CHAPTER 33

ESTIMATION OF WATER PARTICLE VELOCITIES OF SHALLOW WATER WAVES BY A MODIFIED TRANSFER FUNCTION METHOD

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ABSTRACT

This paper is intended to propose a simple, yet highly reliable approximate method which uses a modified transfer function in order to evaluate the water particle velocity of finite amplitude waves at shallow water depth in regular and irregular wave environments. Using Dean's stream function theory,the linear function is modified so as to include the nonlinear effect of finite amplitude wave. The approximate method proposed here employs the modified transfer function. Laboratory experiments have been carried out to examine the validity of the proposed method. The approximate method is shown to estimate well the experimental values, as accurately as Dean's stream function method, although its calculation procedure is much simpler than that of Dean's method.

1. INTRODUCTION

The water surface profile of shallow water waves sharpens at the crest and flattens at the trough, and generally becomes asymmetric due to shoaling on a sloping beach. The asymmetry of the water surface profile gives rise to the unsymmetrical field of the wave kinematics[1)]. So far, no exact theory for asymmetrical waves has been proposed. This has prevented depicting precise characteristics of the kinematics of asymmetric regular and irregular waves. The previous investigations[2)-4)] have pointed out that the Dean's stream function method[5)] (hereafter referred to as DSFM), among several methods, predicts most accurately the water particle velocity of asymmetrical steep and near-breaking waves. However, DSFM requires a complicated iterative calculation as well as a full record of the water surface profile in order to obtain only a maximum water particle velocity. The accurate estimation of the maximum velocity is an important coastal engineering problem. Therefore, DSFM is not necessarily so useful for field engineers to estimate the kinematics of the design wave.

First of all, the difference of the water particle velocity estimated by the transfer function method using the linear wave theory (LTFM) and

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DSFM is investigated. The linear transfer function is then modified by introducing the nonlinear effect of finite amplitude waves with the help of Dean's stream function table[6]. The modified transfer function method (MTFM) which employs the modified transfer function is proposed as a simple yet highly reliable approximate method. The calculation procedure is much simpler than that of DSFM. Lastly, laboratory experiments are carried out to examine the validity of MTFM. The experiments show that MTFM is a highly reliable approximate method to calculate the wave kinematics.

2. MODIFIED TRANSFER FUNCTION

2.1 Modified transfer function for horizontal velocity of water particle

The present authors[7] showed that a time history of the horizontal velocity u(t) is closely correlated to that of the water surface profile and that the transfer function method is an useful one to estimate the horizontal component of the water particle velocity. The transfer function $H_u(h,T,s)$ for the horizontal particle velocity is defined by

$$u(t) = H_u(h,T,s)\eta(t) \; . \qquad (1)$$

Equation (1) shows that the horizontal velocity is predicted from the water surface profile through a filter of the transfer function. In this paper, the water surface profile is not decomposed into Fourier component waves, whether the waves are regular or irregular; the actual water surface profile is employed as $\eta(t)$.

The transfer function $H_u(h,T,s)$ and the horizontal velocity u(t) in case of the linear wave theory are described as follows;

$$H_u(h,T,s) = \frac{2\pi \cosh ks}{T \sinh kh} \; , \qquad (2)$$

$$u(t) = \frac{2\pi}{T}\frac{\cosh ks}{\sinh kh}\eta(t) \; . \qquad (3)$$

One example of comparison between the laboratory experiments and the estimation with Eq.(3) is shown in Fig.1. This figure indicates that the peak value obtained by the experiment agrees comparatively well with Eq.(3) for $\eta<0$. However, in the range of $\eta>0$, the calculated peak value is generally higher than the experimental value. This is a general tendency recognized for many data. The experimental fact implies that the nonlinear effect of finite amplitude waves which is not included in Eq.(3) should be considered in estimating the horizontal velocity, especially for the range of $\eta>0$. Then we will attempt to modify Eq.(2) so as to include the nonlinear effect.

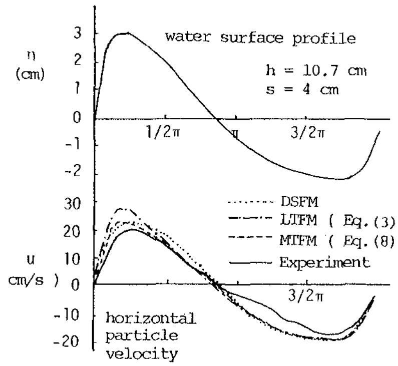


Fig.1 Comparison between measurement and calculations(Regular wave; experimental and DSFM's values are from Nadaoka et al.[4]).

We modify Eq.(2) for $\eta>0$, introducing a correction function f as expressed by

$$H_u(h,T,s) = \left(\frac{2\pi}{T}\frac{\cosh ks}{\sinh kfh}\right); \qquad \eta > 0$$
$$u(t) \doteq \left(\frac{2\pi}{T}\frac{\cosh ks}{\sinh kfh}\right)\eta(t); \qquad \eta > 0 \qquad \text{---} \quad (4)$$

In Eq.(4), fh is used in place of h. The accuracy of u(t) thus evaluated for the finite amplitude waves depends largely upon how accurately f is determined.

The correction function f is formulated by using Dean's stream function table for symmetrical regular waves under various conditions. The nonlinear effect of finite amplitude waves is supposed to be predominant at the phase of wave crest. Therefore, first, the correction function f at the two locations $s=h+\eta^+$(water surface) and s=0(bottom) are determined. That is, the maximum horizontal velocities u^+ at the two locations s=0 and $s=h+\eta^+$ are first read from Dean's stream function table, where η^+ is the wave crest height. Using the second equation of Eq.(4), the values of the correction function f for various combinations of u^+ and η^+ at the locations $s=h+\eta^+$ and s=0 are calculated. The values thus calculated are given in Figs. 2 and 3 in relation to η^+/h with auxiliary parameters h/Lo and H/Lo.

According to Figs.2 and 3, the correction function f can be approximated by Eq.(5) in the range of h/Lo $\leqq 0.2$.

$$\left.\begin{array}{lll} f = 1+\eta^+ & ; & \text{for } s = 0 \\ f = 1 & ; & \text{for } s = h+\eta^+ \end{array}\right\} \text{ for } \eta=\eta^+ . \qquad \text{----------} \quad (5)$$

Further, two assumptions are made here to formulate f for any value of s in the range of $\eta>0$;

(1) Equation (5) is extended to all positive values of η. In other words, we can substitute η (>0) for η^+.

(2) The correction function f at any depth s is expressed by a linear interporation between the values at $s=h+\eta^+$ and s = 0.

Using these assumptions, the correction function f for $\eta>0$ is formulated as

$$f = 1 + \frac{\eta^+}{h}\left(1 - \frac{s}{(h+\eta)}\right). \qquad \text{----------------------} \quad (6)$$

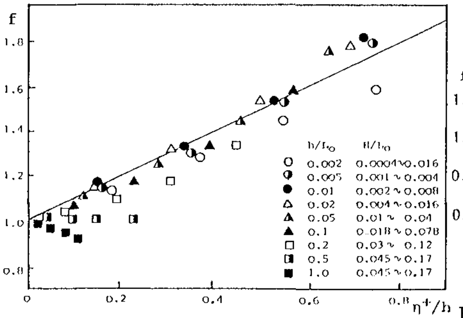


Fig.2 Value of correction function f at s = 0.

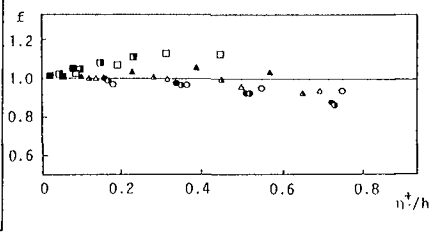


Fig.3 Value of correction function f at $s = h + \eta^+$ (symbols of calculated values are the same as those in Fig.3).

Now, let us discuss the correction function f for $\eta \leqq 0$. As already stated, Eq.(3) predicts comparatively well the experimental values for $\eta \leqq 0$. The same thing has been pointed out by Nadaoka et al. Eq.(2) is, therefore, used as the transfer function for $\eta \leqq 0$ without modification. Based on the above mentioned, the modified transfer function f proposed is given by

$$H_u(h,T,s) = \frac{2\pi}{T} \frac{\cosh ks}{\sinh(h+\eta(1-s/(h+\eta)))} \; ; \eta > 0$$
$$= \frac{2\pi}{T} \frac{\cosh ks}{\sinh kh} \; ; \eta \leqq 0 \quad \text{------- (7)}$$

Using Eqs.(1) and (7), the horizontal velocity u(t) is evaluated by

$$u(t) = \frac{2\pi}{T} \frac{\cosh ks}{\sinh k(h+\eta(1-s/(h+\eta)))} \eta(t) \; ; \eta > 0$$
$$= \frac{2\pi}{T} \frac{\cosh ks}{\sinh kh} \eta(t) \; ; \eta \leqq 0 \quad \text{------ (8)}$$

As seen in Figs.1 and 4, the estimation with the modified transfer function method (MTFM) is in better agreement with DSFM. Then, it can be said that MTFM is much better than the linear transfer function method (LTFM).

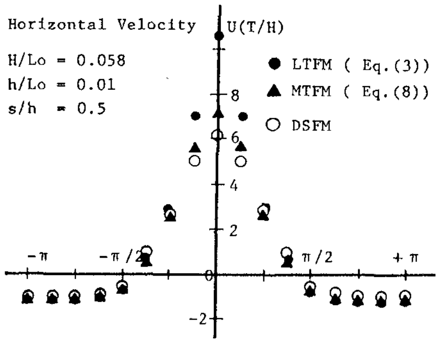


Fig.4 Comparison among DSFM, MTFM and linear transfer function method (LTFM).

2.2 Modified transfer function for vertical velocity of water particle

Judging from the conservation law of mass flux and u(t) given by Eq.(8), it can be supposed that the transfer function for w(t) should include the term $\partial\eta/\partial x$, where x is the horizontal distance. It is not desirable, however, that the transfer function involves the term of $\partial\eta/\partial x$ to avoid a complicated calculation which is not convenient to field engineers. Hence, we try to deduce a simple transfer function for w(t) as follow.

The present authors[7)] have already shown that the time profile of the vertical velocity of water particle resembles that of the water surface with a certain time lag, Δt. In addition, the water surface profile can be transferred to the horizontal velocity profile of water particle by use of Eq.(8). Then, w(t) can be expressed by

$$w(t) = H_w(h,T,s)\eta(t+\Delta t)$$
$$= H_{u-w}(h,T,s)u(t+\Delta t) \quad \text{------------------------ (9)}$$

where, $H_w(h,T,s)$ is a transfer function between w(t) and $\eta(t+\Delta t)$, $H_{u-w}(h,T,s)$ a transfer function between w(t) and $u(t+\Delta t)$ and Δt a time lag from a zero-upcrossing point to the next coming wave crest. The time lag Δt in Eq.(9) can be determined from a record of water surface profile.

In case of the linear wave theory, the transfer function H_{u-w} and Eq.(9) are expressed as Eq.(10) and Eq.(11), respectively.

$$H_{u-w}(h,T,s) = \tanh ks \qquad \text{(11)}$$

$$w(t) = \tanh ks \times u(t + \Delta t)\,. \qquad \text{(11)}$$

One example of comparison between the estimation with Eq.(10) and the values of DSFM is shown in Fig.5. The figure shows that the negative peak value is underestimated by Eq.(10). And, in some another cases, the positive peak value is also apart from the value of DSFM.

Therefore, we try to introduce a correction function g as expressed by

$$\left.\begin{aligned} H_u(h,T,s) &= (\, g \tanh ks \,) \\ w(t) &= (\, g \tanh ks \,)\times u(t + \Delta t) \end{aligned}\right\}. \qquad \text{(12)}$$

The correction function g is formulated by Dean's stream function table, the procedure of which is the same as that of the correction function f for the horizontal velocity.

Figs. 6 and 7 show the calculations of the correction functions g for positive and negative peak values in relation to the parameter of $\eta L_o/h^2$, respectively. In the figures, the correction function g calculated at s/h=0.5 is only shown. The correction functions g at other locations from the bottom up to the water surface were almost equal to that of s/h=0.5.

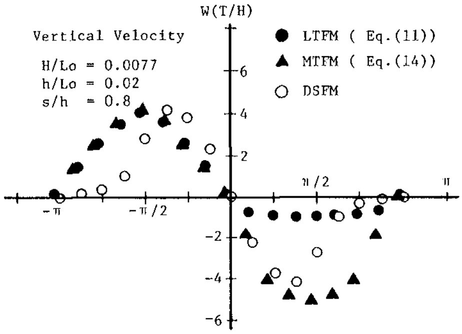


Fig.5 Comparison of vertical velocity of water particle among DSFM, LTFM and MTFM.

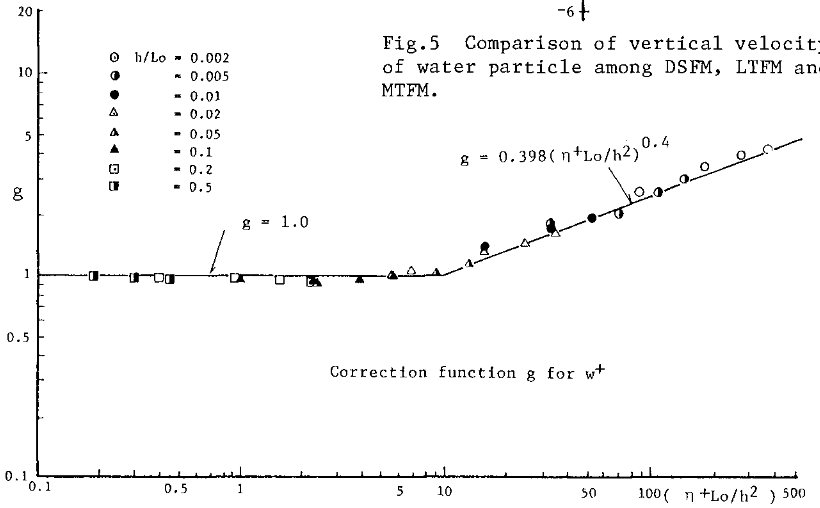


Fig. 6 Value of correction function g for w^+.

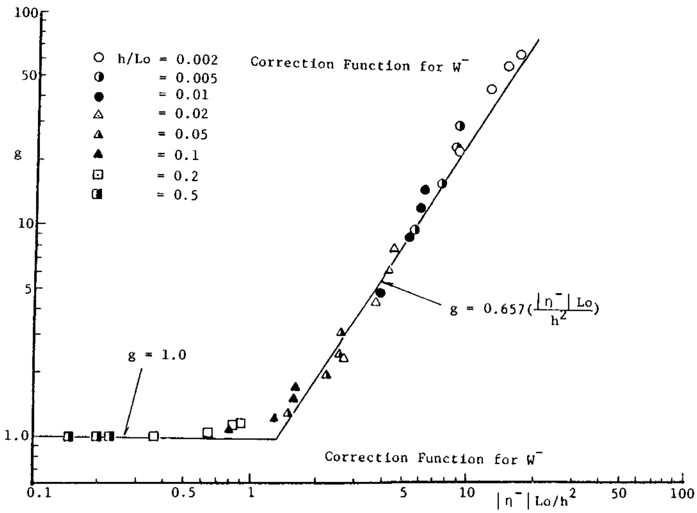


Fig. 7 Value of correction function g for w^-

According to Figs. 6 and 7, the correction function g can be approximated by

$$g = 0.398\left(\frac{\eta Lo}{h^2}\right)^{0.4} \quad ; \quad \eta Lo/h^2 > 10$$
$$= 1 \quad ; \quad -1.3 \leq \eta Lo/h^2 \leq 10 \qquad \text{----- (13)}$$
$$= 0.657\left(\frac{|\eta| Lo}{h^2}\right)^{1.6} \quad ; \quad -1.3 > \eta Lo/h^2 .$$

Using this correction function, the vertical velocity of the modified transfer function method is evaluated by

$$w(t) = 0.398\left(\frac{\eta\, Lo}{h^2}\right)^{0.4} \times \tanh ks \times u(t+\Delta t) \quad ; \quad \eta Lo/h^2 > 10$$
$$= \tanh ks \times u(t+\Delta t) \quad ; \quad -1.3 \leq \eta Lo/h^2 \leq 10 \qquad (14)$$
$$= 0.657\left(\frac{|\eta|\, Lo}{h^2}\right)^{1.6} \times \tanh ks \times u(t+\Delta t) \quad ; \quad \eta Lo/h^2 < -1.3 .$$

Figure 5 shows one example of comparison among DSFM, LTFM and MTFM by Eq.(14). The estimation of MTFM correspods well to that by DSFM , although there is a discrepancy between their time profiles.

Summarizing the above stated, it can be pointed out that MTFM which uses Eqs.(8) and (14) is a highly reliable approximate method to evaluate the wave kinematics.

3. LABORATORY EXPERIMENT

In the experiment, an indoor wave tank of 0.7m in width, 0.95m in depth and 25m in length at Nagoya University was used. At one end of the wave tank, was installed a flap-type wave generator controlled by an oil-pressure servo system. At the other end of the wave tank, beach slope of 1/15 and 1/8 and a horizontal step having a slope of 1/15 at the leading edge were set, as shown in Fig.8. Experimental conditions are given in Table 1.

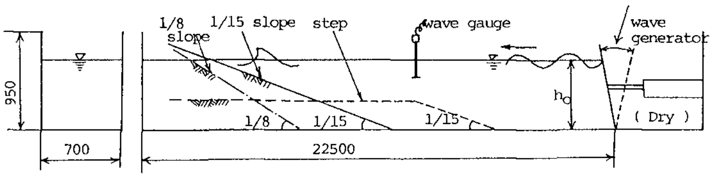


Fig.8 Schematic view of experimental set-up

Table 1 Experimental conditions

run	bottom slope i	wave period T, $T_{1/3}$(s)	wave height H, $H_{1/3}$(cm)	stillwater depth h(cm)	breaker depth h_b(cm)	breaker height H_b(cm)	wave
1 - 1	1/15	1.43	15.4	63	15.0	14.8	regular
1 - 2	"	1.10	10.4	"	10.9	10.9	"
1 - 3	"	0.84	11.5	"	11.3	11.3	"
2 - 1	1/8	1.10	9.7	"	10.4	10.4	"
2 - 2	"	1.00	11.8	"	11.9	11.9	"
3 - 1	step	0.89	9.0	35	10.2	10.2	"
3 - 2	"	1.06	15.4	50	16.0	16.0	"
3 - 3	"	1.00	14.1	44	14.8	14.8	"
4 - 1	1/15	1.08	11.3	63	-	-	irregular
4 - 2	"	1.32	10.3	"	-	-	"

($H_{1/3}$ anf $T_{1/3}$ are significant wave height and period, respectively)

Wave profiles were measured by capacitance-type wave gauges. The water particle velocity was measured by a cantilever type velocimeter newly devised by the present authors[8]. The water particle velocities were measured at many locations from s=0 up to nearly $s=h+\eta^+$ in the vertical direction and from h= 60cm up to h= 1cm in the horizontal direction. Time profiles of water surface and particle velocities were recorded on a magnetic tape over a period of 1 min.

4. RESULTS AND DISCUSSION

In this section, the validity of MTFM is discussed by comparing with DSFM, LTFM and the experimental values. In the calculation of DSFM, the water surface profile was divided into 20 discrete values for one wave cycle. Following Dean's method, with use of the 20 discrete values, the iterative calculation was performed 4 or 5 times until the relative error between the measured and the predicted water surface profiles becomes less than 5%.

4.1 Regular wave

(1) Time profile of water particle velocity

Figure 9 shows time profiles of the water particle velocity estimated by MTFM and DSFM, and experimental values. Figures 9(a), (b) and (c) are the typical examples of the horizontal and vertical velocities for the waves before breaking, just at breaking point and after breaking, respectively. In case of the non-breaking wave, as in Fig.9(a), the water particle velocity predicted by MTFM is very close to that by DSFM, and both estimations are generally in good agreement with experiments. On the other hand, in case of waves just at breaking point and after breaking (see Fig.9(b) and (c)), the horizontal velocity u(t) estimated by MTFM corresponds well to that by DSFM, and the agreement of both calculations with experimantal values is very good. Concerning the vertical velocity w(t), however, a little difference between the two calculations was generally observed. In Figs.9(b) and (c), w(t) estimated by MTFM has a better agreement with experimetal values than that by DSFM.

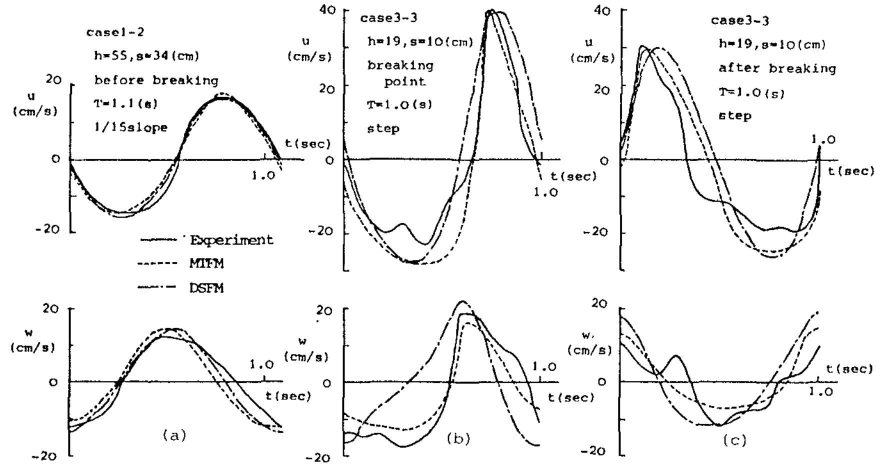


Fig.9 Comparison of time profiles of water particle velocities between calculations and experiments (for regular waves)

(2) Vertical distribution of maximum water particle velocity

Figure 10 shows some examples of the vertical distribution of the maximum horizontal and vertical velocities, where the positive and negative values of $u/\sqrt{gh}$ are, respectively, the non-dimensional onshore and offshore horizontal velocities, and the positive and negative values of $w/\sqrt{gh}$ are the non-dimensional upward and downward vertical velocities, respectively. In Fig.10, the calculated values of DSFM, MTFM and LTFM are drawn to be compared with experimental values. Using Eqs.(8) and (14), the maximum values of $u^+(s)$, $u^-(s)$, $w^+(s)$ and $w^-(s)$ by MTFM are given by

$$\left.\begin{aligned} u^+(s) &= \frac{2\pi}{T}\frac{\cosh ks}{\sinh k(h+\eta^+(1-s/(h+\eta^+)))}\eta^+ \quad \text{(for onshore velocity)} \\ u^-(s) &= \frac{2\pi}{T}\frac{\cosh ks}{\sinh kh}\eta^- \quad \text{(for offshore velocity)} \\ w^+(s) &= \frac{2\pi}{T}\frac{\sinh ks}{\sinh k(h+\eta^+(1-s/(h+\eta^+)))}\eta^+ \quad \text{(for upward velocity)} \\ w^-(s) &= \frac{2\pi}{T}\frac{\sinh ks}{\sinh kh}\eta^- \quad \text{(for downward velocity)} \end{aligned}\right\} \quad (15)$$

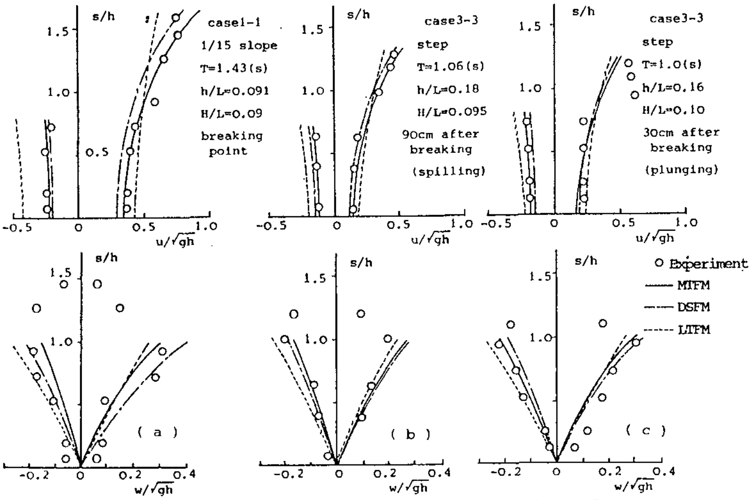


Fig.10 Comparison of the vertical distribution of maximum particle velocity between measurements and calculations (for regular waves).

In Eq.(15), η^+ and η^- are the wave crest and trough elevations, respectively, and it should be stressed that $w^+(s)$ and $w^-(s)$ are derived for $-1.3 \leqq \eta Lo/h^2 \leqq 10$. Therefore, although they cover a wide range of wave conditions, different expressions of $w^+(s)$ and $w^-(s)$ should be used in place of Eq.(15) for the range of $\eta Lo/h^2 \geqq 10$ and $\eta Lo/h^2 \leqq -1.3$.

According to Fig.10, it can be pointed out that the vertical distributions of maximum horizontal and vertical velocities predicted by MTFM and DSFM are in good agreement with the experiments except near the free surface of the plunging breaker and the bottom. The plunging breaker has usually a bore-like character with entrainment of air bubbles. Any of the three methods does not consider the entrainment of air bubbles. This may have caused the discrepancy between the calculations and experiment for the plunging breaker. Figure 10 also shows that $w^+/\sqrt{gh}$ and $w^-/\sqrt{gh}$ near the bottom tend to be underestimated by the three methods. The reason is, as pointed out by Nadaoka et al., that the three theories assume a horizontal bottom and then they cannot exactly express the vertical component of wave motion along the bottom slope.

In Fig.10, calculated values above the stillwater level ($s/h \geqq 1$) are not indicated. The experimental values show that $w^+/\sqrt{gh}$ and $w^-/\sqrt{gh}$ decrease linearly with s/h toward the wave crest. Judging from the comparisons between the calculations and experiments including those in Fig. 10, LTFM seems to be inferior to MTFM and DSFM in predicting the maximum values, $u^+/\sqrt{gh}$, $u^-/\sqrt{gh}$, $w^+/\sqrt{gh}$, and $w^-/\sqrt{gh}$ as well as time profiles of the water particle velocity, u(t) and w(t).

4.2 Irregular wave

In evaluating the velocities of water particle u(t) and w(t), the irregular wave is not decomposed into Fourier component waves, but is treated by the wave-by-wave analysis which uses the zero-downcrossing method. Equations (8) and (14) are applied to the individual waves in a random wave train. In calculating w(t), Δt was given by an average value of time intervals between a zero-upcrossing point and the next-coming wave crest of the wave train.

Figures 11 shows one comparison between the measurements and calculations by means of DSFM and MTFM. First, let us discuss on u(t). As seen in Fig.11, the time profiles estimated by MTFM as well as DSFM correspond well, in general, to the experimental values. However, in Fig.11, DSFM underestimates the positive peak value of u(t) for the particular waves with symbole * for which the mean water level is higher than the stillwater level. On this point, MTFM can be said to be superior to DSFM. Concerning the vertical velocity w(t), both theories (DSFM and MTFM) do not predict well the experimental values. One reason of this is attributed to the method of wave definition employed, i.e. the wave-by-wave analysis like the zero-downcrossing method. The zero-downcrossing method divides the water surface profile into two parts at the zero-downcrossing point. Since the water surface profile near the stillwater level governs sensitively a peak value of w(t), the use of the zero-downcrossing method yields errors in evaluating the peak value. The same thing can be said for the zero-upcrossing method. Therefore, as pointed out by Daemrich at al.[9], the crest-to-crest method may be recommended in order to estimate the vertical velocity.

Lastly, peak values of the horizontal particle velocity u^+ at 2cm above the stillwater level are discussed. Table 2 gives peak values u^+ for successive ten waves in a random wave train, and schematic water surface profiles are shown in the second row for convenience of discussion. Comparing the calculated values with DSFM and MTFM with experimental ones, MTFM can be said to predict the experimental values much better than DSFM in general. However, it should be noted that MTFM as well as DSFM tend to underestimate u^+ of the waves which have two peaks on the water surface above the stillwater level such as No.1, No.4, No.8 and No.9.

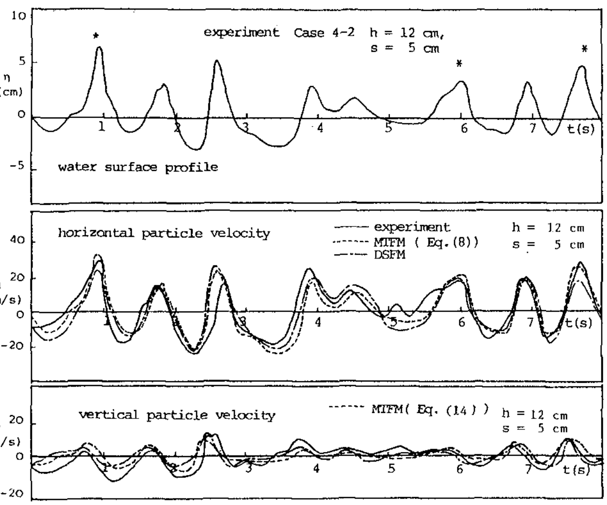
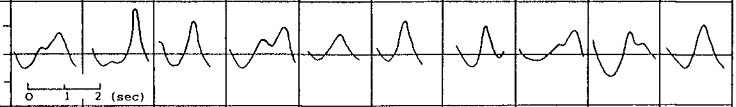


Fig.11 Time profile of water particle velocities for irregular wave (Case 4-2, h=12cm and s=5cm).

Table 2 Comparison between experimental and calculated values of horizontal particle velocity for irregular wave (Case 4-2, h = 21cm and s = 23cm).

Wave No.	1	2	3	4	5	6	7	8	9	10
water surface profile η(t) (cm)										
period T (sec)	1.52	1.39	0.97	1.61	0.97	0.94	0.90	1.68	1.61	1.13
wave height H(cm)	6.4	10.7	8.8	7.5	3.5	7.6	9.2	5.5	9.1	8.7
u^+(Experiment)	40,0	56.1	50.9	46,2	26.8	48.4	46.5	44.7	46.5	43.2
u^+(DSFM)	22.7	51.7	41.3	25.0	18.7	31.0	34.5	21.9	31.0	34.4
u^+(MTFM(Eq.(8))	29.4	62.0	53.4	31.8	25.3	50.5	40.8	31.2	31.4	46.7

u^+(Experiment),u^+(DSFM) and u^+(MTFM(Eq.(8)); unit (cm/s)

Two peaks on the free surface profile will imply the existence at least two different waves. Then, it seems difficult for DSFM and MTFM proposed for a monocromatic wave to evaluate well the particle velocities for such composite waves.

As stated above, the water particle velocity estimated by MTFM is in good agreement with the experimental value of the so-called "quasi-regular waves" in a random wave train. Comparing the modified transfer function method with Dean's stream function method, it can be said that MTFM is never inferior to DSFM in evaluating accurately the water particle velocity of finite amplitude regular or irregular waves. Since the calculation procedure of MTFM is much simpler than that of DSFM, the modified transfer function method proposed in this paper is useful and highly reliable for evaluating the wave kinematics.

5. CONCLUDING REMARKS

The approximate method which uses the modified transfer function has been presented to evaluate the water particle velocity of finite amplitude waves, for engineering purposes. The approximate method (modified transfer function method) is never inferior to Dean's stream function method in evaluating the wave kinematics of asymmetrical or symmetrical finite amplitude waves in regular and irregular wave environments, although its calculation procedure is much simpler than that of Dean's stream function method. In particular, the use of the approximate method is recommended to estimate the maximum water particle velocity of finite amplitude waves in shallow water depth. However, it is found that the predominant large velocity of water particle near the free surface in the surf-zone cannot be estimated accurately by any existing method.

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