CHAPTER 19

DIRECTIONAL SPECTRA IN CURRENT-DEPTH REFRACTION

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ABSTRACT

According to Brink-Kjaer et al.'s discussion(1984), the expression of the wave direction change velocity is modified in our numerical model for the directional wave spectra change due to current-depth refraction (Sakai et al., 1983). The wave reflection and breaking conditions due to current are discussed from a view point of numerical analysis. Effects of the refraction term in the modified wave action equation on the directional spectra change are examined. The relative importance of current and water depth change in the directional spectra change is also examined.

INTRODUCTION

In the design of offshore structures, the wave force is the most predominant force acting on them. At present, it is usual to take into account the frequency spectra of irregular waves in the design. Recently it was pointed out that the frequency spectra were not enough for the design of some kinds of coastal structure. Usually the propagation direction of ocean waves is not uni-directional, but it spreads wide. This directional spreading influences the estimation of wave forces on structures(Battjes, 1982).

The directional spreading of ocean waves is expressed in term of the directional spectra. Several standard forms of directional spectra were already proposed for the deep-water wind waves. However the waves are transformed during propagation, and therefore the directional spectra change. Two main causes of the directional spectra change are the wave refraction due to underwater topography and that due to current. In offshore region, the latter is rather important.

A numerical model was proposed by the authors(Sakai et al., 1983) for the change of directional spectra of irregular waves due to depth-current refraction. Since then, Mathiesen(1984), Brink-Kjaer (1984), Booij et al.(1985) and Yamaguchi et al.(1985) proposed similar numerical models. All models use the following wave action conservation equation as the basic equation for the wave height change of each component composing the irregular waves.

$$\frac{\partial A}{\partial t} + \nabla \cdot \{A(U+c_{gr})\} = 0. \tag{1}$$

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$A(=E/\omega_r$, E is the wave energy of each component, and ω_r is the angular wave frequency relative to the current) is the wave action of each component, t is the time, $\nabla = (\partial/\partial x, \partial/\partial y)$, (x, y) is a horizontal orthogonal coordinate system, $\boldsymbol{U}$ is the vector of the horizontal current velocity, and $\boldsymbol{c}_{gr}$ is the vector of the wave group velocity relative to the current.

In the models of Mathiesen, Brink-Kjaer and Yamaguchi et al., the computation proceeds along the so-called wave ray. On the other hand, in the model of the authors, the basic equations are differentiated directly by a finite difference method. This method has a merit that values can be obtained directly on the grid points, while it is not the case in the ray method.

A steady state is assumed and therefore the local accelaration term $\partial A/\partial t$ can be eliminated in Eq.(1). Instead, a term expressing the spectral density change due to the wave direction change $\partial(A \cdot v_\theta)/\partial\theta$ is added as follows :

$$\frac{\partial}{\partial x}(A \cdot v_x) + \frac{\partial}{\partial y}(A \cdot v_y) + \frac{\partial}{\partial\theta}(A \cdot v_\theta) = 0. \tag{2}$$

This term is added to calculate the changes due to both shoaling and refraction simultaneously, which is called "refraction term" hereafter.

In Eq.(2), v_x and v_y are the x and y components of $\boldsymbol{U} + \boldsymbol{c}_{gr}$ in Eq.(1) and given by

$$v_x = U + c_{gr}\cos\theta, \quad v_y = V + c_{gr}\sin\theta. \tag{3}$$

U and V are the x and y components of $\boldsymbol{U}$, and θ is the angle between the wave direction and the x axis. v_θ is a velocity of wave direction change of each component and given from the irrotational condition of wave number as follows :

$$\begin{aligned} v_\theta = {} & (\partial U/\partial x \cdot \sin\theta - \partial U/\partial y \cdot \cos\theta)\cos\theta + (\partial V/\partial x \cdot \sin\theta - \partial V/\partial y \cdot \cos\theta)\sin\theta \\ & + (\partial c_r/\partial x \cdot \sin\theta - \partial c_r/\partial y \cdot \cos\theta) \\ & + (\partial\theta/\partial x \cdot \cos\theta + \partial\theta/\partial y \cdot \sin\theta)(c_{gr} - c_r), \end{aligned} \tag{4}$$

where, c_r is the wave velocity relative to the current.

In Eq.(4), the spatial derivatives of the wave direction θ are included. The value of these derivatives is not known before the computation. In the authors' model, an approximate method was used to solve such a problem. For this problem, Brink-Kjaer et al.(1984) pointed that the differentiation resulting in Eq.(4) was not complete, and proposed Eq.(5) as a further modified form of Eq.(4).

$$\begin{aligned} v_\theta = {} & \left(\frac{\partial U}{\partial x}\sin\theta - \frac{\partial U}{\partial y}\cos\theta\right)\cos\theta + \left(\frac{\partial V}{\partial x}\sin\theta - \frac{\partial V}{\partial y}\cos\theta\right)\sin\theta \\ & + \omega_r/\sinh(2kh) \cdot \left(\frac{\partial h}{\partial x}\sin\theta - \frac{\partial h}{\partial y}\cos\theta\right), \end{aligned} \tag{5}$$

in which k is the wave number, and h is the water depth. In this equation, the spatial derivatives of wave direction do not appear, so that the approximate method above mentioned is not necessary.

In this paper, at first, the authors' model(Sakai et. al, 1983) is introduced briefly. Subsequently, a comparison is made between the result of the numerical computation using Eq. (5) and that using Eq.(4). The wave reflection condition and breaking condition due to current are discussed from the view point of numerical model for wave directional spectra change due to depth-current refraction. These conditions are not taken account of in several models above mentioned. The relative contribution of the 1st and 2nd terms(convection terms) and the 3rd term(refraction term) in the left hand side of Eq.(2) to the wave directional spectra change is also examined. Finally, the relative effects of the current and the bottom topography on the wave directional spectra change is examined.

NUMERICAL MODEL FOR DIRECTIONAL WAVE SPECTRA CHANGE DUE TO CURRENT-DEPTH REFRACTION(SAKAI ET AL., 1983)

The basic equations for the wave refraction consist of the kinematic equations for the wave direction and the dynamical equation for the wave height. For the case of wave refraction due to current, they are the equation of condition of irrotationality of wave number and the equation of wave energy conservation in current, respectively.

In regular wave refraction, it is enough to solve these two equations separately. In irregular wave refraction, however, not only the wave height change but also the wave direction change modify the directional spectral distribution. It is , therefore, convenient to solve two equations simultaneously. The basic equation for wave height change is, as already mentioned, the equation of wave action conservation (1).

The differentiation of Eq.(2) for the numerical computation is just similar to the differentiation of Nagai, et al.(1974) for the numerical computation of directional spectra change due to underwater topography only. Fig.1 shows the grid and the definition of the quantities in the numerical computation of Eq.(2) by a finite difference method. Now it is assumed that the x-axis is normal to the shore line and all component waves propagate in the positive x direction.

The differential expression of Eq.(2) is as follows:

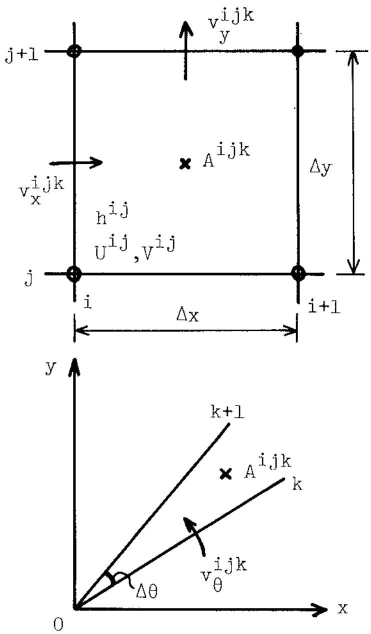

Fig.1 Grid and definition

$$a_1 A^{i\,jk} + a_2 A^{i\,j-1k} + a_3 A^{i\,j+1k} + a_4 A^{i\,jk-1} + a_5 A^{i\,jk+1}$$

$$= bA^{i-1\,jk} = B, \tag{6}$$

in which $a_1 \sim a_6$ and b are functions of v_x, v_y and v_θ and Δx, Δy and $\Delta\theta$. The quantity $A^{i-1\,jk}$ is known because the computation proceeds in the positive x direction. If the frequency of the component waves and $i(x)$ are fixed, and $j = 1 \sim M$ and $k = 1 \sim N$, then a system of $M{\cdot}N$ algebraic equations is obtained. By solving this system of equations , the values of wave action A for all wave directions k and the whole range of $y(j)$ are obtained for the given wave frequency and $x(i)$. A similar computation is repeated for $i = i + 1$ until the shoreward boundary of computation is reached. These computations are repeated for each absolute frequency f of the component waves.

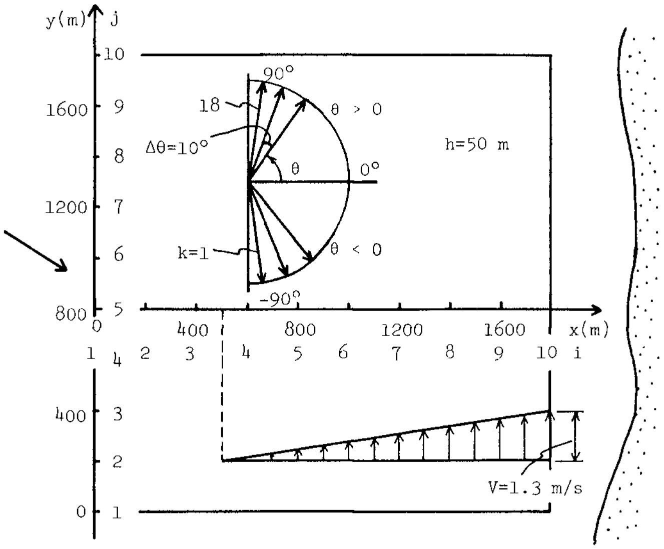

Fig.2 Computation region, water depth, current and division of wave direction

MODIFICATION OF WAVE DIRECTION CHANGE VELOCITY

A numerical computation is done under the same conditions as in the previous paper(Sakai et al., 1983), in order to compare between the computed results by using Eq.(4) with the approximate method and that by using Eq.(5). Fig.2 is the same as Fig.4 in the previous paper, and shows the computation region, the uniform water depth (h = 50m), the profile of current velocity and the division of wave direction.

As in the previous paper, the directional spectrum of Mitsuyasu et al.(1975) is given as the deep-water directional wave spectrum on the y axis. The significant wave period is 7.0sec, the significant wave height is 3.0m and the value of the parameter S_{max} is 10. The wave frequency range from 0.09Hz to 0.71Hz, where the main part of the wave energy is contained, is divided into 19 segments, so that the logarithm of the frequency is divided with equal intervals. The other conditions are same.

Fig.3, (1) shows the distribution of directional spectral density $D(f, \theta)$ at the offshore boundary in the case of the offshore main wave direction θ_{po} = -60°. In this figure, f is the absolute wave frequency. The figure (2) shows the distribution of the transformed spectral density at x = 1,700m calculated by using v_θ of Eq.(4) and the approximate method. An abnormal concentration of wave energy is found at a higher frequency region than 0.5Hz, where no energy was located in the offshore spectra(figure (1)). The figure (3) shows the distribution of the transformed spectral density at x = 1,700m calculated by using the improved wave direction change velocity as pointed by Brink-Kjaer et al.'s discussion(1984)(Eq.(5)). In this case, a slight energy concentration also can be seen in the high frequency region, but the value itself is as small as the value of the marginal part of the original energy distribution.

As seen from Eq.(4) and Eq.(5), the summation of the 3rd and 4th terms in the right hand side of Eq.(4) must be equal to the 3rd term of the right hand side of Eq.(5), which becomes zero when the water depth is constant. The summation of the 3rd and 4th terms in Eq.(4) should also become zero when the water depth is constant. In the computation of Fig.3, the water depth is constant(50m), and therefore no difference should have existed between the results in two figures (2) and (3). The difference between two figures is considered to be due to a wrong estimation of the 3rd and 4th terms in Eq.(4) by using the approximate method(Sakai et al., 1983).

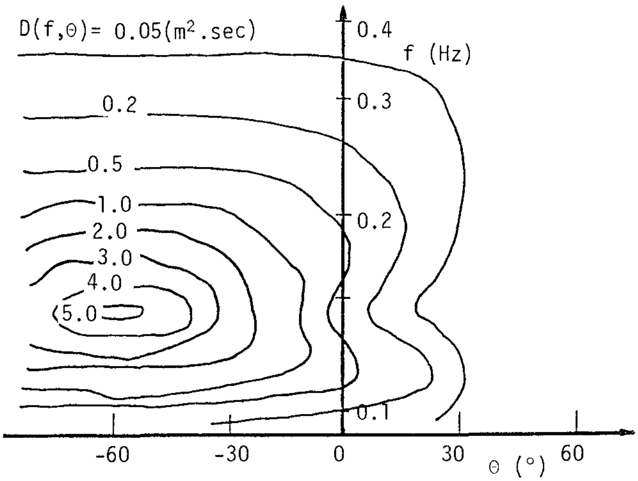

Fig.3,(1) Offshore directional wave spectrum having main wave direction of -60°

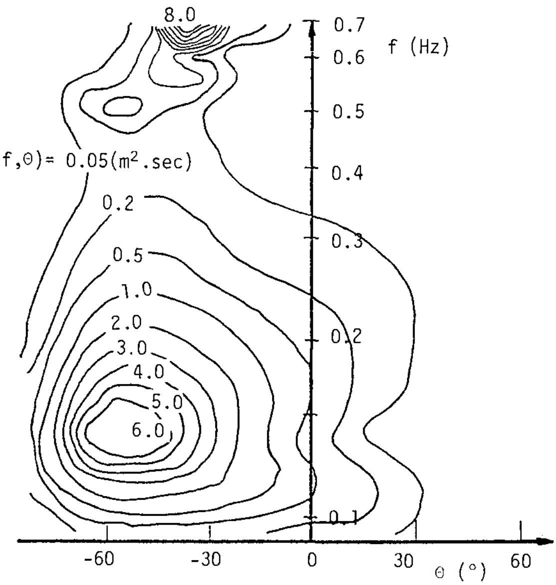

Fig.3,(2) Transformed directional wave spectrum at x = 1,700m calculated by using Eq.(4) and approximate method

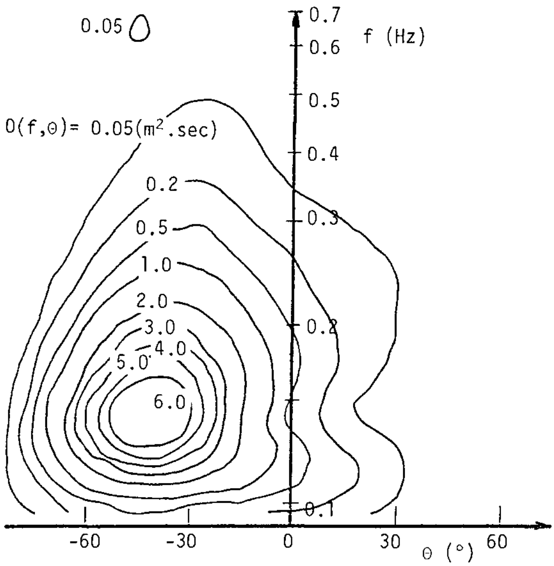

Fig.3,(3) Transformed directional wave spectrum at x = 1,700m calculated by using improved wave direction change velocity(Eq.(5))

In our previous paper, the change of the significant wave height and a representative wave direction was discussed after removing the abnormal energy concentration. Therefore, the change of these two parameters is same between in the case of using Eq.(4) with the approximate method and in the case of using Eq.(5).

REFLECTION AND BREAKING CONDITIONS

Wave reflection due to current

Longuet-Higgins et al.(1961) proposed a maximum velocity of a following current V, beyond which an obliquely incident regular deep-water waves can not penetrate the current. This limit velocity was derived from a condition that an absolute value of sine function does not exceed one in a kinematic equation governing the refraction. In this limiting condition, the wave direction becomes parallel to the current direction. Iwagaki et al.(1977) extended this treatment to a shallow water wave case, and gave the following condition.

$$\frac{V}{c_0} \leq \frac{1-(\sin\theta_0\cdot\tanh\kappa h/\tanh\kappa_0 h_0)^{1/2}}{\sin\theta_0} \tag{7}$$

c is the wave velocity, and the subscript "0" indicates the quantity not in deep-water region but in no-current region. In all numerical models above mentioned, this condition is not taken into account. Tayfun et al.(1976) obtained a similar condition to Eq.(7) in his analytical treatment for the directional spectra change due to a depth and current change in one direction.

As mentioned already, our numerical model(Sakai et al., 1983) differentiates the basic equation (2) directly. The computation does not proceed along the ray of each component. So the condition (7) can not be incorporated directly into this model. In this model, the wave direction is divided as shown in Fig.2. In the computation of Fig.3, the direction of current is parallel to the y axis. This means that, if the reflection corresponds to the fact that the wave direction becomes parallel to the current direction, the wave direction of 90° corresponds to the reflection in this computation. In other words, it is expected that in this model the energy of the component parallel to the current direction at a given point becomes large.

Wave breaking due to current

There exist two kinds of idea as for the breaking of regular waves due to current in the simple theoretical treatment. One is for the case that the wave direction is parallel to the current direction. For this case, Tominaga(1967) derived a condition from a fact that a value of a quantity in a root in an equation giving the wave height change must be positive. In deep water, this condition means that an absolute value of velocity of an opposite current can not exceed the relative wave group velocity.

Another is the case that the the incident waves propagate obliquely into the current. Longuet-Higgins et al. discussed this case. Iwagaki et al. extended their result to a shallow water case. According to Iwagaki et al., the change of wave direction θ is given by Eq.(8).

$$\frac{\sin\theta}{\sin\theta_0} = \frac{1}{(1-V/c_0\cdot\sin\theta_0)^2}\,\frac{\tanh kh}{\tanh k_0h_0}. \tag{8}$$

The change of wave height is given by Eq.(9).

$$\frac{H}{H_0} = \left(\frac{\sin 2\theta}{\sin 2\theta_0}\right)^{-1/2}\cdot\left(\frac{n}{n_0}\right)^{-1/2} \tag{9}$$

n is the ratio of the relative wave group velocity to the relative wave velocity. One of two cases that the wave height becomes infinite is $\theta = 0$. This case corresponds to $V = -\infty$. This means that the solution exists for any finite value of opposite current velocity. In the obliquely incident wave case, therefore, there exists no breaking condition as in the case that the wave direction is parallel to the current direction.

Tayfun et al.(1976) proposed one condition(Eq.(10)) similar to the breaking condition of regular waves propagating parallel to the current(Tominaga, 1967).

$$U\,\frac{k}{k} > -c_{gr}. \tag{10}$$

This condition was derived from a fact that the absolute group velocity $\partial\omega/\partial k$ must be positive in an equation(Eq.(11)) which gives the change of energy density of each component along the ray.

$$D(\omega,\ \theta) = \frac{k\left(\frac{\partial\omega}{\partial k}\right)_0\left(1-\frac{Uk}{\omega}\right)}{k_0\left(\frac{\partial\omega}{\partial k}\right)}D(\omega,\ \theta_0) \tag{11}$$

As seen from above discussion, it can be said that the theoretical wave breaking condition due to current is not yet clearly established. It is also questionable that the breaking condition of regular waves is applicable to each component of irregular waves. Nevertheless it is worthwhile to check the condition (10) of Tayfun et al. in the computation result shown in Fig.3, (3). It is found that component waves which do not satisfy Eq.(10) exist in the region of the slight energy concentration in Fig.3, (3).

CONTRIBUTION OF REFRACTION TERM $\partial(A\cdot v_\theta)/\partial\theta$

From the computation result obtained by using Eq.(5), the relative contribution of the convection terms and refraction term in Eq.(2) is discussed. The conditions are the same as those in Fig.3 except for the offshore main wave direction.

In this case, the water depth is constant, and the phenomena do not change in the y direction. Then Eq.(2) and Eq.(3) become as follow (U = 0 considered):

$$\frac{\partial}{\partial x}(A\cdot v_x) + \frac{\partial}{\partial\theta}(A\cdot v_\theta) = 0, \tag{12}$$

$$v_x = c_{gr}\cos\theta, \tag{13}$$

$$v_\theta = \partial V/\partial x \cdot \sin^2\theta. \tag{14}$$

Since c_{gr} is determined from the wave number conservation equation involving the velocity V, v_x contains the effect of the current indirectly. On the contrary, v_θ contains the effect of the current directly.

Fig.4, (1) ∿ (3) compares the relative contribution of the convection term $\partial(A \cdot v_x)/\partial x$ and the refraction term $\partial(A \cdot v_\theta)/\partial\theta$ in Eq.(12) to the directional spectra change in the case of offshore main wave direction = -30°. The figure (1) shows the directional spectral density distribution at the offshore boundary. The figure (2) shows the transformed density distribution at x = 1,700m in the case that both terms are taken account of. The figure (3) shows the same density in the case that only the convection term is taken account of.

As seen from the figure (2), in the case that both terms are taken account of, the energy grows near -30° (the offshore main wave direction) where the current is opposite to the component waves. It is seen also that the energy shifts totally in the positive wave direction. On the other hand, in the case that only the convection term is taken account of(figure (3)), an abnormal energy concentration occurs in a high frequency and negative wave direction region. Except in this region, the energy distribution does not change so much from the offshore energy distribution (figure (1)).

From these results, it can be said that, at least for the case treated here, the refraction term has an important effect on the wave directional spectra change. This term explains the energy transfer between the wave components having different wave direction.

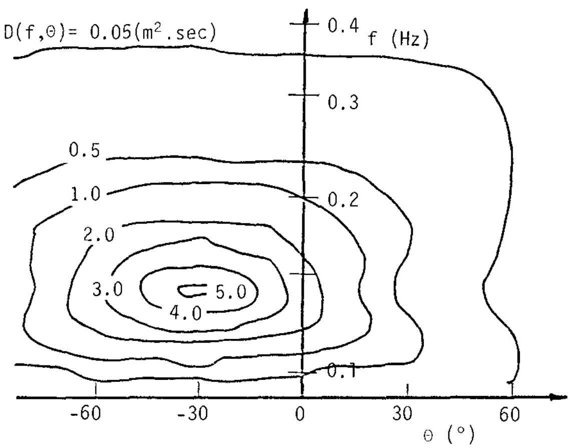

Fig.4,(1) Offshore directional wave spectrum having main wave direction of -30°

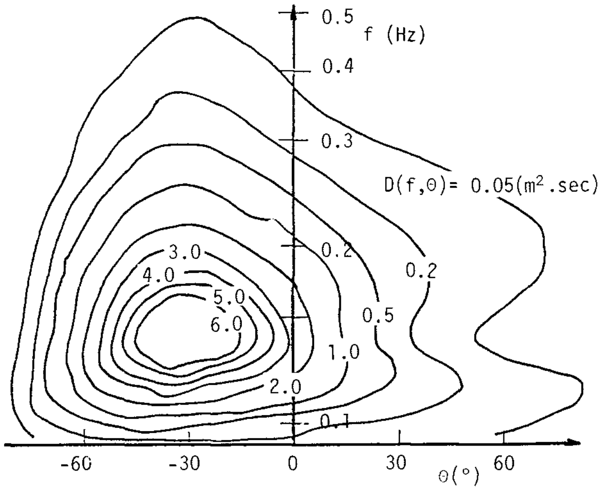

Fig.4,(2) Transformed directional wave spectrum at x = 1,700m in the case that both convection and refraction terms are taken account of

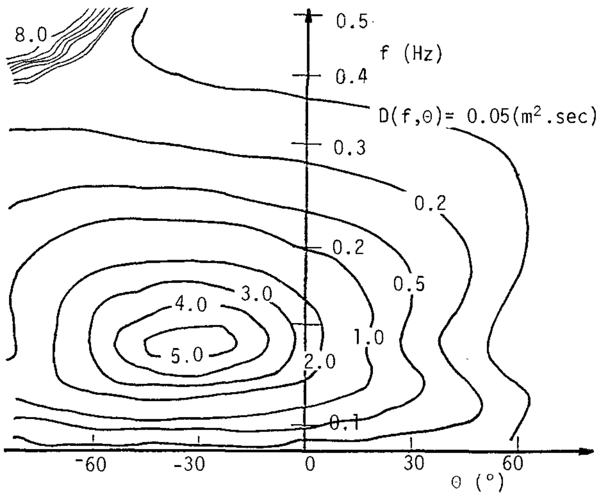

Fig.4,(3) Transformed directional wave spectrum at x = 1,700m in the case that only convection term is taken account of

COMPARISON OF EFFECTS OF CURRENT AND BOTTOM TOPOGRAPHY

Generally speaking, the lower the frequency of a component wave, the earlier the component affected by the bottom topography. It is, therefore, expected that the effect of bottom topography is predominant for lower frequency components and the effect of current is predominant for higher frequency components. To check this, a case of plane beach of 1/40 slope is discussed in addition to the case of current in Fig.2. The water depth at the offshore boundary is 50m, and it decreases in the x direction.

Numerical computations are done for the following three cases : the current in the constant depth(50m)(Fig.2), the plane beach of 1/40 slope without current, and the same current on the beach of 1/40 slope. The offshore main wave direction is -30°. The change of directional spectra at two values of frequency is compared for these three cases.

Fig.5,(1) shows the directional spectra at a frequency f = 0.11Hz(lower than the peak frequency f_p = 1/7.0Hz = 0.14Hz) at x = 1,700m. The directional spectra in the case of current and depth change(broken line) can be explained roughly by the change in the case of depth change only(chain line). The figure (2) shows the directional spectra at a frequency f = 0.19Hz(higher than the peak frequency) at x = 1,700m. The value of the maximum energy in the case of current and depth change(broken line) can be explained by that in the case of current only(solid line), but the wave direction at which the maximum energy occurs can be explained rather by that in the case of depth change only(chain line).

Above mentioned results depends on the water depth, the beach slope, the current velocity and its profile. Still it can be said that the general trend mentioned at the first part of this section does not always hold.

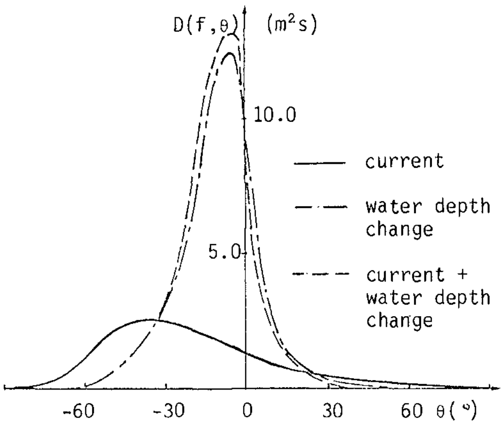

Fig.5,(1) Directional distribution at lower frequency than peak frequency(f = 0.11Hz) at x = 1,700m

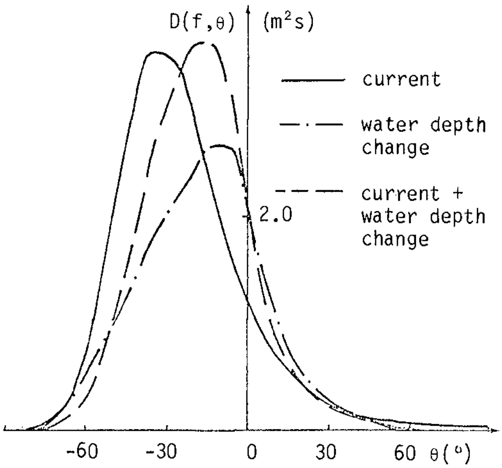

Fig.5,(2) Directional distribution at higher frequency than peak frequency(f = 0.19Hz) there

CONCLUSIONS

The expression of the wave direction change velocity, Eq.(4), in our numerical model for directional wave spectra change due to depth-current refraction(Sakai et al., 1983) was modified according to Brink-Kjaer' et al.'s discussion(1984). The improved expression, Eq.(5), shown by him was used. The wave reflection and breaking conditions due to current were discussed from a view point of numerical analysis. Effects of the refraction term $\partial(A \cdot v_\theta)/\partial\theta$ in the wave action equation, Eq.(2), and the relative importance of current and water depth change in directional spectra change were discussed by using the newly obtained numerical results.

The following conclusions are obtained:

(1) In our previous model where Eq.(4) and an approximate method were used, an abnormal concetration of wave energy occured in a high frequency region. By using Eq.(5) instead of Eq.(4) and the approximate method, this concentration disappears.

(2) The reflection condition of regular waves due to current at first given by Longuet-Higgins and Stewart(1961) can not be incorporated directly into our numerical model. It is expected that the energy density of the component parallel to the local current direction grows rapidly in our model.

(3) There are two theoretical situations for the regular wave breaking on current. One is for waves propagating parallel against the current, where a theoretical breaking condition exists. For waves propagating obliquely into the current, no theoretical limiting condition exists. For irregular waves, Tayfun et al.(1976) showed a similar condition to that for regular waves. In a region where Tayfun et al.'s condition is fulfilled on the frequency direction plane, a slight energy concentration is found in the numerical results.

(4) The refraction term $\partial(A \cdot v_\theta)/\partial\theta$ has a dominant effect on the directional spectra change compared with the convection terms $\partial(A \cdot v_x)/\partial x$ and $\partial(A \cdot v_y)/\partial y$ at least in the example treated here.

(5) Even for a frequency component having a higher frequency than the peak frequency, which is expected to be affected more by the current than by the water depth change, the change of the main wave direction is determined rather by the water depth change, at least in the example treated here.

It is believed that the numerical model proposed here for the directional spectra change of wind waves due to current-depth refraction can be used to predict more accurately the wave condition around offshore structure than the existing wave prediction models. This more accurate prediction of wave conditions will also make the wave force calculation more accurate.

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