compression forces predominate the ratio

(Mach Number) model /(Mach Number) prototype, must be equal to 1, where the March Number = $\dfrac{V}{(\sigma/\rho L)^{1/2}}$

2.2.- Estuary Models

There are many problem areas concerned with tidal model- ling techniques but tidal action provides the major amount of system energy, and as gravitational forces are predominant in tidal flows the Froude Number guides the modelling process.

Due to the large areas that must be simulated the use of distorted scale models is usual but the higher the degree of distortion used in the model, the greater is the roughness which is required in the model.

2.3.- Harbour Wave Action Models

Short period wind waves (T=5-20 sec) are designed in accordance with the Froude model law and are constructed geometrically similar to their prototypes.

When reproducing intermediate and long period waves (T> 20 sec) excessively large scale models are usually required because, otherwise, friction effects may be excessive and it results in excessive bottom friction losses. In such cases distorted linear scales are usually adopted. It also provides easier measurements of wave heights but wave reflection effects are increased.

Similarity of diffraction requires that the linear scales for horizontal distances on the model be equal to the wavelength scale. This ensures a correct simulation of modes of oscillation in the basins for all wave periods and, therefore, the occurrence of resonance at correct wave periods.

For long waves of small amplitude at depths smaller than 0,05 x wavelength distorted scale models reproduce wave refraction, diffraction and resonant periods accurately. For bigger depths the scale distortion has the effect of distorting the wave refraction patterns.

2.4.- Coastal erosion models.

Most of the fluid processes involved are complicated by non-linear fluid behaviour, turbulence and complex boundary conditions. In attempting to develope similitude relations, the idea of reproducing the dominant physical processes may be abandoned and attention turned to an attempt to mantain similitude of the beach profiles and longshore transport rates.

If adequate prototype data ara available and verification procedures in the model are successful we can have confidence in the results of the model although the combination of forces that occur in the prototype cannot always be reproduced exactly in the model.

3.- NUMERICAL MODELLING

3.1.- The "System 21. Mark 8"

The numerical model "System 21 Mark 8" considered here is based on the time-dependent vertically integrated Boussinesq equations of conservation of volume and momentum, assuming constant density. It is able to simulate unsteady two-dimensional flows in vertically homogeneous fluids. The Boussinesq terms account for the deviation from hydrostatic pressures distribution due to vertical accelerations and are of special interest to the short wave simulations because they make possible to consider a large range of water waves without being restricted by linear assumptions. The equations include porosity terms by means of which it is possible to consider partial reflection and wave transmission from and through piers and breakwaters. The model was described in detail by Abbott et al (1978, 1983) and the equations are included in Appendix A.

3.2.- Advantages and drawbacks of numerical modelling.

As mentioned in section 2 a general problem in physical modelling is the distorted scales. In a numerical model this problem does not exist at all and as long as you can describe the phenomena with a set of mathematical equations, prototype simulations can be made no matter how large the model area is. This is a significant advantage considering estuary models.

Often the numerical and the physical models are applied as complimentary tools. The numerical model can be used to establish the current pattern in a large area and to provide boundary conditions for a detailed physical model of a minor area of special interest.

In the numerical as well as in the physical model the accuracy of the solution will depend on the quality of the boundary data. However it is much easier to control the inflow conditions in the numerical model where the required variation of the water level or discharge simply is specified.

Another problem related to wave modelling is to control reflections in the model area. In the physical model it is very difficult to avoid reflections of long waves from the model boundaries and from the paddle generator. This is no problem in a numerical model where the wave energy can be damped out artificially along the closed boundaries or can be allowed to pass through an open boundary without any re--reflection.

Partial reflection from piers and breakwaters can also be simulated in the numerical model. You can either achieve a certain specified reflection coefficient or you can simulate the actual partial reflection from a given vertical rubble mound. Natural breakwater, however, are not always vertical and in this case experimental data is neccessary to estimate the reflection coefficients.

Finally there is the problem of data collection from the model tests. In the physical model you will have to measure for instance the velocity at certain pre-selected locations. If the scale is very distorted this cannot easily be done without affecting the flow you are actually measuring. Furthermore the data collection can only be made in a minor number of pre-selected points. In the numerical model the results will be computed in every single grid point which means that a huge number of information can be stored on a tape for later display by graphics. As an example you can determine significant wave heights in every single grid point leading to a very accurate map of isolines.

On the other hand it should of course be mentioned that the numerical model is limited by the size of the computer and wave simulations longer than 25 minutes real time are seldom made. In some situations this could be a drawback cosidering the statistics made on the results. Therefore you have to make sure that the relative significant wave heights determined do not vary with the length of the simulation.

4.- A CASE STUDY

The extension of the harbour of Bilbao has been subject to intensive studies to obtain its optimum configuration considering social, economical, and technical reasons. The port area considered for the extension covers an area of 6 by 4 km. (See fig. 1)

As part of the complete study, investigations have been carried out at the Danish Hydraulic Institute (DHI) using a mathematical modelling system (System 21 Mark 8) covering the model area with approximately 70.000 computational points.

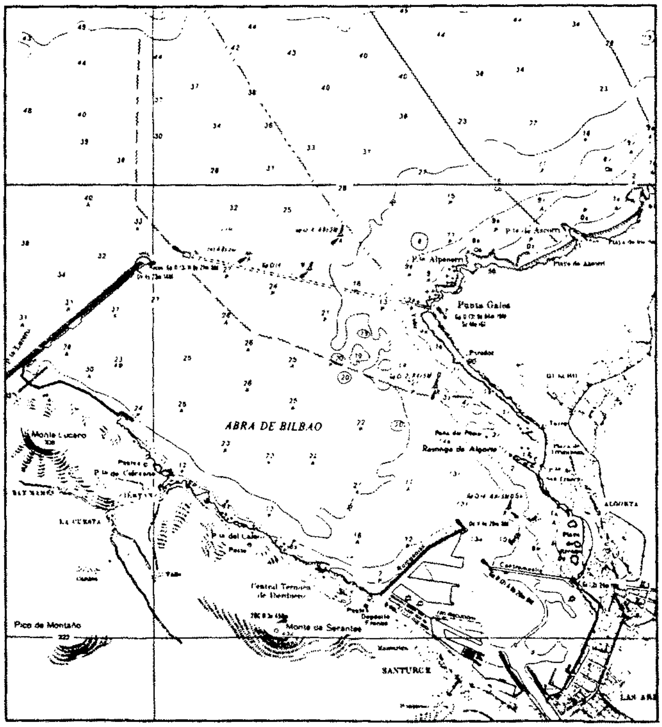


Fig. 1.- Extension Area

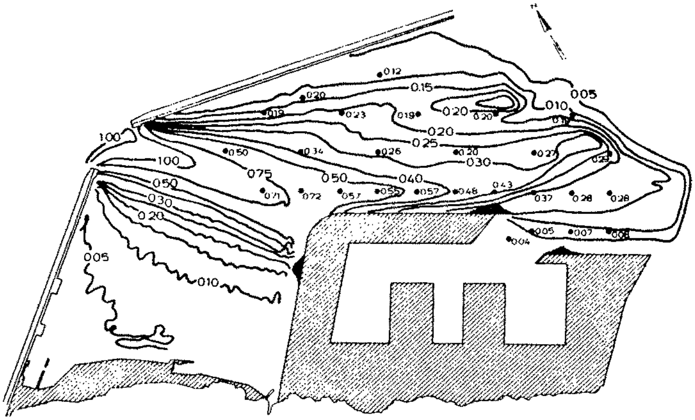


Fig. 2.- Physical and Mathematical Model Results

The grid size used for the present application was 20 m., while the time step was 1.25 seconds. This combination of grid size and time step has been considered a reasonable compromise between high accuracy of the numerical scheme for the type of waves to be simulated and realistic computer requirements.

A simulation period of 20 minutes has been chosen, allowing waves to reach the innermost parts of the harbour and to be reflected an still leave a reasonable amount of data for the following statistical analysis.

Simultaneously, studies using a physical model (scale 1:150) have been carried out at the Centro de Estudios de Puertos y Costas (CEPYC) in Madrid.

It includes the inner harbour and the area covered by the model in the laboratory is 2875 m^2.

Where partial reflection was desired a 0.33 reflection coefficient was chosen and the section obtained in some previous one-dimensional tests was constructed in the model. This section was composed of a concrete slope with stones sticked on it.

Both modelling systems (numerical and physical) have been used to reproduce time series of irregular waves synthesized on the basis of a Jonswap spectrum. A peak period of 19 seconds and a significant wave height of 4.75 m. have been considered.

5.- COMPARISON BETWEEN PHYSICAL AND NUMERICAL MODELLING

5.1.- Introduccion

Two different layouts of the harbour have been considered in this paper.

The first layout is composed of two outer breakwaters and a single operational zone close to the already existing harbour of Bilbao (Fig. 3)

The second layout is composed of the same two outer breakwaters as in the first case, but it also includes a complete development of the western side of the protected area. Additional protection is obtained through the construction of an inner breakwater with a bending of almost 90 degrees (Fig. 4 and Fig. 5)

Two different wave conditions have been considered:

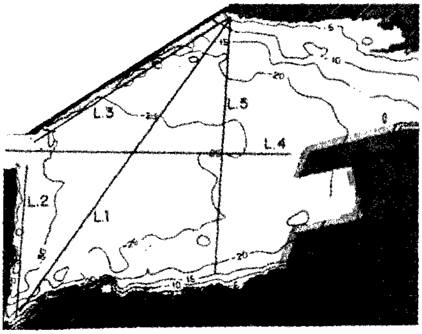


Fig. 3.- Test 1. Bathimetry and comparison lines

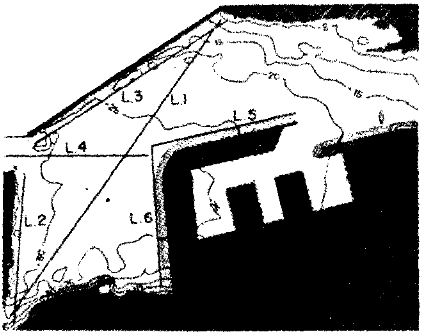


Fig. 4.- Test 2. Bathimetry and comparison lines

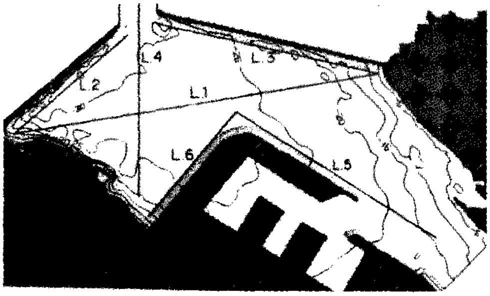


Fig. 5.- Test 3. Bathimetry and comparison lines

- Spectral Moments of order 0, 1, 2 and 4
- Peak frequency (Fp) and Spectral Density at the peak (Smax)
- Peakness Parameter (Qp) as proposed by Goda

We also had to compute the FFT spectra for comparison with the AR algorithm, the spectra computed were: the raw spectrum and smoothed spectra of 8, 16 and 32 degrees of freedom. Two spectral windows, Barlett and Rectangular, were used for each estimation.

The reason behind, this is the dependence of the chosen parameter upon the degree and shape of smoothing. Moreover, in order to get a feeling of the spectral shape, we drew graphs for a few number of wave records with one FFT estimation against the 40 AR calculated spectra, and therefore we could compare the FFT estimation with the 40 orders. Figure 2 shows an example of this kind of graph. An ordinary spectrum was selected and we ould observe some characteristics that come out in all AR estimations:

- A peak appears for high frequencies and slides towards low frequencies for higher orders of the AR model.
- A new peak is born around the 20th. order.
- We notice that we cannot increase indiscriminately the order, since for higher orders both peaks could merge.
- Difficulties in modelling the FFT results for low frequencies of the AR estimations, and reaching the Fp of the FFT for low orders of the models.

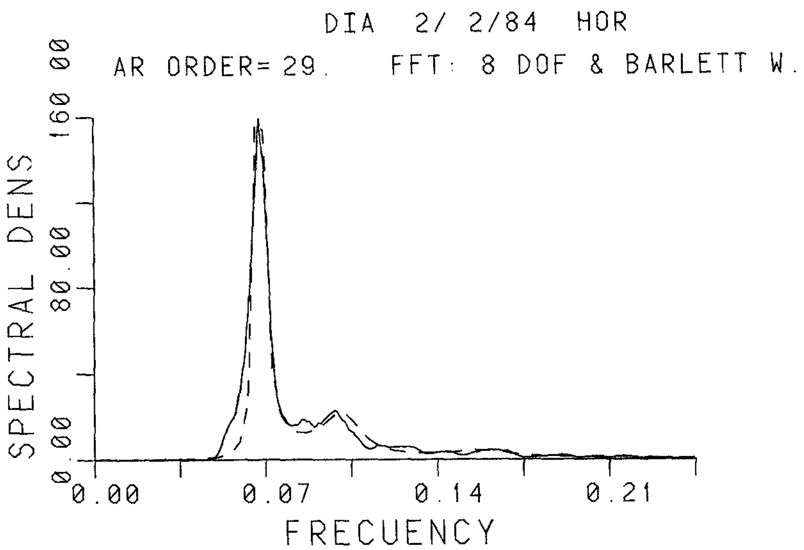


Figure 1.Comparison between FFT spectrum versus Maximum Entropy Spectrum

We could obtain in some cases fittings, as Figure 1 shows, between FFT estimation and an AR model.

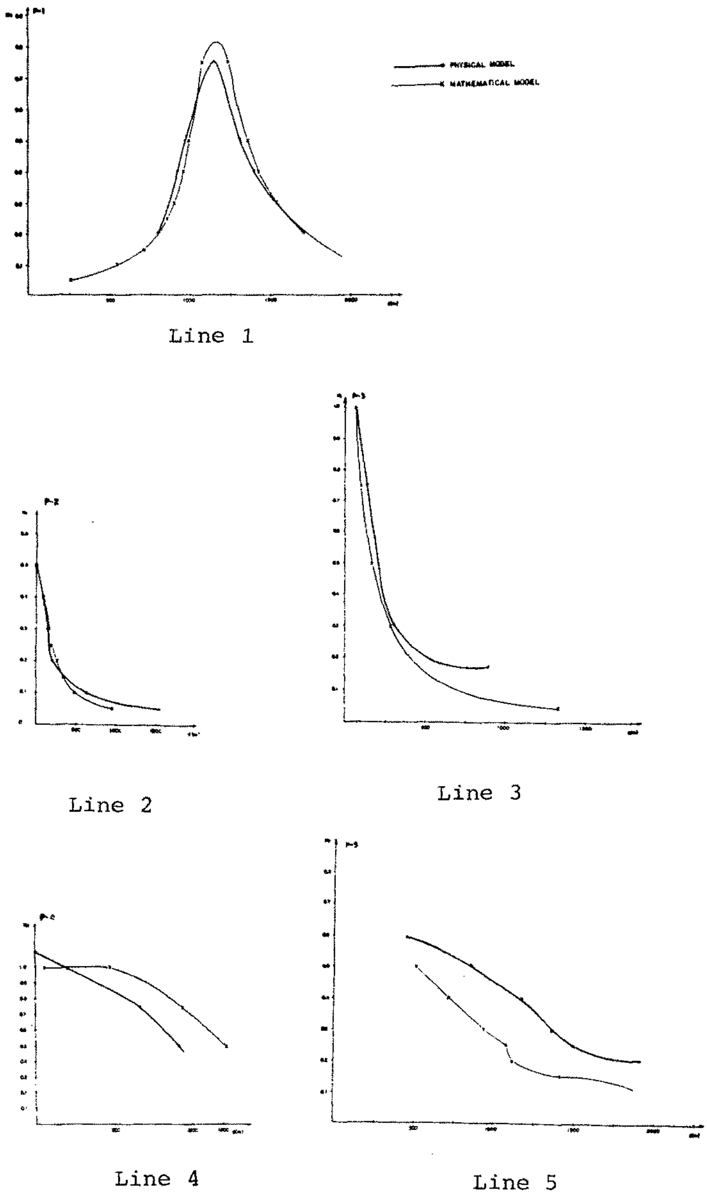
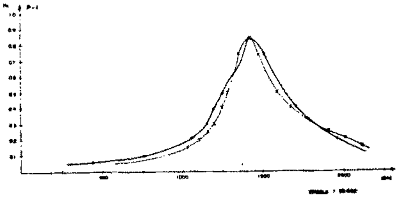


Fig. 6.- Test 1. Comparison of results



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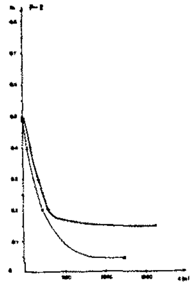

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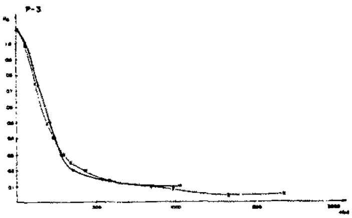

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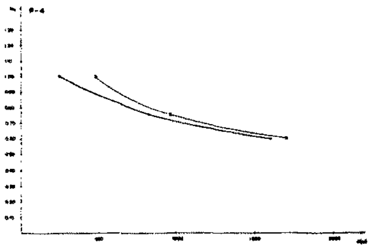

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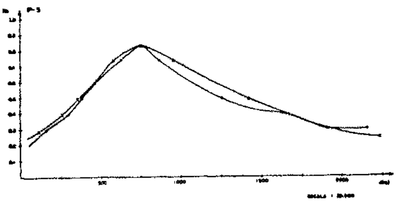

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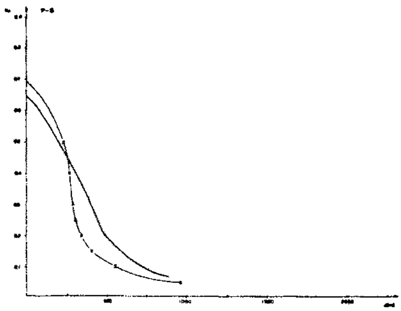

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Fig. 7.- Test 2. Comparison of results

of waves is predominant. In these cases the waves heights obtained from the physical model are higuer than those obtained in the numerical model. Hence it can be concluded that the partial reflection from the piers is different in the two systems of modelling.

5.4.- TEST 3

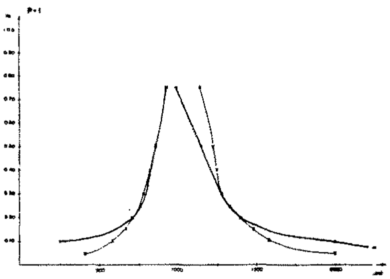
The lines along which comparisons have been made can be inFig. 5 and the comparisons along these lines are shown in fig. 8

As in TEST 2 discrepancies between the numerical and the physical model results appear in the points affected by the partial reflection from the bended inner breakwater in the second layout. Obviously the partial reflection coefficients achieved in the two modelling systems are different. In the physical model some preliminary one-dimensional tests have been carried out using regular waves. A table of reflection coefficients has been established as a function of depth, wave period, wave height, slope and porosity of the structure. Knowing the characteristics of the structure in the prototype some assumtions on the reflection coefficient can be made and modelled in the Laboratory.

In the numerical model the same assumptions can be applied and one can obtain the desired reflection coefficients through the specifications of the rubble mound. The equations solved by the numerical model include porosity which makes it possible to simulat the flow inside vertical porous structures. As shown by Madsen and Warren (1984), the partial reflection from vertical rubble mouds can be simulated very accurately using this method. The reflection will depend on the porosity, the diameter of stones, the width of the rubble mound, the water depth, the wave height and of the wave period. Natural breakwaters, however, are not always vertical and in this case the reflection will be different. In th numerical model, one can still obtain the reflection desired, just by changing the specifications of the vertical rubble mound, but to actually estimate the reflection for a given type of non-vertical construction, experimental data is reequiered.

On the other hand, scale effects will influence the reflections obtained in the physical model. Hence, it becomen important to validate both modelling systems directly against field measurements.

This has not been possible in the present situation and for the time being it can only be concluded that the reflections from the proposed inner breakwater in layout no. 2 are different in the physical and the numerical modelling.



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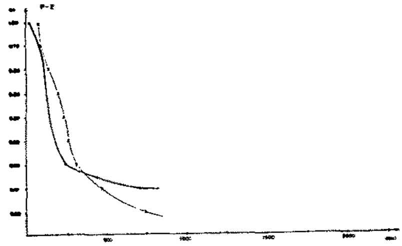

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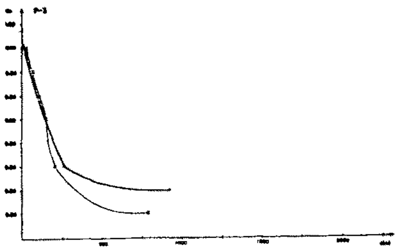

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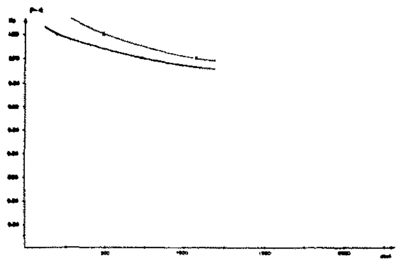

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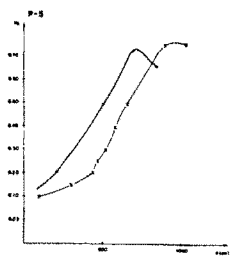

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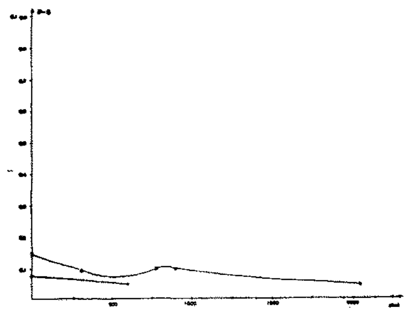

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Fig. 8.- Test 3. Comparison of results

Finally some other general tendencies can be pointed out on the basis of the three cases studied. The results obtained from the physical model are slightly higher in the more inner points at the lee of the breakwaters and slightly lower in the main direction of wave propagation than the results obtained by the numerical model. The explanation for this discrepancy can be found in the way the wave input has been specified in the two types of modelling. Firstly a one-directional input has been applied in the numerical model leading to a perfectly straight wave front at the entrance to the harbour. Secondly some variation across the entrance was observed to take place in the physical model. The reason for this variation is that part of the outer harbour bathymetry was included in the physical model. Hence along the line where the input to the numerical model was supposed to be one-directional, the wave generated in the physical model actually turned out to be directional.

Sand et al (1983) investigated the effect of directional diffraction of waves in a numerical and a physical model. Both modelling systems showed that right in the opening of the harbour the directional wave heights were smaller than the one-directional wave heights but behind the breakwaters the directional waves were clearly higher. This explains the present discrepancy between the physical and numerical model results.

6.- CONCLUSION

The transmission of waves from the sea into a harbour protected by breakwaters is a process which involves shoaling, refraction, diffraction and partial reflection processes.

This paper presents comparisons between numerical model simulations and physical model tests from a practical case study.

It is proven that engineers can confidently apply such models to the study of development projects for harbours and coastal regions. However, until now, numerical wave disturbance models have been validated primarily against results from physical models where scale effects appear. Bottom Friction, wave transmission through pervious structures, and wave reflection are phenomena that with the geometrical similarity are ill-considered in physical models and should be subject to special consideration. Hence, it is essential that both types of modelling systems (physical and numerical) are validated against field measurements.

7.- REFERENCES

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8.- APPENDIX A

The equations solved by the numerical model

The numerical model is based on the time-dependent vertically integrated Boussinesq equations conserving mass and momentum. The model is described in detail by Abbott et al. (1983). The equations are given below:

Continuity

$$n\frac{\partial \zeta}{\partial t} + \frac{\partial p}{\partial x} + \frac{\partial q}{\partial y} = 0$$

x-momentum

$$n\frac{\partial p}{\partial t} + \frac{\partial}{\partial x}\left(\frac{p^2}{h}\right) + \frac{\partial}{\partial y}\left(\frac{pq}{h}\right) + n^2 gh\frac{\partial \zeta}{\partial x} + n^2 p\left(\alpha + \beta\sqrt{\frac{p^2}{h^2} + \frac{q^2}{h^2}}\right)$$

$$-\frac{p^2}{nh}\frac{\partial n}{\partial x} - \frac{pq}{nh}\frac{\partial n}{\partial y} = n\frac{Hh}{3}\left(\frac{\partial^3 p}{\partial x^2 \partial t} + \frac{\partial^3 q}{\partial x \partial y \partial t}\right)$$

y-momentum

$$n\frac{\partial q}{\partial t} + \frac{\partial}{\partial y}\left(\frac{q^2}{h}\right) + \frac{\partial}{\partial x}\left(\frac{pq}{h}\right) + n^2 gh\frac{\partial \zeta}{\partial y} + n^2 q\left(\alpha + \beta\sqrt{\frac{p^2}{h^2} + \frac{q^2}{h^2}}\right)$$

$$-\frac{q^2}{nh}\frac{\partial n}{\partial y} - \frac{pq}{nh}\frac{\partial n}{\partial x} = n\frac{Hh}{3}\left(\frac{\partial^3 q}{\partial y^2 \partial t} + \frac{\partial^3 p}{\partial x \partial y \partial t}\right)$$