CHAPTER 39

Estimating Long-Term Wave Statistics From Long-Term Wind Statistics[†]

Paul C. Liu[‡]

The Wave Climate Synthesis method, which derives long-term wave statistics from long-term wind statistics directly, is tested with four years of wind and wave measurements recorded from eight NOMAD buoys in the Great Lakes during 1981-1984. The results show that it is an excellent method for estimating long-term wave statistics and design wave height. While the method is by no means intended to replace wave hindcasting procedures, it is shown to be a useful additional tool for the coastal engineer.

Introduction

One of the first considerations when solving a coastal engineering problem is the determination of a design wave height. The usual procedure of determining a design wave height where actual wave measurements are not available consists of gathering pertinent long-term wind data, making wave hindcasts from the wind data, and subsequently performing statistical analysis on the hindcast wave data to estimate design waves. Although this conventional approach does provide useful information, it can be time consuming. In this paper we examine an alternative approach called Wave Climate Synthesis (e.g., Andrews et al., 1983) which derives long-term wave statistics directly from long-term wind statistics, bypassing wave hindcasting. This approach simplifies the process of estimating a design wave height. The validity and usefulness of this approach are demonstrated with four years of wind and wave measurements recorded from eight NOMAD buoys in the Great Lakes during 1981-1984.

Approach

The Wave Climate Synthesis method, pioneered by Hogben and collaborators (Andrews, et al., 1983, Hogben, 1987, and Hogben and Dacunha, 1985) at British Maritime Technology (formerly National Maritime Institute), is based on the concept of relating long-term marginal probabilities of significant wave height $P(H_s)$ and wind

---

[†]GLERL Contribution No. 539.

[‡]Oceanographer, NOAA/Great Lakes Environmental Research Laboratory, 2300 Washtenaw Avenue, Ann Arbor, MI 48104.

speed $P(W_r)$ by means of a parametric model of the conditional probability $P(H_s|W_r)$ as

$$P(H_s) = \Sigma P(H_s|W_r)P(W_r). \tag{1}$$

$P(H_s|W_r)$ is assumed to be represented by the Gamma distribution,

$$P(H_s|W_r) = [q^{p+1}/\Gamma(p+1)]H_s^{\,p}\exp(-qH_s), \tag{2}$$

where $\Gamma(.)$ is the Gamma function. The parameters p and q are given in terms of the averages and standard deviations of the wave height in each wind speed interval, H_r and v_r, respectively as

$$p = H_r^{\,2}/v_r^{\,2} - 1, \tag{3}$$

and

$$q = H_r/v_r^{\,2}. \tag{4}$$

H_r and v_r, measured in m, are empirical functions of wind speed W_r, in m/s, given by

$$H_r = [(aW_r^{\,n})^2 + h_2^{\,2}]^{1/2}, \tag{5}$$

and

$$v_r = h_2(b + cW_r + dW_r^{\,2}). \tag{6}$$

In the above formulations the parameters a, b, c, d, h_2, and n are all empirically obtainable from known distributions $P(W_r)$ and $P(H_s)$. From extensive oceanic studies Andrews et al. (1983), using the linear form of (6), developed recommended values of coefficients for open ocean and limited fetch conditions as listed in Table 1. One can make use of the recommended values when actual data are not available

Location	h_2	a	n	b	c
Open Ocean	2.000	0.033	1.480	0.500	0.0125
Limited Fetch	0.500	0.023	1.380	0.750	0.0188

Table 1 Recommended values of coefficients in Equations (5) and (6) according to Andrews et al. (1983)

or derive appropriate values by fitting Equations (5) and (6) to available data. In either case, once these parameters are determined

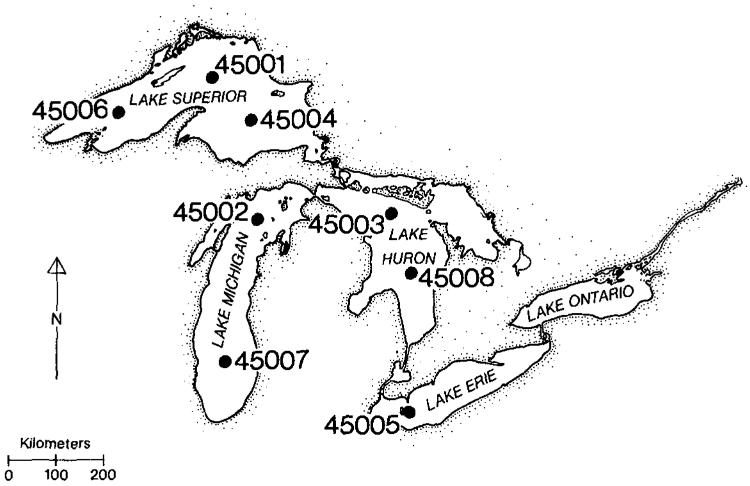

Figure 1 Location map of the eight NOMAD buoys in the Great Lakes

Gage	Lake	Lat.	Long.	Total Data in 1981-1984
45001	Superior C.	48.0 N	87.6 W	16079
45002	Michigan N.	45.3 N	86.3 W	16956
45003	Huron N.	45.3 N	82.8 W	17220
45004	Superior E.	47.2 N	86.5 W	11606
45005	Erie W.	41.7 N	82.5 W	19737
45006	Superior W.	47.3 N	90.0 W	15162
45007	Michigan S.	42.7 N	87.1 W	18448
45008	Huron S.	44.3 N	82.4 W	17286

Table 2 List of location and total data of the eight NOMAD buoys

and a long-term wind speed distribution $P(W_r)$ is given, a corresponding long-term wave height distribution $P(H_s)$ can be readily determined.

Data

To test the applicability of formulations (1) - (6) we use the long-term wind and wave data recorded from NOMAD buoys in the Great Lakes. There have been eight NOMAD buoys moored in the Great Lakes (Figure 1) since 1981 in water depths ranging from 15 m to 250 m. These buoys are boat-shaped, 6 m in length, with an electronic payload for measuring wind speed, wind direction, barometric pressure, air temperature, sea surface temperature, and surface wave spectral data. Most of the meteorological sensors are located 5 m above the water surface. Wind speed and direction, as well as air and surface water temperatures, are measured at 1-s intervals, averaged over 8.5 minutes and reported hourly. The waves are measured with an accelerometer using an on-board Wave Data Analyzer system (Steele and Johnson, 1977) that transmits acceleration spectral data via the UHF GOES satellite to a shore station. Wave frequency spectra with 48 degrees of freedom are calculated from 20 min of measurements each hour. Significant wave heights, H_s, are obtained as four times the square root of the total energy integrated over the calculated wave frequency spectrum.

The data recorded from all eight buoys during 1981 through 1984 were used in this study. The total number of data points available from each buoy ranges from 11,606 for the buoy at Lake Superior E. to 19,737 for the buoy at Lake Erie W., as shown in Table 2.

Applications

An example of applying the Wave Climate Synthesis method as formulated in Equations (1) - (6) to the Great Lakes wind and wave data is shown in Figure 2 for buoy 45004 at Lake Superior E. In Figure 2 there are four sections. The top section is the histogram for wind speed distribution, which is the required input for this method. The second section, obtained from recorded data, presents the joint distribution of wind speed and significant wave height, H_s, with the corresponding histogram for the distribution of H_s shown to the right of the joint distribution. The third section shows, in each wind speed interval, the averages and standard deviations of the wave height, defined as H_r and v_r in Equations (3) -(6), as the circles and dashed lines respectively. Equations (5) and (6) can be fitted, by the least-squares method, to yield the appropriate coefficients; and the corresponding Gamma distributions $P[H_s|H_r(W_r),v_r(W_r)]$ are also plotted. Finally, from the information derived from the above three sections, a joint distribution of wave height and wind speed and a wave height distribution, similar to the second section, can be estimated as shown in the bottom section. In general, only the histogram in the top section and a set of empirical parameters are needed to produce the estimation shown in the bottom section.

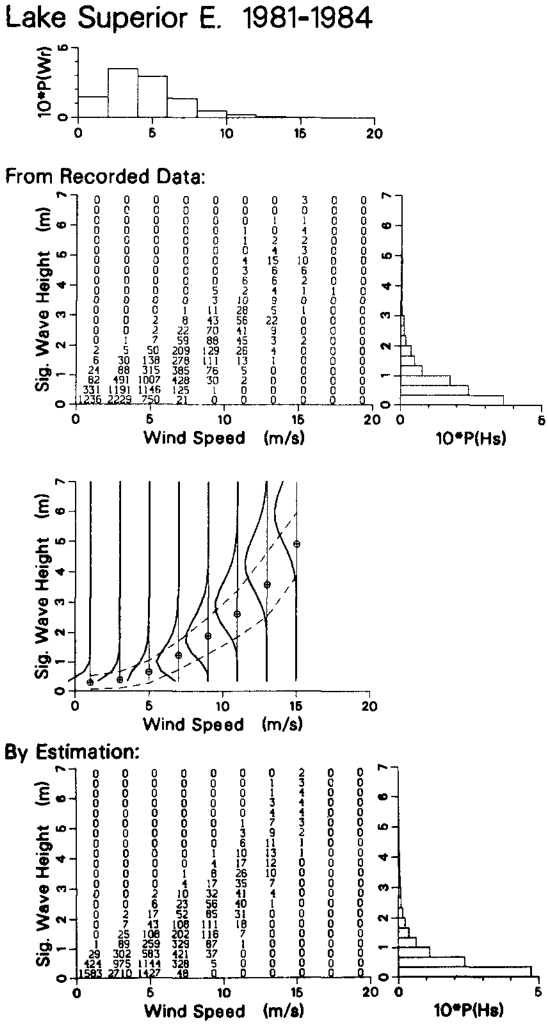

Figure 2 Long-term wind and wave histograms and joint distribution from recorded data and by estimation.

Following this approach, we derived the coefficients in Equations (5) and (6) for the eight buoy locations listed in Table 3. In the oceanic studies, h_2, in m, generally signifies the average swell activity. In the Great Lakes, however, swell is usually less significant, so in this study we simply set h_2 to the average significant wave height corresponding to the lowest wind speed interval (i.e., 0 - 2 m/s). Comparing the results given in Table 3 with the oceanic results in Table 1 indicates that for this study the values of h_2 and coefficient a are smaller while the exponent, n, is larger than those recommended by Andrews et al. (1983) for the oceanic studies. The differences are not significant since they are of the same order of magnitude as the limited fetch cases. A comparison of the fitted curves using the coefficients in Table 3 with recorded data

Gage	h_2	a	n	b	c	d
45001	0.364	0.013	2.294	0.491	0.171	-0.003
45002	0.299	0.013	2.201	0.458	0.192	-0.007
45003	0.289	0.012	2.225	0.450	0.213	-0.008
45004	0.286	0.012	2.302	0.624	0.114	0.007
45005	0.261	0.017	1.879	0.424	0.224	-0.012
45006	0.286	0.011	2.330	0.161	0.398	-0.021
45007	0.348	0.013	2.166	0.403	0.161	-0.003
45008	0.335	0.015	2.143	0.475	0.240	-0.012

Table 3 Derived coefficients for Equations (5) and (6) for all eight buoys.

for all eight buoys is shown in Figure 3. Here the model equations (5) and (6) are shown in solid lines with recorded data in dots. Although the degree of goodness of fit varies among the buoys, the model equations can be considered as providing reasonable representations for the recorded data.

Discussions

In Figure 2 the resemblance between the results from measurement and estimation, as shown in the second and bottom sections, appears to be reasonably close. A further and perhaps more effective comparison can be made by plotting the cumulative significant wave height distribution on a Weibull probability scale. This is shown in Figure 4 for all eight buoys. In this figure the recorded data are given in dots, and estimations based on coefficients in Table 3 are represented by the solid lines. The close comparison between recorded data and estimations over the higher wave heights shows that the Wave Climate Synthesis method is a viable approach for estimating long-term wave height statistics from long-term wind statistics.

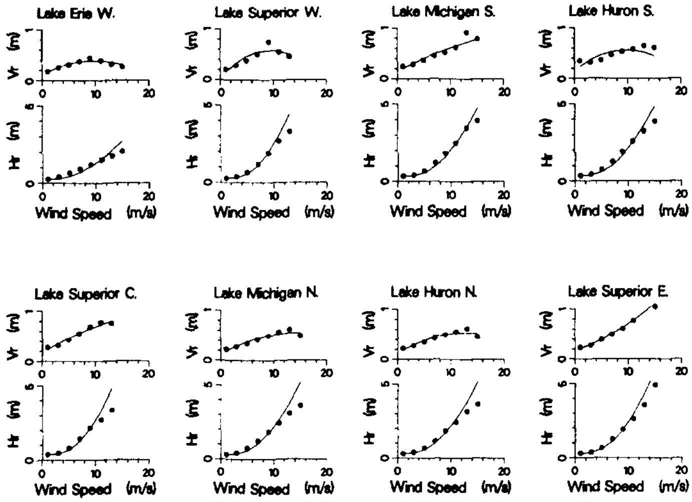

Figure 3 Fitting of Equations (5) and (6) for the eight buoys. Solid lines are estimations, with recorded data shown as dots.

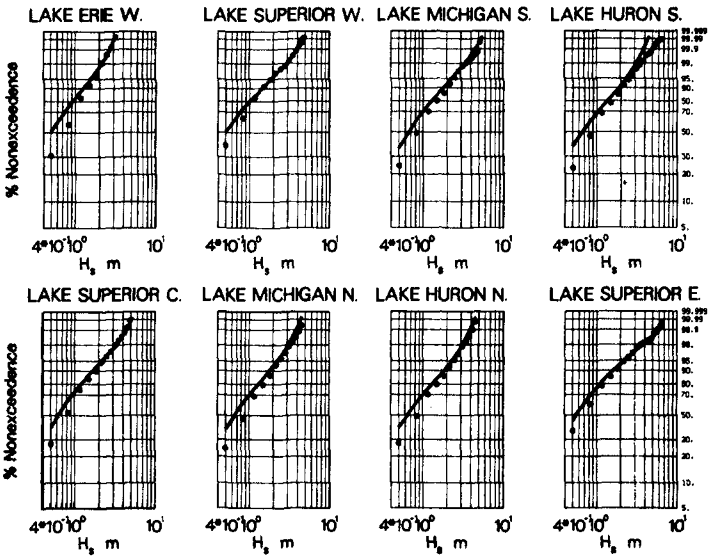

Figure 4 Cumulative distribution of significant wave heights plotted on Weibull probability paper for all eight buoys. Estimation based on Table 2 coefficients appear as solid lines, with recorded data shown as dots.

Gage	h_2	a	n	b	c
45001	↑	↑	↑	↑	0.200
45002					0.091
45003					0.091
45004	0.308	0.013	2.192	0.758	0.215
45005					0.050
45006					0.110
45007					0.099
45008	↓	↓	↓	↓	0.150

Table 4 Modified coefficients for Equations (5) and (6) for all eight buoys.

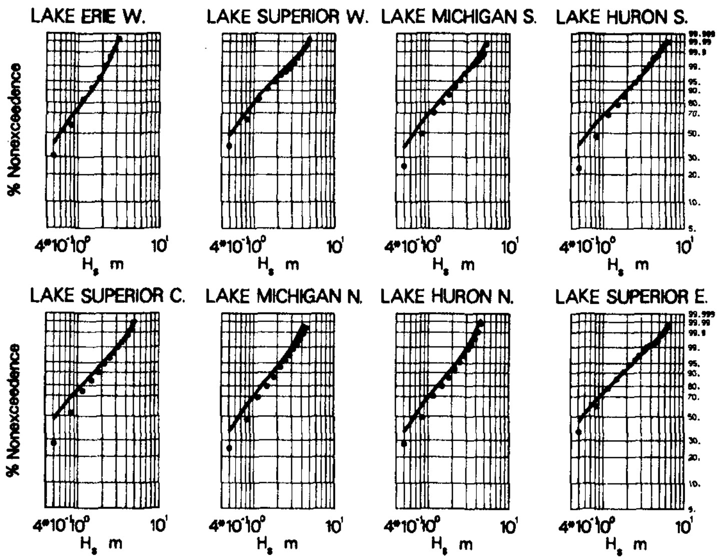

Figure 5 Cumulative distribution of significant wave heights plotted on Weibull probability paper for all eight buoys. Estimation based on Table 4 coefficients appear as solid lines, with recorded data shown as dots.

Closer examination of Figure 4 shows that sometimes the Wave Climate Synthesis method clearly overestimates the recorded data, as in the case of Lake Huron S. Sometimes, as in the cases of Lake Superior C. and Lake Superior E., the method tends to underestimate the recorded data at the high wave height end. The underestimation is especially unacceptable from a design engineer's point of view.

In an effort to rectify these unsettled cases as well as to simplify the use of different coefficients for different buoy locations, we first notice that in Table 3 the values of h_2, a and n for Equation (5) are not significantly different among the different buoys. In addition when we set d = 0 and use only the linear form of representation for the $v_r(W_r)$ function, we found that the coefficients b and c in Equation (6) vary from 0.326 to 1.138 and from 0.013 to 0.225 respectively. Assuming that the variations in b are also not significant, we then choose to use the same average values of h_2, a, n, and b for all eight buoy locations. Thus as shown in Table 4 we allow c to be the only coefficient that is different for different buoy locations. The values of c in the table are determined by trial and error.

Figure 5 presents the resulting cumulative distribution of wave height plots based on the new set of coefficients given in Table 4. This simplified version of coefficients effectively rectifies the excessive overestimation and underestimation and provides satisfactory results for practical applications. Using Table 4, for any given location in the Great Lakes where only wind statistics are available, one needs only to choose an appropriate value of the coefficient c and can then readily proceed to estimate design wave heights with justifiable accuracy.

Concluding Remarks

In this paper we examined applications of the Wave Climate Synthesis method for estimating long-term wave height statistics from given long-term wind statistics. Based on four years of data from each of eight NOMAD buoys in the Great Lakes we found that this method provides reasonably accurate and useful estimations. Many of the results presented here are preliminary in nature. The values of coefficient c in Table 4, for instance, are deduced by trial and error. More detailed analysis and data fitting as well as error analysis will be applied in subsequent studies. The model equations and coefficients will be further examined when additional longer-term data become available. While this method will not replace comprehensive wave hindcasting, it has been shown to be a useful additional tool for estimating design wave heights for coastal engineers.

References

Andrews, K.S., Dacunha, N.M. and Hogben, N., 1983: Wave Climate Synthesis. NMI Report No.R149., National Maritime Institute, Feltham, England, 143 pp.

Hogben, N., 1987: Experience from compilation of global wave statistics. Spring Meetings of the Royal Institution of Naval Architects.

Hogben, N. and Dacunha, N.M., 1985: Wave climate synthesis: Some recent advances. Proceedings, 17th Offshore Technology Conference, (OTC 4938), Vol.2, pp. 355-363.

Steele, K. and Johnson A. Jr., 1977: Data buoy wave measurements. In Ocean Wave Climate, pp.301-316. M. D. Earle and A. Malahoff Eds. Plenum Press, New York..