CHAPTER 26

Applications on Non-linear Wave Combination

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Due to the special bathymetry in Taiwan Strait, the waves off the western coast of Taiwan are considered to be composed of two-source wave system. One propagates from the central part of the Strait named main wave, and the other is generated by the local wind known as local wave which occurs along the shore. After the combination and the transformation procedure from these two-nonlinear-source wave system, the wave height distribution in Taiwan Strait should be modified. A comparison of the wave height distributions based on the present proposed method with the field data indicates that the present method yields a better result than other theorems. Furthermore, the result of application of two non-linear wave theorem to wave prediction are also presented.

Introduction

In a previous paper[11], the author proposed a theoretical expression for the wave height distribution, which was combined by using two linear source-wave system. At about the same time, Chen et.al.[1] investigated the effect of nonlinearity on the distributions of surface elevation and wave heights of random sea. In that study, it was found that the wave height distribution is effected by wave steepness and the tendencies biased toward the left and peaked toward the middle from Rayleigh distribution (δ_r=0) as the value of nonlinear factor δ_r increases. Longuet-Higgins[7] also proposed a wave height distribution which was based on the joint distribution of wave periods and amplitudes in a random wave field. His wave height distribution was affected by the well-known spectral width parameter. Both of their approaches are based on single non-linear wave theorem. However, for the case where the sea wave is composed of different source-wave system, its wave height distribution should be changed. Therefore, following the same procedures as described in[11] , a two non-linear wave combination theorem was proposed.

Non-linear waves combination

According to the single non-linear wave theorem [1], the wave height distribution can be calculated by the following formula:

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$$P(H)=\frac{\pi}{2}D_0^2\cdot H\cdot\left[1+\frac{D_2}{4}\cdot H^2+\left(\frac{3}{64}D_4+\frac{D_2^2}{128}\right)H^4+\frac{1}{512}D_2D_4\cdot H^6\right.$$
$$\left.+\frac{3}{32768}D_4^2\cdot H^8\right]\cdot\exp\left(-\frac{H^2}{8\overline{\zeta^2}}\right) \qquad \cdots\cdots(1)$$

In the above formula, the values of the coefficients D_i, which are functions of the root-mean-sguare of the wave steepness δ_r, are:

$$D_0=\frac{1}{\sqrt{2\pi\overline{\zeta^2}}}\left[\frac{5}{8}+\frac{1}{8}\cdot\frac{3+8(\pi\delta_r)^2[1-(\pi\delta_r)^2]^2}{\{1+(\pi\delta_r)^2[1-(\pi\delta_r)^2]^2\}^2}\right] \qquad \cdots\cdots(2)$$

$$D_1=-\frac{6\sqrt{2\pi}}{(\overline{\zeta^2})^{1/2}}\cdot\frac{\frac{3}{8}(\pi\delta_r)[1-(\pi\delta_r)^2]+\frac{1}{4}(\pi\delta_r)^3[1-(\pi\delta_r)^2]^3}{5+\frac{3+8(\pi\delta_r)^2[1-(\pi\delta_r)^2]^2}{\{1+(\pi\delta_r)^2[1-(\pi\delta_r)^2]^2\}^2}}$$
$$\cdot\frac{1}{\{1+(\pi\delta_r)^2[1-(\pi\delta_r)^2[1-\pi\delta_r)^2]^2\}^{3/2}} \qquad \cdots\cdots(3)$$

$$D_2=-\frac{2}{\overline{\zeta^2}}\cdot(\pi\delta_r)^2[1-(\pi\delta_r)^2]^2\{2-3(\pi\delta_r)^2[1-(\pi\delta_r)^2]^2\}$$
$$/\left[5+\frac{3+8(\pi\delta_r)^2[1-(\pi\delta_r)^2]^2}{\{1+(\pi\delta_r)^2[1-(\pi\delta_r)^2]^2\}^2}\right]\cdot\{1+(\pi\delta_r)^2[1-(\pi\delta_r)^2]^2\}^2 \qquad \cdots(4)$$

$$D_3=-\frac{D_1}{3\overline{\zeta^2}} \qquad \cdots\cdots(5)$$

$$D_4=\frac{D_2}{6\overline{\zeta^2}} \qquad \cdots\cdots(6)$$

From the above eguation, it can be found that as soon as δ_r approaches to zero, the distribution of the wave height will become Rayleigh's.

However, when the combination of two non-linear waves are considered, the above equation can be modified as

$$P(H)=u_{1,i}\cdot H\cdot\exp\left(-\frac{H^2}{u_{6,i}}\right)+u_{2,i}\cdot H^3\cdot\exp\left(-\frac{H^2}{u_{6,i}}\right)$$
$$+u_{3,i}\cdot H^5\cdot\exp\left(-\frac{H^2}{u_{6,i}}\right)+u_{4,i}\cdot H^7\cdot\exp\left(-\frac{H^2}{u_{6,i}}\right)$$
$$+u_{5,i}\cdot H^9\cdot\exp\left(-\frac{H^2}{u_{6,i}}\right) \qquad \cdots\cdots(7)$$

$$u_{1,i} = \frac{\pi}{2} D^2_{0,i} \quad \text{...............(8)}$$

$$u_{2,i} = \frac{1}{4} D^2_{2,i} \cdot u_{1,i} \quad \text{...............(9)}$$

$$u_{3,i} = \left(\frac{3}{64} D_{4,i} + \frac{D^2_{2,i}}{128}\right) \cdot u_{1,i} \quad \text{...............(10)}$$

$$u_{4,i} = \frac{1}{512} D_{2,i} \cdot D_{4,i} \cdot u_{1,i} \quad \text{...............(11)}$$

$$u_{5,i} = \frac{3}{32768} D^2_{4,i} \cdot u_{1,i} \quad \text{...............(12)}$$

$$u_{6,i} = 8\,\overline{\zeta_i{}^2} \quad \text{...............(13)}$$

In the above formulas, $U_{1,i}$, $U_{2,i}$,...., $U_{5,i}$ are the coefficients and $U_{6,i}$ are the wave energy of these two-sour -ce waves where i=1 stands for the refracted wave and i=2 represents the local wave.

Since the wave energy is proportional to the square of the wave height, we have

$$y = H^2 \quad , \quad dy = 2HdH \quad , \quad \frac{dH}{dy} = \frac{1}{2H}$$

After transformation from eq.(7), the distribution of wave energy becomes

$$P(y) = \frac{dH}{dy} P(H) = \frac{1}{2\sqrt{y}} \cdot P(H)$$
$$= \frac{u_1}{2} \exp\left(-\frac{y}{u_6}\right) + \frac{u_2}{2} y \cdot \exp\left(-\frac{y}{u_6}\right) + \frac{u_3}{2} \cdot y^2$$
$$\cdot \exp\left(-\frac{y}{u_6}\right) + \frac{u_4}{2} \cdot y^3 \cdot \exp\left(-\frac{y}{u_6}\right) + \frac{u_5}{2} \cdot y^4 \cdot \exp\left(-\frac{y}{u_6}\right) \text{......(14)}$$

Under the assumption that the total energy of the combined wave is the sum of the energy of the wave associ- ated with each source, that is,

$$z = y_1 + y_2 \qquad y_2 = z - y_1$$

the Jacobian of this transformation is

$$|J| = \begin{vmatrix} \frac{\partial y_2}{\partial z} & \frac{\partial y_2}{\partial y_1} \\ \frac{\partial y_1}{\partial z} & \frac{\partial y_1}{\partial y_1} \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = 1$$

The energy distribution for the refracted and the local wave can then be found, respectively, as

$$P(y_1) = [\frac{u_{1,1}}{2} + \frac{u_{2,1}}{2} y_1 + \frac{u_{3,1}}{2} \cdot y_1^2 + \frac{u_{4,1}}{2} \cdot y_1^3 + \frac{u_{5,1}}{2} \cdot y_1^4] \cdot \exp(-\frac{y_1}{u_{6,1}}) \cdots (15)$$

$$P(y_2) = [\frac{u_{1,2}}{2} + \frac{u_{2,2}}{2} \cdot y_2 + \frac{u_{3,2}}{2} \cdot y_2^2 + \frac{u_{4,2}}{2} \cdot y_2^3 + \frac{u_{5,2}}{2} \cdot y_2^4] \cdot \exp(-\frac{y_2}{u_{6,2}}) \cdots\cdots (16)$$

Since both distributions are mutually independent, we can obtain

$$P(z, y_1) = P(y_1, y_2) \cdot |J| = P(y_1) \cdot P(y_2) = P(y_1) \cdot P(z - y_1)$$

$$P(z) = \int_0^z P(z, y_1)\, dy_1 = \int_0^z P(y_1) \cdot P(z - y_1) \cdot dy_1 \qquad \cdots\cdots\cdots\cdots\cdots (17)$$

Substituting equs.(15) and (16) into eq.(17) , we have

$$P(z) = \int_0^z [\frac{u_{1,1}}{2} + \frac{u_{2,1}}{2} \cdot y_1 + \frac{u_{3,1}}{2} \cdot y_1^2 + \frac{u_{4,1}}{2} \cdot y_1^3 + \frac{u_{5,1}}{2} \cdot y_1^4]$$

$$\cdot [\frac{u_{1,2}}{2} + \frac{u_{2,2}}{2} \cdot y_2 + \frac{u_{3,2}}{2} \cdot y_2^2 + \frac{u_{4,2}}{2} \cdot y_2^3 + \frac{u_{5,2}}{2} \cdot y_2^4]$$

$$\cdot \exp[-\frac{y_1}{u_{6,1}} - \frac{y_2}{u_{6,2}}] \cdot dy_1$$

$$= \int_0^z A_1 \cdot A_2 \cdot \exp(-\frac{z}{u_{6,2}}) \cdot \exp(u \cdot y_1) \cdot dy_1 \qquad \cdots\cdots\cdots\cdots\cdots (18)$$

$$A_1 = \frac{u_{1,1}}{2} + \frac{u_{2,1}}{2} \cdot y_1 + \frac{u_{3,1}}{2} \cdot y_1^2 + \frac{u_{4,1}}{2} \cdot y_1^3 + \frac{u_{5,1}}{2} \cdot y_1^4 \qquad \cdots\cdots\cdots\cdots\cdots (19)$$

$$A_2 = B_1 - B_2 \cdot y_1 + B_3 \cdot y_1^2 - B_4 \cdot y_1^3 + B_5 \cdot y_1^4 \qquad \cdots\cdots\cdots\cdots\cdots (20)$$

$$u = \frac{u_{6,1} - u_{6,2}}{u_{6,1} \cdot u_{6,2}} \qquad \cdots\cdots\cdots\cdots\cdots (21)$$

$$B_1 = \frac{1}{2}(u_{1,2} + u_{2,2} \cdot z + u_{3,2} \cdot z^2 + u_{4,2} \cdot z^3 + u_{5,2} \cdot z^4) \qquad \cdots\cdots\cdots\cdots\cdots (22)$$

$$B_2 = \frac{1}{2}(u_{2,2} + 2u_{3,2} \cdot z + 3u_{4,2} \cdot z^2 + 4u_{5,2} \cdot z^3) \qquad \cdots\cdots(22)$$

$$B_3 = \frac{1}{2}(u_{3,2} + 3u_{4,2} \cdot z + 6u_{5,2} \cdot z^2) \qquad \cdots\cdots(24)$$

$$B_4 = \frac{1}{2}(u_{4,2} + 4u_{5,2} \cdot z) \qquad \cdots\cdots(25)$$

$$B_5 = \frac{1}{2} u_{5,2} \qquad \cdots\cdots(26)$$

$$A_1 \cdot A_2 = \frac{1}{2}[u_{1,1} + u_{2,1} \cdot y_1 + u_{3,1} \cdot y_1^2 + u_{4,1} \cdot y_1^3 + u_{5,1} \cdot y_1^4]$$

$$\cdot [B_1 - B_2 \cdot y_1 + B_3 \cdot y_1^2 - B_4 \cdot y_1^3 + B_5 \cdot y_1^4]$$

$$\doteq c_1 + c_2 \cdot y_1 + c_3 \cdot y_1^2 + c_4 \cdot y_1^3 + c_5 \cdot y_1^4 + c_6 \cdot y_1^5$$

$$+ c_7 \cdot y_1^6 + c_8 \cdot y_1^7 + c_9 \cdot y_1^8 \qquad \cdots\cdots(27)$$

$$c_1 = \frac{1}{2}(u_{1,1} \cdot B_1)$$

$$c_2 = \frac{1}{2}(u_{2,1} \cdot B_1 - u_{1,1} \cdot B_2)$$

$$c_3 = \frac{1}{2}(u_{3,1} \cdot B_1 - u_{2,1} \cdot B_2 + u_{1,1} \cdot B_3)$$

$$c_4 = \frac{1}{2}(u_{4,1} \cdot B_1 - u_{3,1} \cdot B_2 + u_{2,1} \cdot B_3 - u_{1,1} \cdot B_4)$$

$$c_5 = \frac{1}{2}(u_{5,1} \cdot B_1 - u_{4,1} \cdot B_2 + u_{3,1} \cdot B_3 - u_{2,1} \cdot B_4 + u_{1,1} \cdot B_5)$$

$$c_6 = -\frac{1}{2}(u_{5,1} \cdot B_2 - u_{4,1} \cdot B_3 + u_{3,1} \cdot B_4 - u_{2,1} \cdot B_5)$$

$$c_7 = \frac{1}{2}(u_{5,1} \cdot B_3 - u_{4,1} \cdot B_4 + u_{3,1} \cdot B_5)$$

$$c_8 = -\frac{1}{2}(u_{5,1} \cdot B_4 - u_{4,1} \cdot B_5)$$

$$c_9 = \frac{1}{2}(u_{5,1} \cdot B_5) \qquad \cdots\cdots(28)$$

Substituting eq.(27) into eq.(18), we have

$$P(z) = e^{(-z/u_{6,2})} \cdot \{ c_1 \int_0^z e^{(u \cdot y_1)} dy_1 + c_2 \int_0^z y_1 \cdot e^{(u \cdot y_1)} \cdot dy_1$$

$$+ c_3 \int_0^z y_1^2 \cdot e^{(u \cdot y_1)} \cdot dy_1 + c_4 \int_0^z y_1^3 \cdot e^{(u \cdot y_1)} \cdot dy_1$$

$$+c_5\int_0^z y_1^4\cdot e^{(u\cdot y_1)}\cdot dy_1+c_6\int_0^z y_1^5\cdot e^{(u\cdot y_1)}\cdot dy_1+c_7\int_0^z y_1^6\cdot e^{(u\cdot y_1)}\cdot dy_1$$

$$+c_8\int_0^z y_1^7\cdot e^{(u\cdot y_1)}\cdot dy_1+c_9\int_0^z y_1^8\cdot e^{(u\cdot y_1)}\cdot dy_1\}\qquad\cdots\cdots\cdots(29)$$

$$\int_0^z e^{(u\cdot y_1)}dy_1=\frac{e^{u\cdot z}}{u}-\frac{1}{u}$$

$$\int_0^z y_1\cdot e^{(u\cdot y_1)}dy_1=\frac{e^{uz}}{u^2}(uz-1)+\frac{1}{u^2}$$

$$\int_0^z y_1^2\cdot e^{(u\cdot y_1)}\cdot dy_1=\frac{e^{uz}}{u^3}(u^2z^2-2uz+2)-\frac{2}{u^3}$$

$$\int_0^z y_1^3\cdot e^{(u\cdot y_1)}\cdot dy_1=\frac{e^{uz}}{u^4}(u^3z^3-3u^2\cdot z^2+6uz-6)+\frac{6}{u^4}$$

$$\int_0^z y_1^4\cdot e^{(u\cdot y_1)}\cdot dy_1=\frac{e^{uz}}{u^5}(u^4\cdot z^4-4u^3\cdot z^3+12u^2\cdot z^2-24u\cdot z+24)-\frac{24}{u^5}$$

$$\int_0^z y_1^5\cdot e^{(u\cdot y_1)}\cdot dy_1=\frac{e^{uz}}{u^6}(u^5\cdot z^5-5u^4\cdot z^4+20u^3z^3-60u^2\cdot z^2+120uz-120)+\frac{120}{u^6}$$

$$\int_0^z y_1^6\cdot e^{(u\cdot y_1)}\cdot dy_1=\frac{e^{uz}}{u^7}(u^6\cdot z^6-6u^5\cdot z^5+30u^4\cdot z^4-120u^3\cdot z^3+360u^2\cdot z^2$$

$$-720uz+720)-\frac{720}{u^7}$$

$$\int_0^z y_1^7\cdot e^{(u\cdot y_1)}\cdot dy_1=\frac{e^{uz}}{u^8}(u^7\cdot z^7-7u^6z^6+42u^5z^5-210u^4\cdot z^4+840u^3\cdot z^3$$

$$-2520u^2\cdot z^2+5040uz-5040)+\frac{5040}{u^8}$$

$$\int_0^z y_1^8\cdot e^{(u\cdot y_1)}\cdot dy_1=\frac{e^{uz}}{u^9}(u^8\cdot z^8-8u^7\cdot z^7+56u^6\cdot z^6-336u^5\cdot z^5$$

$$+1680u^4\cdot z^4-6720u^3\cdot z^3+2016u^2\cdot z^2-40320uz+40320)$$

$$-\frac{40320}{u^9}$$

In the other hand, substituting equs.(22)to(26)into eq.(28), it becomes

$$c_1=\frac{1}{4}u_{1,1}(u_{1,2}+u_{2,2}\cdot z+u_{3,2}\cdot z^2+u_{4,2}\cdot z^3+u_{5,2}\cdot z^4)$$

$$c_2=\frac{1}{4}\{u_{2,1}(u_{1,2}+u_{2,2}\cdot z+u_{3,2}\cdot z^2+u_{4,2}\cdot z^3+u_{5,2}\cdot z_4)$$

$$-u_{1,1}(u_{2,2}+2u_{3,2}\cdot z+3u_{4,2}\cdot z^2+4u_{5,2}\cdot z^3)\}$$

$$c_3=\frac{1}{4}\{u_{3,1}(u_{1,2}+u_{2,2}\cdot z+u_{3,2}\cdot z^2+u_{4,2}\cdot z^3+u_{5,2}\cdot z^4)$$

$$-u_{2,1}(u_{2,2}+2u_{3,2}\cdot z+3u_{4,2}\cdot z^2+4u_{5,2}\cdot z^3)$$

$$+u_{1,1}(u_{3,2}+3u_{4,2}\cdot z+6u_{5,2}\cdot z^2)\}$$

$$c_4=\frac{1}{4}\{u_{4,1}(u_{1,2}+u_{2,2}\cdot z+u_{3,2}\cdot z^2+u_{4,2}\cdot z^3+u_{5,2}\cdot z^4)$$

$$-u_{3,1}(u_{2,2}+2u_{3,2}\cdot z+3u_{4,2}\cdot z^2+4u_{5,2}\cdot z^3)$$

$$+u_{2,1}\cdot(u_{3,2}+3u_{4,2}\cdot z+6u_{5,2}\cdot z^2)-u_{1,1}(u_{4,2}+4u_{5,2}\cdot z)\}$$

$$c_5=\frac{1}{4}\{u_{5,1}(u_{1,2}+u_{2,2}\cdot z+u_{3,2}\cdot z^2+u_{4,2}\cdot z^3+u_{5,2}\cdot z^4)$$

$$-u_{4,2}(u_{2,2}+2u_{3,2}\cdot z+3u_{4,2}\cdot z^2+4u_{5,2}\cdot z^3)$$

$$+u_{3,1}(u_{3,2}+3u_{4,2}\cdot z+6u_{5,2}\cdot z^2)-u_{2,1}(u_{4,2}+4u_{5,2}\cdot z)+u_{1,1}(u_{5,2})\}$$

$$c_6=-\frac{1}{4}\{u_{5,1}(u_{2,2}+2u_{3,2}\cdot z+3u_{4,2}\cdot z^2+4u_{3,2}\cdot z^3)$$

$$-u_{4,1}(u_{3,2}+3u_{4,2}\cdot z+6u_{5,2}\cdot z^2)+u_{3,1}(u_{4,2}+4u_{5,2}\cdot z)-u_{2,1}\cdot u_{5,2}\}$$

$$c_7=\frac{1}{4}\{u_{5,1}(u_{3,2}+3u_{4,2}\cdot z+6u_{5,2}\cdot z^2)-u_{4,1}(u_{4,2}+4u_{5,2}\cdot z)+u_{3,1}\cdot u_{5,2}\}$$

$$c_8=-\frac{1}{4}\{u_{5,1}(u_{4,2}+4u_{5,2}\cdot z)-u_{4,1}\cdot u_{5,2}\}$$

$$c_9=\frac{1}{4}(u_{5,1}\cdot u_{5,2}) \qquad \cdots\cdots(30)$$

Then, after certain mathematical manipulation, the energy distribution of the combined wave can be found as

$$P(z)=\frac{1}{4u}\{(E_0+E_1\cdot z+E_2\cdot z^2+E_3\cdot z^3+E_4\cdot z^4+E_5\cdot z^5+E_6\cdot z^6+E_7\cdot z^7$$

$$+E_8\cdot z^8)\cdot e^{(-t/u_{6,1})}-(E_0+F_1\cdot z+F_2\cdot z^2+F_3\cdot z^3+F_4\cdot z^4)\cdot e^{(-t/u_{6,2})}\} \quad \cdots(31)$$

$$E_0=u_{1,1}\cdot u_{1,2}-\frac{1}{u}(u_{2,1}\cdot u_{1,2}-u_{1,1}\cdot u_{2,2})$$

$$+\frac{2}{u^2}(u_{3,1}\cdot u_{1,2}-u_{2,1}\cdot u_{2,2}+u_{1,1}\cdot u_{3,2})-\frac{6}{u^3}(u_{4,1}\cdot u_{1,2}$$

$$-u_{3,1}\cdot u_{2,2}+u_{2,1}\cdot u_{3,2}-u_{1,1}\cdot u_{4,2})+\frac{24}{u^4}(u_{5,1}\cdot u_{1,2}-u_{4,1}\cdot u_{2,2}$$

$$+u_{3,1}\cdot u_{3,2}-u_{2,1}\cdot u_{4,2}+u_{1,1}\cdot u_{5,2})+\frac{120}{u^5}(u_{5,1}\cdot u_{2,2}-u_{4,1}\cdot u_{3,2}$$

$$+u_{3,1}\cdot u_{4,2}-u_{2,1}\cdot u_{5,2})+\frac{720}{u^6}(u_{5,1}\cdot u_{3,2}-u_{4,1}\cdot u_{4,2}+u_{3,1}\cdot u_{5,2})$$

$$+\frac{5040}{u^7}(u_{5,1}\cdot u_{4,2}-u_{4,1}\cdot u_{5,2})+\frac{40320}{u^8}(u_{5,1}\cdot u_{5,2}) \quad \cdots\cdots(32)$$

$$E_1=u_{2,1}\cdot u_{1,2}+\frac{1}{u}(u_{2,1}\cdot u_{2,2}-2u_{3,1}\cdot u_{1,2})+\frac{2}{u^2}(u_{2,1}\cdot u_{3,2}$$
$$-2u_{3,1}\cdot u_{2,2}+3u_{4,1}\cdot u_{1,2})+\frac{6}{u^3}(u_{2,1}\cdot u_{4,2}-2u_{3,1}\cdot u_{3,2}$$
$$+3u_{4,1}\cdot u_{2,2}-4u_{5,1}\cdot u_{1,2})+\frac{24}{u^4}(u_{2,1}\cdot u_{5,2}-2u_{3,1}\cdot u_{4,2}+$$
$$+3u_{4,1}\cdot u_{3,2}-4u_{5,1}\cdot u_{2,2})-\frac{120}{u^5}(2u_{3,1}\cdot u_{5,2}-3u_{4,1}\cdot u_{4,2}$$
$$+4u_{5,1}\cdot u_{3,2})+\frac{720}{u^6}(3u_{4,1}\cdot u_{5,2}-4u_{5,1}\cdot u_{4,2})-\frac{20160}{u^7}u_{5,1}\cdot u_{5,1} \cdots(33)$$

$$F_1=u_{1,1}\cdot u_{2,2}-\frac{1}{u}(u_{2,1}\cdot u_{2,2}-2u_{1,1}\cdot u_{3,2})+\frac{2}{u^2}(u_{3,1}\cdot u_{2,2}-2u_{2,1}\cdot u_{3,2}$$
$$+3u_{1,1}\cdot u_{4,2})-\frac{6}{u^3}(u_{4,1}\cdot u_{2,2}-2u_{3,1}\cdot u_{3,2}+3u_{2,1}\cdot u_{4,2}+4u_{1,1}\cdot u_{5,2})$$
$$+\frac{24}{u^4}(u_{5,1}\cdot u_{2,2}-2u_{4,1}\cdot u_{3,2}+3u_{3,1}\cdot u_{4,2}-4u_{2,1}\cdot u_{5,2})$$
$$+\frac{120}{u^5}(2u_{5,1}\cdot u_{3,2}-3u_{4,1}\cdot u_{4,2}+4u_{3,1}\cdot u_{5,2})+\frac{720}{u^6}(3u_{5,1}\cdot u_{4,2}$$
$$-4u_{4,1}\cdot u_{5,2})+\frac{20160}{u^7}u_{5,1}\cdot u_{5,2} \quad \cdots\cdots(34)$$

$$E_2=u_{3,1}\cdot u_{1,2}+\frac{1}{u}(u_{3,1}\cdot u_{2,2}-3u_{4,1}\cdot u_{1,2})+\frac{2}{u^2}(u_{3,1}\cdot u_{3,2}$$
$$-3u_{4,1}\cdot u_{2,2}+6u_{5,1}\cdot u_{1,2})+\frac{6}{u^3}(u_{3,1}\cdot u_{4,2}-3u_{4,1}\cdot u_{3,2}+$$
$$+6u_{5,1}\cdot u_{2,2})+\frac{24}{u^4}(u_{3,1}\cdot u_{5,2}-3u_{4,1}\cdot u_{4,2}+6u_{5,1}\cdot u_{3,2})$$
$$+\frac{360}{u^5}(u_{4,1}\cdot u_{5,2}+2u_{5,1}\cdot u_{4,2})-\frac{15840}{u^6}(u_{5,1}\cdot u_{5,2})$$
$$+\frac{20160}{u^7}(u_{5,1}\cdot u_{5,2}) \quad \cdots\cdots(35)$$

$$F_2=u_{1,1}\cdot u_{3,2}-\frac{1}{u}(u_{2,1}\cdot u_{3,2}-3u_{1,1}\cdot u_{4,2})+\frac{2}{u^2}(u_{3,1}\cdot u_{3,2}-$$
$$-3u_{2,1}\cdot u_{4,2}+6u_{1,1}\cdot u_{5,2})-\frac{6}{u^3}(u_{4,1}\cdot u_{3,2}-3u_{3,1}\cdot u_{4,2}+$$

$$+6u_{2,1}\cdot u_{5,2})+\frac{24}{u^4}(u_{5,1}\cdot u_{3,2}-3u_{4,1}\cdot u_{4,2}+6u_{3,1}\cdot u_{5,2})$$

$$+\frac{360}{u^5}(u_{5,1}\cdot u_{4,2}-2u_{4,1}\cdot u_{5,2}+12u_{5,1}\cdot u_{5,2}) \quad \cdots\cdots(36)$$

$$E_3=u_{4,1}\cdot u_{1,2}+\frac{1}{u}(u_{4,1}\cdot u_{2,2}-4u_{5,1}\cdot u_{1,2})+\frac{2}{u^2}(u_{4,1}\cdot u_{3,2}$$

$$-4u_{5,1}\cdot u_{2,2})+\frac{6}{u^3}(u_{4,1}\cdot u_{4,2}-4u_{5,1}\cdot u_{3,2})+\frac{24}{u^4}(u_{4,1}\cdot u_{5,2}$$

$$-4u_{5,1}\cdot u_{4,2})+\frac{6240}{u^5}u_{5,1}\cdot u_{5,2}-\frac{6720}{u^7}u_{5,1}\cdot u_{5,2} \quad \cdots\cdots(37)$$

$$F_3=u_{1,1}\cdot u_{4,2}-\frac{1}{u}(u_{2,1}\cdot u_{4,2}-4u_{1,1}\cdot u_{5,2})+\frac{2}{u^2}(u_{3,1}\cdot u_{4,2}$$

$$-4u_{2,1}\cdot u_{5,2})-\frac{6}{u^3}(u_{4,1}\cdot u_{4,2}-4u_{3,1}\cdot u_{5,2})+\frac{24}{u^4}(u_{5,1}\cdot u_{4,2}$$

$$-4u_{4,1}\cdot u_{5,2})+\frac{480}{u^5}u_{5,1}\cdot u_{5,2} \quad \cdots\cdots(38)$$

$$E_4=u_{5,1}\cdot u_{1,2}+\frac{1}{u}(u_{5,1}\cdot u_{2,2})+\frac{2}{u^2}(u_{5,1}\cdot u_{3,2})-\frac{54}{u^3}(u_{5,1}\cdot u_{4,2})$$

$$-\frac{1656}{u^4}(u_{5,1}\cdot u_{5,2})+\frac{1680}{u^7}(u_{5,1}\cdot u_{5,2}) \quad \cdots\cdots(39)$$

$$F_4=u_{1,1}\cdot u_{5,2}-\frac{1}{u}(u_{2,1}\cdot u_{5,2})+\frac{2}{u^2}(u_{3,1}\cdot u_{5,2})-\frac{6}{u^3}(u_{4,1}\cdot u_{5,2})$$

$$+\frac{24}{u^4}(u_{5,1}\cdot u_{5,2}) \quad \cdots\cdots(40)$$

$$E_5=\frac{336}{u^3}\cdot u_{5,1}\cdot u_{5,2}-\frac{336}{u^7}u_{5,1}\cdot u_{5,2} \quad \cdots\cdots(41)$$

$$E_6=-\frac{56}{u^2}u_{5,1}\cdot u_{5,2}+\frac{56}{u^7}u_{5,1}\cdot u_{5,2} \quad \cdots\cdots(42)$$

$$E_7=\frac{8}{u}u_{5,1}\cdot u_{5,2}-\frac{8}{u^7}u_{5,1}\cdot u_{5,2} \quad \cdots\cdots(43)$$

$$E_8=-u_{5,1}\cdot u_{5,2}+\frac{1}{u^7}u_{5,1}\cdot u_{5,2} \quad \cdots\cdots(44)$$

Since $Z=H^2$ and $dZ=2HdH$, the combined wave height distribution is proposed as

$$P(H)=\frac{dz}{dH}P(z)$$

$$= \frac{H}{2u} \{ (E_0 + E_1 \cdot H^2 + E_2 \cdot H^4 + E_3 \cdot H^6 + E_4 \cdot H^8 + E_5 \cdot H^{10} + E_6 \cdot H^{12} + E_7 \cdot H^{14} + E_8 \cdot H^{16}) \cdot e^{(-H^2/\kappa_{6,1})} - (E_0 + F_1 \cdot H^2 + F_2 \cdot H^4 + F_3 \cdot H^6 + F_4 \cdot H^8) \cdot e^{(-H^2/\kappa_{6,2})} \} \qquad \cdots\cdots(45)$$

Comparison with the measured data and the other theorems

In a study of non-linear wave height distribution, Longuet-Higgins[7] proposed a probability density function from the joint distribution of wave periods and amplitudes in a random wave field as

$$P(x) = 2x \cdot \exp(-x^2) \cdot N(\nu) \cdot F(x/\nu) \qquad \cdots\cdots(46)$$

$$F(x/\nu) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{x/\nu} \exp(-\beta^2)\, d\beta \qquad (\text{error function}) \qquad \cdots\cdots(47)$$

Where $x = H/H_r$, ν is a spectral width parameter; $N(\nu)$ is a normalization factor.

In a similar study, Chen et. al.[1] applied non-linear effect wave theory to obtain a dimensionless wave height distribution.

Comparison between the wave height distributions computed from the above-mentioned formulas and the field data is shown in Fig.1. It indicates that the present proposed method yields a better result.

Application

In order to apply the non-linear wave combination theorem to the prediction of wave in Taiwan Strait, the weather forecasting data, such as the weather chart, wind velocity and its direction must be available. Then, the prediction of wave can be completed using the following steps.

1. Waves from the East China Sea and Taiwan Strait are evaluated by Liang's[4] element wave prediction model.
2. The local wave height is computed by Tang's[9] shallow water wave prediction formula.
3. Since the wave steepness of two-source wave system can be calculated independently, the distribution of wave height of the combined wave using two non-linear wave combination theorem could be obtained.

The preliminary result of this prediction model is shown in Fig.2. It can be found that the results obtained by the non-linear two-source wave system are in agreement with those obtained by the two-linear source-wave system

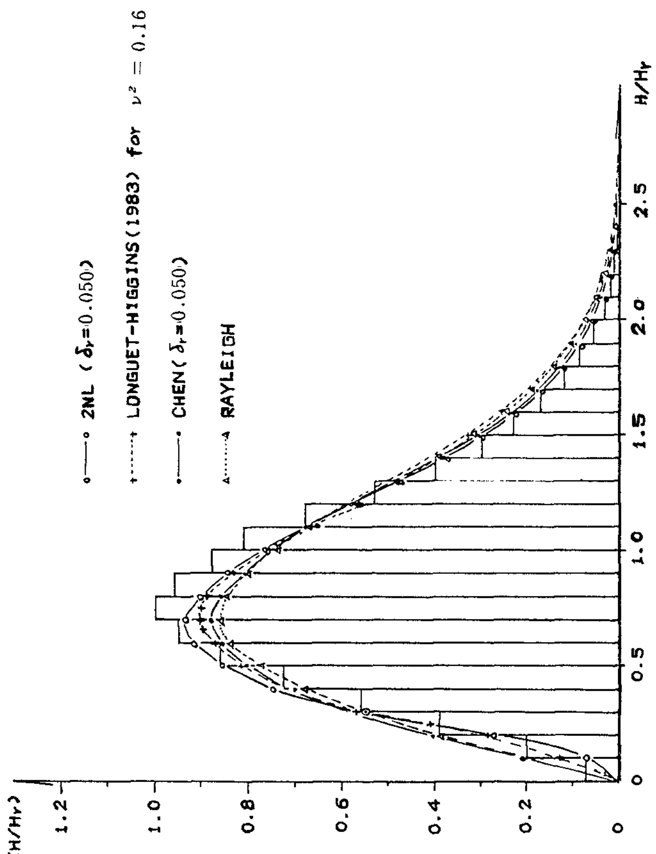

Fig.1 Comparison of the wave height distributions

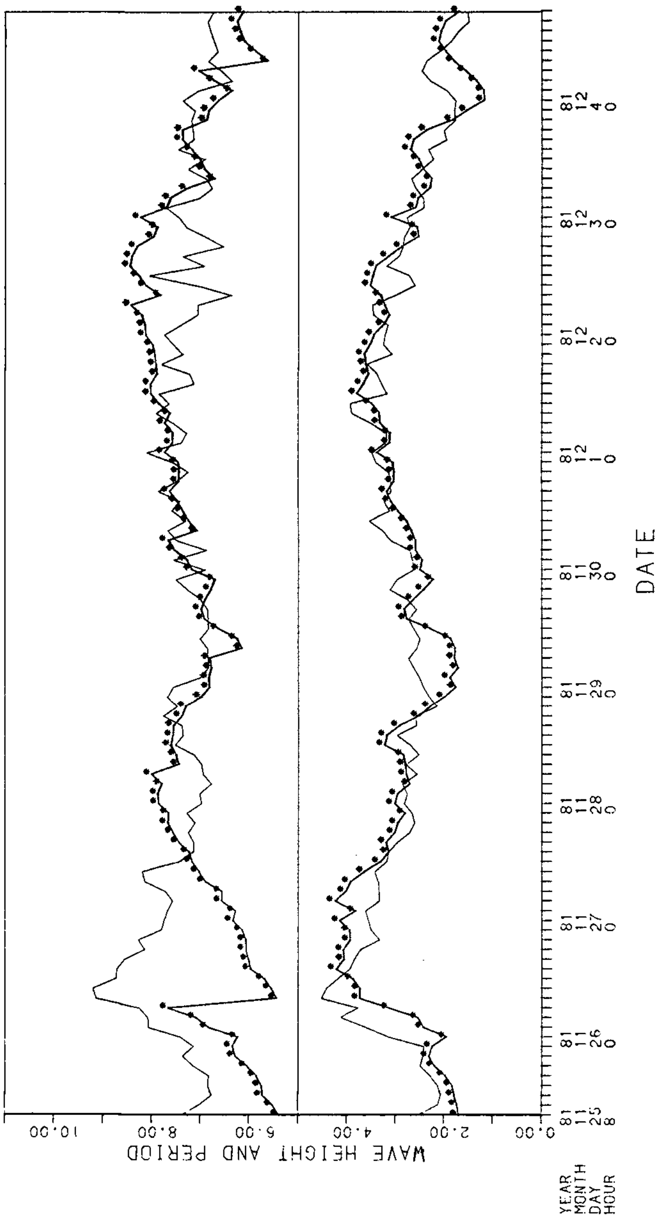

Fig.2 Results of wave prediction

including the energy loss effect. This implies that the higher-order terms in the non-linear two-source wave system are related to the energy loss.

Conclusion

In this study, the distribution of wave-height for two nonlinear source wave system is proposed, which yields the same result as those of two linear source-wave system as the wave steepness approaches zero.

From the field data, it was found that due to wave-wave interaction, some of the short-period wave will disappear which causes the mean wave height to increase and the maximum wave height to decrease. The proposed distribution can describe such a phenomenum.

Finally, it should be pointed out that although the proposed model is slightly complicated, as compared with that proposed by Chen et. al., the model is still simple to use and the computational effort does not increase significantly.

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