CHAPTER 5

Numerical Simulation of 1964 Tsunami Across the Pacific Ocean

Michael H. Chen [1]

A two dimensional numerical longwave model using an appropriate open sea boundary condition has been developed. The use of the open-sea boundary condition makes it possible to simulate longwave propagation using a smaller region without covering the entire ocean. The numerical model is used to predict the arrival time of tsunamis resulting from the 1964 Alaskan earthquake at various stations with reasonable success.

Introduction

Tsunami is a seawave generated by a near shore shallow-water undersea ground movement. It usually causes major damage to the coastal area not only next to the epicenter of the earthquake, but also at a distance away. Due to its nature, studies of the tsunami can be classified into three categories: (1) generation, (2) propagation and dispersion across the ocean and (3) response at the coastal region. A very lengthy summary of references regarding every aspect of tsunami phenomena was presented by Wiegel (1980) in his study of tsunamis in the Philippines.

Most analytical models related to tsunamis are either limited to a one dimensional model or a two dimensional model with a constant water assumption. Mathematical models are also available for water depths which vary in some specific manner.Using an integral transform technique, Carrier (1966) developed a one dimensional analytical model to relate the wave height at the source region to a location far away. The generation of a tsunami in the region adjacent to the earthquake epicenter has been studied extensively by Hammack (1972) in his theoretic and experimental work.

Apart from the theoretic approach, many numerical models have been developed to study the longwave phenomena. Using a space-staggered implicit finite difference method, Leendertse (1967) gave a detailed discussion on the aspects of a numerical model for the longwave equation. Hwang et al. (1973) developed a numerical longwave model in spherical coordinates using methods described by Leendertse to examine the effect of the earth's curvature on the propagation of a seawave. A similiar two dimensional numerical model was also presented by Skovgaard and Jonsson (1980). In their studies, Homma's mathematical model for a longwave over a parabolic island was included.

[1]Member of Technical Staff, TRW Defense System Group, One Space Park, Redondo Beach, CA 90278

In the present study, a numerical longwave model in spherical coordinates has been developed using an alternated direction. explicit finite difference method proposed by Loomis (1973). A suitable open sea boundary condition based on the concept of matched impedance is adapted. Applying this open sea boundary condition, it is possible to investigate wave response at a particular area in the middle of Pacific Ocean without including the entire ocean.

The performance of the numerical model is verified by performing a simulation of a wave impinging upon a circular island over a constant water depth, and a real time tsunami simulation resulting from the 1964 Alaskan earthquake.

Governing Equations

With the assumption that the Coriolis force and convective terms can be neglected and the wave height, η, is small compared to the original water depth, d, the linearized longwave equations in spherical coordinates can be expressed as:

$$\frac{\partial u}{\partial t} = -\frac{g}{R}\frac{\partial \eta}{\partial \theta} \tag{1}$$

$$\frac{\partial v}{\partial t} = -\frac{g}{R sin\theta}\frac{\partial \eta}{\partial \phi} \tag{2}$$

$$\frac{\partial \eta}{\partial t} = -\frac{1}{R sin\theta}\left[\frac{\partial (d u sin\theta)}{\partial \theta} + \frac{\partial (d v)}{\partial \phi}\right] \tag{3}$$

where u and v are the velocity components in the x and y direction, respectively; d and η are the original water depth and wave amplitude, respectively; θ and ϕ are the angles related to the local latitude and longitude respectively; R is the radius of the earth and t is for the time.

The above set of equations are then expressed in their finite difference forms to construct the numerical model for investigaing the longwave propagation and its response at the coastal area.

Boundary Treatment

In addition to the stability problem related to the numerical scheme, the specification of the boundary condition at the artificial boundary where the computation terminated is also very important. In his survey articles on acoustic radiation, Shaw (1970) related several physical boundary conditions to their corresponding mathematical forms. The most suitable boundary condition at the open sea is the matched impedance condition where the wave is assumed to be totally transmitted.

Applying linearized wave theory with the assumption of constant depth beyond the computational domain, the matched impedance condition becomes

$$\frac{\partial \eta}{\partial t} = -\frac{c}{R sin\theta}\left[cos\delta\,\frac{\partial \eta}{\partial \theta} + sin\delta\,\frac{\partial \eta}{\partial \phi}\right] \tag{4}$$

where c is the wave speed and δ is the angle between the wave direction and the local open boundary.

This is equivalent to the radiation condition given by Sommerfeld (1949) at the far field for the scattered wave. Equation (4) will be used at the open sea boundary where the time derivative of the wave amplitude can be related to its spatial derivative.

In addition to the open-sea condition, a treatment at the solid boundary is also required. This treatment is relatively quite simple and can be achieved by appling the null nomal derivative conditions for all flow quantities. The condition for the wave amplitude at a solid boundary is:

$$\vec{n} \cdot \left[\frac{\partial\eta}{\partial\theta}\hat{i} + \frac{\partial\eta}{\partial\phi}\hat{j}\right] = 0 \tag{5}$$

where $\hat{i}$ and $\hat{j}$ are unit vector in the θ and ϕ direction respectively and $\vec{n}$ is the local outward normal at the shoreline.With these boundary treatments, the numerical longwave model can then be formulated.

Finite Difference Scheme

The numerical model is constructed by using a space and time staggered method. The flow quantities are assigned at a location half a grid space apart. The vector quantities such as u and v are assigned at the edge of the grid element. On the other hand, the scalar quantities, d and η, are assigned in the middle of the grid element. The value at the location where a particular quantity is not computed is approximated by its average value among its four surrounding points.

Using the above arrangement for the flow quantities, the finite difference equations of the linearized longwave equations are:

$$u_{i,j}^{n+1} = u_{i,j}^{n} - \frac{g\Delta t}{R\Delta\theta}\left[\eta_{i,j}^{n+1} - \eta_{i,j-1}^{n+1}\right] \tag{6}$$

$$v_{i,j}^{n+1} = v_{i,j}^{n} - \frac{g\Delta t}{R sin\theta_j \Delta\phi}\left[\eta_{i,j}^{n+1} - \eta_{i-1,j}^{n+1}\right] \tag{7}$$

$$\begin{aligned}\eta_{i,j}^{n+1} &= \eta_{i,j}^{n} - \frac{\Delta t}{2R sin\theta_j}\Big\{\left[(d_{i+1,j}^{n} + d_{i,j}^{n})v_{i+1,j}^{n+1} - (d_{i,j}^{n} + d_{i-1,j}^{n})v_{i,j}^{n+1}\right]/\Delta\phi \\ &+ \left[(d_{i,j+1}^{n} + d_{i,j}^{n})u_{i,j+1}^{n+1} sin\theta_{j+1/2} - (d_{i,j}^{n} + d_{i,j-1}^{n})u_{i,j}^{n+1} sin\theta_{j-1/2}\right] \\ &/\Delta\theta\Big\}\end{aligned} \tag{8}$$

Equations (6), (7) and (8) are the basis of the current numerical model. In conjunction with the treatment of solid and open boundary, the propagation of a tsunami over an ocean can then be studied.

The actual integration in time is achieved in two stages. First, the wave amplitude, η, at $(n+1)^{th}$ step is integrated using its value at n^{th} step and the newly calculated velocities, u and v through equation (8). With this newly obtained wave amplitude, $\eta_{i,j}^{n+1}$, the velocities, $u_{i,j}^{n+1}$ and $v_{i,j}^{n+1}$, are then computed using their values at n^{th} step using equations (6) and (7), respectively. This method of integration is not exactly an explicit method but is crucial in the numerical stability. A completely explicit method is usual not stable.

Verification

It is desired to verify the performance of a numerical model by comparing the computed result with either an analytical solution or experimental data. A general solution for the wave height at the shore due to a series of monochromatic waves is given by Morse and Feshback (1953).

Such a solution for a monochromatic wave with an amplitude of one meter and a period of 4 minutes impinging upon a circular island having a radius of 18 km among a uniform depth of 4 km was presented by Vastano and Reid (1966) in their study of tsunami response at an island. A similar simulation has been performed using the present numerical model. The results of three models are shown in Figure 1.

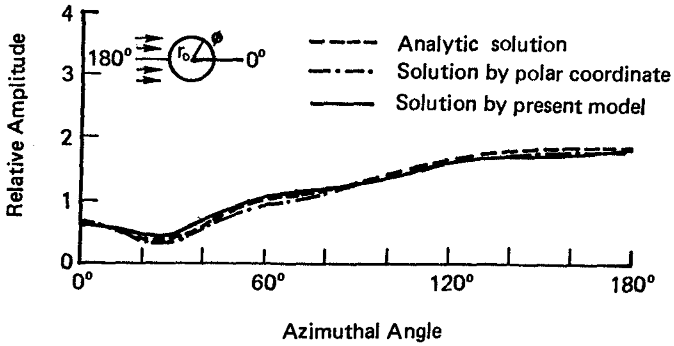

Figure 1. The comparison of maximum wave amplitude at shoreline among three models.

The predicted results by both numerical models agree quite well with the analytic solution. The use of the matched impedance condition at the open boundary transmits the progress wave outward to the open sea successfully. A similiar calculation was also presneted by Skovgaard and Jonsson (1980). In their model, the treatment on the open sea boundary is different. A known solution from the analytical model was used at the open sea. The open-sea boundary treatment presneted in the current study is not adapted in their model.

Results and Discussions

The current numerical longwave model has been applied to simulate the destructive tsunami resulting from the 1964 Alaskan Earthquake. The original water depth is obtained from the world hydrographic chart prepared by the Scripps Institutes of Oceanography. The initial tsunamis distribution is derived from the work of Hwang and Divoky (1970) which uses the ground displacement data of the 1964 Alaskan earthquake summarized by Plafker (1969) as its data to compute the near field tsunami wave height. The computational basin is 91 $\times$ 63 (Longitude by Latitude) which covers the area from the North American continent to the Ilusian Island and from the Alaska to the equator. The grid size is 1^o by 1^o which is relatively large.

The ground displacement aftershock was illustrated by Plafker (1969). It had a broad crustal warping along a northeast - southwest trend hinge line from Prince William Sound, Alaska to Kodiak, Alaska, USA. He suggested that the axis of seabed uplift was also the axis of the source region for the generated tsunami. This axis determined the direction of the main wave. His assertion was clearly demonstrated by the numerical result. The recorded tide gage at several stations along the Pacific Ocean were presented by Spaeth and Berkman (1967). These valuable records are used to verify how well the numerical model performed.

The reported duration for the completion of ground motion was approximately 150 seconds which is very short. In other words, the initial tsunami spatial distribution could be approximated by the net undersea ground displacement, instantaneously. This assumption was adopted by Hwang and Divoky (1970) in their nearfield tsunami calculation in the earlier stage. The surface elevation at 200 seconds in their report is then converted into a 1^o by 1^o grid in spherical coordinates and is used as the initial condition in the current numerical simulation.

This initial surface height used in the current simulation is given in Table 1.

TABLE 1. THE SIMULATED INITIAL WAVE HEIGHT OF TSUNAMIS DUE TO the 1964 ALASKAN EARTHQUAKE
(unit: meter)

	152^o [1]	151^o	150^o	149^o	148^o	147^o	146^o	145^o	144^o	143^o	142^o
59^oN	⋆[2]	⋆	⋆	⋆	⋆	3.01	1.86	1.43	⋆	⋆	⋆
58^oN	⋆	⋆	.402	2.45	5.86[3]	4.61	2.16	1.43	.772	.319	.038
57^oN	⋆	-.49	2.68	5.19	2.55	1.09	.765	.151	.025	.001	0.00
56^oN	.646	1.80	4.99	1.76	.691	.047	.003	0.00	0.00	0.00	0.00
55^oN	.391	1.28	1.12	.243	.025	.000	0.00	0.00	0.00	0.00	0.00

notes:

1. It is the local longtudinal value on the western hemisphere.
2. The symbol, ⋆, represents a zero wave height at the land.
3. The maximum initial wave height is 5.8644 meter.

The computed wave patterns show a strong seiche phenomenon in the Gulf of Alaska, resulting in very strong wave activity for as long as 10 hours after the shock. This agrees very well with the recorded result. As asserted by Plafker, the main wave propagated in a southeast direction toward the North American continent. Therefore, a portion of the wave was reflected and trapped and formed an oscillating pattern within the Gulf of Alaska. Eventually, this seiche phenomenon died out because part of wave energy was radiating outward into the open sea.

After passing through the Gulf of Alaska, the main wave continued its southeast direction and propagated toward the west coast of the North American continent. The current simulated results predict a higher maximum height at the location where there is a higher angle between the incoming wave and the shoreline which agrees with the recorded tide data. According to the tide-gage records resulting from the 1964 Alaskan earthquake, summarized by Spaeth and Berkman (1967), both Crescent City and Hilo experienced a considerably high wave activity.

The predicted wave heights are smaller than the recorded values. For comparison, the predicted and recorded tide marigrams at Crescent City, California are shown in figures (2) and (3). The predicted wave height at this location merely reaches 1 meter which is very small in comparison with the recorded value of 4 meters. This big difference is partly due to the exclusion of the contribution of the astronomical tide in the numerical simulation.

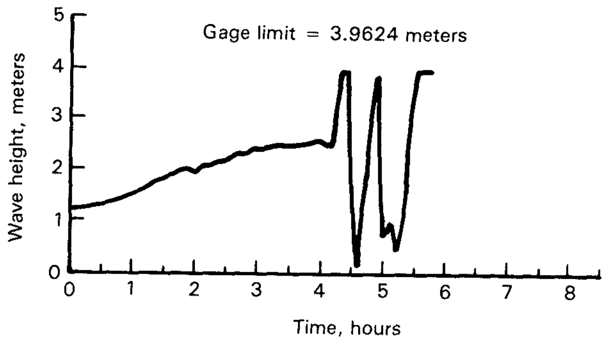

Figure 2. The tide gage record of tsunami at Crescent City, California due to the 1964 Alaskan Earthquake.

From the tide gage record, it indicates the astronomical tide is around 2.6 meters at the moment of the arrival of tsunami. If this value is used as the basis to compute the actual tsunami contribution, the estimated peak tsunami height will not exceed 2 meters. Consequently, the discrepancy between the predicted and record results is reduced considerably.

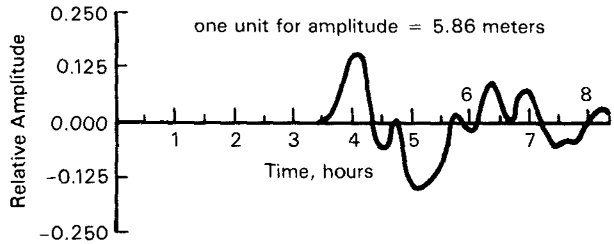

Figure 3. The simulated tsunami time history at Crescent City, California due to the 1964 Alaskan Earthquake.

From the tide marigram, the tsunami arrival time can also be calculated. The arrival time can be determined by the time either when the wave height starts to deviate from its normal state or when the first peak arrives. The former is defined as the first arrival time; the latter is defined as the first peak arrival time. Both arrival times are useful information in the tsunami warning system. The first arrival times at several stations are tabulated in Table 2.

TABLE 2. THE FIRST ARRIVAL TIME OF TSUNAMI RESULTING FROM THE 1964 ALASKAN EARTHQUAKE

Station	Long.° W	Lat.° N	Observed Hr. Min.	Predicted Hr. Min.
Yakutat, Alaska	139	59	1. 24.	0. 20.
Sikta, Alaska	135	57	1. 30.	0. 41.
Crescent City, CA	124	42	4. 03.	3. 28.
San Francisco, CA	122	38	5. 06.	4. 03.
Midway Islands	177	28	4. 51.	4. 21.
Oahu, Hawaii	160	21	5. 09.	4. 40.
Johnston Island	170	17	6. 03.	5. 33.

The difference between the recorded and computed first arrival time is approximately 30 minutes for the island stations and much higher for the stations closer to the source region. If the arrival of the first peak is compared between the computed and the recorded results, the difference reduces to less than 15 minutes at Crescent City, California. In fact, the difference at the other stations is also reduced considerably.

In spite of a use of relatively large grid spacing, 1^o by 1^o; and a very rough treatment in the initial tsunami height distribution, the numerical model still predicts results comparable to the recorded data. The performance of the numerical model can be considered very good.

Conclusion and Recommandations

The major objective of the present study is to develop a suitable boundary condition for the open-sea boundary resulting from the partition of a larger area into a smaller region. With the use of this open sea treatment the large-scale propagation of a tsunami across an ocean can be investigated properly. The matched impedance condition in acoustic radiation is adapted to describe the condition at the two-dimensional open-sea boundary. This open sea boundary condition leads to a satisfactory agreement with the analytic steady-state solution for a monochromatic wave impinging upon a circular island of uniform water depth.

Using 1^o by 1^o spatial grid element and the bathymetry data from the world hydrographic chart, the staggered alternate-direction, explicit scheme of linearized long-wave theory is applied to simulate the 1964 tsunami. In general, the predicted first arrival time is earlier than the recorded values which is due to the difficulty in the treatment of coastal areas, especially at the source region. However, if the arrival of the maximum is used to compared the simulated and recorded data, their differences are much smaller. It indicates the use of the first arrival time as a criterion to determine the performance of the numerical model may not be appropriate. Also, we can conclude that use of a 1^o by 1^o grid element and linear longwave is sufficient for the prediction of tsunamis arrival time.

As pointed out by Goto and Shuto (1980), in their tsunami runup study, the predicted tsunami runup height by the nonlinear model is about 20% above that by the linear model. Based on their findings and the current result, a much finer and localized investigation should be performed using a higher order longwave equations in order to improve the predictive results at the locations such as Crescent City, California and Hilo, Hawaii where the wave amplification is important.

The conventional procedures for a small scale calculation are performed in two stages. First, a large scale calculation covering a much bigger area is performed. In this stage of calculation, all necessary flow quantities at the boundary of the smaller computational basin are stored. Second, these stored results are used as input boundary conditions in the small scale calculation. In this method, the exact effect to the computed flow quantities due to the reflection from a wall condition at the land area is not known. This uncertainity can be removed if both small and large scale calculation are performed, simultaneously.

Acknowledgments

The author wishes to thank the late G. R. Miller of the Joint Tsunami Research Effort, the University of Hawaii. This research was sponsored by a Resident Research Associateship jointly by National Research Council and National Oceanic and Atmsopheric Administration, USA.

References

1. Wiegel, R.L. (1980). Tsunamis along west coast of Luzon, Philippines. Proceedings of the 17th Coastal Engineering Conference, Sydney, Australia, The American Society of Civil Engineers, Vol. 1, pp. 652-671.

2. Carrier, G.F. (1966). Gravity waves on water of variable depth. Journal of Fluid Mechanics, Vol. 24, pp. 641-659.

3. Hammack. J.L. (1972). Tsunami generation and near field propagation. Ph.D. thesis, California Institute of Technology, Pasadena, 261 pp.

4. Leendertse, J.J. (1967). Aspects of a computational model for long-period water wave propagation. The Rand Corporation. Memorandum RM-5294-PR.

5. Hwang, L.S. et al. (1973). Rate Island tsunami model: Generation and open sea characteristics. Bull. Seismol. Soc. America, Vol. 62, pp. 1579-1596.

6. Loomis, H.G. (1973). A package program for time-stepping long waves into coastal regions with application to Haleiwa Harbor, Oahu. U.S. Department of Commerce, Technical Report NOAA-JTRE-79.

7. Skovgaard, O. and Jonsson. I.G. (1980). Transient Finite-Difference Tsunami Calculations. Proceedings of the 17th Coastal Engineering Conference, Sydney, Australia, The American Society of Civil Engineers, Vol. 1, pp. 123-137.

8. Shaw, R.P. (1970). An integral equation approach to acoustic radiation p. 143-180 in Topics in Ocean Engineering, Vol. 2, Edited by C. I. Bretschneider Houston, Gulf Publications.

9. Sommerfeld, A. (1949) Partial Differential Equations. New York, Academic Press, pp. 333-352.

10. Morse, P.M. and Feshbeck, H. (1953). Method of Theoretical Physics. New York, McGraw-Hill.

11. Vastano, A.C. and Reid, R.O. (1966). A numerical study of the tsunami response at an island. Texas A&M Univ. Technical Report No. 66-2T, 141 pp.

12. Plafker, G. (1969). Tectonics of the March 27, 1964, Alaskan Earthquake. Geological Survey Professional Paper, Vol. 543-I.

13. Spaeth, M.G. and Berkman, S.C. (1967). The tsunami of March 28, 1964, as recorded at tide stations. U.S. Department of Commerce, ESSA Technical Report CSGS33.

14. Hwang, L.S. and Divoky, D. (1970). Tsunami generation. Journal of Geophical Reviews, Vol. 75, pp. 6802-6817.

15. Goto, C. and Shuto, N. (1980). Run-up of Tsunamis by Linear and Nonlinear Theories. Proceedings of the 17th Coastal Engineering Conference, Sydney, Australia, The American Society of Civil Engineers, Vol. 1, pp. 693-707.

Notation

c = the wave speed for a longwave.
d = the original water depth.
g = the acceleration coefficient of the gravitational force.
R = the radius of the earth.
t = the time.
u = the velocity component in the θ direction.
v = the velocity component in the ϕ direction.
η = the water surface elevation.
δ = the angle between wave direction and local open boundary.
ϕ = the angle for its longitude value on earth at each location.
θ = the angle between the north pole and each location.
$\hat{i}$ = the unit vector in the θ direction.
$\hat{j}$ = the unit vector in the ϕ direction.
$\vec{n}$ = the unit outward normal vector at the shoreline.