# CHAPTER 7

## THE INFLUENCE OF WAVES ON CURRENT PROFILES

by

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### ABSTRACT

A simple model is presented for steady current profiles in the presence of waves. The current reduction and apparent roughness increase caused by the waves are shown to depend mainly on one dimensionless parameter $\tilde{u}_*/\bar{u}_*$, i.e. the ratio between the friction velocity amplitude due to the waves and the time averaged friction velocity. The model recognises the need to apply different eddy viscosities to different flow components. Also, the thickness of the wave influenced layer near the bed is comceptually separated from the vertical scale of the wave boundary layer.

### INTRODUCTION

The water motion in coastal and estuarine areas is generally a combination of wave motion and currents which can be considered steady compared to the waves. Waves and currents influence each other in mainly two ways. Firstly, variability in current strength will cause wave refraction. Secondly, vigorous wave-induced mixing close to the bed will change the current profile. In the following we shall deal only with the latter type of of interaction.

The problem was studied theoretically by Lundgren (1972), who realised that the waves change the current profile by increasing the eddy viscosity, $\nu_c$, felt by the current in a thin layer near the bed (Figure 1), while outside this thin layer ($z > L$) the waves introduce no mixing. Hence the outer current profile is logarithmic

$$\bar{u}(z) = \frac{\bar{u}_*}{\kappa} \ln (z/z_1) \qquad \text{for } z > L \qquad (1)$$

with the only difference being that the usual zero intercept $z_0 = r/30$ has been replaced by the larger $z_1$; $r$ is the Nikuradse roughness of the bed and $\bar{u}_*$ is the time averaged friction velocity. Thus the wave effect on the outer current profile amounts to a constant shift:

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$$\Delta\bar{u} = \frac{\bar{u}_*}{\kappa} \ln (z_0/z_1) \qquad \text{for } z > L \qquad (2)$$

or an apparent increase in bed roughness from $30z_0$ to $30z_1$.

A complete description of the velocity shift or $z_1/z_0$ must contain the three basic, independent variables: $Aw/\bar{u}_*$, $A/r$, and $\varphi$ i.e.

$$z_1/z_0 = F(A\omega/\bar{u}_*,\ A/r, \varphi) \qquad (3)$$

where A is the horizontal semi-excursion of the wave motion near the bed, $\omega$ is the angular frequency $2\pi/T$ and $\varphi$ is the angle between current and wave propagation. Lundgren evaluated $\ln(z_1/z_0)$ on the basis of wave eddy viscosity measurements by Jonsson and Carlsen (1976) and assuming that the eddy viscosity felt by the current is the one caused by current alone plus the one caused by waves alone, added geometrically.

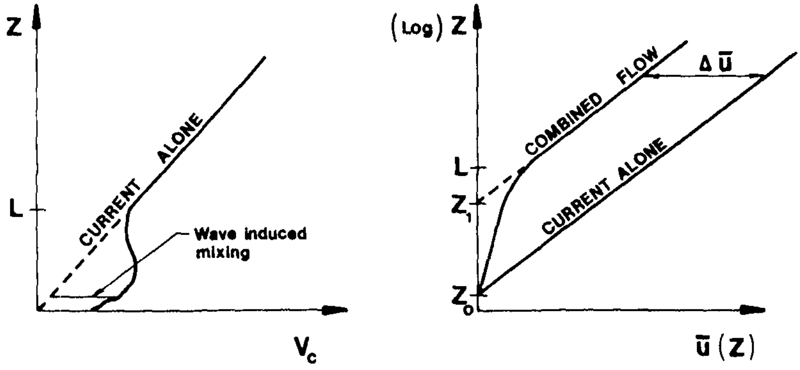

Figure 1 : Waves increase the eddy viscosity $\nu_c$ felt by the current inside a thin layer ($z<L$) near the bed. The resulting current profile is logarithmic for $z>L$, so the wave induced change to the outer profile amounts to a shift u or in other words, to an apparent roughness increase (from $30z_0$ to $30z_1$).

Lundgren did not consider changes in the wave boundary layer structure due to the current. Such changes were estimated later on theoretical grounds by Grant and Madsen (1979) and Christoffersen (1982). Such changes are however very minor according to empirical data and are probably without practical consequence unless the current is very strong ($\bar{u}_* \gtrsim \hat{U}_*$), see Nielsen (1985).

The models of Grant and Madsen and of Christoffersen both assume that the same eddy viscosity applies to both waves and currents, an assumption which seems altogether reasonable a priori. However, surprisingly it is not valid in general. Laboratory measurements by van Doorn (1981) show that the eddy viscosities felt by waves and currents at the same point of the same flow can differ by as much as a factor 4, see Coffey and Nielsen (1984). The fact that different eddy viscosities must be applied to different flow components has also been observed by Bakker and van Kesteren (personal communication). In heuristic terms the phenomenon can be explained as follows : The eddy viscosity can be seen as the product of a turbulent velocity and a vertical length scale i.e.

$$\nu_T = V_\nu L_\nu \tag{4}$$

The turbulent velocity $V_\nu$ is probably equally effective with respect to both waves and currents, but that is not the case for the vertical length scale $L_\nu$. The length scale relevant to the current shear stress is known to grow proportionally to the distance from the bed; but the length scale relevant to the oscillatory shear stress cannot grow beyond a certain fraction of the wave boundary layer thickness or the equivalent Stokes length $\sqrt{2\nu_T/\omega}$. Hence the currents feel a larger eddy viscosity than the waves.

The implication of this is that while the eddy viscosity concept is useful as a formal parameter in simplistic flow models, the interpretation into physical terms is not as straight forward as previously imagined.

## CHOSING MODEL STRUCTURE

The following section is concerned with choosing the appropriate form of the eddy viscosity felt by the current on the basis of empirical evidence. Obviously, following the remarks above, the same eddy viscosity is not expected to apply to the wave boundary layer structure.

Apart from the fairly complicated, empirical curve suggested by Lundgren (1972), two simple forms have been suggested for the eddy viscosity felt by the current inside the wave dominated layer. Christoffersen (1982) suggested a constant eddy viscosity through the bottom layer while Grant and Madsen (1979) recommended a linearly growing eddy viscosity. Both models will in general result in a discontinuity of $\nu_c$ and thus of the current gradient at the top of the wave influenced layer ($z = L$). See Figure 2.

For both models, it is fairly easy to obtain expressions for $z_1/z_0$ by using the terminology of Figure 2 and the usual assumptions: $\tau/\rho = \bar{u}_*^2$ and $\bar{u}(z_0) = 0$.

For the "Christoffersen type model" (Figure 2A) we have :

$$\nu_c = \begin{cases} K\bar{u}_* z_0 F & , \quad z < L \\ K\bar{u}_* z & , \quad z > L \end{cases} \tag{5}$$

and hence :

$$\frac{d\bar{u}}{dz} = \frac{\bar{\tau}/\rho}{\nu_c} = \begin{cases} \dfrac{\bar{u}_*}{\kappa z_o F}, & z < L \\ \dfrac{\bar{u}_*}{\kappa z}, & z > L \end{cases} \tag{6}$$

which with $\bar{u}(z_o) = 0$ gives :

$$\bar{u}(z) = \begin{cases} \dfrac{\bar{u}_*}{\kappa F}(z/z_o - 1), & z < L \\ \dfrac{\bar{u}_*}{\kappa}\ln(z/z_1), & z > L \end{cases} \tag{7}$$

The value of $z_1$ is found by matching the two expressions at $z = z_oF$. We find

$$z_1/z_o = \frac{L}{z_o}\, e^{(1-L/z_o)/F} \tag{8}$$

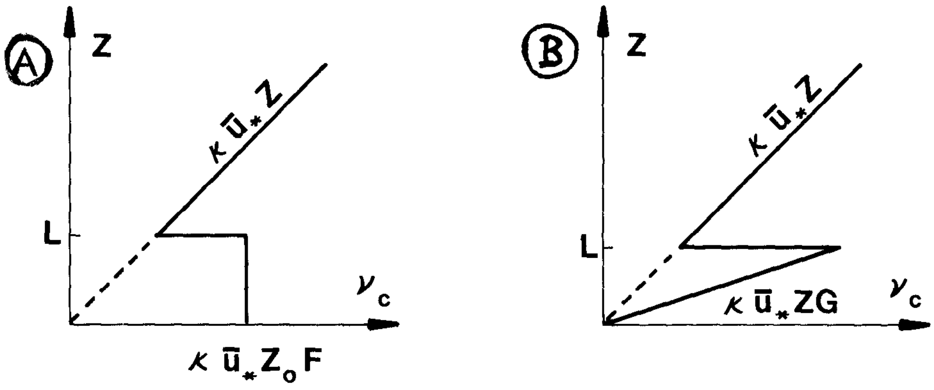

Figure 2 : Two simple previous models for $\nu_c$ in combined flows and the resulting apparent roughness change.

The analogous calculations for the model outlined in Figure 2B lead to :

$$z_1/z_o = \left(\frac{L}{z_o}\right)^{1-\frac{1}{G}} \tag{9}$$

The two results are interestingly similar in that both expressions are assympthotically proportional to $L/z_o$ for large values of the wave-induced eddy viscosity (large F or G). This is very interesting in view of the fact that both Christoffersen and Grant and Madsen hypothesised that L should be closely related to the wave boundary layer thickness $\delta$ . If that was true, the formulae (8) and (9) would indicate that $z_1/z_o$ depend mainly on $\delta/z_o$. The available laboratory data do not support this. See Figure 3.

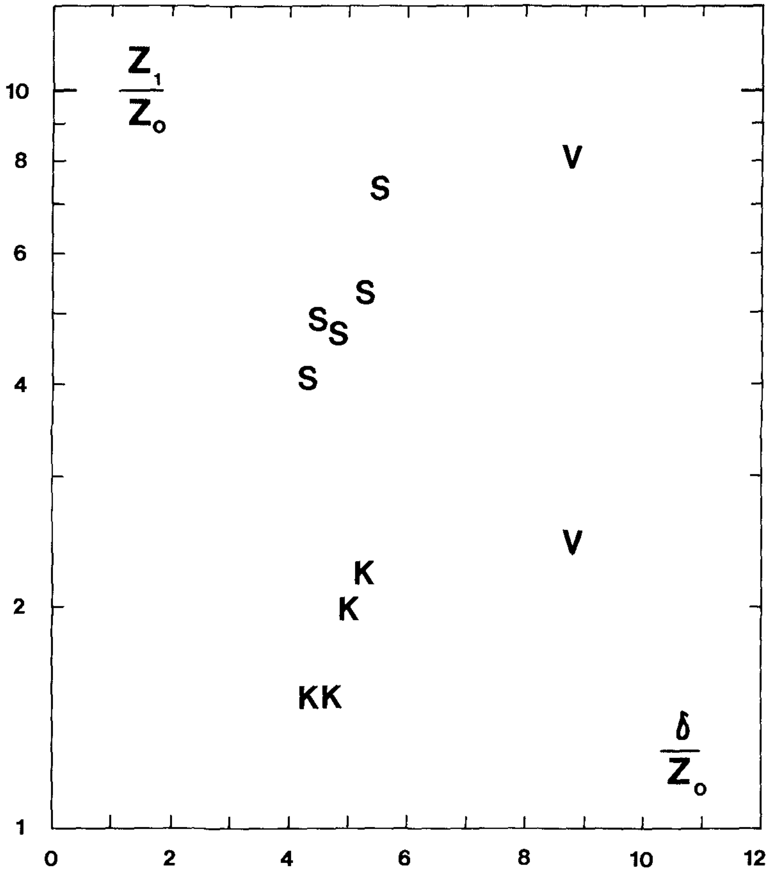

Figure 3 : Apparent roughness increase as function of dimensionless wave boundary layer thickness.

However, as Figure 4 shows, $z_1/z_0$ depends almost exclusively on the friction velocity ratio $\hat{u}_*/\bar{u}_*$. Such dependence will result if the thickness of the wave influenced layer has the form :

$$L = z_0 F(\tilde{u}_*/\bar{u}_*) \tag{10}$$

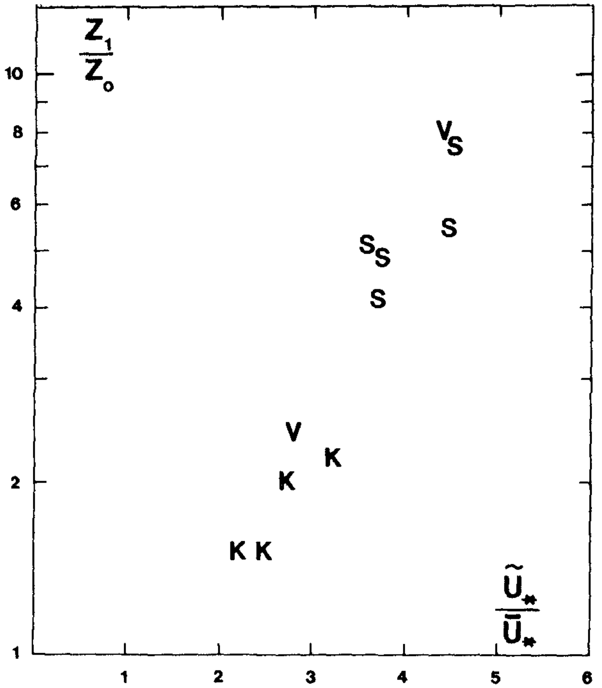

Figure 4 : Apparent roughness increase as function of the friction velocity ratio.

In the following we shall develop a simple model which is in accordance with this empirical observation and in good agreement with the available laboratory data in general. The experimental data used are summarised in Table 1.

## AN EMPIRICAL MODEL

The empirical observations above support the eddy viscosity model outlined in Figure 5.

The square toe has been chosen primarily because it allows $\nu_c$ and thus the current gradient to be continuous functions of z. But also because there is some evidence (Kemp and Simons, 1982, and van Doorn, 1981, 1982) that it is somewhat more realistic than the triangular one, at least for relative roughnesses (r/A) corresponding to natural sand ripples. (See Figure 6.)

| **Author** | **Symbol** | **T** **(s)** | **A** **(m)** | **r** **(m)** | $\bar{u}_*$ **(m/s)** | $z_1$ **(m)** |
|---|---|---|---|---|---|---|
| Van Doorn (1981) | V | 2.0 | 0.085 | 0.021 | 0.016 | 0.0056 |
| | V | 2.0 | 0.085 | 0.021 | 0.026 | 0.0017 |
| Kemp & Simons (1982) | K | 1.0 | 0.011 | 0.025 | 0.018 | 0.0013 |
| | K | 1.0 | 0.015 | 0.025 | 0.019 | 0.0013 |
| | K | 1.0 | 0.018 | 0.025 | 0.020 | 0.0017 |
| | K | 1.0 | 0.021 | 0.025 | 0.019 | 0.0019 |
| Kemp & Simons (1983) | S | 1.0 | 0.011 | 0.025 | 0.011 | 0.0034 |
| | S | 1.0 | 0.013 | 0.025 | 0.012 | 0.0041 |
| | S | 1.0 | 0.016 | 0.025 | 0.013 | 0.0039 |
| | S | 1.0 | 0.021 | 0.025 | 0.013 | 0.0044 |
| | S | 1.0 | 0.024 | 0.025 | 0.014 | 0.0061 |

Table 1 : Experimental data

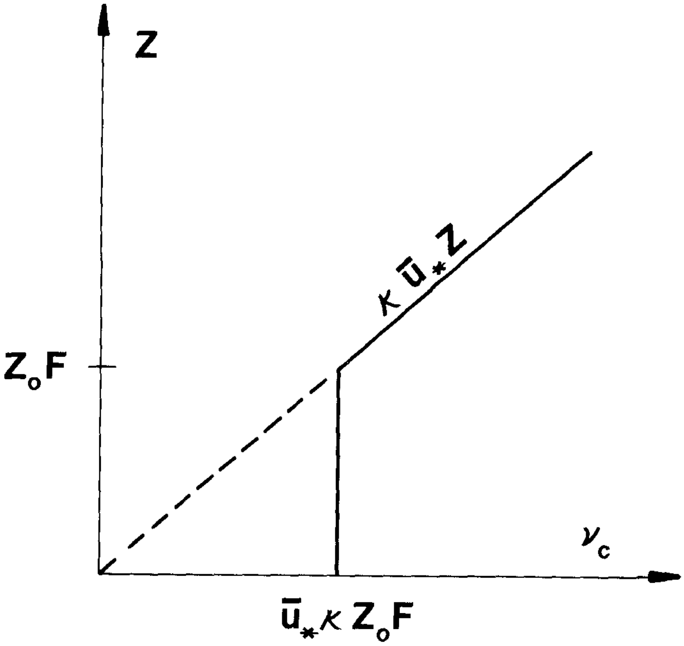

Figure 5 : Proposed distribution of the eddy viscosity felt by a steady current in the presence of waves. F is a function of the friction velocity ratio ($\tilde{u}_*/\bar{u}_*$) and possibly of the angle between current and wave propagation.

The values of $\nu_c$ observed in the horizontally homogenous layer ($z > 0.4$ centimetres) show good agreement with the form hypothesised in Figure 5.

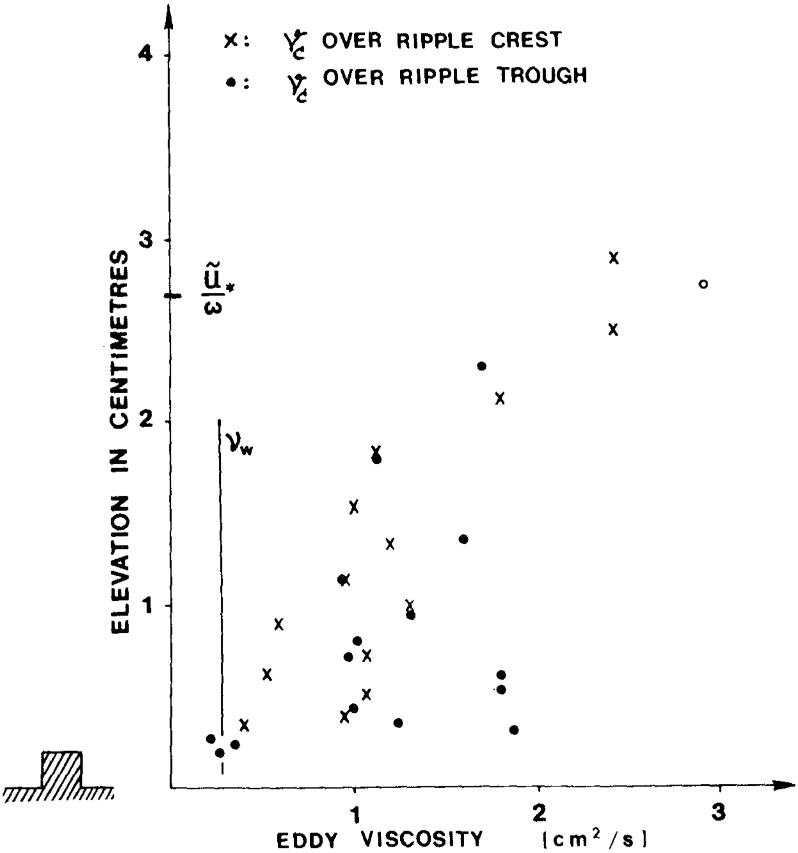

Figure 6 : Observed eddy viscosities from van Doorn (1982), Test S10. The eddy viscosity applicable to the waves ( $\nu_w^t$) is derived in accordance with Nielsen (1985) eqs 31-34.

A finite value of the eddy viscosity at the theoretical (or averaged) bed level is appropriate for rough beds. See e.g. Kajiura (1968).

With the eddy viscosity given by :

$$\nu_c^t = \begin{cases} K\,\bar{u}_* z_{0F} & \text{for} \quad z < z_{0F} \\ K\bar{u}_* z & \text{for} \quad z > z_{0F} \end{cases} \tag{11}$$

and assuming $\overline{\tau} = \overline{u}_*^2$ , we get :

$$\frac{du}{dz} = \begin{cases} \overline{u}_*/(Kz_oF) & \text{for} \quad z < z_oF \\ \overline{u}_*/(Kz) & \text{for} \quad z > z_oF \end{cases} \tag{12}$$

and with $\overline{u}(z_o) = 0$, we get :

$$\overline{u}(z) = \begin{cases} \frac{\overline{u}_*}{KF}(z/z_o - 1) & \text{for} \quad z < z_oF \\ \frac{\overline{u}_*}{K}\ln(z/z_1) & \text{for} \quad z > z_oF \end{cases} \tag{13}$$

from which $z_1$ can be found by matching the two expressions at $z = z_oF$. We find

$$\frac{z_1}{z_o} = F e^{\frac{1}{F}-1} \tag{14}$$

Values of the function $F = F(\tilde{u}_*/\overline{u}_*)$ which must tend towards unity for small $\tilde{u}_*/\overline{u}_*$ have been plotted in Figure 7 and for predictive purposes the curve

$$F = 1 + \frac{1}{6}(\tilde{u}_*/\overline{u}_*)^3 \tag{15}$$

has been fitted to the data.

The form of (15) is rather different from what would result from the hypotheses of Lundgren and Grant and Madsen for combined eddy viscosity, and the underlying mechanics are not understood in detail so applicability outside the experiemental range $\tilde{u}_*/\overline{u}_* < 5$ cannot be guaranteed. Also, future experimental data with different angles between current and wave propagation may call for incorporation of $\varphi$-dependence, i.e. $F = F(\tilde{u}_*/\overline{u}_*, \varphi)$.

## PRACTICAL APPLICATION

In most practical cases, the problem is to estimate the current profile $\overline{u}(z)$ from knowledge of the bed roughness r and the velocities $A\omega$ and $\overline{u}(z_r)$ at only one reference level $z_r$, normally one metre above the bed.

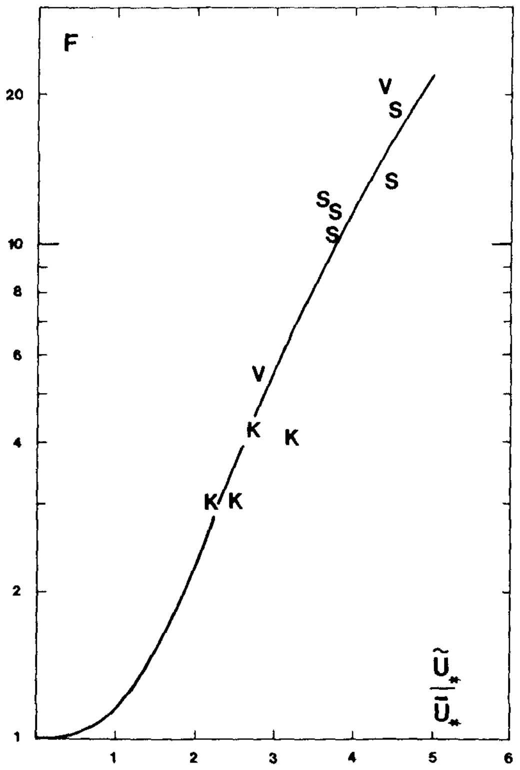

Figure 7 : Observed values of F. The curve corresponds to equation 15.

Thus $\bar{u}_*$ is unknown and must be determined iteratively from :

$$\bar{u}(z_r) = \frac{\bar{u}_*}{K} \ln \frac{z_r}{z_1} \tag{16}$$

with the use of (14) and (15) or the direct approximation

$$z_1/z_o = 1 + 0.06\ (\tilde{u}_*/\bar{u}_*)^3 \tag{17}$$

which leads to

$$\bar{u}(z_r) = \frac{\bar{u}_*}{K} \ln \frac{z_r}{z_o\ (1 + 0.06\ (\tilde{u}_*/\bar{u}_*)^3)} \tag{18}$$

## EXAMPLE

As an example, consider the experiment V10 from van Doorn (1981): The Nikuradse bed roughness is $r = 0.021$m and measurements at $z_r = 0.05$m give $A\omega = 0.257$ m/s and $u(0.05) = 0.090$ m/s. The wave period is 2 seconds so we find $A = 0.082$m and hence :

$$f_w = 0.37\ (r/A)^{2/3} = 0.149 \tag{19}$$

(Kajiura 1968) and :

$$\tilde{u}_* = \sqrt{0.5f}\ A\omega = 0.070 \text{ m/s} \tag{20}$$

Iterative solution of (18) then gives $u_* = 0.0152$ m/s, corresponding to $\tilde{u}_*/\bar{u}_* = 4.60$ and $F = 17.2$. The apparent roughness increase is

$$z_1/z_0 = F \exp\left(\frac{1}{F} - 1\right) = 6.7 \tag{21}$$

and the shift of the outer velocity profile

$$\Delta\bar{u} = \frac{\bar{u}_*}{\kappa}\ln(z_0/z_1) = -0.07 \text{ m/s} \tag{22}$$

The velocity profile based on (13) and the above results is shown in Figure 8 together with the measured profile.

## DISCUSSION

It has been shown that the apparent roughness increase $(z_1/z_0)$ shown by the logarithmic part of current profiles in the presence of waves is mainly a function of the friction velocity ratio $\tilde{u}_*/\bar{u}_*$.

An empirical model has been established which enables estimation of the lower and the logarithmic part of a current profile from knowledge of the wave conditions, the bed roughness plus either the current friction velocity $\bar{u}_*$ or the current speed at a single evaluation $\bar{u}(z_r)$.

The central idea of the model is the same as used by Lundgren (1972), Grant and Madsen (1979) and Christoffersen (1982), namely that the waves reduce the current gradients near the bed by introducing extra mixing (eddy viscosity) below a certain level L near the bed. The effect of this, relative to a situation without waves is a constant reduction $\Delta\bar{u}$ for $z > L$ which can also be interpreted as an apparent increase of bed roughness, from $30z_0$ to $30z_1$. See Figure 1.

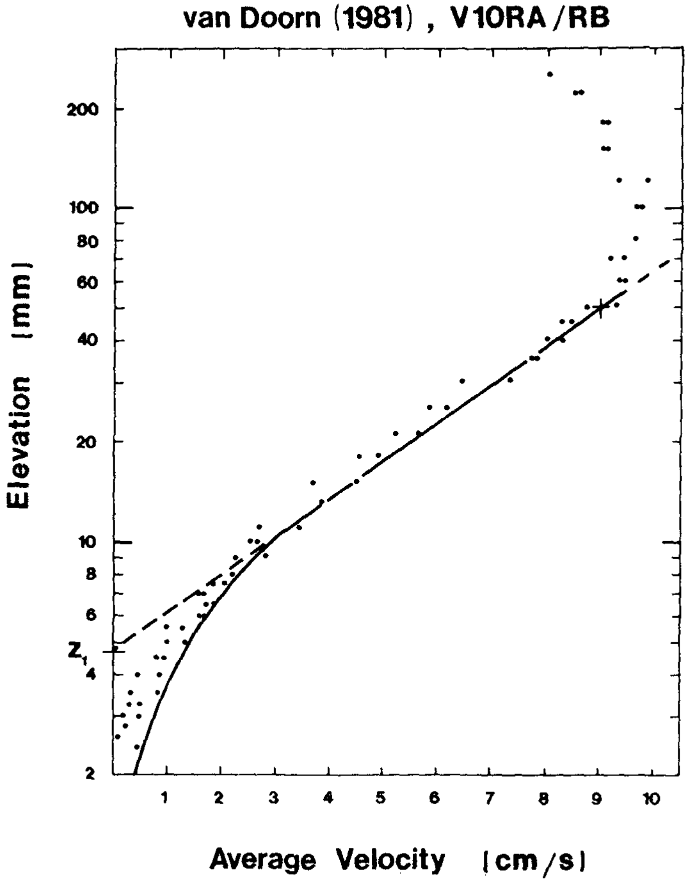

Figure 8 : Current profile predicted from the bed roughness and the velocity at a single point, $\bar{u}(0.05m) = 0.09$ m/s. Measurements by van Doorn (1981). Note that the upper part ($z > 0.07m$) of the velocity distribution is not logarithmic and therefore not covered by the model. The water depth in the experiement was 0.30 metres.

The previous idea of the bottom layer thickness L being proportional to the wave boundary layer thickness (Grant and Madsen 1979 and Christoffersen 1982) has been replaced by :

$$L = z_0 \, F(\tilde{u}_*/\bar{u}_*) \tag{23}$$

a step which is supported by the empirical data as shown in Figures 3 and 4. The conceptual separation of the wave boundary layer thickness as defined by the waves themselves and the thickness of the layer which is felt by the current to be wave influenced is essential. It is related to the fact that the vertical length scales relevant to the corresponding shear stress components are different. The length scale relevant to $\bar{u}$ is Kz, and it can grow to a considerable fraction of the flow depth. On the other hand the length scale relevant to $\tilde{u}$ is restricted to be some fraction of the wave boundary layer thickness.

Dependence of $z_1/z_o$ on the angle between current and wave propagation has not been incorporated because experimental verification is not possible at the moment.

Also, the forces driving the boundary layer mass transport under progressive waves have been neglected. These are likely to dominate the profiles of weak currents (for large $\tilde{u}_*/\bar{u}_*$) but the presently available data indicate that they can be ignored for $\tilde{u}_*/\bar{u}_* \lesssim 5$.

The model is suitable for practical use within the experimental range $\tilde{u}_*/\bar{u}_* \lesssim 5$ because it is considerably simpler than previous models and in good agreement with the physical reality.

NOTATION

| | | |
|---|---|---|
| $A$ | (m) | Water particle semi-excursion. |
| $F$ | - | Empirical function of $u_*/u_*$, see Figure 5. |
| $f_w$ | - | Wave friction factor. |
| $L$ | (m) | Thickness of wave influenced layer. |
| $r$ | (m) | Nikuradse roughness. |
| $\tilde{u}$ | (m/s) | Wave induced velocity. |
| $\tilde{u}_*$ | (m/s) | Friction velocity for waves alone. |
| $\bar{u}$ | (m/s) | Current velocity |
| $\bar{u}_*$ | (m/s) | Current friction velocity. |
| $\Delta\bar{u}$ | (m/s) | Wave induced current reduction. |
| $z$ | (m) | Vertical co-ordinate. |
| $z_r$ | (m) | Reference level. |
| $z_o$ | (m) | Zero intercept height for pure current, $r/30$. |
| $z_1$ | (m) | Zero intercept height, Figure 1. |
| $\delta$ | (m) | Wave boundary layer thickness. |
| $K$ | - | von Karman's constant. |
| $\nu_c$ | ($m^2/s$) | Eddy viscosity felt by current |
| $\rho$ | ($kg/m^3$) | Fluid density. |
| $\bar{\tau}$ | ($N/m^2$) | Current bed shear stress. |
| $\varphi$ | - | Angle between waves and current. |
| $\omega$ | (rad/s) | Wave angular velocity $2\pi/T$. |

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