CHAPTER 43

WAVE GROUP PROPERTY OF WIND WAVES FROM MODULATIONAL INSTABILITY

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ABSTRACT

This paper discusses wave grouping of wind waves from the physical viewpoint of wave modulational instability. Amplitude modulation periods obtained from the smoothed instantaneous wave energy history (SIWEH) of the observed data are compared with the predicted values by the modulational instability theory using the Zakharov equation for a finite constant water depth derived by Stiassnie and Shemer(1984). The modulation period normalized by the typical wave period corresponds to the length of total run. It is shown that the amplitude modulation periods of the observed data agree satisfactorily with the predicted values. Thus we conclude that the modulational instability is a hydrodynamical cause of grouping of high waves.

INTRODUCTION

In recent years wave grouping has been recognized by coastal engineers as an important factor for the stability of rubble mound breakwaters, the fluctuation of wave overtopping quantity of seawalls, the slow drift oscillation of moored vessels and floating structures, the surf beat and so on. Johnson, Mansard and Ploeg(1978) have found by laboratory experiments that even if power spectra of random waves are the same, the degree of damage of a rubble mound breakwater is not the same but depends on the degree of wave grouping; that is, grouped waves cause more severe damage of the breakwater than non-grouped waves.

There are three kinds of theoretical approaches to the statistical properties of run length and total run length of wave heights. One is the theory by Goda(1970) for waves of which successive wave heights are independent, the second is the wave envelope theory by Nolte and Hsu(1973) and Ewing(1973) for waves with narrow-banded spectra, and the third is the Markov process theory by Kimura(1980) for waves of which successive wave heights are mutually correlated with the property of the Markov chain. Elgar, Guza and Seymour(1984)

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reviewed these existing theories and showed that the theories have internal inconsistencies in their assumptions and treatments.

Studies on statistical properties of wave groups of ocean waves have been carried out by Wilson and Baird(1972), Rye(1974), Chakrabarti and Snider(1974), Goda(1976), Burcharth(1980), Goda(1983), Elgar et al.(1984), Mase and Iwagaki(1984) and Battjes and van Vledder(1984). In these studies, observations of wave group statistics have been compared with the theoretical predictions or with the numerical simulation data. It seems from these studies that among the theories of run length Kimura's theory is most applicable to waves with not only narrow-banded but also wide-banded spectra if the correlation parameter which appears in the two-dimensional Rayleigh distribution and is related to the correlation coefficient of successive wave heights is chosen adequately. However, when the correlation coefficient is larger than 0.4, the mean length predicted by the theory of Kimura is always shorter than the observed one (Mase and Iwagaki, 1984).

Most of the studies concerning wave groups including theories are based on a statistical viewpoint. The existing theories are not based on hydrodynamics of wave motion but consider only statistical wave properties, such as the wave energy spectrum or the wave height distribution. Wave groups are also significant in coastal engineering from a viewpoint of hydrodynamics. One is the modulational instability and the evolution of finite amplitude surface waves. The other is a new model of wind waves such as the envelope soliton model proposed by Mollo-Christensen and Ramamonjiarisoa(1978) (wind wave fields are not composed of linear component waves but composed of envelope solitons which are formed by Stokes waves) and the modulated nonlinear wave model shown by Lake and Yuen(1978) (wind wave fields are considered as a modulated nonlinear wave train with a single carrier wave).

The grouping of high waves can be explained by two different viewpoints. One is the statistical viewpoint in which wave group statistics are discussed in a framework that wave fields are thought to be the superposition of independent linear component waves. The other is the hydrodynamical viewpoint such as the modulational instability. Benjamin and Feir(1967) have made it clear that a uniform wave train in deep water is unstable and that it evolves into a modulated wave train. Lake and Yuen(1978) compared the theoretical modulation frequency by Benjamin and Feir(1967) with the observed one of laboratory wind waves. It was found that both agree qualitatively.

Even if a parameter representing wave groups is a statistical quantity, the characteristic dominated by hydrodynamics is probably contained in the statistical quantity. The objective of this paper is to discuss the wave group property of natural wind waves from the hydrodynamical viewpoint. Concretely, we compare the observed amplitude modulation periods (or repetition periods of groups of high waves) with the predicted ones from the modulational instability theory using the Zakharov equation. The modulation periods normalized by the typical wave period correspond to the lengths of total runs.

ANALYZED WAVE DATA

Wave data used in this paper were collected at Hikone-Aisei of

Lake Biwa in Shiga Prefecture, Japan, where eleven wave gauges of capacitance type were installed. Wave observations at Hikone-Aisei Wave Observatory and Nagahama Wave Observatory were started by Iwagaki et al.(1976) in March, 1975, for the duration of a year to examine properties of fetch-limited wind waves.

Wave data analyzed for amplitude modulation periods are four continuous records of storm waves for five hours from 11:27, 18:00, and 23:00 on October 5, and from 18:30 on December 16, 1975, recorded by the wave gauge N-9 installed in water depth of 4m. The first two have been analyzed in the previous paper (Mase and Iwagaki, 1984) from the statistical viewpoint. The predominant wave direction was NW when the waves analyzed here were recorded, which was nearly perpendicular to the shoreline. The slope of the beach was nearly uniform and approximately 1/50.

The records were digitized at a sampling interval of 0.04 s and recorded on a magnetic tape. These continuous wave records were divided into fourteen segments of twenty minutes long. Fig.1 shows the time series of the significant wave height, $H_{1/3}$, the significant wave period, $T_{1/3}$, the mean wave height, $\overline{H}$, and the mean wave period, $\overline{T}$. The numbers of individual waves contained in each wave record were about 300 to 600.

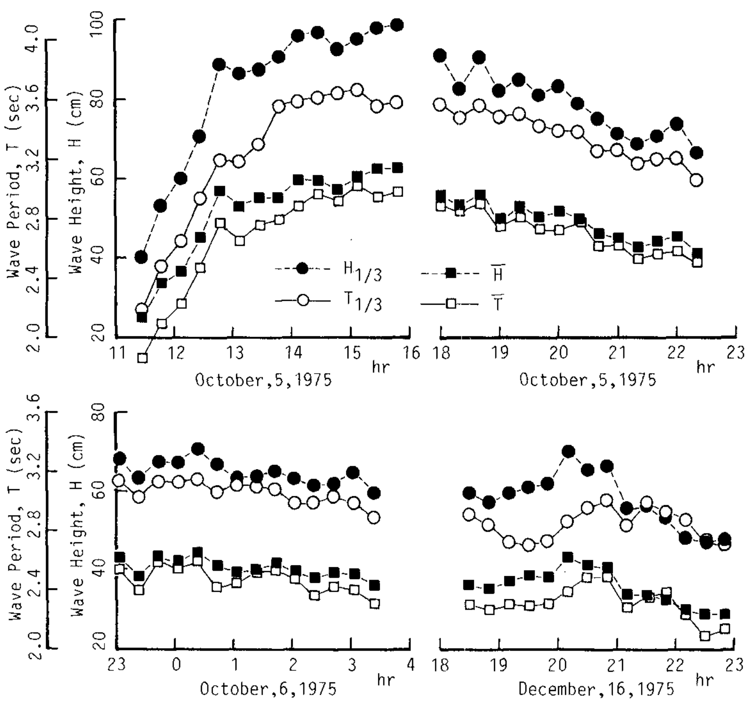

Fig.1. Time series of significant wave height, significant wave period, mean wave height and mean wave period.

AMPLITUDE MODULATION PERIOD BY MODULATIONAL INSTABILITY THEORY

As for the analysis of modulational instability of waves, it has been shown by Crawford, Lake, Saffman and Yuen(1981) that the Zakharov equation first derived by Zakharov(1968) is a useful equation which can consider the effect of finite amplitude. In this section, we describe the theory of modulational instability using the Zakharov equation according to Crawford et al.(1981) and Stiassnie and Shemer(1984), and later we compare the amplitude modulation period predicted by the theory with the observed one. Crawford et al.(1981) used the third order Zakharov equation for infinite deep water, and Stiassnie and Shemer(1984) derived the Zakharov equation to the fourth order for constant (finite or infinite) water depth and investigated the Class I and Class II instabilities by using the modified Zakharov equation. In this study, we use the Zakharov equation up to the third order for finite water depth derived by Stiassnie and Shemer(1984).

Let $B(\boldsymbol{k},t)$ be a kind of amplitude spectrum. The Zakharov equation to the third order is expressed as follows (Crawford et al. and Stiassnie et al.):

$$i\frac{\partial B(\boldsymbol{k},t)}{\partial t} = \iiint_{-\infty}^{\infty} T(\boldsymbol{k},\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)B^*(\boldsymbol{k}_1,t)B(\boldsymbol{k}_2,t)B(\boldsymbol{k}_3,t)$$
$$\times\delta(\boldsymbol{k}+\boldsymbol{k}_1-\boldsymbol{k}_2-\boldsymbol{k}_3)\exp[i\{\omega(\boldsymbol{k})+\omega(\boldsymbol{k}_1)-\omega(\boldsymbol{k}_2)$$
$$-\omega(\boldsymbol{k}_3)\}t]\,d\boldsymbol{k}_1 d\boldsymbol{k}_2 d\boldsymbol{k}_3, \tag{1}$$

where $*$ denotes the complex conjugate, $\boldsymbol{k}=(k_x,k_y)$ is the wavenumber vector, ω is the angular frequency related to the wavenumber as $\omega(\boldsymbol{k})=(g|\boldsymbol{k}|\tanh|\boldsymbol{k}|h)^{1/2}$, and δ denotes the delta function. Eq.(1) represents the interaction of amplitude spectra or the slow evolution of the dominant components of waves. The kernel $T(\boldsymbol{k},\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$ (abbreviated as $T_{0,1,2,3}$ hereafter) is shown in the paper of Stiassnie and Shemer(1984). Since there are some misprints in the expression of the kernel, we used the correct expression informed directly by Dr. Stiassnie. The kernel $T_{0,1,2,3}$ is seen in Mase and Iwagaki(1986).

In the case of a uniform wave train with the wavenumber vector $\boldsymbol{k}_0=(k_0,0)$, the solution of Eq.(1) is

$$B_0(\boldsymbol{k}_0,t) = b_0\exp(-iT_{0,0,0,0}b_0^2 t), \tag{2}$$

where $T_{0,0,0,0}b_0^2$ is the Stokes corrected frequency due to the nonlinearity and b_0 is related to the actual amplitude a_0 as follows:

$$b_0 = \pi(\frac{2g}{\omega_0})^{1/2}a_0. \tag{3}$$

When disturbances with the wavenumber vectors $\boldsymbol{k}_1=\boldsymbol{k}_0-\boldsymbol{K}$ and $\boldsymbol{k}_2=\boldsymbol{k}_0+\boldsymbol{K}$ with the amplitudes $B_1(\boldsymbol{k}_1,t)$ and $B_2(\boldsymbol{k}_2,t)$ $(|B_1|, |B_2| \ll |B_0|)$ are imposed on the uniform wave train, the time evolutions of B_1 and B_2 are written from Eq.(1) neglecting the squares of small quantities by

$$i\frac{dB_1}{dt} = 2\,T_{1,0,1,0}b_0^2B_1 + T_{1,2,0,0}B_2\,b_0^2\exp(-i\tilde{\omega}t), \tag{4}$$

$$i\frac{dB_2}{dt} = 2\,T_{2,0,2,0}b_0^2B_2 + T_{2,1,0,0}B_1\,b_0^2\exp(-i\tilde{\omega}t), \tag{5}$$

here, $\tilde{\omega} = (2\,\omega_0 - \omega_1 - \omega_2) + 2\,T_{0,0,0,0}b_0^2$. Assuming a solution of the form

$$B_1 = b_1\exp\{-i(0.5\,\tilde{\omega} - \Omega)t\}, \tag{6}$$

$$B_2 = b_2\exp\{-i(0.5\,\tilde{\omega} + \Omega)t\}, \tag{7}$$

the following equation is obtained using the condition that the non-trivial solutions of b_1 and b_2 exist:

$$\Omega = (T_{2,0,2,0} - T_{1,0,1,0})b_0^2 \pm\sqrt{\{0.5\,\tilde{\omega} - (T_{1,0,1,0} + T_{2,0,2,0})b_0^2\}^2 - T_{1,2,0,0}T_{2,1,0,0}b_0^4}. \tag{8}$$

When Ω is not real, disturbances grow exponentially with time and amplitude modulations occur, which means the instability of waves.

For the two-dimensional case such as $K=(K_x,0)$, the non-dimensional perturbation wavenumber κ is defined by K_x/k_0. Fig.2 shows the non-dimensional growth rate $\mathrm{Im}(\Omega)/(\omega_0k_0^2a_0^2/2)$ as a function of $\kappa/2k_0a_0$ for various values of k_0a_0 and for five values of k_0h. Fig.2(a) is the same result as given by Crawford et al.(1981). In experiments with deep water waves by Lake, Yuen, Rungaldier and Ferguson(1977), it was found that even if a uniform wave train is generated by a wave-making paddle the wave train modulates with increase in the propagation distance due to the growth of the most unstable mode which corresponds to the peak of each curve in Fig.2. For example, when waves are generated for which $k_0a_0=0.1$, disturbances with $\kappa/2k_0a_0=0.87$ grow in the case of deep water, see Fig.2(a). It is seen from Fig.2 that the domain of the non-dimensional wavenumber $\kappa/2k_0a_0$ becomes narrow and the non-dimensional growth rate of disturbances $\mathrm{Im}(\Omega)/(\omega_0k_0^2a_0^2/2)$ decreases with decrease in k_0h for the same value of k_0a_0, when $k_0a_0\leqq 0.3$. The modulational instability cannot occur if the non-dimensional water depth k_0h is smaller than 1.36.

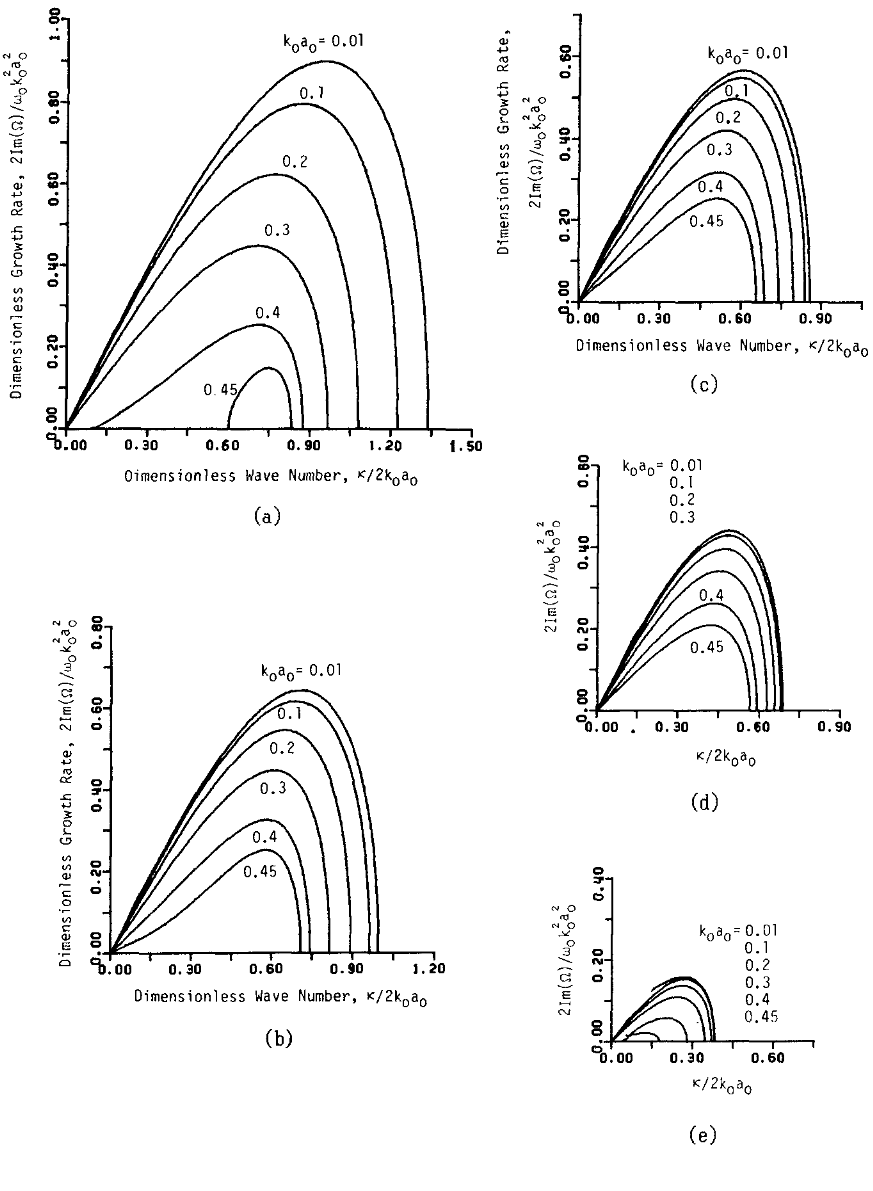

Fig.2. Wave instability diagram. (a) k_0h=10.0; (b) 3.0; (c) 2.5; (d) 2.0; (e) 1.5.

The amplitude modulation period to be compared with the observed modulation period is calculated as follows. For given values of $k_0 a_0$ and $k_0 h$, the value of κ corresponding to the most unstable mode is found. The frequencies of disturbances are given as function of κ by

$$\omega_1' = \omega_1 + 0.5\,\tilde{\omega} - \mathrm{Re}(\Omega), \tag{9}$$

$$\omega_2' = \omega_2 + 0.5\,\tilde{\omega} + \mathrm{Re}(\Omega), \tag{10}$$

where $\mathrm{Re}(\Omega)$ denotes the real part of Ω. The frequency of the dominant wave including the Stokes correction is

$$\omega = \omega_0 + T_{0,0,0,0} b_0^2. \tag{11}$$

The non-dimensional difference between frequencies of the dominant wave and disturbances becomes

$$\Delta = (\omega - \omega_1')/\omega = (\omega_2' - \omega)/\omega$$

$$= \{0.5(\omega_2 - \omega_1) + \mathrm{Re}(\Omega)\}/\omega. \tag{12}$$

Finally, the amplitude modulation period is given by

$$T_g = 2\pi/\Delta\omega. \tag{13}$$

Until now we have used the wavenumber k_0 and the amplitude a_0 of a carrier wave. Wave characteristics obtained by experiments and field observations are not a_0 and k_0, but the wave height H and the wave period T. Therefore, we have to estimate a_0 and k_0 by using H and T. In the estimation of a_0 and k_0 from H, T and the water depth h, we can use the third order Stokes wave theory. Since, however, waves observed in fields are not uniform, and it is not known what quantity we should use as the carrier wave. Lake and Yuen(1978) used the average wave steepness in comparing the experimental results of modulation frequencies with the theoretical values of Benjamin and Feir(1967). In this study, we adopt the significant waves or the mean waves as the carrier waves for the time being, and use $H/2$ as a_0 and the wavenumber determined from T and h by the small amplitude wave theory as k_0.

Fig.3 shows the time series of values of ka and kh obtained from the significant wave (designated with subscript '1/3') and from the mean wave (designated with subscript 'm').

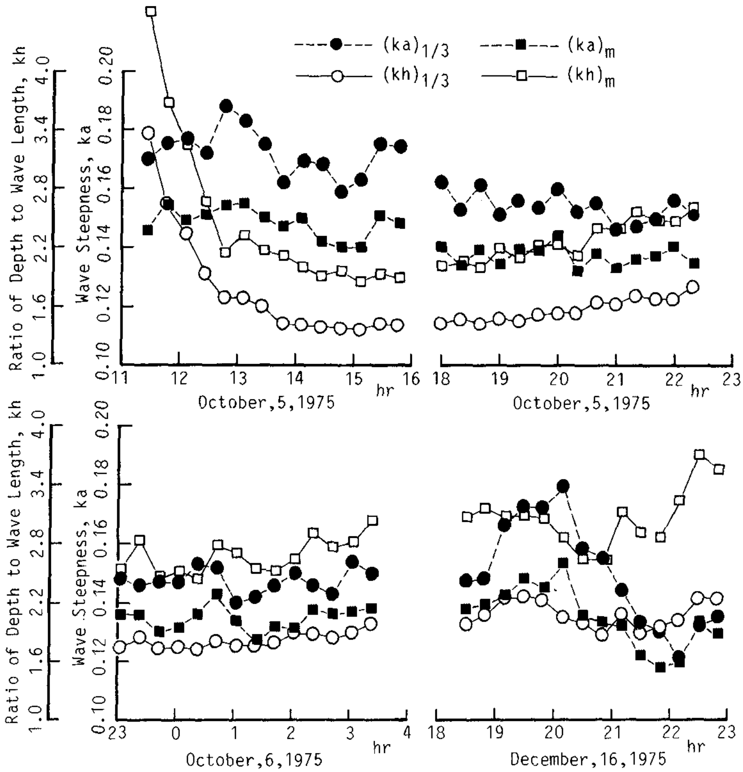

Fig.3. Time series of wave steepness ka and dimensionless water depth kh.

COMPARISON OF OBSERVED AMPLITUDE MODULATION PERIOD WITH PREDICTED ONE BY MODULATION INSTABILITY THEORY

We use the smoothed instantaneous wave energy history (SIWEH) proposed by Funke and Mansard(1979) to calculate the amplitude modulation period of field data. The SIWEH, $E(t)$, is described by

$$E(t) = \frac{1}{T_p}\int_{-\infty}^{\infty}\eta^2(t+\tau)Q(\tau)d\tau, \tag{14}$$

$$Q(\tau) = \begin{cases} 1-|\tau|/T_p & |\tau| < T_p \\ 0 & |\tau| \geqq T_p \end{cases}, \tag{15}$$

where T_p is the peak period of an energy spectrum, $\eta(t)$ the water surface variation, and τ the time lag.

The amplitude modulation period can be estimated from the SIWEH by various methods. One is to use the peak period of the energy spectrum of SIWEH, $(T_g)_{MP}$. The other is to use the mean value of the zero-up-crossing times of $\{E(t)-\bar{E}\}$, $(T_g)_{MF}$, in which the SIWEH is modified so that the component waves of which frequencies are lower than $0.5/(T_g)_{MP}$ and higher than $1.5/(T_g)_{MP}$ are removed by using the Fast Fourier Transform technique. The amplitude modulation periods $(T_g)_{MP}$ and $(T_g)_{MF}$ almost agree, see Mase and Iwagaki(1986).

Fig.4 shows the comparison of the observed modulation periods $(T_g)_{MP}$ and $(T_g)_{MF}$ with the predicted ones $(T_g)_{CS}$ and $(T_g)_{CM}$ where $(T_g)_{CS}$ is calculated from the significant wave and $(T_g)_{CM}$ from the mean wave by Eq.(13). The predicted values larger than 40.0 s are plotted at 40.5 s in the figure. The values of $(T_g)_{CS}$ are large compared with the observed values except from 11:30 to 12:10 on

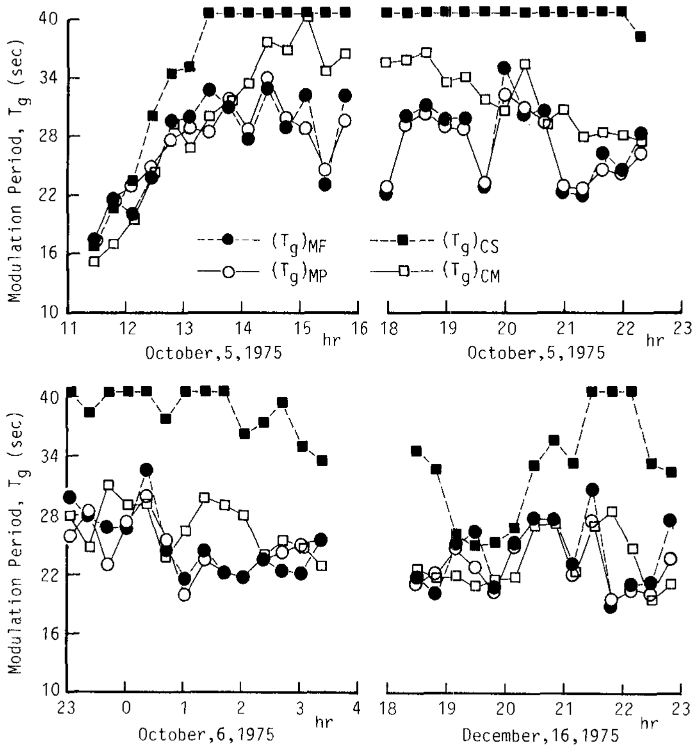

Fig.4. Time series of observed and predicted amplitude modulation periods.

October 5 and from 19:10 to 20:10 on December 16 where the value of $(kh)_{1/3}$ is larger than 2.0. On the other hand, the values of $(T_g)_{CM}$ agree fairly well with the observed data in a wide range. It is seen from the figure that we should better use the mean wave as the carrier wave than the significant wave. However, the values of $(T_g)_{CM}$ are always larger than the observed ones when the value of $(kh)_m$ is smaller than 2.1 (14:30 to 19:30 on October 5).

Fig.5 shows the comparison of $(T_g)_{MF}$ and $(T_g)_{CM}$ in a different form. This figure indicates that the agreement between both values is satisfactory as far as the mean values are concerned. When the non-dimensional water depth $(kh)_m$ is small, the difference between the predicted values and observed ones becomes large the reason is explained in the following discussion.

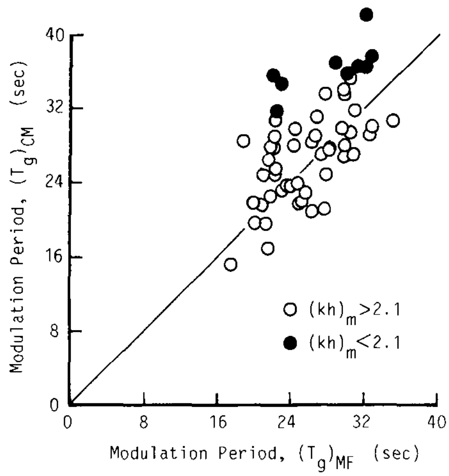

Fig.5. Comparison of observed amplitude modulation period with predicted one.

DISCUSSION

The Zakharov equation derived by Stiassnie and Shemer(1984) was applied to the present study. In spite of the sloping beach of 1/50, the modulational instability theory by the Zakharov equation can predict the amplitude modulation periods of wind waves sufficiently when the non-dimensional water depth is large ($(kh)_m$>2.1 according to the present study). However, there are slight differences between the observed and predicted values of amplitude modulation periods in the case when the water depth is relatively shallow. As shown in Fig.2, the growth rate of disturbances becomes small and the difference between frequencies of the dominant wave and the disturbances becomes small with decrease in the water depth. Thus, for waves propagating from deep water into a region with small water depth (as $(kh)_m<2.1$), it is considered that amplitude modulations to be formed do not take place sufficiently due to the small growth rate of sideband modes, and that the deep-water amplitude modulation period remains dominant at the shallow-water observation point.

In the experiments of evolutions with the fetch or with the propagation distance of laboratory wind waves by Hatori(1984) and of mechanically generated random waves by Mase, Furumuro and Iwagaki(1984), it was found that there are several significant spikes, or sideband modes around the spectral maximum. The existence of sideband modes indicates that a phenomenon of wave modulational instability occurs in wave fields. Furthermore, Fig.5 in this paper and Fig.13 in the paper of Lake and Yuen(1978) both show that the modulational instability is an important factor in grouping of high waves.

At present, random waves used in experiments are simulated so that their energy spectrum matches a certain target spectrum. However, the sequence of waves, or wave groups, must be considered to simulate the more realistic sea waves. Funke and Mansard(1979) proposed a new technique of random wave simulation which simulates waves to match a target spectrum and a target SIWEH. Mase, Kita and Iwagaki(1983) used the same technique to simulate random waves. From the present study, it is found that the agreement between the amplitude modulation periods obtained from SIWEH of field data and predicted ones by the modulational instability theory is fairly good. This implies that the mean value of repetition periods or the peak period of the target SIWEH used in the random wave simulation must coincide with the theoretical modulation period determined from the wave height and the wave period.

CONCLUSIONS

The grouping of high waves can be explained by two different viewpoints, the statistical and physical viewpoints. Most studies concerning wave groups including run theories depend on the statistical viewpoint. In this paper, we discussed the wave group property of wind waves from the physical viewpoint of wave modulational instability. We chose the amplitude modulation period as a representative factor of the wave group property. The modulation period was calculated from the smoothed instantaneous wave energy history. The theoretical modulation period was calculated by the modulational instability theory using the Zakharov equation.

A comparison of the observed and theoretical modulation periods showed acceptable agreement if the mean wave was chosen as the carrier wave. In particular, when the non-dimensional water depth is as large as $(kh)_m>2.1$, both periods agree well. However, there was a slight difference between both values in the case of shallow water depth, which is attributed to the small growth rate of sideband modes and the effect of the remaining modulation period dominant in the deeper water depth. Thus we conclude that the modulational instability is a hydrodynamical cause of grouping of high waves.

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REFERENCES

Battjes,J.A. and van Vledder,G.Ph., 1984. Verification of Kimura's theory for wave group statistics. Proc.19th Coastal Eng. Conf., pp.642-648.

Benjamin,T.B. and Feir,J.E., 1967. The disintegration of wave trains on deep water, Part 1. Theory. Jour. Fluid Mech., Vol.27, pp.417-430.

Burcharth,H.F., 1980. A comparison of natural waves and model waves with special reference to wave grouping. Proc.18th Coastal Eng. Conf., pp.303-318.

Chakrabarti,S.H. and Snider,R.H., 1974. Mean length of ocean waves. Jour. Geophys. Res., Vol.79, No.36, pp.5665-5667.

Crawford,D.R., Lake,B.M., Saffman,P.G. and Yuen,H.C., 1981. Stability of weakly nonlinear deep water waves in two and three dimensions. Jour. Fluid Mech., Vol.105, pp.177-191.

Elgar,S., Guza,R.T. and Seymour,R.J., 1984. Groups of waves in shallow water. Jour. Geophys. Res., Vol.89, No.C3, pp.3623-3634.

Ewing,J.A., 1973. Mean length of run of high waves. Jour. Geophys. Res., Vol.78, No.12, pp.1933-1936.

Funke,E.R. and Mansard,E.P.D., 1979. Synthesis of realistic sea states in a laboratory flume. National Res. Council of Canada, Hydraulics Lab. Rept., LTR-HY-66, 54p.

Goda,Y., 1970. Numerical experiments on wave statistics with spectral simulation. Rept. Port and Harbour Res. Inst., Vol.9, No.3, pp.3-57.

Goda,Y., 1976. On wave groups. Proc. Int. Conf. on Behavior of Offshore Structures, pp.115-128.

Goda,Y., 1983. Analysis of wave grouping and spectra of long-travelled swell. Rept. Port and Harbour Res. Inst., Vol.22, No.1, pp.1-41.

Hatori,M., 1984. Nonlinear properties of laboratory wind waves at energy containing frequencies. Jour. Oceanographical Soc. Japan, Vol.40, pp.12-18.

Iwagaki,Y., Tuchiya,Y., Sakai,T., Yamaguchi,M., Shibano,T., Kimura,A., Yasuda,K. and Serizawa,S., 1976. Wave observation in Lake Biwa. Disaster Prevention Res. Inst., Annuals, Vol.19, pp.361-379 (in Japanese).

Johnson,R.R., Mansard,E.P.D. and Ploeg,J., 1978. Effects of wave grouping on breakwater stability. Proc.16th Coastal Eng. Conf., pp.2228-2243.

Kimura,A., 1980. Statistical properties of random wave groups. Proc.17th Coastal Eng. Conf., pp.2955-2973.

Lake,B.M., Yuen,H.C., Rungaldier,H. and Ferguson,W.E.Jr., 1977. Nonlinear deep-water waves: Evolution of a continuous wave train. Jour. Fluid Mech., Vol.83, pp.49-74.

Lake,B.M. and Yuen,H.C., 1978. A new model for nonlinear wind waves. Jour. Fluid Mech., Vol.88, pp.33-62.

Mase,H., Kita,N. and Iwagaki,Y., 1983. Random wave simulation

considering wave groups. Coastal Eng. in Japan, JSCE, Vol.26, pp.61-75.
Mase,H. and Iwagaki,Y., 1984. An analysis of wave data for wave grouping. Coastal Eng. in Japan, JSCE, Vol.27, pp.83-96.
Mase,H., Furumuro,K. and Iwagaki,Y., 1984. The change of characteristics of wave groups with wave traveling. Proc.31th Coastal Eng. Conf., JSCE, pp.158-162 (in Japanese).
Mase,H. and Iwagaki,Y., 1986. Wave group analysis of natural wind waves based on modulational instability theory. Coastal Eng., Vol.10 (in print).
Mollo-Christensen,E. and Ramamonjiarisoa,A., 1978. Modeling the presence of wave groups in a random wave field. Jour. Geophys. Res., Vol.83, pp.4117-4122.
Nolte,K.G. and Hsu,F.H., 1973. Statistics of ocean wave groups. Prepr. 4th OTC, No.1688, pp.139-146.
Rye,H., 1974. Wave group formation among storm waves. Proc.14th Coastal Eng. Conf., pp.164-183.
Stiassnie,M. and Shemer,L., 1984. On modifications of the Zakharov equation for surface gravity waves. Jour. Fluid Mech., Vol.143, pp.47-67.
Wilson,J.R. and Baird,W.F., 1972. A discussion of some measured wave data. Proc.13th Coastal Eng. Conf., pp.113-130.
Zakharov,V.E., 1968. Stability of periodic waves of finite amplitude on the surface of a deep fluid. Jour. Appl. Mech. Tech. Phys. (Engl. Trans.) 2, pp.190-194.