CHAPTER 40

EVALUATION OF A MODIFIED STRETCHED LINEAR WAVE THEORY

by

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Introduction

Many experimental investigations of the drag and inertia force coefficients have relied on the determination of water particle kinematics from measured wave forms. Since the pioneering work of Airy (1845), Stokes (1847, 1880) and others, a number of wave theories have been developed for predicting water particle kinematics. Clearly, the use of a certain wave theory will lead to corresponding force coefficients. Therefore, a wave theory that provides more accurate water particle kinematics is very important.

Reid (1958) developed the simple superposition method for predicting water particle kinematics from a measured sea surface that could be either random or periodic. The method is based upon linear long-crested wave theory. Borgman (1965, 1967, 1969a, 1969b) introduced the linearized spectral density of wave force on a pile due to a random Gaussian sea. The drag force component has been approximated in the simplest form by a linear relation. This method, however, cannot calculate properties of the wave field and wave force above the mean water level.

Wheeler (1969) applied simple superposition with a stretching factor in the vertical coordinate position for hurricane-generated wave data during Wave Project II. With this method it was possible to evaluate the wave force above the mean water level.

Hudspeth, et al. (1974) compared the wave forces computed by simple superposition and irregular stream function methods. By using the simple superposition and a stretched vertical coordinate to calculate water particle kinematics, the comparison between measured and calculated forces indicates the stream function method was generally better than the linear wave theory method. The magnitudes of the difference between the theories, however, is not consistent. For maximum forces, the differences are not great. They noted that the linear

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wave theory provides greater facility in carrying out certain mathematical operations than is possible using nonlinear wave theories.

The advantage of using the simple superposition method is that the entire measured sea surface data can be analyzed without the need to consider only one extreme wave using a nonlinear symmetric wave theory or irregular stream function theory. This paper will introduce a new mathematical stretching factor similar to the stretching factor used by Wheeler (1969). For several wave conditions, the symmetrical stream function wave theory, the stretched linear wave theory, and the new approach will be compared.

Simple Superposition

To simplify the problem, only two-dimensional wave motion will be considered. A long, irregular sequence of linear waves can be represented as an infinite sum of simple harmonic waves with closely spaced frequencies and random phase angles, i.e.,

$$\eta(x,t) = \sum_n a_n \cos (k_n x - \sigma_n t + \alpha_n) \tag{1}$$

where η is the water surface displacement; $a_n = \sqrt{2P_\eta(\sigma_n)\Delta\sigma}$; $P_\eta(\sigma_n)$ is the (one-sided) energy density spectrum of the irregular sea, varying with angular frequency σ_n; k_n is the wave number for the nth wave component ($k_n = 2\pi/L_n$); L_n is the wave length of the nth wave component; x is distance and α_n is the phase angle for the nth wave component.

When the small amplitude wave theory is used to estimate the flow regime in a wave system from the surface profile (Eq. 1), the velocity potential may be expressed by

$$\phi(x,z,t) = \sum_n \frac{a_n g}{\sigma_n} \frac{\cosh k_n(h+z)}{\cosh k_n h} \sin (k_n x - \sigma_n t + \alpha_n) \tag{2}$$

where z is referenced to the mean water surface and is positive upwards. The dispersion relationship is given by

$$\sigma_n^2 = gk_n \tanh k_n h \tag{3}$$

The horizontal water particle velocity component is given by

$$u(x,z,t) = -\frac{\partial\phi}{\partial x} = \sum_n \frac{a_n k_n g}{\sigma_n} \frac{\cosh k_n'(h+z)}{\cosh k_n h} \cos(k_n x - \sigma_n t + \alpha_n) \tag{4}$$

From Eq. 4, the horizontal water particle kinematics at any given time or any given position can be found very easily. But when the position is close to the free surface, the results are not in agreement with laboratory or field data. Mathematically, Eqs. 2 and 3 are determined individually, component by component, using the small amplitude wave theory approach assuming that all wave components are independent of each other. Fig. 1 shows a high-frequency wave component superimposed upon a low frequency wave component. Evaluating Eq. 4 at a time when both components are maximum, the vertical surface coordinate is $z = a_1 + a_2$, and it is found that the contribution of the high-frequency component to the kinematics is exaggerated by the vertical displacement due to the large low-frequency wave.

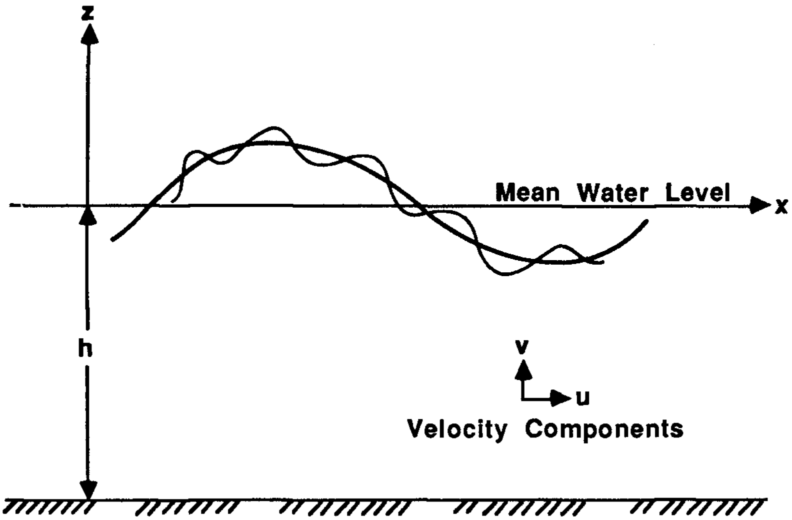

Fig. 1. Superposition of a high frequency wave on a low frequency wave.

Stretched Linear Wave Theory (SLWT)

The simple superposition technique was first developed by Reid in 1958. Wheeler applied the technique to hurricane-generated wave data in 1969. The simple superposition technique, however, predicts unreasonably large wave kinematics when it applies to locations that are above the mean water level. Therefore, Wheeler introduced an intuitive stretching factor to predict the wave kinematics by using the simple superposition method. If kinematic predictions are desired at elevation $S = h + z$ above the bottom (where h is the mean water depth), then the stretching results in calculations being carried out at elevation S' above the bottom, where

$$S'(x,z,t) = \alpha(x,t)S \tag{5}$$

and

$$\alpha(x,t) = \frac{h}{h+\eta(x,t)} \tag{6}$$

Here $\eta(x,t)$ is the instantaneous water surface displacement measured from the mean water level. This vertical stretching is particularly significant near the free surface. It is of interest to note that the velocity potential that is stretched in this manner no longer exactly satisfies the Laplace equation.

By using the stretched linear wave theory, the kinematics (and other quantities) calculated at the actual free surface are the same as those evaluated at the mean water level in an unstretched system, thus eliminating the exaggeration of the high frequency component described previously.

Stream Function Wave Theory (SFWT)

The stream function wave theory, as described by Dean (1965), can be used to represent both theoretical symmetrical and irregular waves. In application, the theory is used to generate a symmetric wave with given parameters, such as wave height, period, and water depth. It is this symmetric form of the stream function wave theory that will be used in the comparison.

Taking a frame of reference moving with wave celerity C, the problem is reduced to a steady form. Then the kinematic free surface boundary condition becomes

$$v = \frac{\partial \Psi}{\partial x} = (u - C)\,\frac{\partial \eta}{\partial x} = -\,\frac{\partial \Psi}{\partial y}\,\frac{\partial \eta}{\partial x} \text{ on } z = \eta(x) \tag{7}$$

where v is the vertical velocity component and Ψ is the stream function. Eq. 7 is satisfied exactly by the equation for the stream function:

$$\Psi(x,z) = \frac{L}{T}\, z + \sum_{n=1}^{NN} A(n)\, \sinh\!\left[\frac{2n\pi}{L}\,(h+z)\right] \cos\!\left(\frac{2n\pi}{L}\, x\right) \tag{8}$$

Evaluating this expression on the free surface, i.e., setting $z = \eta$, the free surface is

$$\eta = \frac{T}{L}\,\Psi_n - \frac{T}{L} \sum_{n=1}^{NN} A(n)\, \sinh\!\left[\frac{2n\pi}{L}\,(h+\eta)\right] \cos\!\left(\frac{2n\pi}{L}\, x\right) \tag{9}$$

Since the coordinate system is moving with the wave celerity, and the pressure is zero on the free surface, the dynamic free surface boundary condition is

$$\frac{1}{2g}\left((u-C)^2 + v^2\right) + \eta = Q \text{ on } z = \eta(x) \tag{10}$$

where Q is the total head or "Bernoulli constant" on the free surface.

The numerical problem of establishing the wave theory is thus one of determining values of A(1), A(2),...., A(NN), L, Ψ_η, such that the dynamic free surface boundary condition is satisfied as closely as possible. These coefficients are determined iteratively employing a nonlinear least-squares procedure.

Modified Stretched Linear Wave Theory (MSLWT)

The SLWT was reported to be based on wave tank studies; Lo (1979) introduced a new stretching factor by using the free surface boundary conditions. Assuming that the pressure is zero on the free surface, the dynamic free surface boundary condition is

$$\frac{1}{2g}(u^2 + v^2) + z - \frac{1}{g}\frac{\partial\phi}{\partial t} = C(t) \quad , \quad z = \eta \tag{11}$$

The kinematic free surface boundary condition is

$$\frac{\partial\eta}{\partial t} + u\frac{\partial\eta}{\partial x} = v \quad , \quad z = \eta \tag{12}$$

Eqs. 11 and 12 are nonlinear partial differential equations, and the nonlinearities come from two major sources. One is the product terms in the equations, i.e., u^2, v^2 and $u\ \partial\eta/\partial x$; the other is that the equation applies to the actual free surface, $z = \eta$. The most popular technique for solving these equations is by using the Taylor series expansion to expand the value of the condition from $z = 0$ to $z = \eta$ (mean water level, a known location), then by using the perturbation procedure to solve the equations order by order. For example, the small amplitude wave theory is the solution of the first order, and the Stokes theories of various orders are extensions of the approach.

The MSLWT will be solved to the first order by using Eqs. 11 and 12 only on the free surface $z = \eta$. The solution is:

$$\eta(x,t) = a\cos(kx - \sigma t) \tag{13}$$

$$\phi_m(x,y,t) = -\frac{ag}{\sigma}\frac{\cosh k\,(h+z)}{\cosh k\,(h+\eta)}\sin(kx - \sigma t) \tag{14}$$

$$\sigma^2 = gk\tanh k(h+\eta) \tag{15}$$

Eqs. 13, 14 and 15 satisfy the boundary conditions (without the product terms in Eqs. 11 and 12) exactly. But, like the solution of the stretched linear wave theory, they no longer satisfy the Laplace equation. The effective wave number k now depends on water surface displacement. Comparing the solution for small amplitude wave theory with that for the modified stretched linear wave theory, the new stretching factor found for the modified stretched linear wave theory is given by

$$\text{Stretching Factor} = \frac{\cosh kh}{\cosh k\,(h+\eta)} \tag{16}$$

This stretching factor was established by using the boundary conditions of the wave system. For a given time, position and mean water depth, this factor will be a constant through the vertical coordinate. But under the same conditions, the stretching factor for the stretched linear wave theory (Equations 5 and 6) is a function of depth, and it has a maximum value at the free surface, a minimum value at the bottom.

To describe the statistical properties of a random water surface, it is convenient to decompose the variable of interest into Fourier components. The Fourier series representation of the water surface displacement is given by the simple superposition method as Eq. 1.

When the MSLWT is used to estimate the flow regime in a wave system from the surface profile, the velocity potential may be expressed by

$$\phi_m(x,z,t) = -\sum_n \frac{a_n g}{\sigma_n} \frac{\cosh k_n(h+z)}{\cosh k_n(h+\eta)} \sin\,(k_n x - \sigma_n t + \alpha_n) \tag{17}$$

where the dispersion relationship is given by

$$\sigma_n^2 = g\,k_n \tanh k_n(h+\eta) \tag{18}$$

and the first order horizontal and vertical water particle velocities are derived from Eqs. 17 and 18, as follows:

$$u_m(x,z,t) = \sum_n a_n\,\sigma_n \frac{\cosh k_n(h+z)}{\sinh k_n(h+\eta)} \cos(k_n x - \sigma_n t + \alpha_n) \tag{19}$$

and

$$v_m(x,z,t) = \sum_n a_n\,\sigma_n \frac{\sinh k_n(h+z)}{\sinh k_n(h+\eta)} \sin\,(k_n x - \sigma_n t + \alpha_n) \tag{20}$$

Comparison Between SLWT and MSLWT

Ohmart and Gratz (1978) presented a comparison of measured and predicted ocean wave kinematics. They found by comparing linear wave theory, Stokes Fifth Order Wave Theory and the Irregular Stream Function Wave Theory (IRSF) that IRSF yields better results. The field data were measured from the test structure (the CAGC Eugene Island 266F platform), located in the Gulf of Mexico. Mean water depth was 177 feet. A wave staff was mounted on one corner of the structure. The current meters were mounted at elevations of 5 and 20 feet below mean water level.

A 34 minute block of data was recorded each two hours or four hours depending upon the severity of the storm. The data sampling rate was at 0.5 second intervals.

Two sets of Ohmart and Gratz's data were selected for comparison with IRSF, the MSLWT, and the SLWT. Both sets were measured during storm Delia in September 1973 and in the deep water region. Figs. 2-5 present a representative comparison of the horizontal water particle velocities among the data, the IRSF theory, the MSLWT and the SLWT. The comparison shows that both the MSLWT and SLWT agree reasonably well with the data. The SLWT yields a better result (4% closer to the data) than that of the MSLWT near the crest phase angle.

Stream function wave theory (SFWT) results are known to provide a reasonably good fit with laboratory and field data. Therefore, a comparison between the SLWT, MSLWT and SFWT was also studied.

The SFWT is a nonlinear wave theory, thus the wave components given by Eq. 1 for the SLWT and MSLWT cannot be regarded as independent linear components. Rather these components are nonlinear harmonics of the fundamental. Therefore, the wave number k cannot be solved component by component using the dispersion relationship (Eq. 3) for the SLWT and Eq. 15 for the MSLWT for this condition. Only the first component can be solved for the wave number based on Eqs. 3 and 15; in general, the wave number of the nth component is nk.

Six sets of dimensionless wave conditions were selected for tabulation and evaluation from Dean's stream function tabulations (1974). Each case is characterized by values of h/L_o and H/L_o. In the present study, three values of the parameter h/L_o ranging from 0.002 to 2.0 were selected and include the relative depth range of shallow, intermediate, and deep water. The parameter H/L_o includes wave steepness ratios: 0.25 and 1.0 of the breaking wave steepness for each of the 3 h/L_o values. Fig. 6 shows the dimensionless wave conditions selected for evaluation and also indicates the referencing notation for the cases. Tables I and II present the horizontal water particle velocity comparison between the MSLWT and SFWT, and SLWT and SFWT for wave case No. 2. The wave steepness (H/L_o) for wave case 2 is 0.001564, and the relative depth (h/L_o) is 0.002 (Note: This wave is at breaking).

The percent values listed in the tables are the differences between the SFWT and the MSLWT (or the SLWT), defined as

$$\text{Percent} = \frac{\text{MSLWT (or SLWT)} - \text{SFWT}}{\text{SFWT}} \times 100 \tag{21}$$

The main body of the table lists the dimensionless horizontal water particle velocities of the MSLWT (or SLWT). The row labelled "surface" represents the dimensionless velocities evaluated at the free surface; the percentage differences for velocities are calculated as defined above (Eq. 21). The remaining part of the table represents the dimensionless velocities and percentage differences evaluated on a grid of $(\Theta, S/h)$. The lack of entries for the higher S/h and larger Θ values (right side of page) result from the fact that the wave profile

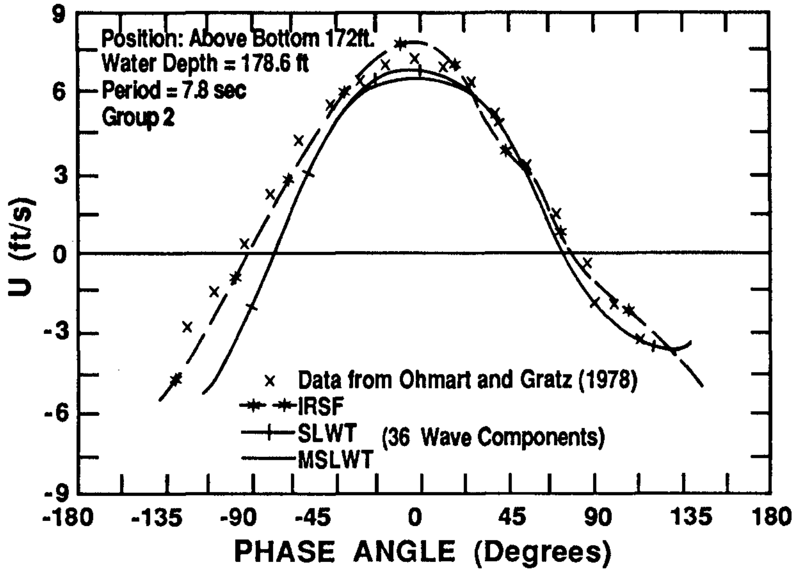

Fig. 2. Horizontal water particle velocity at a water depth of 172 feet for wave condition Group 2. Comparison of various wave theories with data by Ohmart and Gratz.

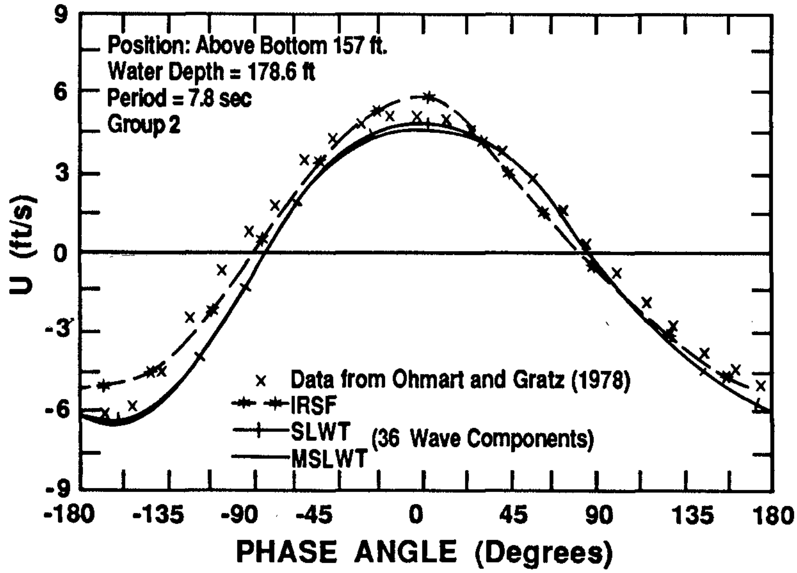

Fig. 3. Horizontal water particle velocity at a water depth of 157 feet for wave condition Group 2. Comparison of various wave theories with data by Ohmart and Gratz.

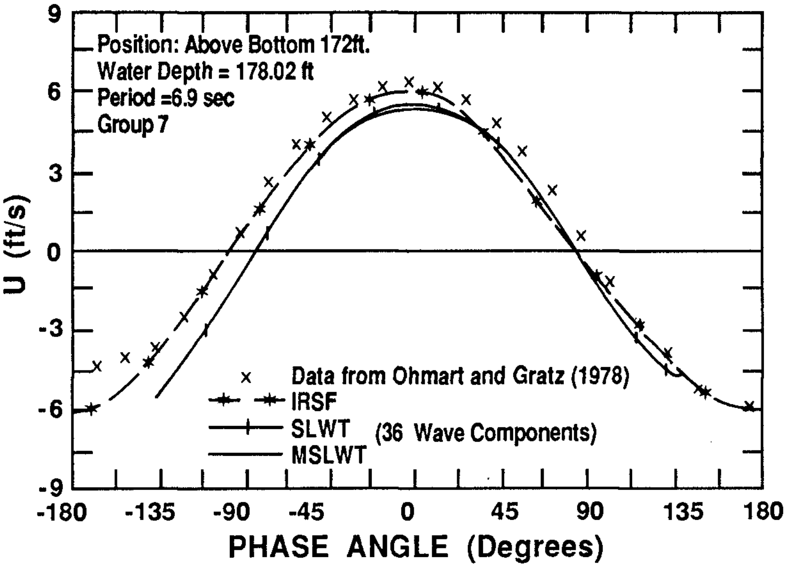

Fig. 4. Horizontal water particle velocity at a water depth of 172 feet for wave condition Group 7. Comparison of various wave theories with data by Ohmart and Gratz.

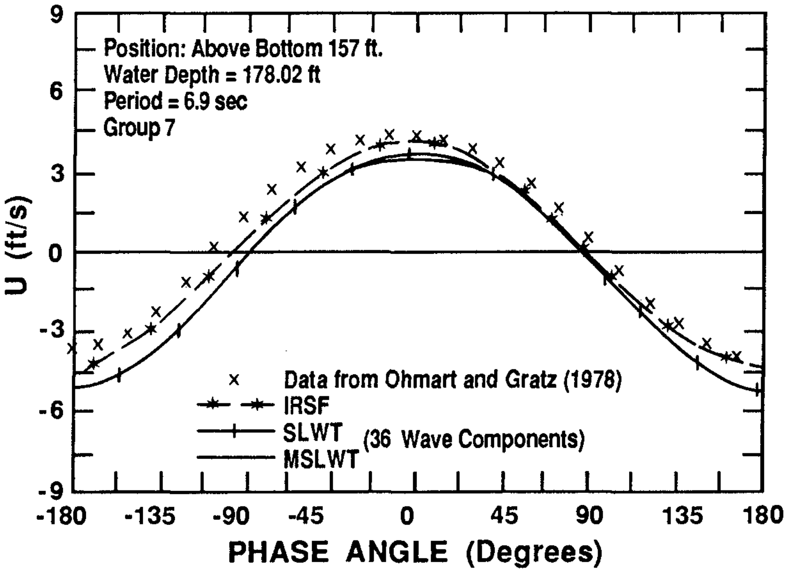

Fig. 5. Horizontal water particle velocity at a water depth of 157 feet for wave condition Group 7. Comparison of various wave theories with data by Ohmart and Gratz.

TABLE I-DIMENSIONLESS HORIZONTAL VELOCITY COMPONENT FIELD : MSLWT VS SFWT

THETA =	0.0	10.0	20.0	30.0	50.0	75.0	100.0	130.0	180.0
ETA/HEIGHT=	0.916	0.201	-.017	-.040	-.044	-.040	-.040	-.041	-.041
	-4.5%	-4.1%	18.5%	3.2%	1.2%	-0.6%	0.4%	0.3%	0.2%
SURFACE	62.477	2.874	-2.648	-2.785	-2.760	-2.237	-2.269	-2.342	-2.439
	7.8%	-76.4%	184.4%	30.4%	15.4%	10.8%	16.0%	14.5%	13.9%
S/DEPTH=1.7	61.565								
	9.5%								
S/DEPTH=1.6	56.519								
	6.6%								
S/DEPTH=1.5	52.191								
	4.0%								
S/DEPTH=1.4	48.472								
	1.8%								
S/DEPTH=1.3	45.272								
	-0.2%								
S/DEPTH=1.2	42.518								
	-1.9%								
S/DEPTH=1.1	40.149	4.212							
	-3.5%	-66.4%							
S/DEPTH=1.0	38.115	6.230							
	-4.8%	-52.1%							
S/DEPTH=0.9	36.373	7.906	-2.197	-2.660	-2.692	-2.259	-2.276	-2.340	-2.418
	-5.9%	-41.1%	205.8%	27.1%	14.1%	11.5%	15.7%	14.3%	13.6%
S/DEPTH=0.8	34.890	9.287	-1.751	-2.516	-2.611	-2.283	-2.285	-2.339	-2.394
	-6.9%	-32.5%	251.9%	23.4%	12.6%	12.2%	15.3%	14.0%	13.3%
S/DEPTH=0.7	33.638	10.417	-1.374	-2.408	-2.551	-2.301	-2.294	-2.339	-2.376
	-7.7%	-25.8%	359.3%	20.9%	11.7%	12.7%	15.0%	13.8%	13.1%
S/DEPTH=0.6	32.593	11.330	-1.055	-2.326	-2.507	-2.313	-2.301	-2.340	-2.363
	-8.4%	-20.6%	748.4%	19.1%	11.1%	13.0%	14.8%	13.7%	13.0%
S/DEPTH=0.5	31.738	12.056	-0.790	-2.263	-2.474	-2.322	-2.308	-2.340	-2.353
	-8.9%	-16.6%	******%	17.8%	10.8%	13.2%	14.7%	13.6%	12.9%
S/DEPTH=0.4	31.057	12.619	-0.575	-2.216	-2.450	-2.328	-2.313	-2.341	-2.345
	-9.4%	-13.5%	-486.1%	17.0%	10.6%	13.3%	14.6%	13.5%	12.9%
S/DEPTH=0.3	30.539	13.039	-0.409	-2.182	-2.433	-2.332	-2.317	-2.341	-2.340
	-9.7%	-11.3%	-266.3%	16.4%	10.4%	13.4%	14.5%	13.5%	12.9%
S/DEPTH=0.2	30.175	13.329	-0.290	-2.159	-2.421	-2.335	-2.320	-2.342	-2.336
	-10.0%	-9.8%	-192.0%	16.0%	10.4%	13.4%	14.4%	13.4%	12.9%
S/DEPTH=0.1	29.958	13.498	-0.219	-2.145	-2.415	-2.336	-2.321	-2.342	-2.334
	-10.1%	-9.0%	-161.3%	15.8%	10.4%	13.4%	14.4%	13.4%	12.9%
S/DEPTH=0.0	29.887	13.554	-0.195	-2.141	-2.413	-2.337	-2.322	-2.342	-2.334
	-10.2%	-8.7%	-152.6%	15.7%	10.4%	13.4%	14.4%	13.4%	12.9%

Table I. Horizontal velocity comparison between the MSLWT and SFWT for wave case No. 2.

TABLE II-DIMENSIONLESS HORIZONTAL VELOCITY COMPONENT FIELD : SLWT VS SFWT

THETA =	0.0	10.0	20.0	30.0	50.0	75.0	100.0	130.0	180.0
ETA/HEIGHT=	0.916	0.201	-.017	-.040	-.044	-.040	-.040	-.041	-.041
	-4.5%	-4.1%	18.5%	3.2%	1.2%	-0.6%	0.4%	0.3%	0.2%
SURFACE	70.899	4.015	-2.653	-2.753	-2.719	-2.199	-2.232	-2.304	-2.401
	22.4%	-67.1%	184.8%	28.9%	13.7%	8.9%	14.1%	12.7%	12.1%
S/DEPTH=1.7	70.202								
	24.9%								
S/DEPTH=1.6	66.275								
	25.0%								
S/DEPTH=1.5	62.803								
	25.2%								
S/DEPTH=1.4	59.735								
	25.4%								
S/DEPTH=1.3	57.027								
	25.7%								
S/DEPTH=1.2	54.639								
	26.0%								
S/DEPTH=1.1	52.539	5.262							
	26.3%	-58.1%							
S/DEPTH=1.0	50.700	7.159							
	26.7%	-45.0%							
S/DEPTH=0.9	49.098	8.751	-2.198	-2.626	-2.650	-2.222	-2.239	-2.303	-2.380
	27.0%	-34.8%	205.9%	25.5%	12.3%	9.7%	13.8%	12.4%	11.8%
S/DEPTH=0.8	47.712	10.077	-1.750	-2.480	-2.568	-2.247	-2.248	-2.302	-2.356
	27.4%	-26.7%	251.6%	21.7%	10.8%	10.4%	13.4%	12.1%	11.5%
S/DEPTH=0.7	46.525	11.173	-1.370	-2.371	-2.503	-2.264	-2.257	-2.301	-2.338
	27.7%	-20.4%	358.0%	19.0%	9.8%	10.9%	13.2%	12.0%	11.3%
S/DEPTH=0.6	45.522	12.068	-1.049	-2.288	-2.463	-2.276	-2.265	-2.302	-2.324
	28.0%	-15.4%	743.6%	17.2%	9.2%	11.2%	13.0%	11.8%	11.1%
S/DEPTH=0.5	44.693	12.785	-0.782	-2.226	-2.430	-2.285	-2.271	-2.303	-2.314
	28.2%	-11.5%	*******%	15.9%	8.8%	11.4%	12.9%	11.7%	11.1%
S/DEPTH=0.4	44.028	13.346	-0.566	-2.179	-2.406	-2.291	-2.277	-2.303	-2.307
	28.4%	-8.6%	-479.9%	15.0%	8.6%	11.5%	12.8%	11.7%	11.1%
S/DEPTH=0.3	43.517	13.767	-0.398	-2.144	-2.389	-2.295	-2.280	-2.304	-2.301
	28.6%	-6.4%	-262.0%	14.4%	8.5%	11.6%	12.7%	11.6%	11.1%
S/DEPTH=0.2	43.157	14.058	-0.279	-2.121	-2.376	-2.298	-2.283	-2.304	-2.298
	28.8%	-4.9%	-188.4%	14.0%	8.4%	11.6%	12.6%	11.6%	11.1%
S/DEPTH=0.1	42.942	14.230	-0.207	-2.107	-2.372	-2.299	-2.285	-2.304	-2.296
	28.8%	-4.0%	-158.0%	13.7%	8.4%	11.6%	12.6%	11.6%	11.1%
S/DEPTH=0.0	42.871	14.287	-0.183	-2.103	-2.370	-2.300	-2.285	-2.305	-2.295
	28.9%	-3.7%	-149.4%	13.7%	8.4%	11.6%	12.6%	11.6%	11.1%

Table II. Horizontal velocity comparison between the SLWT and SFWT for wave case No. 2.

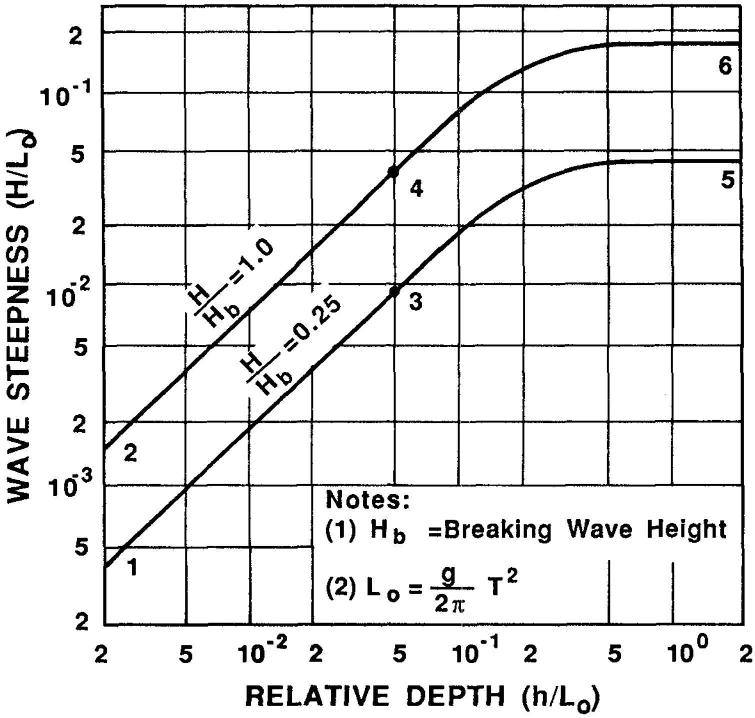

Fig. 6. Dimensionless wave characteristics selected for tabulation.

in the trough region is lower than in the crest region (left side of page). If asterisks appear, it means that the percentage differences are very large and cannot be covered by the computer output. This avoided the tabulation of very large percentages that would have been the result of division by a small number. Generally, although the asterisks indicate a large percentage error, they occur at locations where the denominator in Eq. 21 is near zero and are associated with small absolute differences. Finally, it is noted that the small percentage differences in relative water surface displacement are due to the stream function profile being used to develop a truncated Fourier series representation as an input water surface profile to both of the stretched wave theories. Figs. 7 and 8 present a comparison of the horizontal water particle kinematics between the SFWT, SLWT, and MSLWT under the crest. From this comparison, the MSLWT provides a better fit to the SFWT than for the SLWT.

Summary and Conclusions

A new stretching factor was developed based on the first order free surface boundary conditions. The simple superposition method and the modified stretched linear wave theory, provide reasonable agreement with the water particle kinematics from the Stream Function Tables. This technique is useful and convenient for the following reasons:

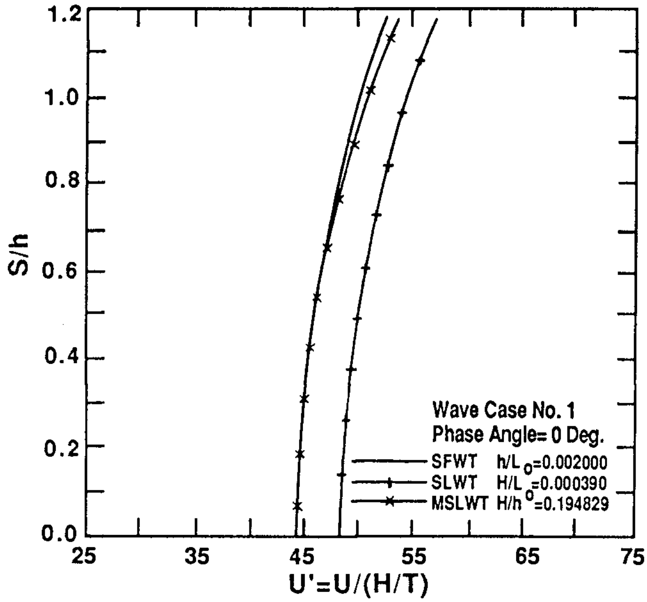

Fig. 7. Dimensionless horizontal water particle velocity distribution over depth (under the crest).

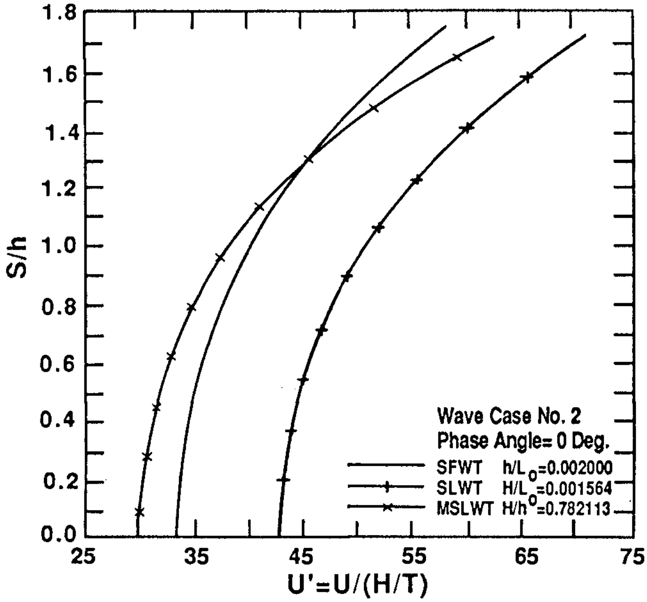

Fig. 8. Dimensionless horizontal water particle velocity distribution over depth (under the crest).

1. The simple superposition method and the MSLWT can be used to analyze any length of irregular wave records for design and dynamic studies of offshore structures. Through the superposition method, a given wave record (usually available as a function of time only) can be represented as an infinite sum of simple harmonic waves with closely spaced frequencies and appropriate phase angles.

   When the MSLWT is used to estimate the flow regime in a wave system from the surface profile, Eqs. 16, 18, 19 and 20 allow the calculation of the potential function and the kinematics.

   This method is convenient for the analysis of long and irregular wave profiles which can be characterized by power spectra and thus related to statistical wave prediction techniques that relate wave spectra to weather history for a given location.

2. Most nonlinear wave theories require a periodic wave profile, but by using the simple superposition method and the MSLWT, aperiodic wave profiles can be analyzed.

3. Based on the MSLWT, the spectral and probabilistic approaches to wave force prediction can be extended to include calculations from the sea floor to the free surface without neglecting the contribution from the mean water surface to the free surface.

4. In laboratory studies of nonlinear waves, the simple superposition method and the MSLWT will yield reasonable kinematic predictions.

5. A more extensive comparison between field data and the result of using the simple superposition method and the MSLWT is desirable and will be pursured when additional data are available. It is planned to extend the MSLWT to account for nonlinear terms included in the complete free surface boundary conditions.

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