

## CHAPTER 29

### LATERAL MIXING AND WAVE DIRECTION IN THE WAVE-CURRENT INTERACTION REGION

Kyoung Ho Kim<sup>1</sup>, Sawaragi Toru<sup>2</sup>, Deguchi Ichiro<sup>3</sup>

#### ABSTRACT

The lateral mixing coefficient, in which the wave energy dissipation by wave breaking is taken account and the assumption of Richardson's 4/3 power law is involved, is derived for the surf zone and the diffusion of tracers injected in the wave-current interaction region is discussed experimentally to investigate the proposed lateral mixing coefficient. Furthermore measurements of velocity field on the three dimensional sloping beach of plane wave flume have been made by a bidirectional electromagnetic current meter. The results were used to investigate the characteristics of the structure of on-offshore and alongshore mean currents and the techniques for the determination of wave angle in the surf zone

#### 1. INTRODUCTION

There are many problems which have to be solved in analyzing the nearshore currents system in the surf zone, such as the expressions for the bottom shear stress, lateral mixing, wave direction, and radiation stress and so on. Of them, especially, the lateral mixing and wave direction problems have not been clarified sufficiently.

The studies of oceanic and estuarine mixing have been performed a lot experimentally and theoretically, however, for the lateral mixing which designates the turbulent exchange of momentum few detailed studies have been made because the turbulence generated due to breaking waves is not universally defined. Wave directions have been computed on the basis of the law of conservation of waves together with the irrotationality of wave number. In laboratory, in general, by taking the angle between the shoreline and the wave crest line wave angles are determined, however, this has many problems in the surf zone because of the difficulties in the de-

1. Assistant professor, Dept. of Civil Engineering, Chungbuk National University, Cheongju, Korea
2. Professor, Dept. of Civil Engineering, Osaka University, Suita, Osaka, Japan
3. Assistant professor, Dept. of Civil Engineering, Osaka University, Suita, Osaka, Japan

termination of the wave crest line .

In this paper, the equation for the lateral mixing coefficient in which the wave energy dissipation is taken account is proposed. Using the experimental results for long-shore currents and diffusion of tracers injected in the wave-current interaction region, the validity of proposed lateral mixing coefficient is verified. The methods of the determination of wave directions are also examined by using the horizontal velocity components and the results are compared with the values obtained from other methods.

## 2. LATERAL MIXING

The lateral mixing term corresponding to the horizontal turbulent momentum exchange in the wave-current interaction region is an important parameter for determining the spatial distribution of longshore current with distance to the shoreline and it has been said to be dependent on the local gradient of mean currents.

In the paper, lateral mixing coefficient is derived by using the wave energy dissipation rate due to wave breaking. The dependence of the lateral mixing coefficient on the wave steepness and bottom slope are discussed. The principle assumptions are used herein as follows.

- (1) Lateral mixing in the surf zone can be expressed by Richardson's 4/3 power law.
- (2) The mean dissipation rate of turbulence energy is in balance to that of wave energy due to wave breaking( Battjes, 1975)
- (3) The mixing scale is the same in order as mean water depth( see Iwata and Sawaragi, 1982).

From the assumption (1), lateral mixing coefficient  $K$  becomes following expression.

$$K \sim \epsilon^{1/3} (\sqrt{\bar{Y}^2})^{4/3} \quad (1)$$

where  $\epsilon$  is the mean rate of turbulent energy dissipation and  $\sqrt{\bar{Y}^2}$  is the mixing scale. The mean dissipation rate of turbulent energy and the mixing scale are expressed from the assumptions (2) and (3)

$$\epsilon \sim D/\rho d \quad (2)$$

$$\sqrt{\bar{Y}^2} \sim d \quad (3)$$

where  $D$  is the mean rate of wave energy dissipation, which by hypothesis equals the rate of production of turbulent energy.  $\rho$  is the water density and  $d$  is the mean water depth.

Sawaragi et. al.(1984) already reported that the mean dissipation rate of wave energy  $D$  can be expressed as Eq.(3) using the experimental results for the breaking waves on the sloping beach.

$$D = cF\rho^{-1/2}d^{-3/2}E^{3/2} \quad (4)$$

where  $c$  is the dimensionless constant,  $E$  is the wave energy density and  $F$  is the wave decay coefficient, which is expressed by surf similarity parameter  $\xi_s$  and the bottom slope  $S$ .

$$F = \begin{cases} 5.30 - 3.30\xi_s - 0.07S^{-1}, & \text{in the surf zone} \\ 0, & \text{outside the surf zone} \end{cases} \quad (5)$$

Substituting Eqs. (2)- (4) into Eq. (1), we obtain the following relation

$$K = \begin{cases} AF^{1/3}\sqrt{gd} H, & \text{in the surf zone} \\ 0, & \text{outside the surf zone} \end{cases} \quad (6)$$

where  $A$  is the dimensionless constant and  $H$  is the wave height. Considering the variation of mean water level and using the coefficient  $\gamma$ , which is the empirical proportionality factor between the wave height and the mean water depth, Eq. (6) is transformed into Eq.(7).

$$K = \begin{cases} AF^{1/3}\gamma S^*x'\sqrt{gd}, & \text{in the surf zone} \\ 0, & \text{outside the surf zone} \end{cases} \quad (7)$$

where  $S^*$  is the modified bottom slope and  $x'$  is the distance from the mean shoreline.

Perhaps the most well-known and frequently used expression for  $K$  was given by Longuet-Higgins(1970b). He suggested Eq.(8) for  $K$  by using Prandtl's mixing length theory.

$$K = N x \sqrt{gd} \quad (8)$$

where  $N$  is the constant which has the range of  $0 < N < 0.016$ . On the other hand, Battjes(1975) proposed Eq.(9) for  $K$  assuming the turbulent energy generated due to breaking waves.

$$K = (5/16\gamma^2)^{1/3} S^{4/3} M \sqrt{gd} \quad (9)$$

where  $M$  is the constant which is similar to  $N$  of Eq.(8).

Comparing Eq.(7) with Eqs.(8) and (9), we find that  $K$  in Eq.(7) comprises the wave steepness  $H_0/L_0$ , wave height-water depth ratio  $\gamma$  and the bottom slope  $S$  which represent

the wave characteristics. Accordingly it is thought that the constant A in Eq.(7) is more universal than the constants M and N.

To examine how K in Eq.(7) depends on wave steepness and the bottom slope, the relationship between  $S^*F^{1/3}$  and  $H_o/L_o$  is shown in Fig. 1. The values of  $S^*F^{1/3}$  are almost the function of S only and are not directly related with the wave steepnesses. Eq.(7) is similar to Eq.(9) proposed by Battjes.

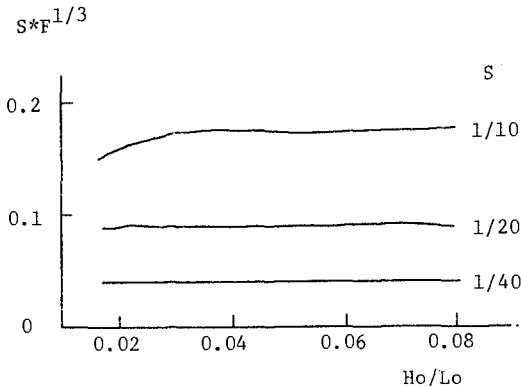


Fig. 1. Value of  $S^*F^{1/3}$  versus  $H_o/L_o$ .

### 3. WAVE DIRECTION

Wave angle just prior to wave breaking may be the most important parameter in the surf zone for the determination of longshore currents and sediment transport. According to Longuet-Higgins( 1970 a,b ) good estimates of wave angle are critical for the calculation of the alongshore component of radiation stresses, particularly when the angle of incident wave is small. Sherman et. al. (1986) reported that when the wave angle is  $5^\circ$ , a  $1^\circ$  error in the angle estimate results in a 20% error in the radiation stress computation.

In the numerical calculation of nearshore current, the wave angle is computed by the law of conservation of waves together with the irrotationality of wave number. On the other hand, in the field or laboratory wave directions have been determined by using either the horizontal velocity components measured with a current meter or the angle between wave crest line and shoreline. In laboratory, in general, the latter has been used, however, this has many problems because of the difficulties in the determination of obvious

wave crest line in the surf zone. Thus the validity of the above-mentioned methods has been performed not by the comparison of the computed results with the measured results but by how the calculated currents can revive the real currents.

In the present paper, the horizontal velocity components were measured by a bidirectional electromagnetic current meter in the fixed bed of plane flume and the problems in the measurements and the calculation of wave angle are examined by comparing the results from the above-mentioned methods. The wave angle is defined by the angle relative to the shore-normal as shown in Fig. 2.

If  $u$  and  $v$  are the onshore( $x$ ) and alongshore( $y$ ) components of velocity, respectively, two methods for determining the wave angle are derived as follows.

$$\theta_p = \tan^{-1} [2\overline{(u-U_o)}\overline{(v-V_o)} / \{\overline{(u-U_o)}^2 - \overline{(v-V_o)}^2\}] / 2 \quad (10)$$

$$\theta_t = \tan^{-1}(v_w/u_w) \quad (11)$$

where "\_\_\_" represents the time mean values and the subscripts  $o$  is corresponding to the time mean currents and  $w$  corresponding to the components of fluctuation.

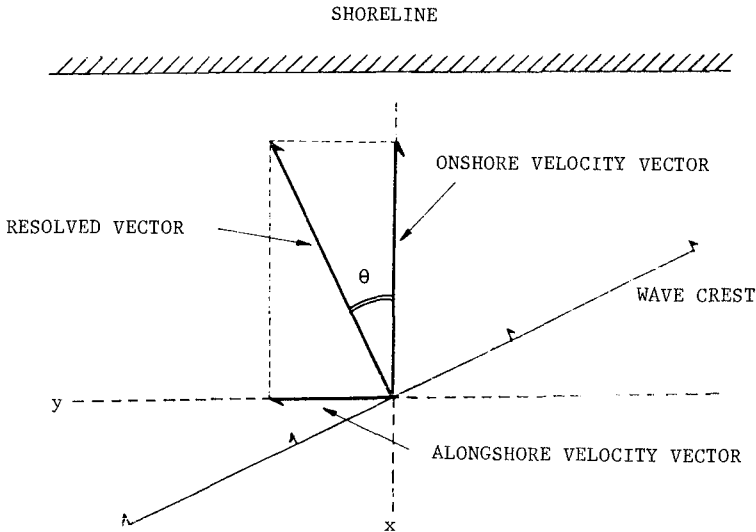


Fig. 2 Coordinate system for the wave angle.

4. EXPERIMENTS AND RESULTS

4.1 Experimental set-up and procedures

The wave facility at Osaka university was used to conduct the experiments. The plane flume is 20m long, 10m wide and 50cm deep. The waves broke on the bottom slope 1/10 ( Fig. 3) and the shoreline was made in order to form the angle of 30° against the wave generating board. The wave conditons are shown in Table. 1.

Table. 1 Wave conditions

bottom slope S	wave period T(sec)	wave steepness $H_0/L_0$	wave height $H_0$ (cm)	incident wave angle $\theta$ (degree)
1/10	0.91	0.052	6.70	30
	1.15	0.033	6.68	30
	1.44	0.015	6.70	30

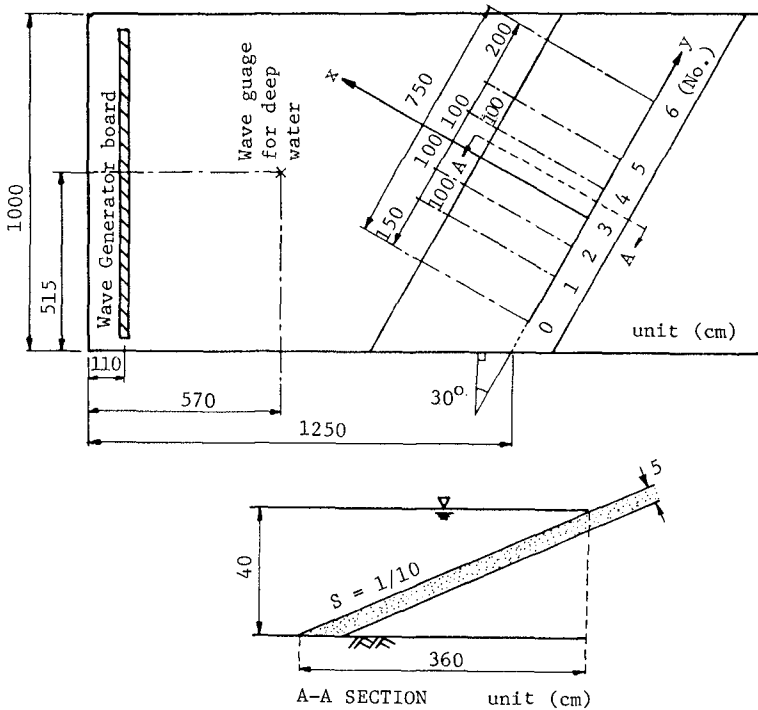


Fig. 3 Experimental set-up.

Wave profiles were measured over the surf zone at positions with an interval of 10cm or 20cm to the onshore direction(x) by means of the capacity type wave gauges. velocities were measured at 1-3 points of the vertical directions with the same measuring points of wave profiles by a bidirectional electromagnetic current meter. Simultaneously the wave crest and the diffusion of tracers injected in the flume were took photographs by a 16mm cine-camera to determine the wave direction and the coefficient of diffusion of tracer (L).

The wave profiles and the velocities were recorded as electric signals with a data recorder and they were destised at 100 hz by computer over 40 or 50 waves per each measueing point and the computer techniques were utilized to obtain the ensemble mean values. In Fig. 4-6, the example of the experi- mental results are shown. The upper figure shows the distri- butions of wave height H and the variation of mean water level  $\bar{\eta}$  and the lower is the distribution of longshore cur- rent with the distance to the shoreline together with the computed values of them.

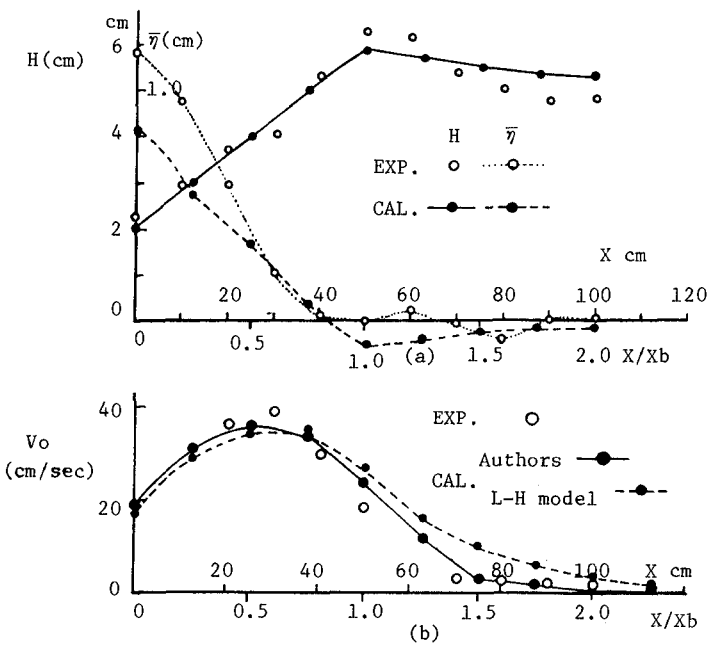


Fig. 4 Wave height, wave set-up and longshore current. ( $H_o/L_o = 0.015$ )

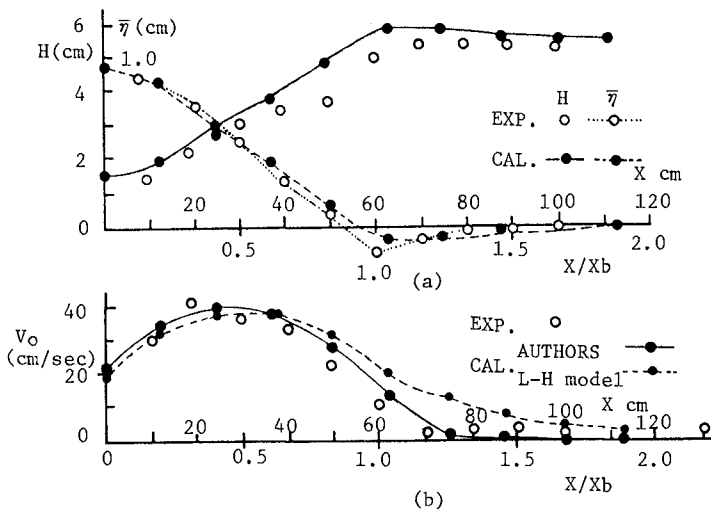


Fig. 5 wave height, wave set-up and longshore current. ( $H_o/L_o = 0.033$ )

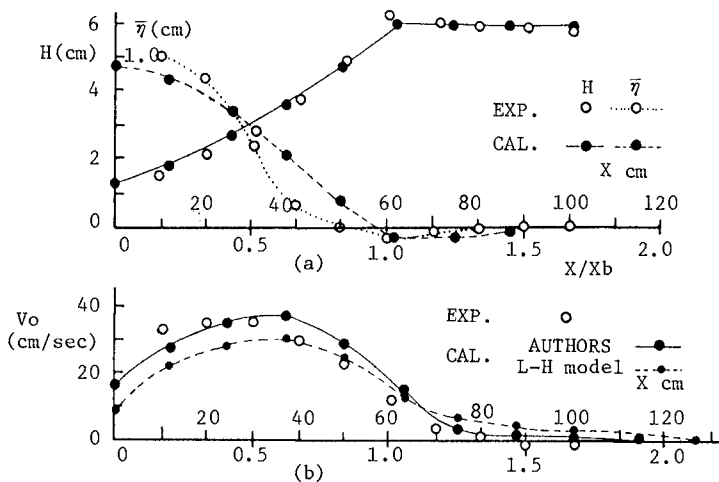


Fig. 6 Wave height, wave set-up and longshore current. ( $H_o/L_o = 0.052$ )



4.2 Time mean on-offshore and longshore current

Fig. 7 shows the vertical distributions of time mean on-offshore( $U_o$ ) and longshore current( $V_o$ ) measured in the case in which the wave steepness  $H_o/L_o = 0.015$ . As shown in Fig. 7(a) the longshore current is almost uniform vertically at least under the wave trough. The values of longshore currents in Fig. 4-6 are the mean values of these ones.

On the other hand, the on-offshore current  $U_o$  in Fig. 7 (b) is facing the offshore directions and the magnitude of them is larger in the upper zone and smaller in the lower zone. Whether this offshore current is a kind of rip current or a compensating one for the mass transport is not clear yet. In two-dimensional sloping beach, however, Nadaoka et. al.(1982) and Svendsen(1984) already reported that steady flows compensating for the mass transport exist. The values of  $U_o$  increase as they go towards the shoreline and the maximum value of it appears within the surf zone.

Fig. 8 shows the comparisons of the offshore mean current  $U_o$  in the present experiments which is measured on the

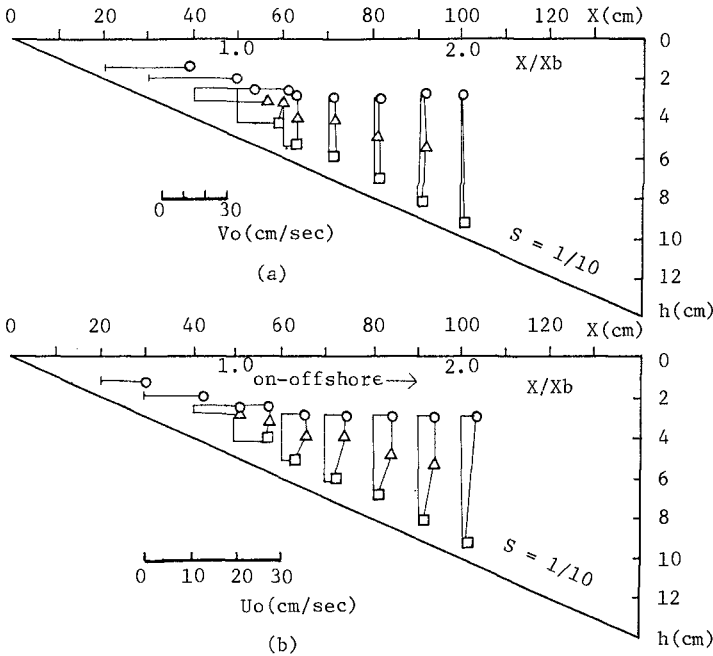


Fig. 7 Vertical distribution of on-offshore and longshore currents.

three-dimensional sloping beach with those measured on the two-dimensional sloping beach by Nadaoka et. al. (1984). The normalized values of  $U_o$  by  $\sqrt{gh_b}$  ( $h_b$  is the water depth corresponding to the breaking point) are plotted with the distance to the shoreline. The results are favourably good in magnitude and direction except for the neighbourhood of the shoreline. This mean current was also measured for the cases of  $H_o/L_o = 0.033$  and  $0.052$ , and its maximum value is nearly  $U_o/\sqrt{gh_b} \approx 0.2$  regardless of wave steepnesses. This makes us to guess that compensating flow for the mass transport exists even in the three-dimensional flow field where longshore currents occur.

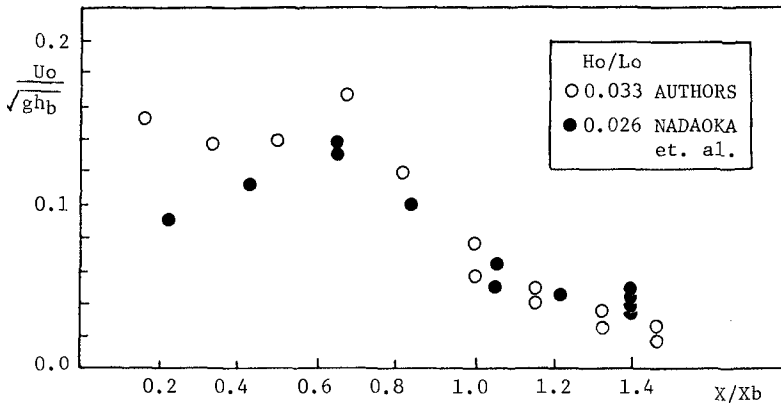


Fig. 8 Comparison of offshore current.

#### 4.3 Diffusion coefficient

The diffusion coefficient ( $L$ ) is shown in Fig. 9, which is obtained from the time variation of tracers continuously injected at positions  $x/x_b = 1/3, 2/3, 1$  and  $3/2$ , when the longshore currents occur ( $x_b$  is the distance to the breaking point). The diffusion coefficient somewhat decreases as the wave steepnesses increase, however, the variation is small and it is in proportion to the power of  $3/2$  of the distance from the shoreline. It is almost zero outside the surf zone.

It was already shown that the present model for the lateral mixing in section 2 is in proportion to the bottom slope  $S$ , wave height-depth ratio  $\gamma$ , and the distance from the shoreline  $x$ . In the present experiments the value of  $\gamma$ , has the range of 0.8-1.2 except for the vicinity of the shoreline and the breaking point. Its local variation is larger rather than that due to wave steepnesses, that is, the diffusion coefficient may not have the obvious dependency on the wave steepness. since it is thought that the

mechanisms of the lateral mixing can be assumed to be the same, the present model for the lateral mixing, Eq.(7) is reasonable. To conform this fact, the numerical calculation for the distributions of longshore currents with the distance to the shoreline and wave deformation was performed by using the conservation equations of mass, momentum and energy together with Eq.(4) for wave energy dissipation rate and Eq.(7) for the lateral mixing coefficient. The calculated results are illustrated in Fig.4- 6 together with the experimental results. In the calculation, radiation stresses, which are computed by the small amplitude wave theory for long waves, were multiplied by the reduction coefficient of 0.6-0.7 according to Stive and Wind (1984). In the figures the computed profiles of longshore currents using Eq.(8) for the lateral mixing coefficient proposed by Longuet-Higgins are also shown for comparisons. In the calculation of them the effect of wave-current interaction was considered. The computed longshore currents with Eq.(7) is in good accordance with the experimental results, particularly, outside the surf zone.

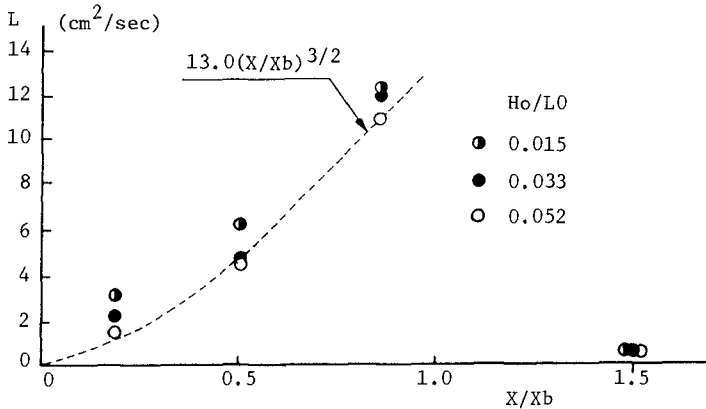


Fig. 9 Diffusion coefficient.

#### 4.4 Wave direction

Fig. 10 shows the comparisons of wave angles obtained by using the methods mentioned in section 3.  $\theta_m$  is the angle obtained by taking between the shoreline and the wave crest line taken photographs by 16mm cine-camera.  $\theta_p$  is the one computed by Eq.(10). The computed values obtained by the numerical analysis are also shown for comparisons. From Fig. 10, it is found that  $\theta_p$  shows a good accordance with the calculated values by the law of conservation of waves outside the surf zone. Within the surf zone the values of  $\theta_p$  largely varies, however, on the average they are nearly in accordance with the calculated values considering the effect of wave-current interaction. The wave angles obtained from

the wave crest line are generally overestimated.

In Fig. 11 and 12, the phase mean velocity components of wave fluctuation  $u_w$  and  $v_w$  are vectorally plotted for the regions of  $x/x_b=0.6$  in the surf zone and of  $x/x_b=2.2$  outside the surf zone, respectively. In the figure(a) of each the vector  $(u_w, v_w)$  and the wave angles  $\theta_p$  and  $\theta_t$  are shown. Figure (b) shows the profiles of the phase mean values of the displacement of water level  $\eta_w$  and those of water particle velocities due to fluctuation  $(u_w, v_w)$ . In the surf zone the phases of  $\eta_w$ ,  $u_w$  and  $v_w$  do not accord well one another. Though the values of  $v_w$  were taken over thirty waves, the turbulence of it is very large. Thus the wave direction  $\theta_t$  determined by the vector  $(u_w, v_w)$  greatly varies according to the phases with the range of about  $\pm 80^\circ$ . On the other hand, outside the surf zone the phases of  $\eta_w$ ,  $u_w$  and  $v_w$  show a good accordance one another, especially, the accordance of the phases between  $u_w$  and  $v_w$  is so good. Accordingly, the change of  $\theta_t$  is small and it varies within the range of about  $\pm 15^\circ$ . The values of  $\theta_p$  is almost the same as those of  $\theta_t$  in the phase in which  $\eta_w$  is less than zero.

From these results we see that outside the surf zone where longshore current and turbulence are not predominant, the wave direction can be determined by  $\theta_p$  and  $\theta_t$ , however, in the surf zone where the turbulence due to breaking and longshore current are predominant, it can not be always determined by them. The large variation of  $\theta_p$  in the surf zone is thought to be caused by the turbulence of  $v_w$  and the interaction between the water particle velocities due to fluctuation and the mean currents.

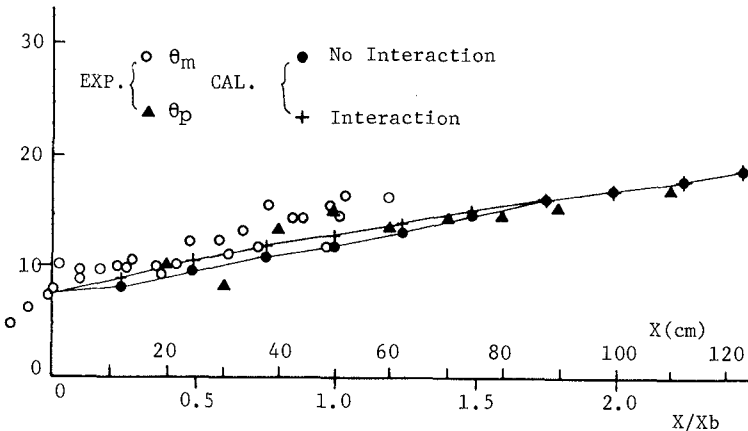


Fig. 10 Comparisons of wave angles.

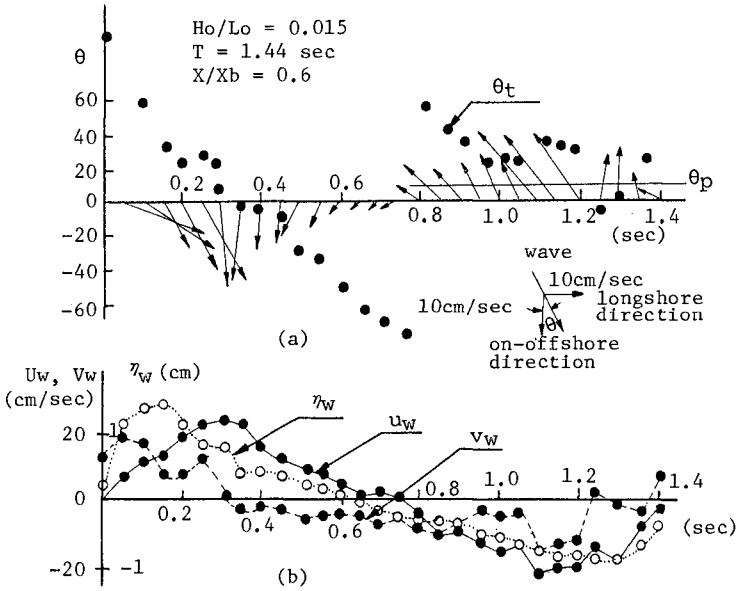


Fig. 11 Water particle velocity vector in the surf zone.

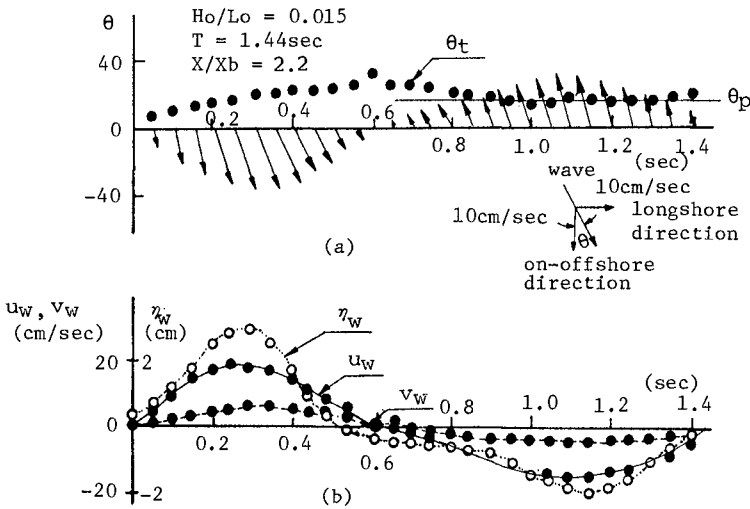


Fig. 12 Water particle velocity vector outside the surf zone.

## 5. DISCUSSION

In this paper, the lateral mixing coefficient, wave direction and the characteristics of on-offshore( $U_0$ ) and longshore current( $V_0$ ) are discussed. Firstly the lateral mixing coefficient is derived for the three-dimensional sloping beach where longshore current occurs. Secondly experiments are carried out to examine the wave direction, the structure of mean current in the surf zone and the diffusion of tracers injected.

The lateral mixing coefficient on the basis of wave energy dissipation is shown to be applicable for the surf zone. Wave angle  $\theta_p$  calculated by the horizontal velocity components and the mean currents show good estimates outside the surf zone. Within the surf zone the variation of  $\theta_p$  is large, however, on the average its values are nearly in accordance with those by traditional refraction analysis of deep water waves considering the effect of wave-current interaction. Wave angles obtained by the photographic interpretation of wave crest are generally overestimated. Furthermore it is found that even on the three-dimensional sloping beach where longshore current occurs the compensating flow may exist.

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